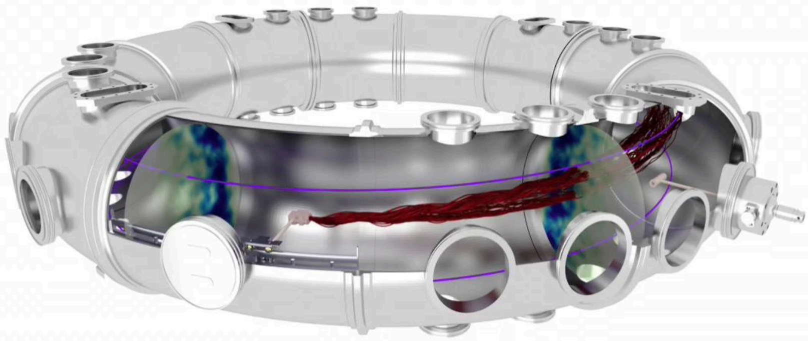
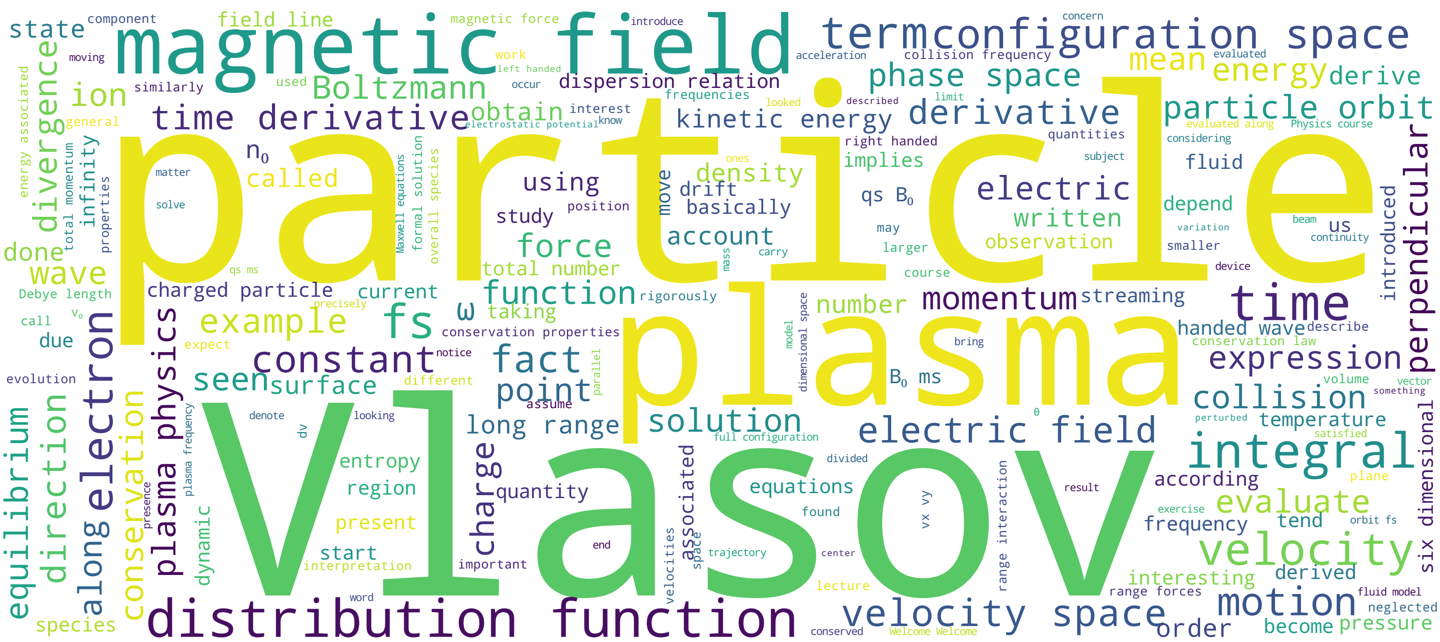


The Vlasov equation

Plasma Physics and Application to Fusion Energy, Astrophysics and Industry

Lecture 2d

Paolo Ricci



Search MOOC

Video

Introduction

- From Boltzmann to Vlasov equation
- Conservation laws
- Interpretation of Vlasov equation
- Solution of Vlasov equation

Plasma

Welcome. Welcome to the Plasma Physics course of EPFL. Today, we're going to talk about the Vlasov equation. We will derive the Vlasov equation from the Boltzmann equation. We will deduce the conservation laws that can be applied to the Vlasov equation. We will interpret the Vlasov equation and we will give some formal solution of it.

Notes

Summary

0m 05s

From Boltzmann to Vlasov equation

$$\frac{\partial f_s}{\partial t} + \vec{v}\cdot\frac{\partial f_s}{\partial \vec{r}} + \frac{q_s}{m_s}\left(\vec{E}^{\,l.r.} + \vec{v}\times \vec{B}^{\,l.r.}\right)\cdot\frac{\partial f_s}{\partial \vec{v}} = \left(\frac{\partial f}{\partial t}\right)_c$$

If collisions can be neglected $(n\lambda_D^3 \ggg 1)$ $\Rightarrow$ $\left(\frac{\partial f}{\partial t}\right)_c = 0$

$$\boxed{\frac{\partial f_s}{\partial t} + \vec{v}\cdot\frac{\partial f_s}{\partial \vec{r}} + \frac{q_s}{m_s}\left(\vec{E}^{\,l.r.} + \vec{v}\times \vec{B}^{\,l.r.}\right)\cdot\frac{\partial f_s}{\partial \vec{v}} = 0}$$ Vlasov equation

For simplicity of notation $\vec{E}^{\,l.r.} \to \vec{E}$, $\vec{B}^{\,l.r.} \to \vec{B}$

Plasma

So, the Boltzmann equation states that the derivative with respect to time of the distribution function is due to the streaming of particles in configuration space, plus the streaming in velocity space, due to the long-range electric and magnetic forces, and also taking to account the short-range forces that are associated with collisions. Now as we have seen, collisions can typically play a less crucial role in determining the dynamics of a plasma than the long-range forces. Therefore, if collisions can be neglected, which is the case where the number of particles in a Debye cube is really, really large, larger than what is required by the definition of a plasma. So large that the collisions can be completely neglected. Then we can set the collision operator to zero. We obtain therefore an equation, an equation that is called the *Vlasov equation* and which is one of the fundamental equations in plasma physics. Now, for simplicity of notation, we will denote the electric field due to long range interactions simply as E, and similarly the magnetic field due to long-range interaction, as B. We won't forget however, that the electric and magnetic field that enter in the Vlasov equation are the ones due to long-range interactions.

Notes

Summary

0m 28s

From Boltzmann to Vlasov equation

$$\frac{\partial f_s}{\partial t} + \vec{v}\cdot\frac{\partial f_s}{\partial \vec{r}} + \frac{q_s}{m_s}\left(\vec{E}^{e.e.} + \vec{v}\times B^{e.e.}\right)\cdot\frac{\partial f_s}{\partial \vec{v}} = \left(\frac{\partial f}{\partial t}\right)_c$$

If collisions can be neglected $(n\lambda_D^3 \ggg 1) \Rightarrow \left(\frac{\partial f}{\partial t}\right)_c = 0$

$$\boxed{\frac{\partial f_s}{\partial t} + \vec{v}\cdot\frac{\partial f_s}{\partial \vec{r}} + \frac{q_s}{m_s}\left(\vec{E}^{e.e.} + \vec{v}\times \vec{B}^{e.e.}\right)\cdot\frac{\partial f_s}{\partial \vec{v}} = 0}$$ Vlasov equation

For simplicity of notation $\vec{E}^{e.e.} \to \vec{E}$, $\vec{B}^{e.e.} \to \vec{B}$

Vlasov equation is coupled to Maxwell equations

$$\rho = \sum_s q_s \int d\vec{v}\, f_s(\vec{r},\vec{v},t) \qquad \vec{j} = \sum_s q_s \int d\vec{v}\, \vec{v}\, f_s(\vec{r},\vec{v},t)$$

Plasma

The Vlasov equation has then to be coupled to Maxwell's equations. And in fact, one can define the charge density ρ as the sum overall species present in our system of qs, the charge of each particle, times the integral of the velocity space of fs(r, v, t) and similarly for the current density we will have that j will be given by the sum over all species, of the charge of each species, times the integral of the velocity of the particles, averaged according to their distribution.

Notes

Summary

3m 08s

Conservation laws 1/2

$$\frac{\partial f_s}{\partial t} + \vec{v} \cdot \frac{\partial f_s}{\partial \vec{r}} + \frac{q_s}{m_s} \left(\vec{E} + \vec{v} \times \vec{B} \right) \cdot \frac{\partial f_s}{\partial \vec{v}} = 0$$

- Conservation of total number of particles

$$\int d\vec{r} \int d\vec{v} \ \text{Vlasov} \Rightarrow \underbrace{\int d\vec{r} \int d\vec{v} \frac{\partial f_s}{\partial t}}_{\frac{\partial}{\partial t} \int d\vec{r} \int d\vec{v} f_s} + \int d\vec{r} \int d\vec{v} \ \vec{v} \cdot \frac{\partial f_s}{\partial \vec{r}} + \int d\vec{r} \int d\vec{v} \frac{q_s}{m_s} \left(\vec{E} + \vec{v} \times \vec{B} \right) \cdot \frac{\partial f_s}{\partial \vec{v}} = 0$$

Plasma

What are the properties of Vlasov equation? Let's give a look at the conservation properties for the distribution function as it's evolved by the Vlasov equation. The Vlasov equation that we have just derived can be written as.... this, and then, there are some conservation properties that are satisfied by the Vlasov equation, that we can study. First of all, there is the conservation of the total tumber of particles that is satisfied by the Vlasov equation. In fact, if we integrate over all the configuration space, and over all velocities, the Vlasov equation, then we obtain for the time derivative, the integral over all space, overall velocities, of ∂fs/∂t. Plus for the term related to the streaming in configuration space, we have the integral over all space, over all velocities of v·∂fs/∂r And then, there is the streaming term in velocity space. Now, let's consider the three terms here separately. Let's start from the first one. We can take out the time derivative from the two integral signs. and we will in fact that ∂/∂t over first the configuration space and over the velocity space of fs.

Notes

Summary

4m 15s

Conservation laws 1/2

$$\frac{\partial f_s}{\partial t} + \vec{v}\cdot\frac{\partial f_s}{\partial \vec{r}} + \frac{q_s}{m_s}\left(\vec{E} + \vec{v}\times\vec{B}\right)\cdot\frac{\partial f_s}{\partial \vec{v}} = 0$$

- Conservation of total number of particles

$$\int d\vec{r}\int d\vec{v}\ \text{Vlasov} \Rightarrow \underbrace{\int d\vec{r}\int d\vec{v}\,\frac{\partial f_s}{\partial t}}_{\frac{\partial}{\partial t}\int d\vec{r}\int d\vec{v}\, f_s = \frac{d}{dt}N_s} + \underbrace{\int d\vec{r}\int d\vec{v}\ \vec{v}\cdot\frac{\partial f_s}{\partial \vec{r}}}_{\int d\vec{r}\int d\vec{v}\,\frac{\partial}{\partial \vec{r}}\cdot(\vec{v} f_s) = 0} + \underbrace{\int d\vec{r}\int d\vec{v}\,\frac{q_s}{m_s}\left(\vec{E} + \vec{v}\times\vec{B}\right)\cdot\frac{\partial f_s}{\partial \vec{v}}}_{\int d\vec{r}\int d\vec{v}} = 0$$

This double integral actually is equal to the total number of particles and therefore this is equal to the time derivative of Ns, the total number of particle Ns, For what concern the term related to the streaming in phase space. Well, we have seen that we can bring the velocity term inside the ∂/∂r term. Velocity and configuration space are two independent variables in coordinates of the phase space. Therefore this can be written as the integral over dr and the integral over dv of the divergence of v fs. [∂/∂r · (v fs)] Now we can use the divergence theorem to carry out the integral over the configuration space, and this integral will be reconducted to an integral over the surface that extends to infinity, and if we assume that the distribution function -once we push our surface of interest to infinity- decays to zero, then this integral will be equal to zero. Finally, the integral related to the streaming of particles in velocity space, will be given by the double integral over the configuration space and the velocity space... Now let me do that immediately we can bring as we have seen, the first term inside the ∂/∂v operator because E is independent of v and (v × B) is perpendicular to v.

Notes

Summary

6m 44s

Conservation laws 1/2

$$\frac{\partial f_s}{\partial t} + \vec{v} \cdot \frac{\partial f_s}{\partial \vec{r}} + \frac{q_s}{m_s}(\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial f_s}{\partial \vec{v}} = 0$$

- Conservation of total number of particles

$$\int d\vec{r} \int d\vec{v} \text{ Vlasov} \Rightarrow \underbrace{\int d\vec{r} \int d\vec{v} \frac{\partial f_s}{\partial t}}_{\frac{\partial}{\partial t}\int d\vec{r}\int d\vec{v} f_s = \frac{d}{dt}N_s} + \underbrace{\int d\vec{r} \int d\vec{v}\, \vec{v} \cdot \frac{\partial f_s}{\partial \vec{r}}}_{\int d\vec{r}\int d\vec{v} \frac{\partial}{\partial \vec{r}} \cdot (\vec{v} f_s) = 0} + \underbrace{\int d\vec{r} \int d\vec{v} \frac{q_s}{m_s}(\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial f_s}{\partial \vec{v}}}_{\int d\vec{r}\int d\vec{v} \frac{\partial}{\partial \vec{v}} \cdot \left[\frac{q_s}{m_s}(\vec{E}+\vec{v}\times\vec{B}) f_s\right] = 0} = 0$$

$$\Rightarrow \frac{dN_s}{dt} = 0$$

- Conservation of momentum

$$\vec{P}_{tot} = \vec{P}_{particles} + \vec{P}_{fields} = \sum_s m_s \int d\vec{r} \int d\vec{v}\, \vec{v} f_s + \varepsilon_0 \int d\vec{r} (\vec{E} \times \vec{B}) \quad \text{is conserved}$$

Plasma

Therefore I can write this as the divergence in velocity space of qs/ms (E + v × B) fs And here, here we can again use the divergence theorem to evaluate this volume integral, volume in velocity space as an integral over a surface that extends to infinity and as there are no particles which have an infinite velocity, the surface integral of this term here, of this expression will be zero, and therefore the whole expression here will be equal to zero. The conclusion is that just one term survives, that is dNs/dt which is equal to zero. Particles are conserved. Analogously, we wont prove it rigorously, but let me just mention that this Vlaslov equation implies the conservation of momentum. In fact, if we consider total momentum of our system, that is equal to the sum of the momentum of the particles, associated with particles, plus the momentum of the field that is for the momentum of particles equal to the sum of overall species of ms, the integral over the full configuration space and the full velocity space of the momentum of the particles taking into account v according to how they are distributed in free space. Plus, the momentum associated with the electric field, that is the momentum of the Poynting vector. that has to be integrated over all configuration space. All this, this total momentum is a conserved quantity.

Notes

Summary

8m 54s

Conservation laws 2/2

- Conservation of energy

$$E_{tot} = E_{particles} + E_{fields} = \sum_s \int d\vec{r} \int d\vec{v} \, \frac{1}{2} m_s v^2 f_s \quad + \frac{1}{2} \int d\vec{r} \left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0} \right) \qquad \text{is conserved}$$

- Entropy is conserv

Plasma

Besides the conservation of the total number of particles and the conservation of the total momentum, there are other quantities that are conserved by the Vlasov equation. In particular, energy and entropy. Let's give a look at those. For what concerns the conservation of energy, we have to define the total energy, which is the sum of the kinetic energy of the particles, plus the energy associated with the electric and magnetic fields. The total kinetic energy will be given by the sum over all species of the integral of the distribution function over the full configuration space velocity space of one half the kinetic energy of particles with velocity v, weighted according to the distribution function. Then we will have to add to this the energy of the electric and magnetic fields. That is the integral over the full configuration space of the energy associated with the electric field plus the energy associated with the magnetic field. and this is a conserved quantity. We won't prove it rigorously, but let me point that this property that is fairly fundamental. And finally, let me also state another property of the Vlasov equation. That is the entropy is conserved.

Notes

Summary

11m 36s

Conservation laws 2/2

- Conservation of energy

$$E_{tot} = E_{particles} + E_{fields} = \sum_s \int d\vec{r} \int d\vec{v} \, \frac{1}{2} m_s v^2 f_s \; + \frac{1}{2} \int d\vec{r} \left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0} \right) \quad \text{is conserved}$$

- Entropy is conserved

$$S = - \sum_s \int d\vec{r} \int d\vec{v} \; f_s \ln f_s$$

$$\frac{dS}{dt} = - \sum_s \int d\vec{r} \int d\vec{v} \left(\frac{\partial f_s}{\partial t} \ln f_s + \frac{\partial f_s}{\partial t} \right) = 0$$

$\Rightarrow$ Vlasov equation describes time-reversible processes (collisions are neglected)

Plasma

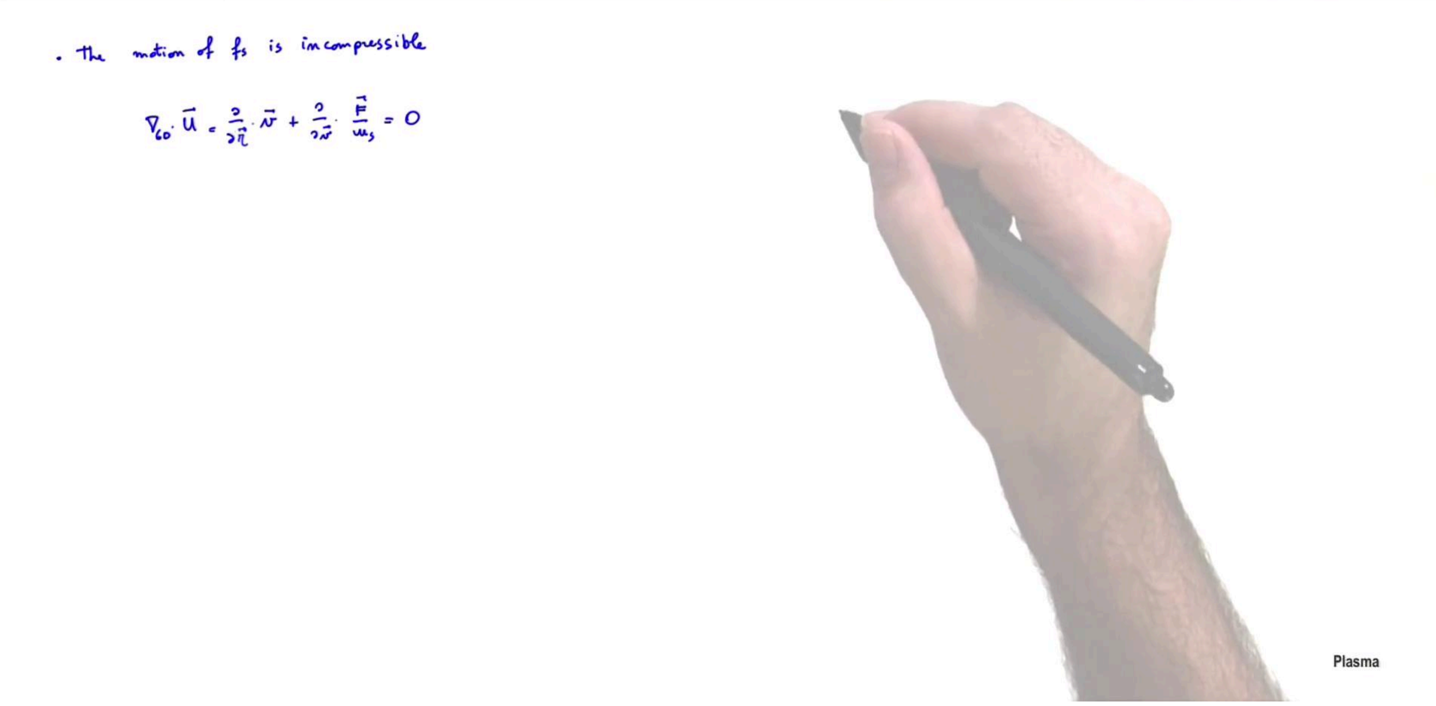
We define entropy, the total entropy of the system, according to the typical definition that is given in the information theory. Therefore as minus the sum overall species of that integral over all the configuration space, all velocities of fs, -the distribution function- times the natural logarithm of fs. Now, we won't prove it rigorously, but let me state that if we evaluate the time derivative of this function with respect to time, of the total entropy, which is therefore, -if you take the derivative inside the integral- (∂fs/∂t ln(fs) + ∂fs/∂t). If you carry out this calculation, taking into account that fs satisfies the Vlasov equation then what you find is that this dS/dt is equal to zero. Therefore, the Vlasov equation describes time-reversible processes. Why is that? Fundamentally because dissipative processes, -collisional processes- have been neglected in the Vlasov equation.

Notes

Summary

13m 30s

Interpretation of Vlasov equation



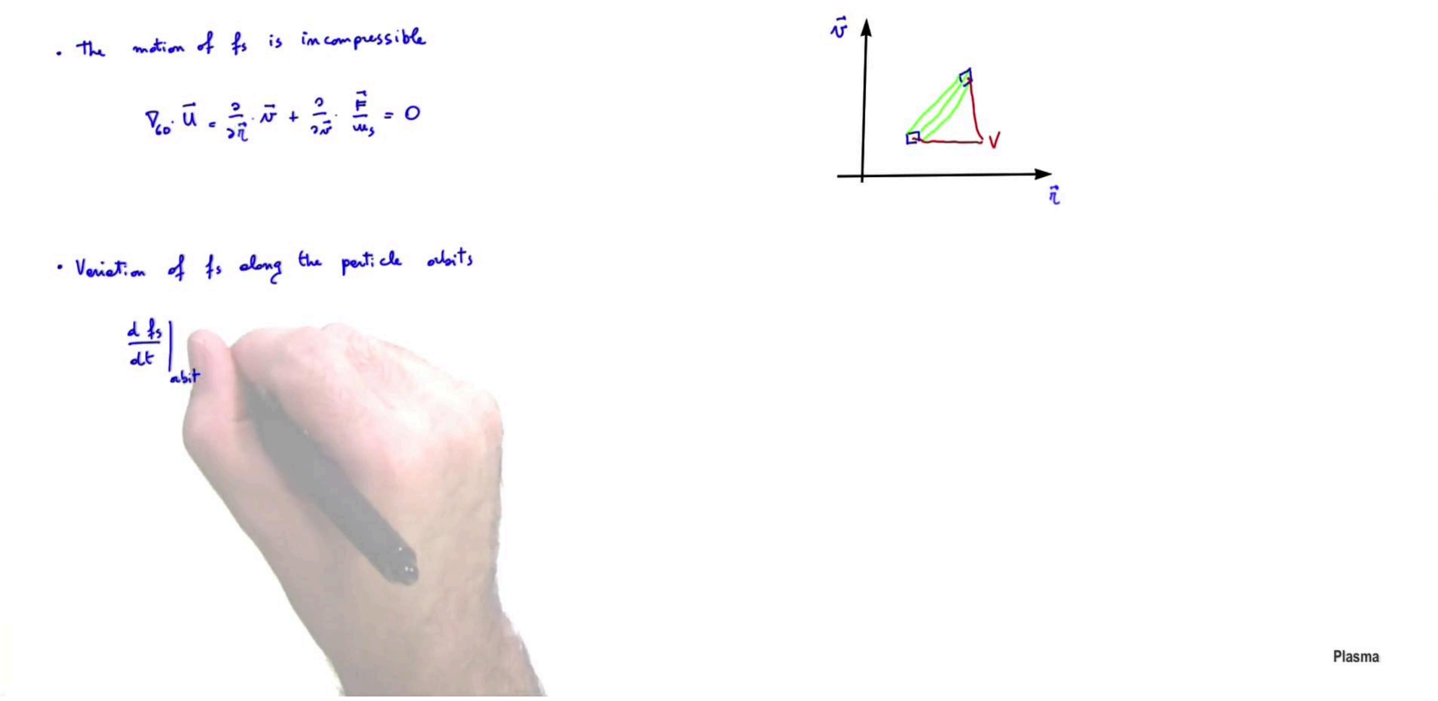
Having focused on the conservation properties implied by the Vlasov equation, we can now give a look at how the Vlasov equation can be physically interpreted. There are actually two interpretations, two physical interpretations of the Vlasov equation that I would like to point out. The first one is associated with the fact that as it's described by the Vlasov equation, the motion of fs -the distribution function- is incompressible. In fact if we take the divergence in the six-dimensional space of u, where u is the six-dimensional velocity that we have introduced in the module where we derived the Boltzmann equation, that will be ∂/∂r · v the divergence of v, plus the divergence in the velocity space of the acceleration that will be equal to zero. This is because the velocity as we have said is a phase space coordinate independent from the configuration space, and the force is given by the sum of the electric and magnetic forces. The electric field does not depend on the v, the Lorentz force is perpendicular to v and therefore also this divergence term is equal to zero. What does it mean in practice?

Notes

Summary

15m 24s

Interpretation of Vlasov equation



Well, if we look at the evolution of the particle distribution function in [the phase] space that I am considering here and where I project the six-dimensional space over the 2-D plane that we can deal with. Therefore, when we consider here the projection of the configuration space coordinate and velocity coordinate. So, if we consider a small volume in the phase space, the particles that are contained in this volume will be displaced as a function of time. And if we look at the volume that these particles will occupy after a certain time, we may find that it may have changed its shape, however, the volume that contains these particles will be the same. The motion is incompressible. There is another interpretation that I would like to point out of the Vlasov equation. In order to look at this interpretation, we have to look at the variation of the distribution function along the particle orbit. More precisely what we want to evaluate is the time derivative of the distribution function as seen by a particle traveling over its orbit.

Notes

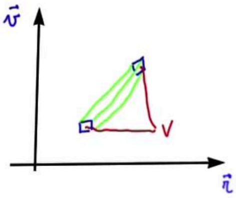
Summary

17m 07s

Interpretation of Vlasov equation

- The motion of f_s is incompressible

$$\nabla_{6D}\cdot\vec{U} = \frac{\partial}{\partial\vec{r}}\cdot\vec{v} + \frac{\partial}{\partial\vec{v}}\cdot\frac{\vec{F}}{m_s} = 0$$



- Variation of f_s along the particle orbits

$$\left.\frac{d f_s}{dt}\right|_{orbit} = \frac{\partial f_s}{\partial t} + \left.\frac{d\vec{r}}{dt}\right|_{orbit}\cdot\frac{\partial f_s}{\partial\vec{r}} + \left.\frac{d\vec{v}}{dt}\right|_{orbit}\cdot\frac{\partial f_s}{\partial\vec{v}} =$$

$$= \frac{\partial f_s}{\partial t} + \vec{v}\cdot\frac{\partial f_s}{\partial\vec{r}} + \frac{\vec{F}}{m_s}\cdot\frac{\partial f_s}{\partial\vec{v}} = \frac{\partial f_s}{\partial t} + \vec{v}\cdot\frac{\partial f_s}{\partial\vec{r}} + \frac{q_s}{m_s}\left(\vec{E}+\vec{v}\times\vec{B}\right)\cdot\frac{\partial f_s}{\partial\vec{v}} \underset{\text{Vlasov}}{\overset{\wedge}{=}} 0$$

Plasma

This will be given by -taking into account the definition of the convective derivative that we have here-, will be given by explicit time derivative of fs, plus, -by using the chain rule-, the derivative of the position and configuration of space with respect to time, along a particle orbit, · ∂fs/∂r, plus, the derivative with respect to time of the velocity along a particle orbit · ∂fs/∂v. This is equal to -- okay, we won't to work on the explicit time derivative, it's fine as it is. So what is the derivative with respect to time of the particle position if we are moving along with the particles? This will be given by the velocity of the particle · ∂fs/∂r, plus... -- okay, what is the derivative with respect to time of the velocity when we are moving with the particles? That will be equal to the acceleration of the particle. That is, F/ms · ∂fs/∂v. This therefore will be equal to ∂fs/∂t + v·∂fs/∂r +... plus we can explicitly write the force term that is given by the sum of the electric and magnetic forces. ... + qs/ms (E + v × B) · ∂fs/∂v And the interesting thing is that here what you have obtained is actually the Vlasov equation and therefore as fs is evolving according to Vlasov equation this -because of Vlasov- has to be equal to zero.

Notes

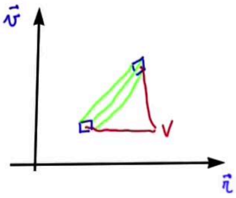
Summary

18m 56s

Interpretation of Vlasov equation

- the motion of f_s is incompressible

$$\nabla_{6D}\cdot \vec{U} = \frac{\partial}{\partial \vec{r}}\cdot\vec{v} + \frac{\partial}{\partial\vec{v}}\cdot\frac{\vec F}{m_s} = 0$$



- Variation of f_s along the particle orbits

$$\left.\frac{d f_s}{dt}\right|_{orbit} = \frac{\partial f_s}{\partial t} + \left.\frac{d\vec r}{dt}\right|_{orbit}\cdot\frac{\partial f_s}{\partial \vec r} + \left.\frac{d\vec v}{dt}\right|_{orbit}\cdot\frac{\partial f_s}{\partial\vec v} =$$

$$= \frac{\partial f_s}{\partial t} + \vec v\cdot\frac{\partial f_s}{\partial\vec r} + \frac{\vec F}{m_s}\cdot\frac{\partial f_s}{\partial \vec v} = \frac{\partial f_s}{\partial t}+\vec v\cdot\frac{\partial f_s}{\partial \vec r} + \frac{q_s}{m_s}(\vec E+\vec v\times\vec B)\cdot\frac{\partial f_s}{\partial\vec v} \underset{\text{Vlasov}}{=} 0$$

$\Rightarrow$ f_s, as measured when moving along a particle trajectory, is constant

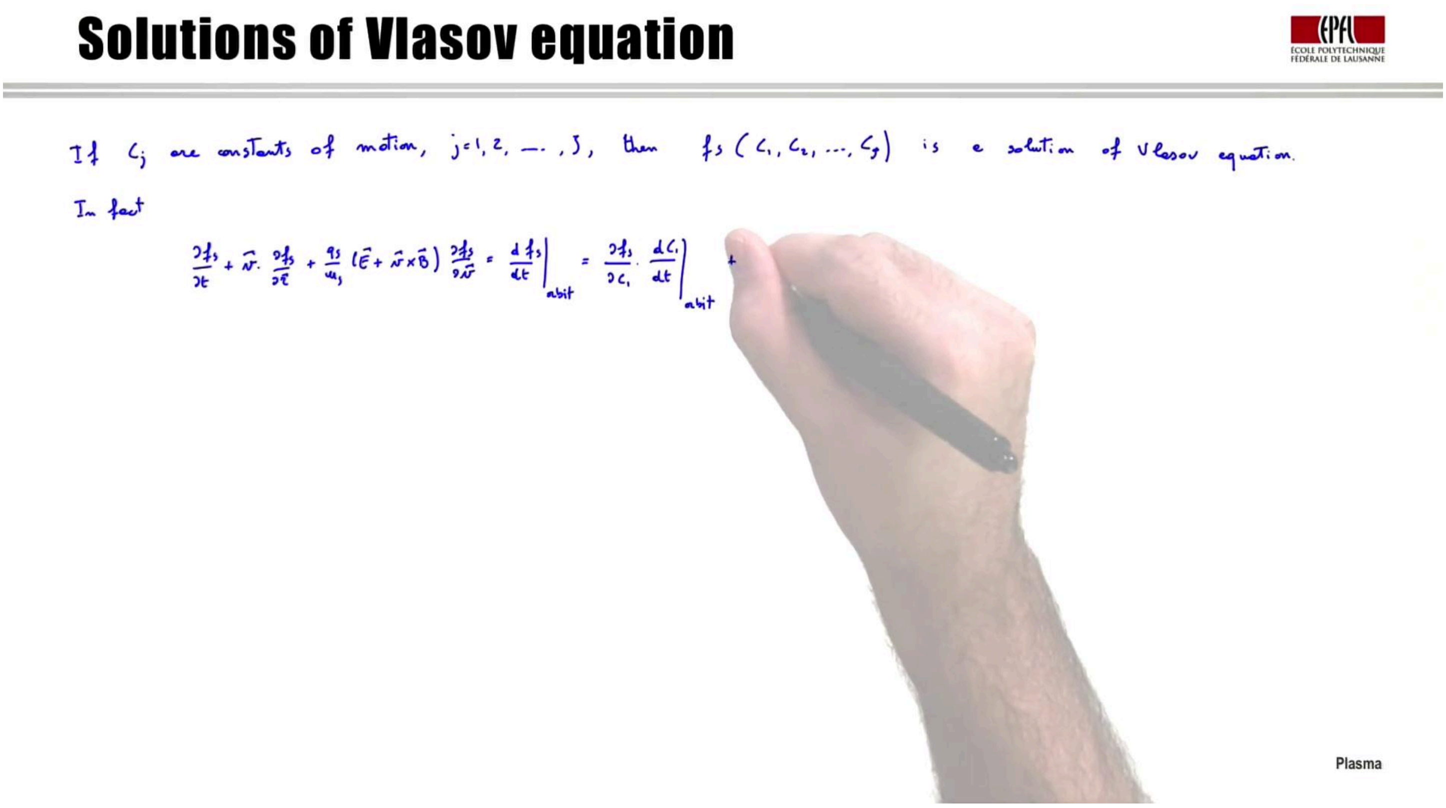
Plasma

What does it mean? It means that according to the Vlasov equation, fs, the distribution function, as measured when moving along the particle trajectory is constant.

Notes

Summary

21m 15s



We have looked at the conservation properties associated with the Vlasov equation. We have looked at the possible interpretation of the Vlasov equation. Now, its time to give a look if there are solutions to the Vlasov equation. Surprisingly, there are many solutions of the Vlasov equation. In fact, if Cj are constant of motion, for j = 1, 2,... to a big number J then what I'm going to show is that fs, a distribution function that is a function of these Cj, fs(C1, C2,..., CJ) is a solution of the Vlasov equation. In fact, if we try to inject this distribution function into the Vlasov equation, and therefore, we evaluate the time derivative of fs streaming in the configuration space and in velocity space which as we have just seen, is equal to the time derivative of the distribution function as seen from the particle orbit. Then what we have is, -by using the chain rule- that the derivative with respect to time of the distribution function along the particle orbit will be equal to the derivative of fs with respect to C1 [∂fs/∂C1] times the derivative of C1 with the respect to time [∂C1/∂t] as evaluated along the particle orbit, ...

Notes

Summary

21m 42s

Solutions of Vlasov equation

If C_j are constants of motion, $j=1,2,\dots,J$, then $f_s(C_1, C_2, \dots, C_J)$ is a solution of Vlasov equation.

In fact

$$\frac{\partial f_s}{\partial t} + \vec{v}\cdot\frac{\partial f_s}{\partial \vec{r}} + \frac{q_s}{m_s}(\vec{E}+\vec{v}\times\vec{B})\frac{\partial f_s}{\partial \vec{v}} = \frac{d f_s}{dt}\Big|_{orbit} = \frac{\partial f_s}{\partial C_1}\cdot\underbrace{\frac{dC_1}{dt}\Big|_{orbit}}_{=0} + \frac{\partial f_s}{\partial C_2}\cdot\underbrace{\frac{dC_2}{dt}\Big|_{orbit}}_{=0} + \dots + \frac{\partial f_s}{\partial C_J}\cdot\underbrace{\frac{dC_J}{dt}\Big|_{orbit}}_{=0} = 0$$

Examples

- $\vec{E}=\vec{B}=0 \Rightarrow v_x, v_y, v_z$ are constants of motion $\Rightarrow f_s = f_s(v_x, v_y, v_z)$ is solution of Vlasov
- $\vec{E}=0, \vec{B}=B_0\vec{e}_z \Rightarrow v_z, v^2, v_y + \frac{q_s B_0}{m_s}x, v_x - \frac{q_s B_0}{m_s}y$ are constants $\Rightarrow f_s = f_s\left(v_z, v^2, v_y + \frac{q_s B_0}{m_s}x, v_x - \frac{q_s}{}\right.$

Plasma

+ ∂fs/∂C2 ∂C2/∂t as evaluated along the particle orbit, plus all the other derivatives until ∂fs/∂CJ ∂CJ/∂t along the particle orbit. Now, as C1, C2, C3,..., CJ are constants of motion, the time derivatives of these functions as evaluated along the particle orbit is equal to zero. Therefore, all this quantity will be equal to zero. In other terms, the distribution function that we have just defined satisfies the Vlasov equation. It's a solution of the Vlasov equation. To make things a bit more concrete, let us give some examples. For example, let's consider the case of a system where there is no magnetic nor electric field. What we will have in this case is that vx, vy and vz are constants of motion which implies that any function fs(vx, vy, vz) is a solution of the Vlasov equation. Maybe a more interesting case is the one of E = 0, a magnetic field that is uniform along the z direction, then in this case, we have a number of constants of motion: For example, the velocity in the z direction is a constant of motion. The kinetic energy of the particles, [is a constant of motion] and then if you look at the particle orbits you find that vy + (qs B_0/ms) x and vx - (qs B_0/ms) y are all constants of motion. This implies that any function fs (vz, v^2, vy + (qs B_0/ms) x,......

Notes

Summary

23m 59s

Solutions of Vlasov equation

If C_j are constants of motion, $j=1,2,\dots,J$, then $f_s(C_1, C_2, \dots, C_J)$ is a solution of Vlasov equation.

In fact

$$\frac{\partial f_s}{\partial t} + \vec{v}\cdot\frac{\partial f_s}{\partial \vec{r}} + \frac{q_s}{m_s}(\vec{E}+\vec{v}\times\vec{B})\frac{\partial f_s}{\partial \vec{v}} = \frac{d f_s}{dt}\Big|_{orbit} = \frac{\partial f_s}{\partial C_1}\cdot\underbrace{\frac{dC_1}{dt}\Big|_{orbit}}_{=0} + \frac{\partial f_s}{\partial C_2}\cdot\underbrace{\frac{dC_2}{dt}\Big|_{orbit}}_{=0} + \dots + \frac{\partial f_s}{\partial C_J}\cdot\underbrace{\frac{dC_J}{dt}\Big|_{orbit}}_{=0} = 0$$

Examples

- $\vec{E} = \vec{B} = 0 \Rightarrow v_x, v_y, v_z$ are constants of motion $\Rightarrow f_s = f_s(v_x, v_y, v_z)$ is solution of Vlasov
- $\vec{E}=0, \vec{B} = B_0\vec{e}_z \Rightarrow v_z, v^2, v_y + \frac{q_s B_0}{m_s}x, v_x - \frac{q_s B_0}{m_s}y$ are constants $\Rightarrow f_s = f_s\left(v_z, v^2, v_y + \frac{q_s B_0}{m_s}x, v_x - \frac{q_s B_0}{m_s}y\right)$ is solution of Vlasov

Observations

- Contrast to Boltzmann equation, where Maxwellian is the only stationary solution
- Very difficult to find the constants of motion

Plasma

vx - (qs B_0/ms) y) is a solution of Vlasov. Let me make a couple of observations. The first observation is that the behavior of the Vlasov equation is fairly different from that one can expect from the Boltzmann equation. Where because of the presence of collisions in the Boltzmann equation, we expect that only the Maxwellian distribution function is a stationary solution. Here with Vlasov, things are very different because whatever function for example in this case of vx, vy, and vz is a solution of Vlasov. Therefore, it can be very far from the thermodynamic equilibrium. A second thing to notice is that, in general, it's very difficult to find the constants of motion in realistic cases, the ones that are of interest to us for practical application.

Notes

Summary

27m 12s

Here we are at the end of this of lecture. What we have done was to derive from the Boltzmann equation the Vlasov equation. That is done simply by neglecting the collision operator that is present in the Vlasov equation. And then what we have done was to study and to look at the properties of the evolution of the distribution function in phase space. We have derived one conservation law, the conservation of the total number of particles. There are other conservation laws that I have just stated: the ones of momentum, energy and the conservation of entropy. It takes a bit time, and I leave it as an exercise to do, it's an interesting calculation to do by yourself, if you would like, but let me point out that the Vlasov equation has these nice properties. And then what we have done is to provide some new interpretation, some interpretation of the Vlasov equation, which has actually allowed us to find a formal solution of Vlasov equation. Unfortunately, formal solutions do not work in the cases of practical interest, and therefore, one has to solve the Vlasov equation or in general the kinetic equations by using a numerical approach.

Notes

Summary

28m 52s