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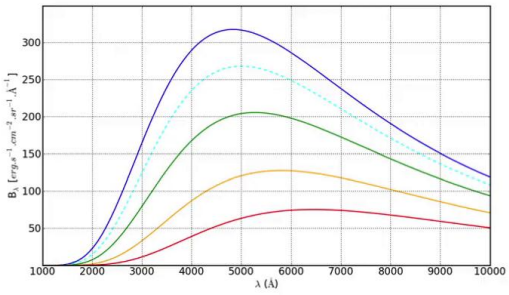

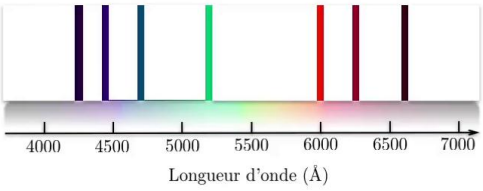

Black Body / Atomic Lines

Introduction to Astrophysics; Chapter 2.2

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In this lecture, we will see more details about black-body radiation and the spectral lines produced for example by an atomic or molecular gas. The black body, first of all, caracterises heated bodies that re emit radiation at all wavelengths. For instance, that is the case of a light bulb filament that converts electric energy into light. That is the case of glowing embers which, due to the heating by the flames, will radiate at all wavelengths. It is also the case of lava ejected by volcanoes.

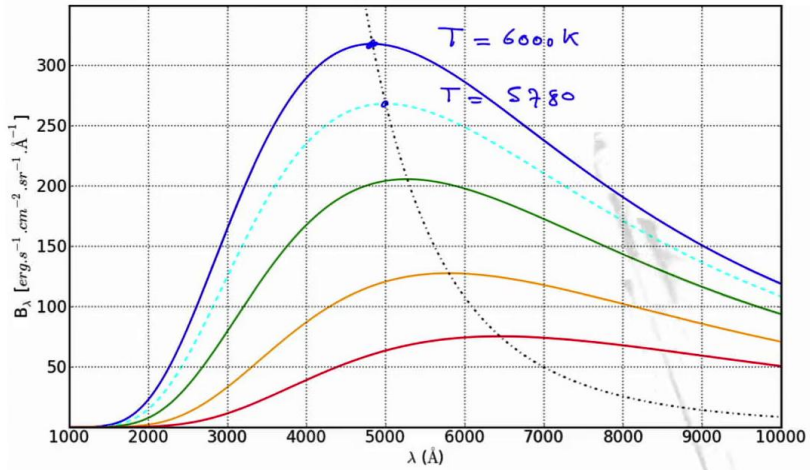
Notes

Summary

0m 06s

Black Body Radiation

Wavelength of the maximum emission (Wien displacement law)



$$B_\lambda = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1}$$

$$\lambda_{\max} T = 2,9 \times 10^{-3} \text{ [m.K]}$$

$$\lambda_{\max} T = 2,9 \times 10^{7} \text{ [Å.K]}$$

Introduction to Astrophysics

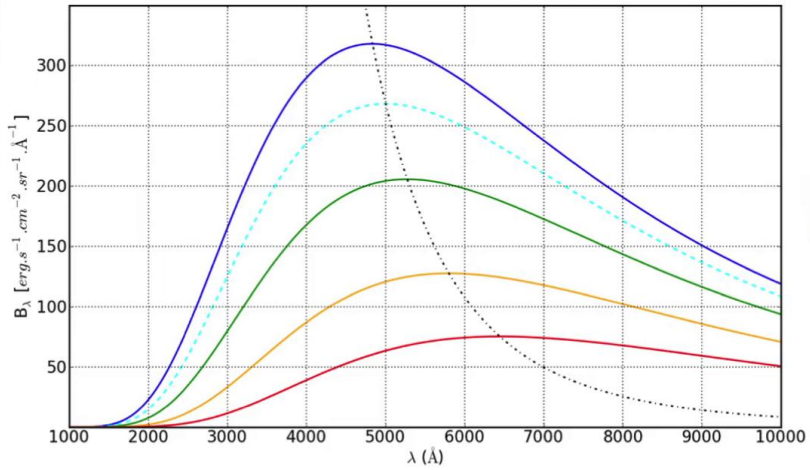
This blackbody radiation is characterised by what we call the Planckian curve which gives the energy distribution as a function of wavelength or as a function of frequency. The units of this luminance (or specific intensity) is a power by surface unit, by steradian, for radiation that is emitted in all directions and optionally by wavelength unit as shown here on that axis, or by frequency unit. These Planckian curves have a bell shape with an increasing and decreasing slope and so also a maximum. The position of this maximum can be calculated by deriving the function which gives the luminance. If we do it here, as a function of wavelength we will find that the maximum of the curve will occur at a position that depends only upon the temperature. We have therefore a linear relation between the position of the maximum of the black body curve and the black body temperature. So here we have a constant in m.K. We can have it in Å.K, which is more useful with the units we use in astrophysics, in the wavelength range used in astrophysics. We can see here the position of the maximun of the black body curve for a black body temperature of 6000K. Or here for a temperature of 5780K.

Notes

Summary

0m 38s

Wavelength of the maximum emission (Wien displacement law)



$$\lambda_{\max} T = 2,9 \times 10^{7} \ [\text{Å.K}]$$

$$\nu_{\max}/T = 5,9 \times 10^{10} \ [\text{Hz.K}^{-1}]$$

$(\lambda_{max} \cdot T)(\nu_{max} / T) = \lambda_{max} \cdot \nu_{max} = c$

$B_\nu = B_\lambda \cdot \frac{d\lambda}{d\nu} = B_\lambda \cdot \frac{\lambda^2}{c} = 0.57\, c$

That is the surface temperature of the sun for example. and these curves show different temperatures, here 5500K, 5000K and 4500K. Stars are excellent black bodies. So we can immediately see that λmax decreases when T increases. We can indeed observe it here. A high temperature yields a peak emission in the short wavelengths, the bluer ones. And lower temperatures result in emission peaks that lie at higher wavelengths, the redder ones. Since we have red on this side of the spectrum and blue on that side, in the optical wavelengths range A hot star will appear blue while a cold stellar object will appear red. So of course this Wien's displacement law which gives the position of the maximum of the black body peak as a function of temperature can be expressed in wavelengths or in frequencies. depending on whether we express the luminance as a funtion of wavelength or frequency. And looking at the two equations we tend to feel that (λmax x T) x (νmax/T) should be equal to λmax x νmax of course, but also equal to c. Actually that is not the case. If we take the numerical values here, we can see that we have 0.57 * c and that comes from the fact that Bν is not equal to Bλ, but is equal to Bλ * dλ/dν as we have shown in a previous lecture and so is equal to Bλ x λ²/c.

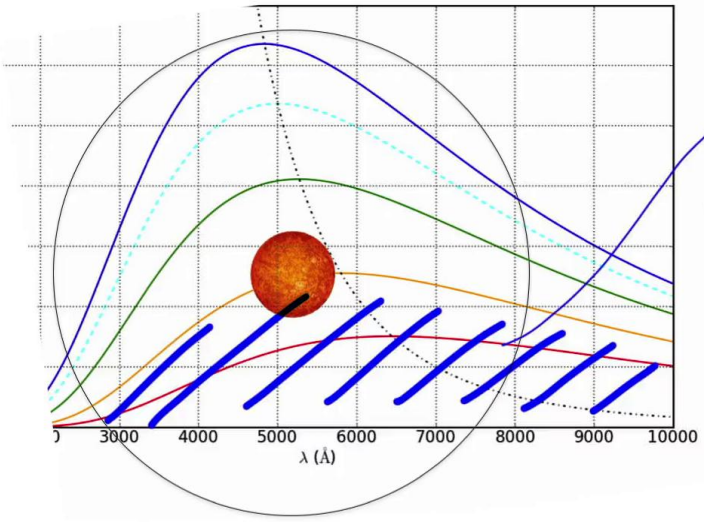
Notes

Summary

2m 14s

Black Body Radiation

Link with stellar luminosity ...a température T (densité de flux d'énergie)



$$M = \sigma T^4$$

$$\sigma = 5,67 \times 10^{-11}\ \mathrm{erg.s^{-1}.cm^{-2}}$$

M : puissance développée à la surface du corps noir.

Another feature which will interest us, is of course the total energy, the energy flux density developed by a stellar object which radiates black body radiation, in other words the whole of the luminance at all wavelengths or at all frequencies. So once the integration is done the available energy once we have integrated the black body for example here, under the orange curve, the result of the integral is the density of the energy flux which is simply proportional to the temperature to the fourth. So we have here a proportionality constant which is of course a power by surface unit and by Kelvin to the minus 4. As soon as we have the black body temperature we can determine the radiated power by surface unit at the surface of the black body. So M, is simply the power emitted by a unit surface at the surface of the black body.

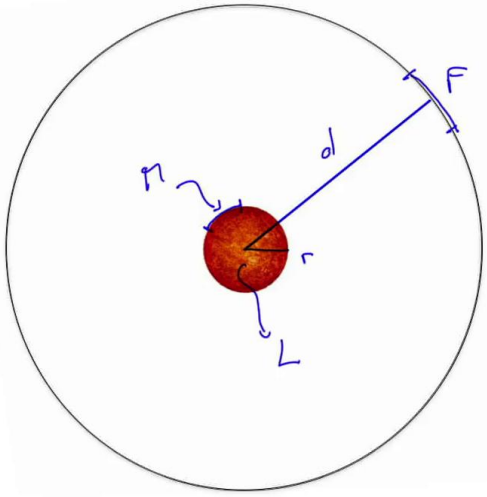
Notes

Summary

4m 32s

Link with stellar luminosity

L en [erg. s^{-1}]



$$M = \sigma T^4$$

$L = 4\pi r^2 . M$

$= 4\pi r^2 . \sigma T^4$

$F = \frac{L}{4\pi d^2}$ → reçu sur Terre

↓

[erg. s^{-1}. cm^{-2}. A^{-1}]

Introduction to Astrophysics

So now, what is the relation with stellar luminosity? The luminosity, is here the total energy freed by the star, The luminosity will be given in erg/s. This luminosity, will be the surface of the star multiplied by M. So $4\pi r^2 * \sigma T^4$, where we have here the radius of the star. The flux M will be integrated on the whole surface. And M is the available power by surface unit at the surface of the black body here of the star. So now, we at the level of the earth, we receive only part of that energy, at a certain distance d. If we are situated at a certain distance d, we will recive a flux by surface unit: the flux F here which will be L diluted on a sphere of radius $4\pi d^2$. So the flux we are receiving, we on the ground, is $L / 4\pi d^2$, that is receivend on earth. For this, the unit of F will be erg by second by cm squared because it is diluted on a sphere of radius d, and also by Angstrom in case we are interested in the spectroscopy where we disperse the light as a function of wavelength.

Notes

Summary

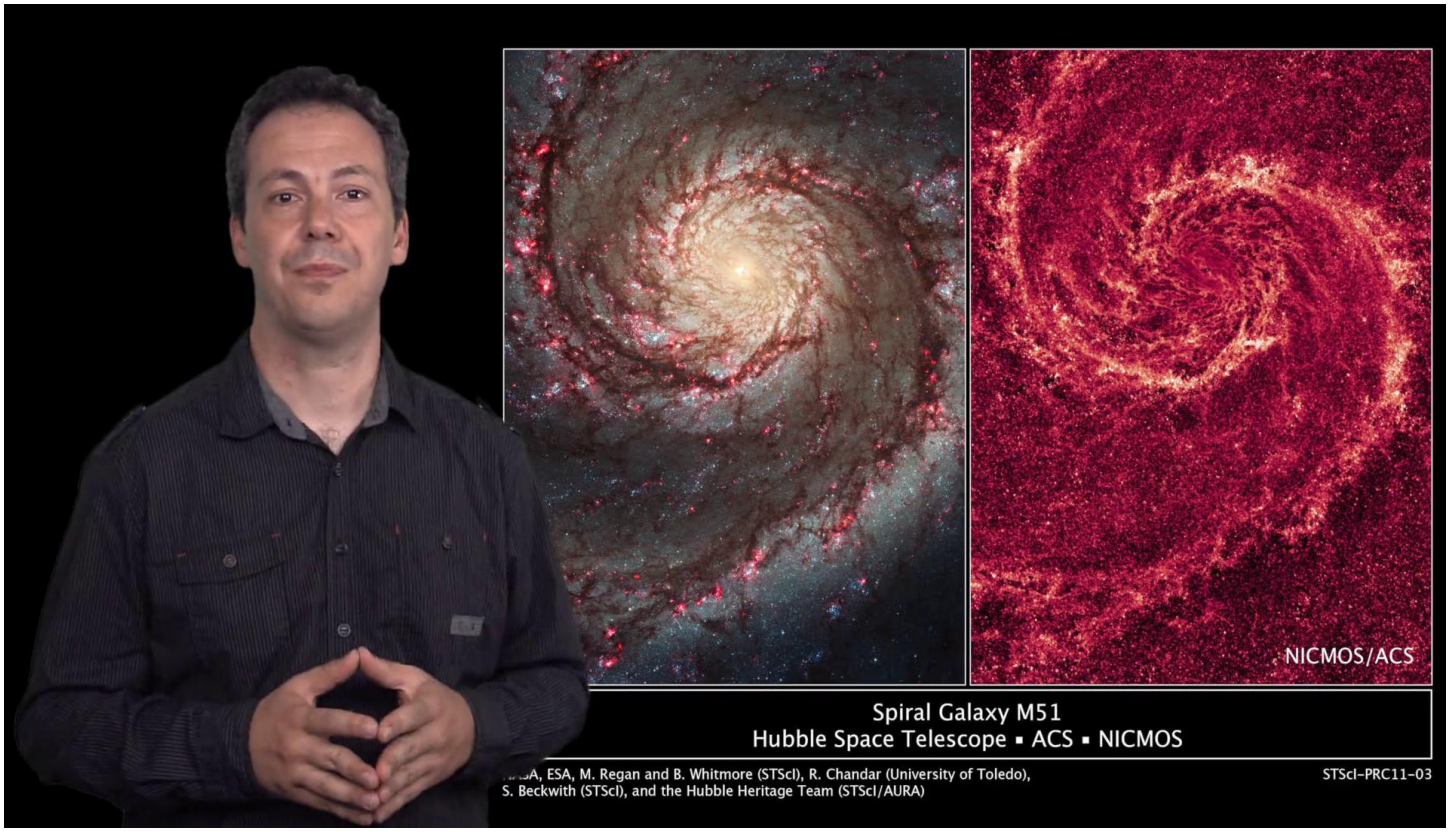
5m 42s

A lot of stellar objects emit black body radiation. Our Sun to begin with, whose surface emits in the visible wavelength range the energy produced at the center of the Sun by nuclear reactions.

Notes

Summary

7m 32s

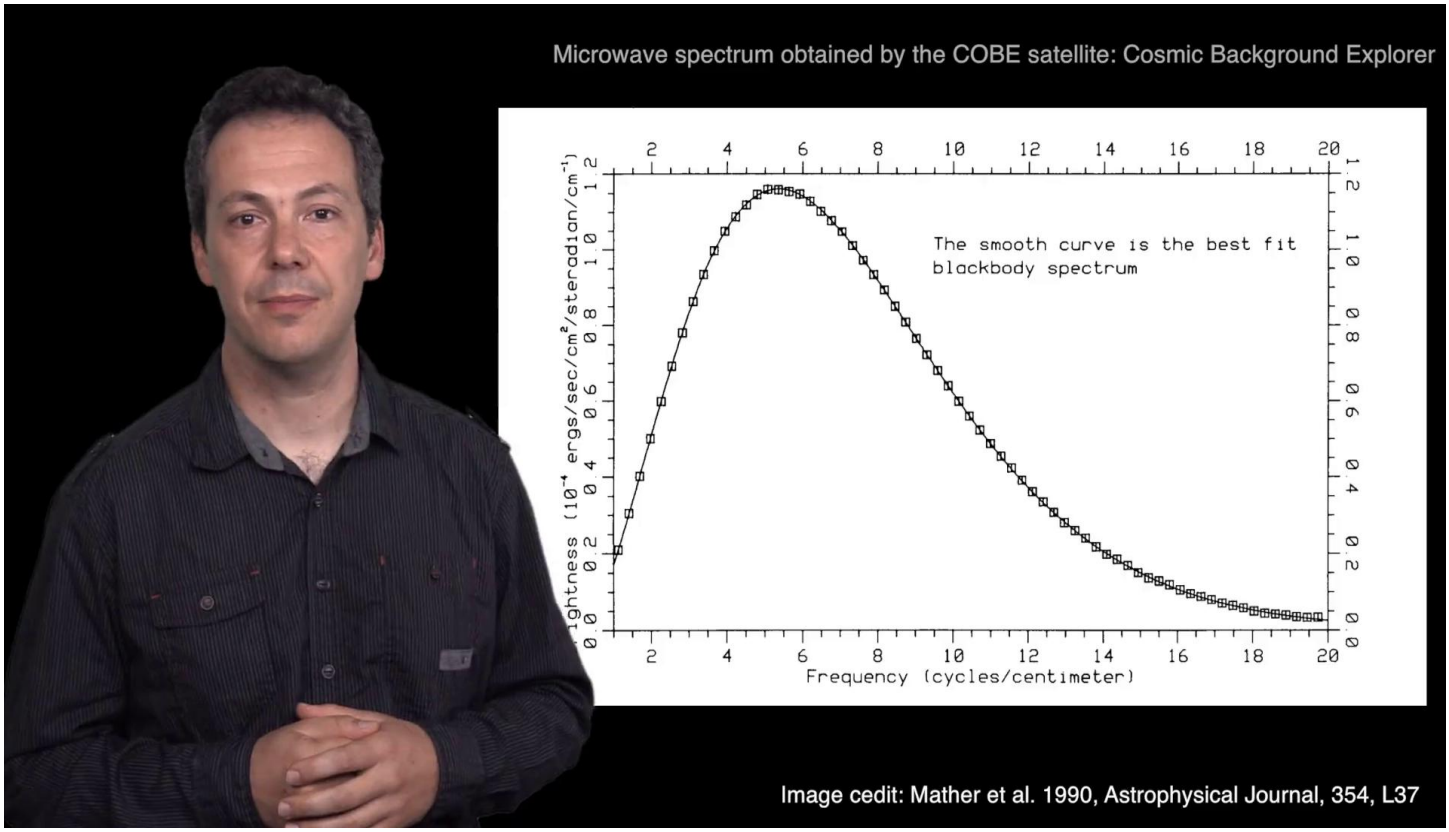


The dust in the disks of spiral galaxies is heated by young stars, found in the spiral arms, visible here on the right in the optical image of the galaxy. This energy emitted by the stars, is absorbed by interstellar dust and re emitted in the form of black body radiation whose peak is situated in the infra-red wavelength as can be seen here on the image to the right, of the same galaxy but taken in the infra-red wavelengths.

Notes

Summary

7m 46s



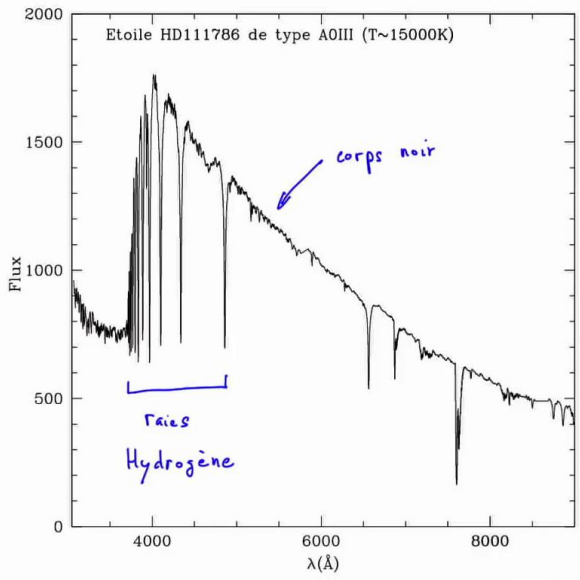
One of the most beautiful black bodies is probably the cosmological black body whose spectrum is shown here as a function of frequency. It is an isotropic micro-wave radiation which is present everywhere in the universe and whose origin lies in the Big Bang. The universe which was extremely hot extended itself and became colder, emitting an ambiant radiation which corresponds now, 13,7 billion years after the big bang to a black body whose temperature is very close to the absolute zero : 2,73K. What you see here are the measurement points in black, and the theoretical curve as a solid line. You see that theory and measurements perfectly agree. Black body radiation is certainly one of the most common types of radiation in astrophysics. It produces a continuous spectrum. The other type of frequently encountered spectrum is the line spectrum which we will see in a moment.

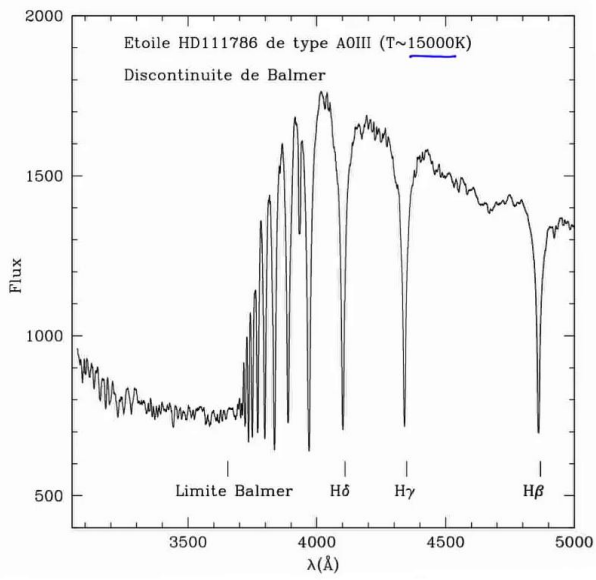
Notes

Summary

8m 16s

Atomic Lines





We had already talked about atomic lines in a previous chapter. We had taken the example of a relatively hot star, here at 15000K. Now that we have learned about black bodies, we see that a hot star essentially radiates in the blue wavelengths. That is what happens here. We have an extremely steep spectrum in the blue and we have a continuous one, which is the black body spectrum which is described by the Planckian curve. Now, on top of that, we have a line spectrum, with absorption lines in this case, which are due to hydrogen atoms present in the atmosphere, in the upper layers of the stellar atmosphere. Let's see in more details the line spectrum produced in this case here by the hydrogen atoms.

Notes

Summary

9m 12s

Hydrogen Atom

With the two previous equations we can write the total energy of the electron

$$E_{\text{Tot}} = \underbrace{\frac{1}{2}mv^2}_{E_c} - \underbrace{\frac{1}{4\pi\epsilon_0}\frac{e^2}{r}}_{E_p}$$

As a function of **n** and **h** instead of **r** and **v**

$$E_{\text{Tot}} = -\frac{me^4}{32\pi^2\epsilon_0^2\hbar^2} \cdot \frac{1}{n^2}$$

For that, let's go back to Bohr's atomic model where the electron is in orbit around the proton. We obviously have an equilibrium between the Coulomb force. We have a proton and then an electron in orbit around the proton, namely, a hydrogen atom. This is the mass times acceleration. This is just the Newtons law here. Quantum mechanics tells us that kinetic momentum is quantified and that the kinetic momentum takes an integer value multiplied by a constant. Here we have an integer value times a constant, which is the h bar constant namely the Planck constant divided by 2π, so h / 2π. The two equations which we have written in the previous slide are simply Newton's law and the quantification of kinetic momentum, i.e. the fact that the kinetic momentum of the electron around the proton can take only very specific values. These two equations will permit us to express the total energy of the electron. So the kinetic energy here and the potential energy here, as a function of n and the Planck constant, so an integer value and the Planck constant, rather then as a function of the radius and the velocity of the electron. So we have two equations with two unknowns, the solution is trivial.

Notes

Summary

10m 02s

Hydrogen Atom

The energy of the electron is **quantized**

$$E_{\text{Tot}} = -\underbrace{\frac{me^4}{32\pi^2\epsilon_0^2\hbar^2}} \cdot \frac{1}{n^2}$$

$$C = -2.18 \times 10^{-18}\ \text{J}$$
$$C = -13.6\ \text{eV}$$

eV: électron-Volt. Energy gained by an electron at rest when accelerated by a difference of potential 1 V.

We come back to the total energy of the electron, which will be negative since the system is bound. The total energy will depend upon a constant, is negative and will depend on the energy level characterised by n, which we defined earlier and which is responsible for the quantification of the kinetic momentum. The energy of the electron is quantified. The total energy will depend on an integer value and a constant. We recognise here a lot of physical constants: the Planck constant divided by 2π, the mass and charge of the electron and the electrostatic permittivity constant of vacuum ε0. If we express all that in numerical terms we find that this constant is 2.18 x 10^-18 Joules It is an energy. If we convert the result into a more useful energy unit for particle physics or astrophysics or also for atomic physics, we find -13,6 eV. So for the record, an electron-volt is the energy acquired by one electron at rest when it is accelerated by a voltage difference of 1 Volt. That is the definition of an electron-volt.

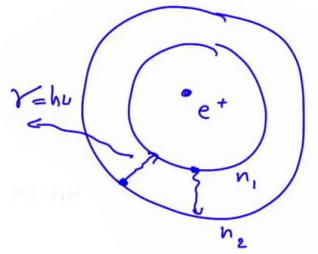
Notes

Summary

11m 34s

Hydrogen Atom

Energy levels $E_n = -13.6 \times \left(\frac{1}{n^2}\right)\ \mathrm{eV}$



Each orbit “n” is characterized by a given **angular momentum** and a given **energy**

The differences in energy between the orbital levels correspond to the energy of the photons emitted or absorbed

$$E_{n_1} - E_{n_2} = -13.6 \times \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\ \mathrm{eV} = h\nu$$

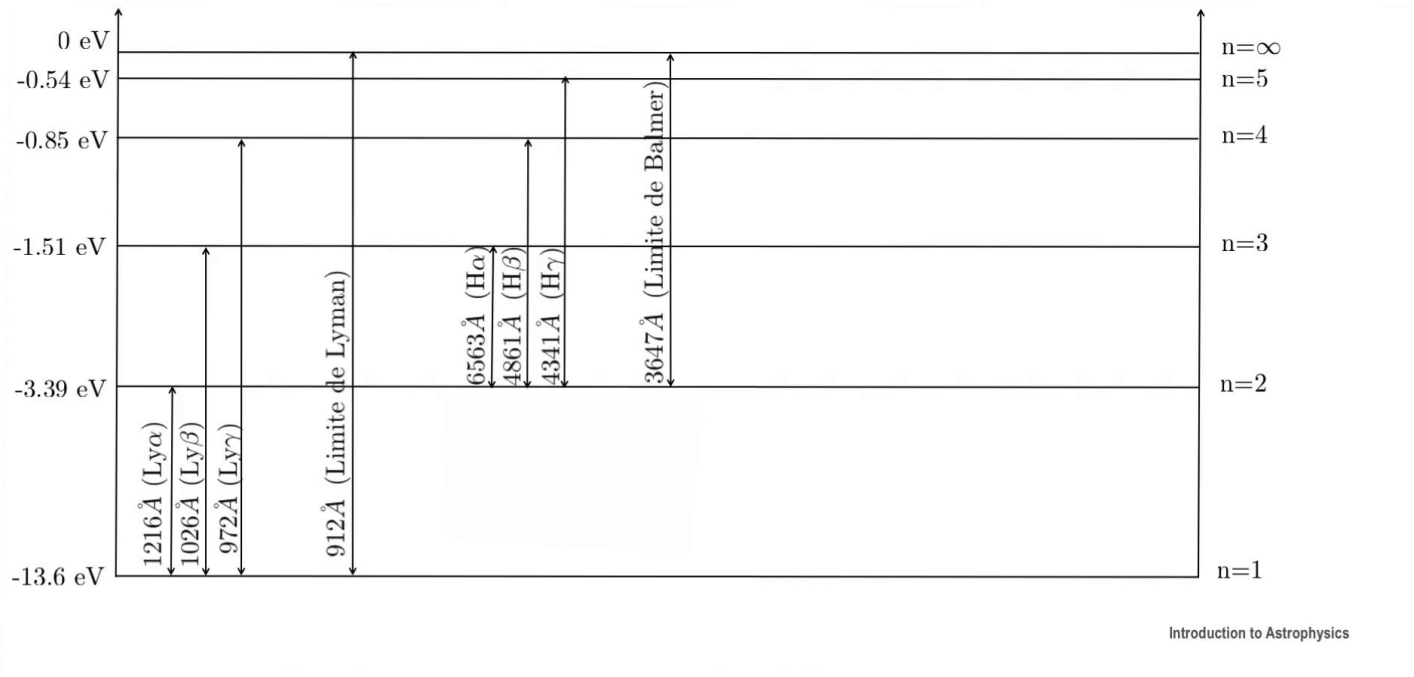
Introduction to Astrophysics

We saw that the total energy is quantified, so we have an energy that is a function of the integer value n and depends on the constant -13,6eV * (1/n²) where in fact n characterises the orbit on which moves an electron around the proton. We have a proton here and then we have an electron that might be on an orbit n1 or on an orbit n2. The electron will be able to go from one orbit to another by gaining or losing energy. If for example a photon is absorbed by an atom the electron will be able to go from the n1 layer to the n2 layer by gaining energy. Conversely, we can also have electrons that are in the n2 layer, an "excited" layer, and then will go back to the n1 layer. The atom will lose energy, by emitting a photon with a certain energy hv. What the frequency of this photon will be ? We can compute it by simply calculationg the energy difference between the atomic layers of the hydrogen atom. For example, the energy difference between layers 1 and 2 can be expressed using the equation we have seen before. -13,6 times the difference of the numerical constants here. So (1/n1² -1/n2²) is of course in electron-volts. It is also the energy that the photons will have once emitted by the hydrogen atoms that will go down from the layer n2 to n1. It is also the energy one has to bring to an hydrogen atom to raise an electron from layer n1 to layer n2.

Notes

Summary

12m 52s

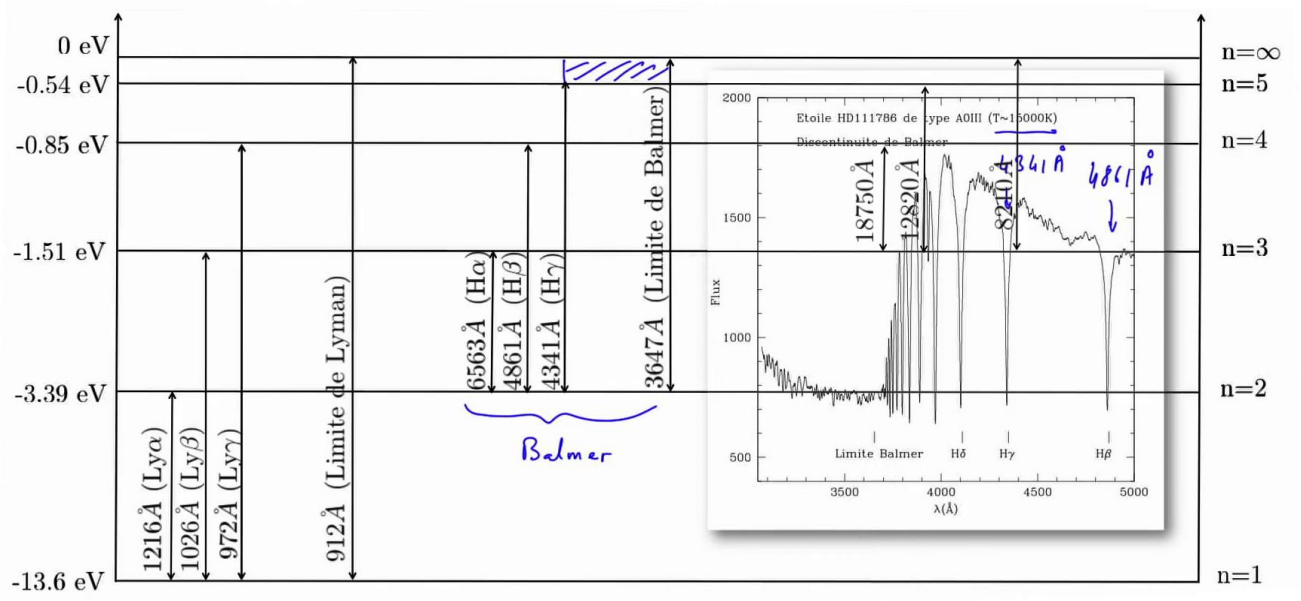


We can summarize here in a figure what happens in a hydrogen atom. We said that we have multiple possible energy levels that correspond each time to an integer value, from 1 to the limit, infinity. We could not represent here all the energy levels between the layer 5 and infinity. But if we could zoom in we would see more and more layers. We can have all possible transitions between a starting layer and arrival layers. For example to go from the fundamental layer, here, to higher energy layers, one needs energies that will be characterised by photons whose wavelengths are written here. So, if we want energies, high energy differences, we need photons with short wavelengths so with high frequencies and therefore high energy differences. Here have the first series of lines called the Lyman series, which corresponds to all possible transitions between the fundamental layer and all other atomic layers. So if we supply an atom with an energy higher then 13.6eV, the atom will get ionized. In other words, it will lose its electron. So an ionized gas is a gas transformed into a mix of ions and electrons. So for the Lyman series, we see that the wavelengths are in the ultraviolet.

Notes

Summary

14m 54s



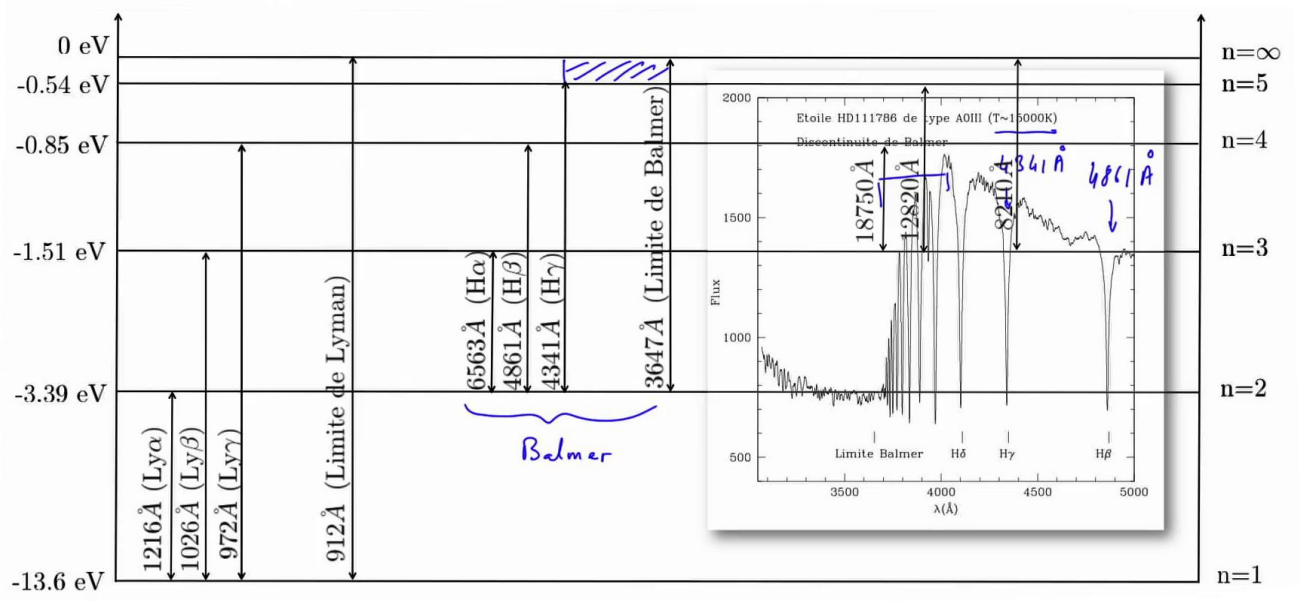
If we play the same game starting from layer 2 to all layers above it, we get the so-called Balmer series which, this time, is in the optical range. Finally, if we start a bit further still, from the next energy level, we will find the Paschen series, which is in the infrared. Of course there are other series, going further and further in wavelength and to lower and lower energy differences. Now lets go back to the example of a star we had before the star with a black body spectrum of 15000K and whose radiation peak is located in the blue part of the spectrum. So we can recognise here some lines of the Balmer series. The Balmer series is here and we can see if we want to predict the wavelengths of the different transitions here that we have for example a line at 4861Å which is actually the Hβ line. So this is 4861Å. So if we look here we see exactly the wavelength predicted by this formula. If we look at the following line, we have Hγ at 4341Å which will be here. The Hα line lies outside of the wavelength range of this spectrum, so it is not visible. If we went further and further down the blue side of the spectrum, we would have a whole series of lines, here, from layer 2 to all the layers that we couldn't represent and that would be in this box here.

Notes

Summary

16m 36s

$$E_{n_1} - E_{n_2} = -13.6 \times \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \text{eV} = h\nu$$



We haven't shown them here, since there would be an infinity of arrows here. That said, we can see them in the spectrum of the star. This multitudes of lines has a limit, here we have a limit that corresponds to the ionisation of the atom. So if an electron is on layer 2 and we provide it with a higher energy, than the energy so to -3,39 eV, the atom will lose its electron and so be ionised. No lines will be formed since the atom doesn't exist any more as an atom. We have a proton/electron pair, so an ionised gas. Here are all line series we can directly observe in the spectrum of the star.

Notes

Summary

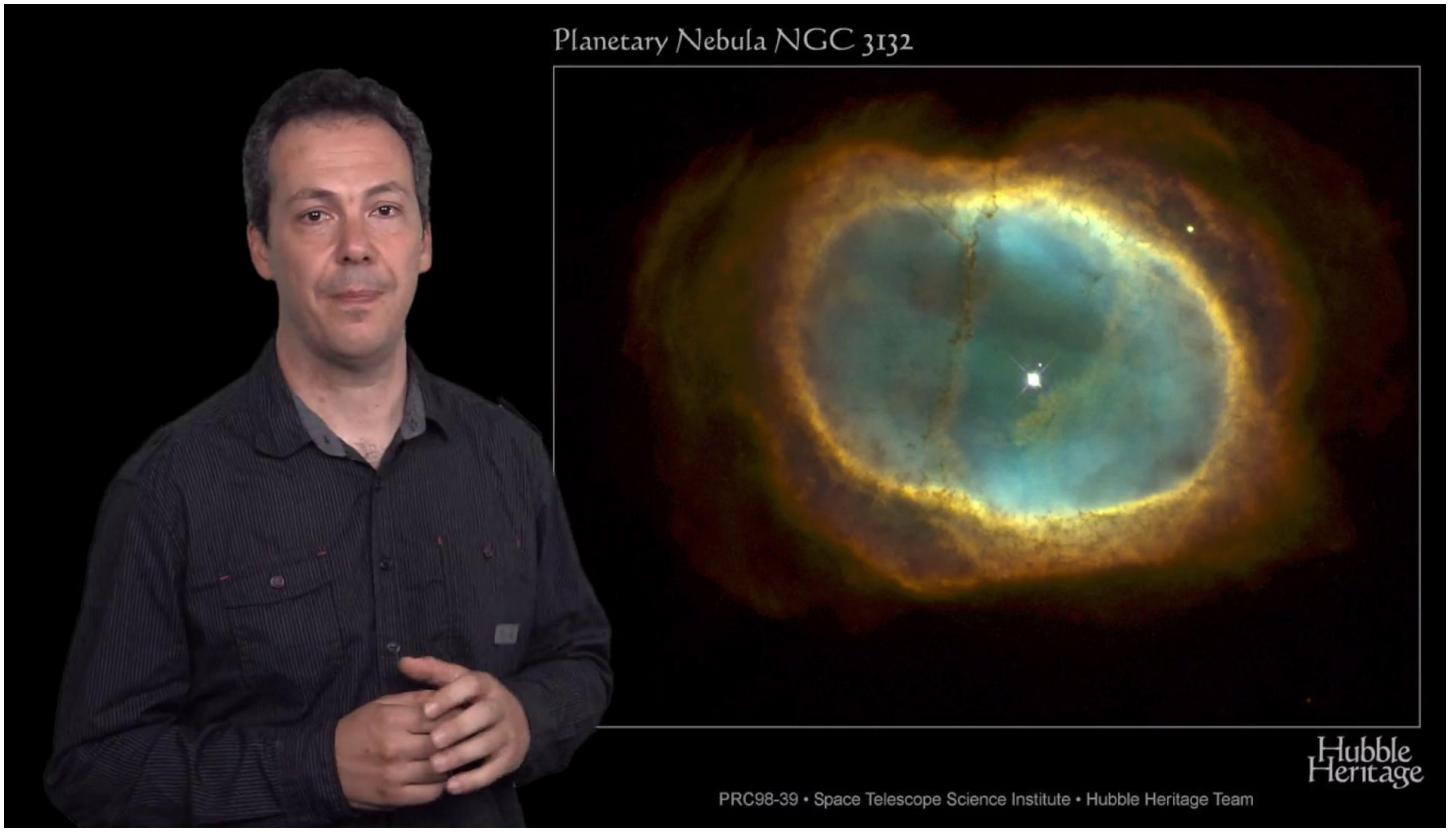
18m 35s

There exists a wide variety of astronomical objects characterized by a line spectrum. The line spectrum is often typical of the interstellar medium where the atoms excited by stellar radiation produce a spectrum consisting almost exclusively in lines. The latter inform us about the cheamical composition of the gas of which nebulae are made.

Notes

Summary

19m 15s



The same is true of planetary nebulae where the differents gas layers are excited by the radiation of the white dwarf star lying at the centre of the nebula.

Notes

Summary

19m 35s

And, closer to us, the polar aurorae we can see during strong solar activity display also a line spectrum. We can see one here from the International Space Station. In this case the atoms in the upper layers of the terrestial atmosphere are excited by collisions with the protons and electrons from the Sun that are captured in the terrestrial magnetic field. It is by losing energy through de-excitation that the atoms emit the polar auroras which have a magnificent line spectrum.

Notes

Summary

19m 44s