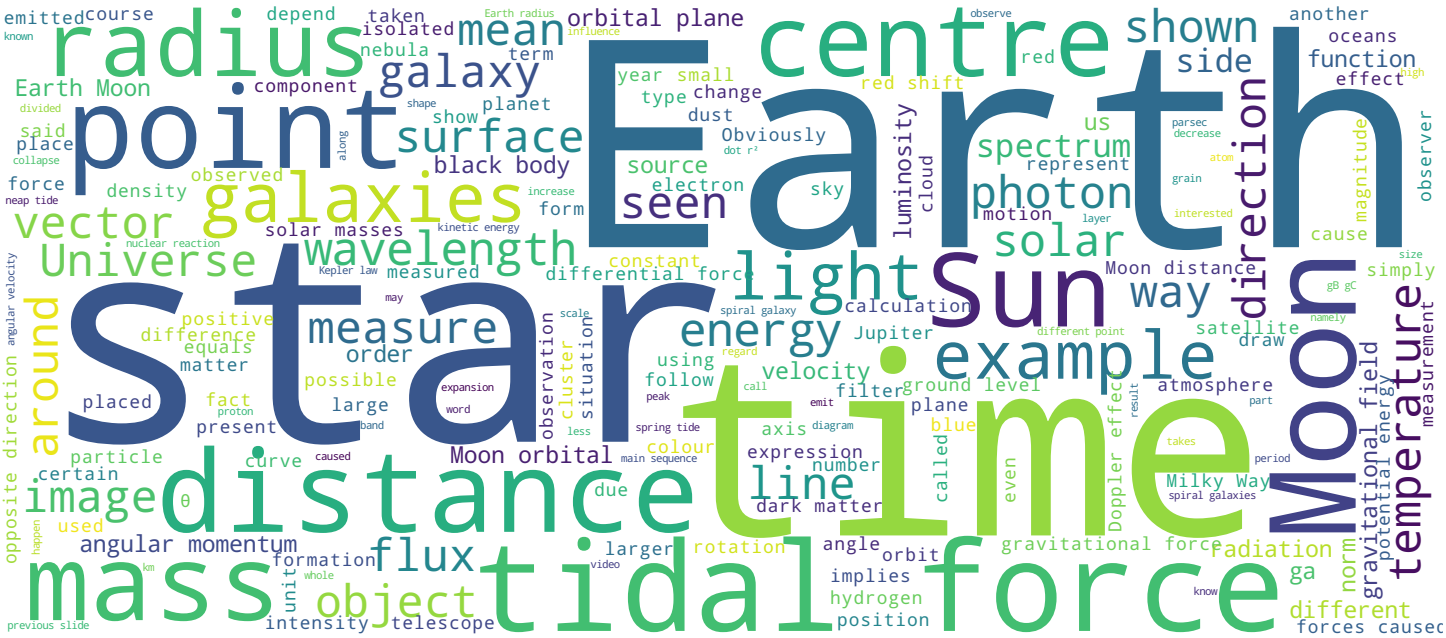
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Tidal Forces and Earth-Moon Distance

Tides at Mont Saint-Michel
Image: wikipedia

Introduction to Astrophysics; Chapter 3.3

Frédéric Courbin



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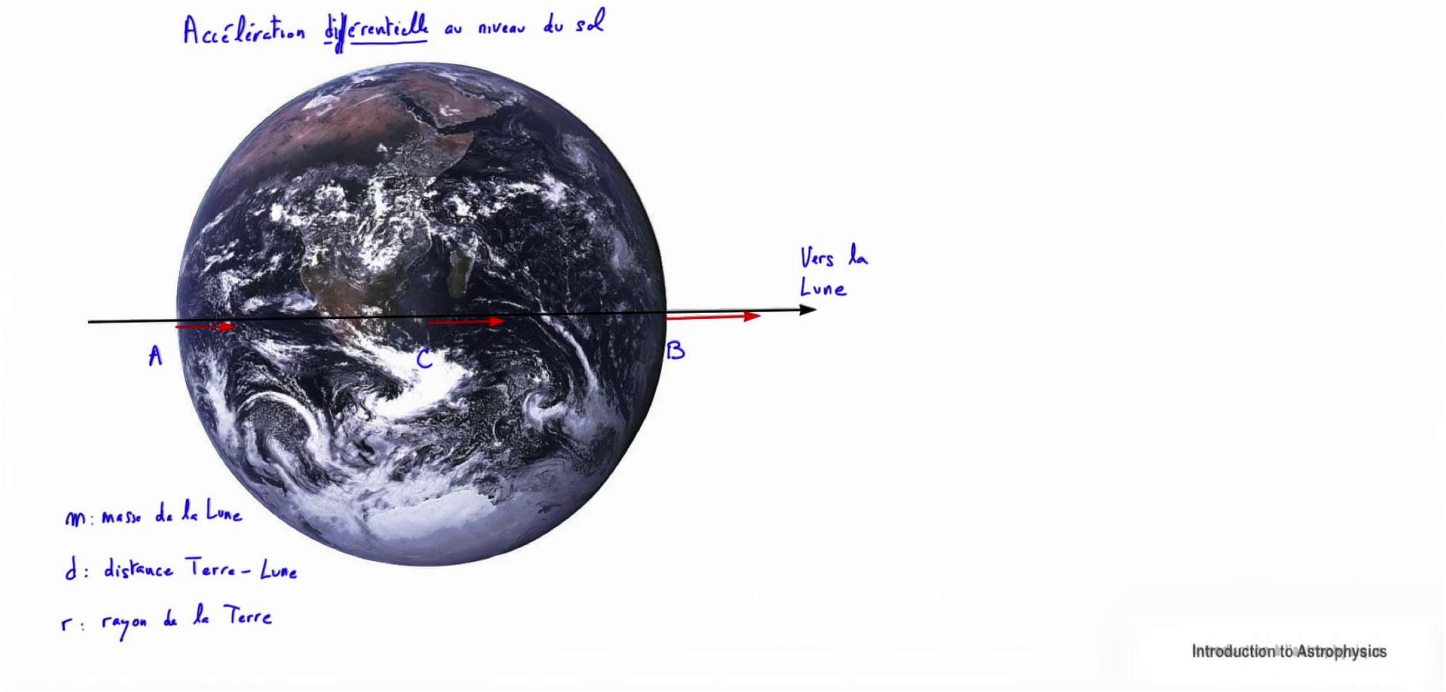
Tidal forces play a role in the formation and destruction of stars. They are present in all spatial scales: in planets, comets, stars and stellar clusters. It is known that tidal forces can deform objects as large as galaxies. We are familiar with the tidal forces on the Earth through the periodic motion of the oceans: the tides, which gave their name to this force. There are some dramatic manifestations of the forces in places such as the Breton site Mont Saint-Michel, or when they cause the tidal bore to the delight of surfers. In this lecture, the origins of the tidal forces will be described along with some illustrations of their manifestation on Earth, but also elsewhere in the Solar System and in the Universe.

Notes

Summary

0m 05s

Tidal Forces: Principle



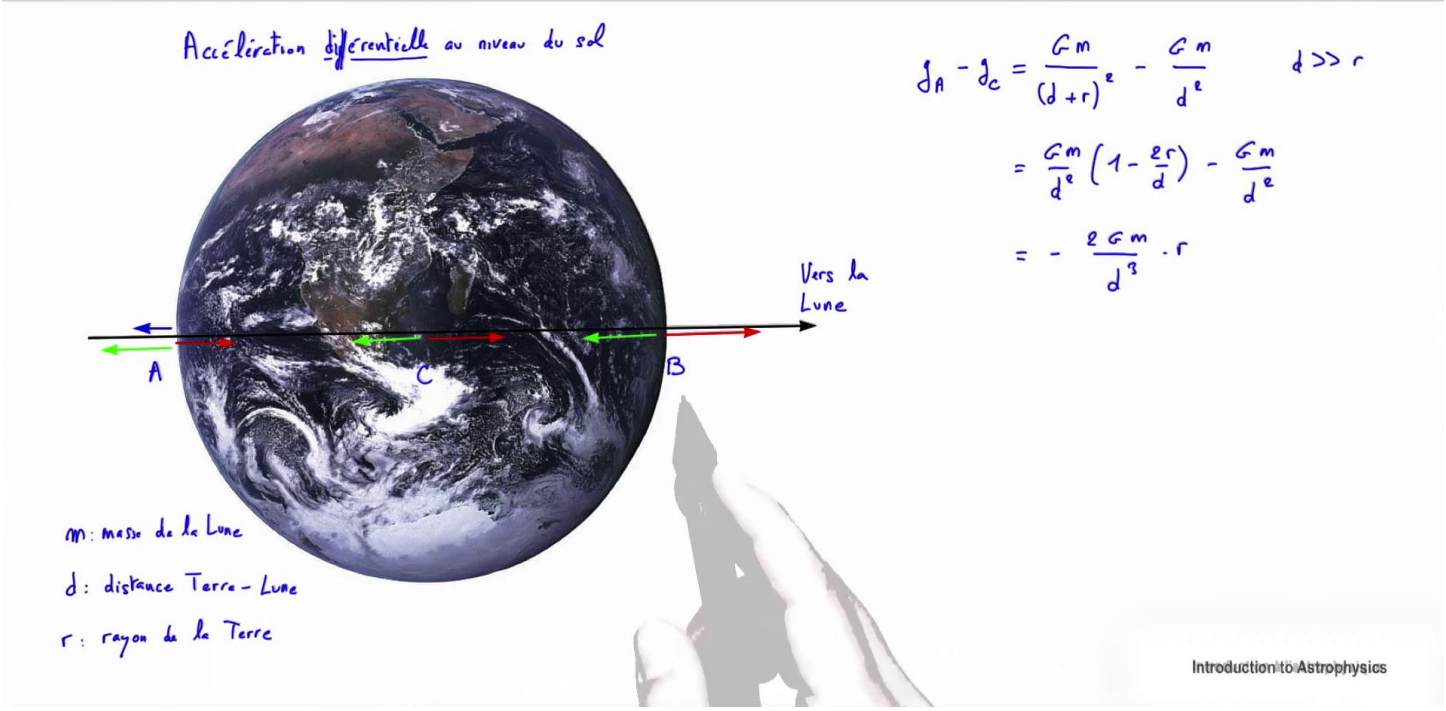
How do the tidal forces work? It is a differential acceleration at ground level. A differential acceleration due to the Moon experienced at ground level. We will seek how the Moon's gravitational force varies at the ground level with respect to another point, this reference point being the centre of the Earth. In order to illustrate the situation, let's consider the Earth, and the Moon with a mass m. The moon is at the distance d from the Earth, r is the radius of the Earth. Let's calculate the tidal forces at two different points on the Earth. The observer is placed on the Moon's orbital plane. The arrow points in the direction of the Moon that has a gravitational influence on the Earth. The closer the Moon, the stronger the gravitational field. If the Moon is in this direction, it attracts this side of the Earth with some force, and the other side of the Earth with a force that is much smaller. The centre of The Earth is attracted with a force that has an intermediate norm. Three points are taken: A, B and C for the centre of the Earth. Now, we look for the Moon's differential gravitational field at a given point with regard to the same gravitational field at a different point.

Notes

Summary

0m 55s

Tidal Forces: Principle



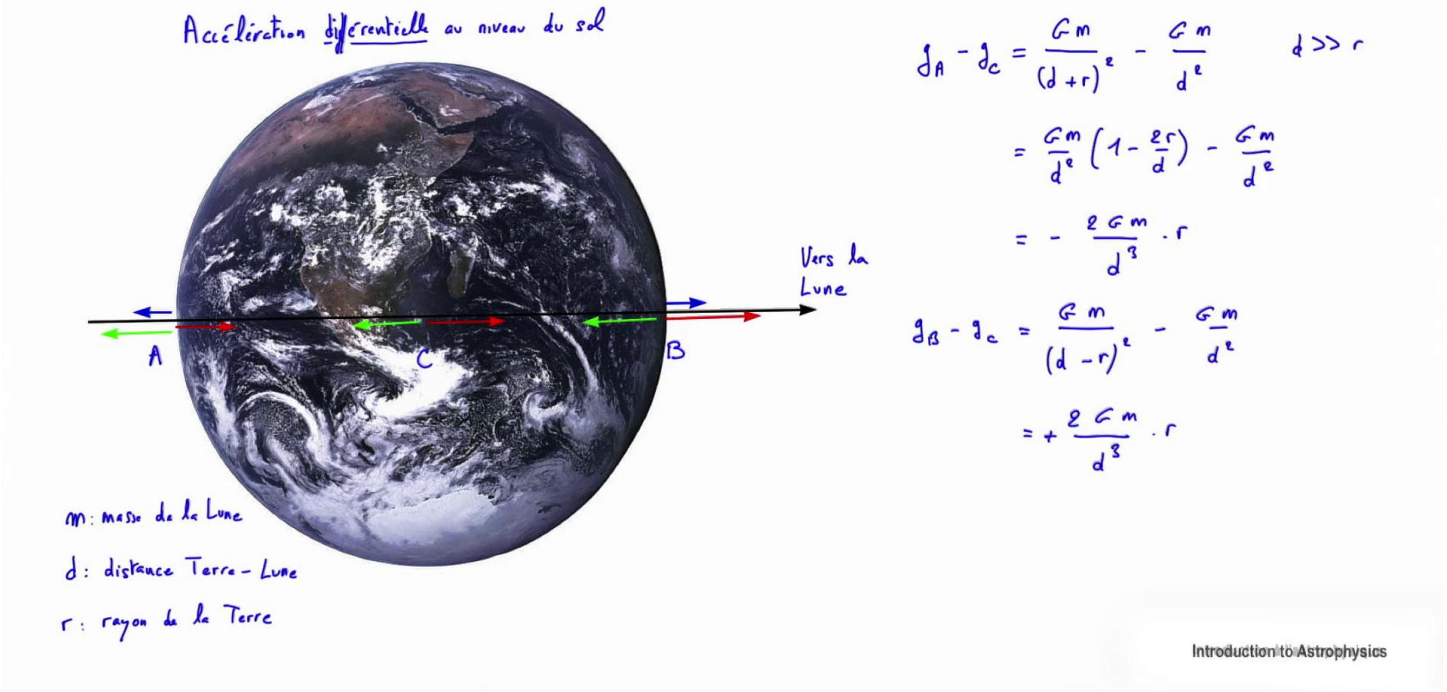
It means that we look for the gravitational field g caused by the Moon at the point A minus g caused by the Moon at C, for example. gA - gC can be represented using vectors. This is gA, minus gC, that can be placed at this point with the same norm as at point C. Minus gC is also here, and one can see that gC - gC is equal to zero. It can also be represented at this point. Now one can calculate gA - gC and gB - gC which is the tidal force, the differential force that describes the acceleration experienced at this point with respect to the centre of Earth, or at this point also with respect to the centre of Earth. Rewriting the equation using m - mass of the Moon $gA = Gm/(d + r)^2$, A is at distance d (Moon - Earth distance) plus r - Earth's radius, minus $gC = Gm/d^2$. Earth - Moon distance is much larger than the Earth's radius so this expression can be developed into a Taylor series factorising the Gm/d^2 term. With some further calculations one soon finds - $(2Gm/d^3) * r$. The expression for the tidal force at A is negative, so the vector of this force is directed outwards, it can be marked in this way. This is the tidal force at A. The same calculations can be done for B.

Notes

Summary

2m 57s

Tidal Forces: Principle



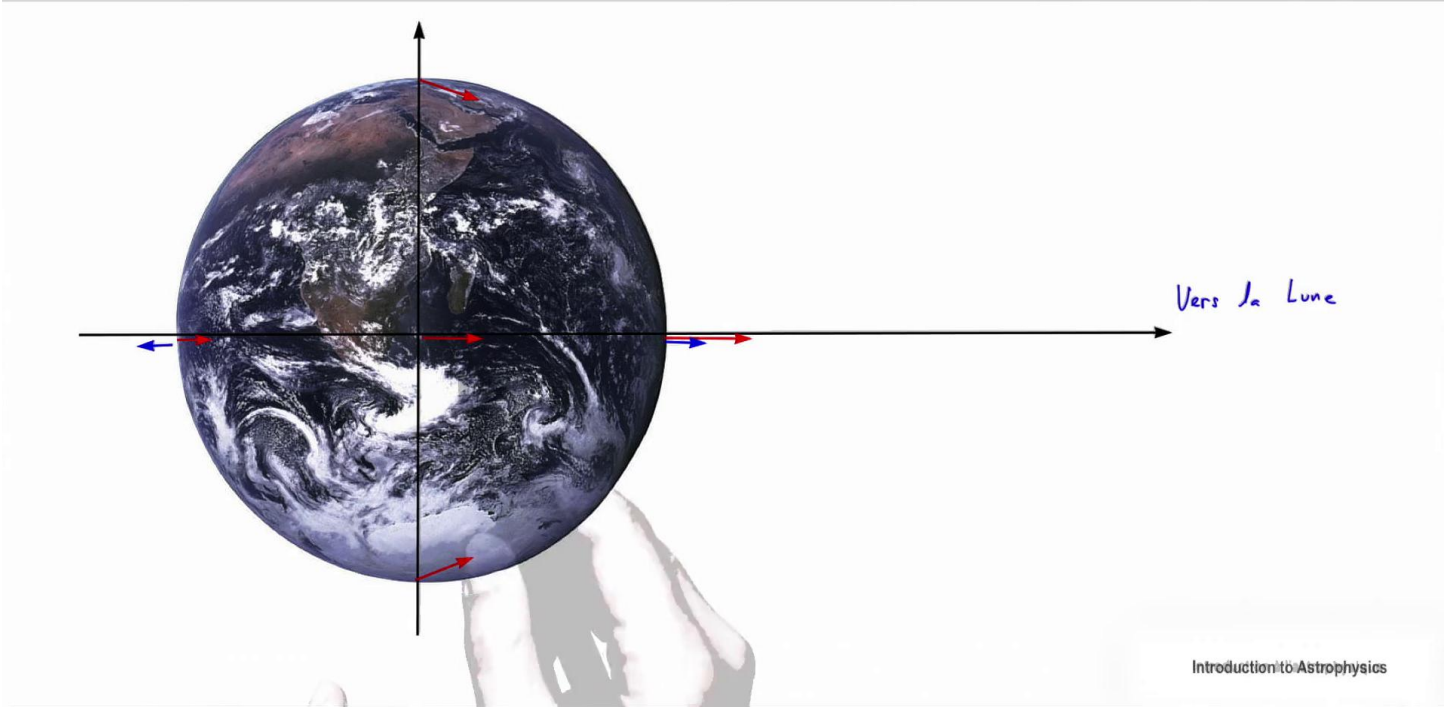
It can be noticed that for gB - gC, the force will be directed toward the Moon. Its norm can be calculated in the same way as before, hence, gB - gC = Gm / (d - r)² Before, there was a plus in this place, now the other side of the Earth is considered. so it is -GC / d², it stays the same. Computing the Tylor series gives (2Gm/d³) * r with the plus sign. Plus means the direction toward the Moon, minus - the opposite direction. There are two differential forces that pull each side of the Earth. Obviously, the differential force with regard to C is equal to zero. To a first approximation, as the expansion was limited to the first order, those two forces have the same norm but opposite directions.

Notes

Summary

5m 03s

Tidal Forces: Principle



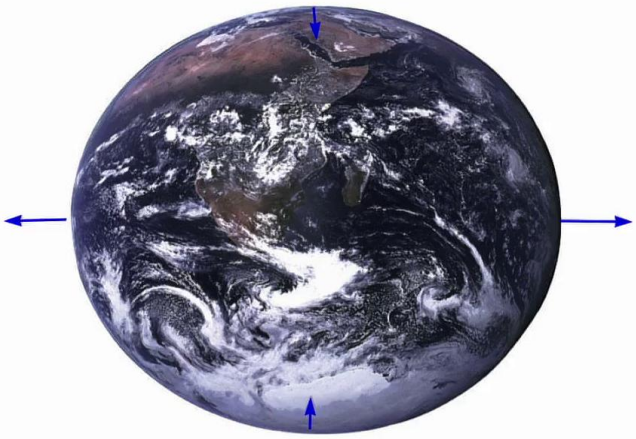
The intensity of the tidal forces in the Moon's orbital plane was shown. Here we represent in black the Moon's orbital plane. This is the direction of the Moon. Now, let's represent the Moon's gravitational force at different points of the globe. One force is exerted here, a force at the centre is weaker because of the greater distance to the Moon, another one is even smaller. Obviously, the vectors are drawn larger in order to illustrate better the differences. Then, we are interested in the points lying on a line perpendicular to the Moon's orbital plane, a line like this. One can draw the gravitational forces at those points, the vectors are tilted in this way. Each arrow has several components. If the difference between each vector at the border of the Earth and the vector at the centre is taken one can find the differential forces that were shown in the previous slide: both directed outward with equal norm but with opposite directions. Those are the tidal forces in the Moon's orbital plane. Now, let's consider those three points. This vector at the centre has a single component along the Earth - Moon direction. Those two vectors have two components.

Notes

Summary

6m 17s

Tidal Forces: Principle



Introduction to Astrophysics

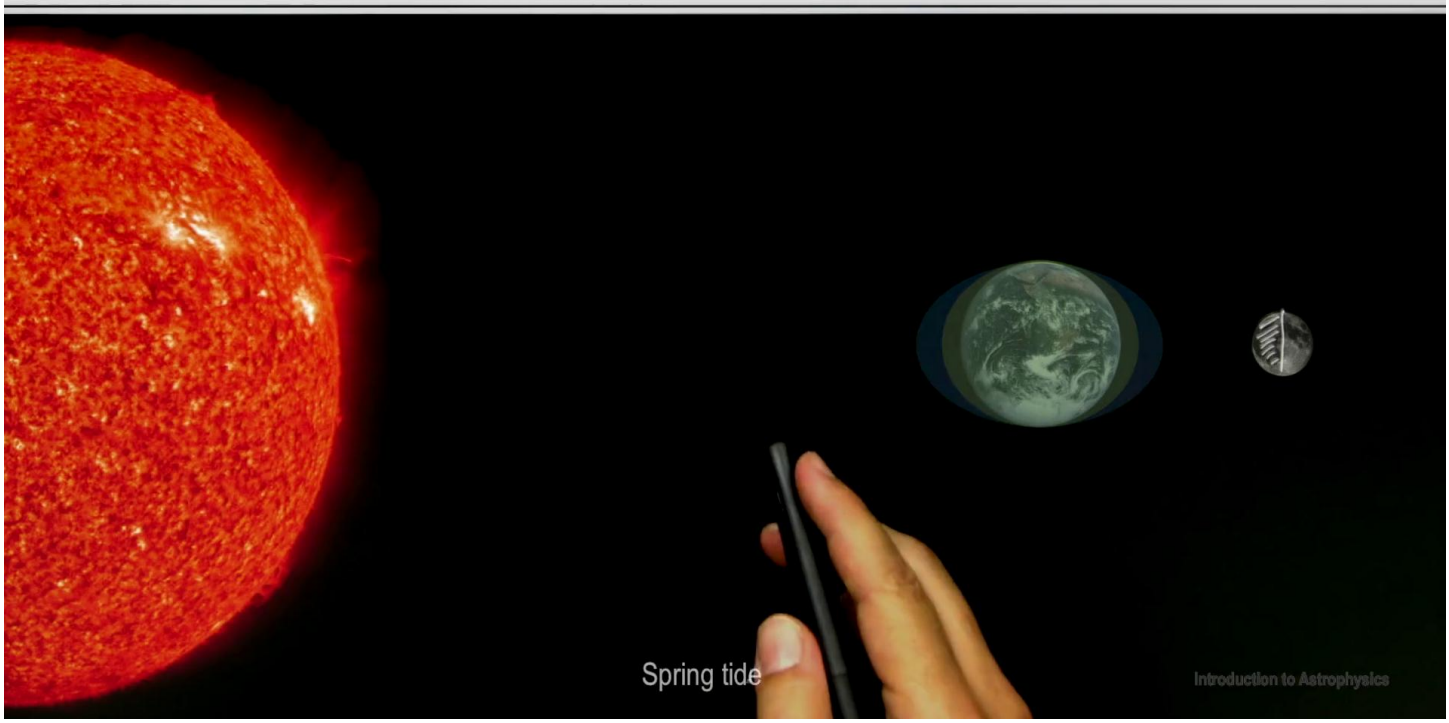
Each of those three vectors is at the same distance to the Moon. Placing the x axis here and y axis there, one can see that their x components are the same, meaning that this red vector has the same norm as those two. If one takes the difference between those two vectors, let's say that here we have the points D and E, point C at the centre, the gravitational field of the Moon at point D minus the same at C, as it was shown on the previous slide, the resulting vector has only the y component of the gravitational force of the Moon at point D. The resulting vectors can be represented like this. The Moon influences the Earth in such a way that the Earth is stretched in the Moon's orbital plane and then "crushed" in the direction perpendicular to the Moon's orbital plane. The Earth is flattened, the tidal forces have an influence on the Earth's crust, especially on the fluids on its surface - the sea. The Moon is placed in this plane, and so the oceans are stretched on both sides of the Earth, they undergo the force toward the centre of the Earth in the direction perpendicular to the Moon's orbital plane. This has a direct influence on the motions of the oceans around the Earth.

Notes

Summary

8m 04s

Combined Effects



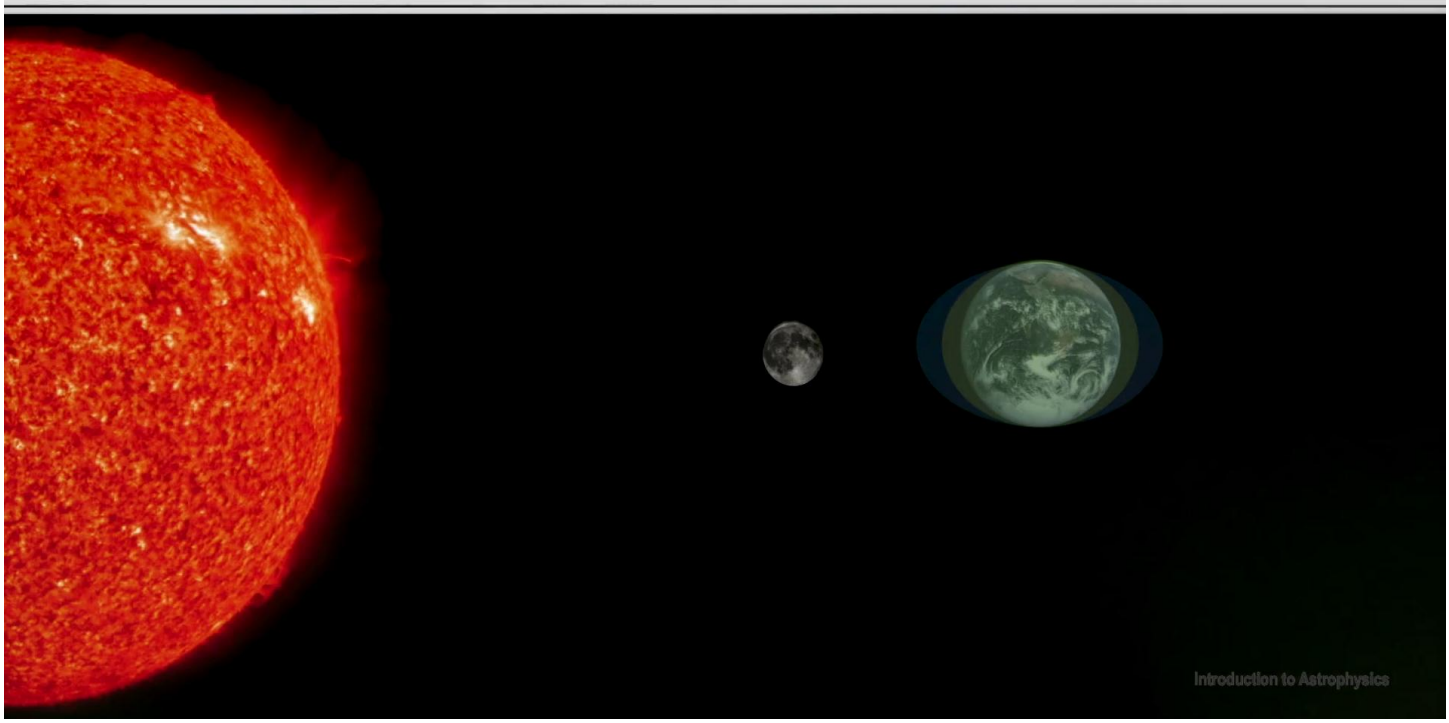
If the Earth-Moon system was isolated, the tidal forces would have the directions as indicated, and obviously, the oceans would follow this motion - there would be two liquid lobes on the Earth's surface. The description was for the tidal forces due to the Moon but, obviously, the same effect is present due to the Sun. The intensity of the tidal forces caused by the Sun is weaker. Clearly, depending on the position of the Moon with respect to the Earth and the Sun, the intensity of the tides changes. In this situation, one has what is called neap tide caused by the combination of the tidal forces originating from the Moon and the Sun, both effects being perpendicular. Moon rotates around the Earth in 28 days, Earth around the Sun in 365 days. It means that the changes in the tidal forces caused by the Moon are much faster than the changes of the tidal forces caused by the Sun. In this situation, the combined effect of the tidal forces caused by the Moon and the Sun creates spring tide. Both lobes created by the tidal forces of the Moon and the Sun overlap. Once more the neap tide, and then the situation with the full moon, as only this side of the Moon is illuminated, we have a spring tide once again.

Notes

Summary

10m 10s

Decelerating Earth Rotation Speed



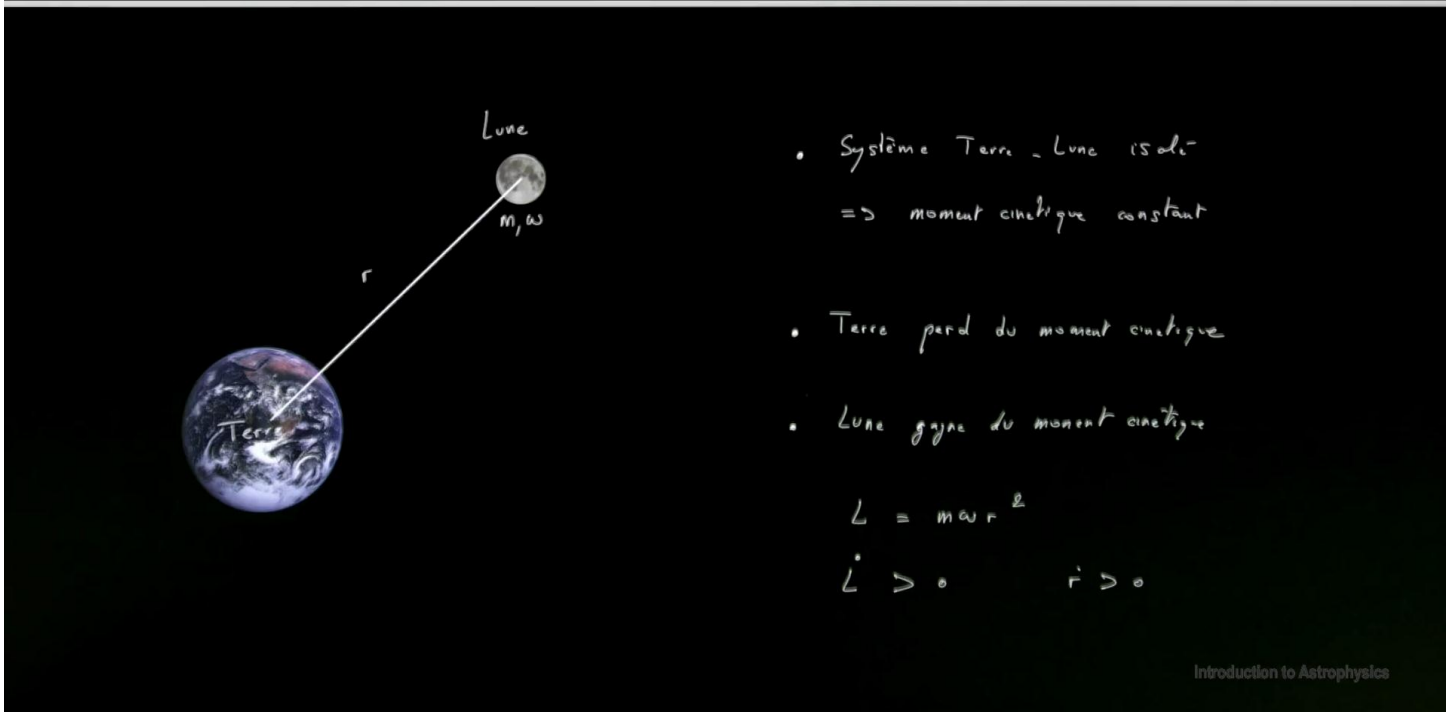
For the full moon and new moon we have spring tides. For the waxing and waning quarter moon there is a neap tide. The tidal forces influence the motions of the oceans which in turn influence the Earth's rotation. Indeed, when the Moon rotates around the Earth the oceans tend to "follow" it. Due to the tidal forces the oceans take an ellipsoidal shape that follow the Moon's trajectory. In this motion, the oceans rub the earth's crust, causing the release of energy in the form of heat. The Earth looses rotational energy, and so angular momentum. This implies that the Earth slows down its rotation. This is not the only consequence; while the Earth is slowing down, the Earth - Moon distance increases.

Notes

Summary

11m 56s

Increasing Distance to the Moon



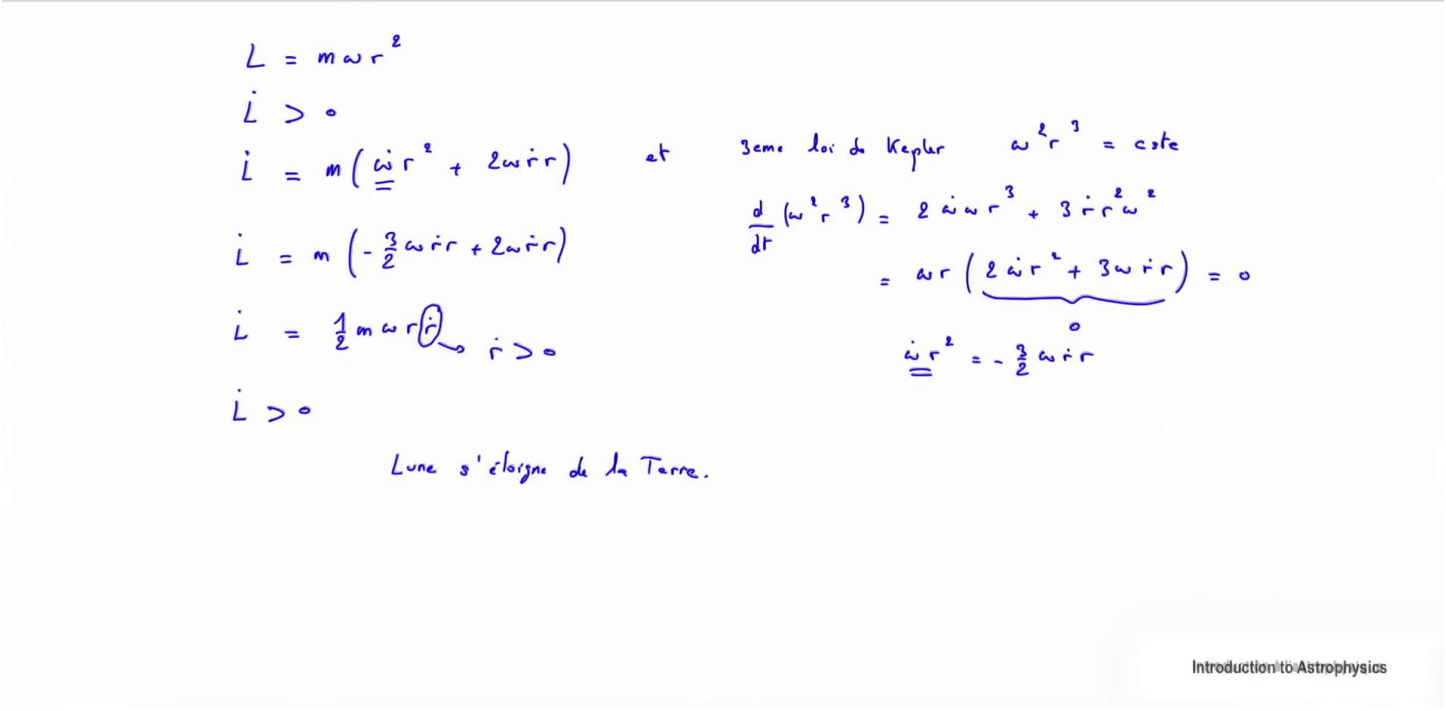
This effect is shown on the next slide. Considering the Earth - Moon system as isolated, the Earth at this place, and the Moon here. Let r be the Earth - Moon distance, m the mass of the Moon and ω its angular velocity around the Earth. This system is isolated meaning that its total angular momentum is constant. Due to the friction of the oceans upon the terrestrial crust, the Earth looses angular momentum increasing at the same time the angular momentum of the Moon, as the total angular momentum is conserved. The Moon's angular momentum is equal to $L= m\omega r^2$. As it was said before, the angular momentum of the Moon increases so L dot is positive. Now, it will be shown how the fact that L dot is positive implies that r dot is positive, meaning that the Moon increases its distance from the Earth.

Notes

Summary

13m 00s

Increasing Distance to the Moon

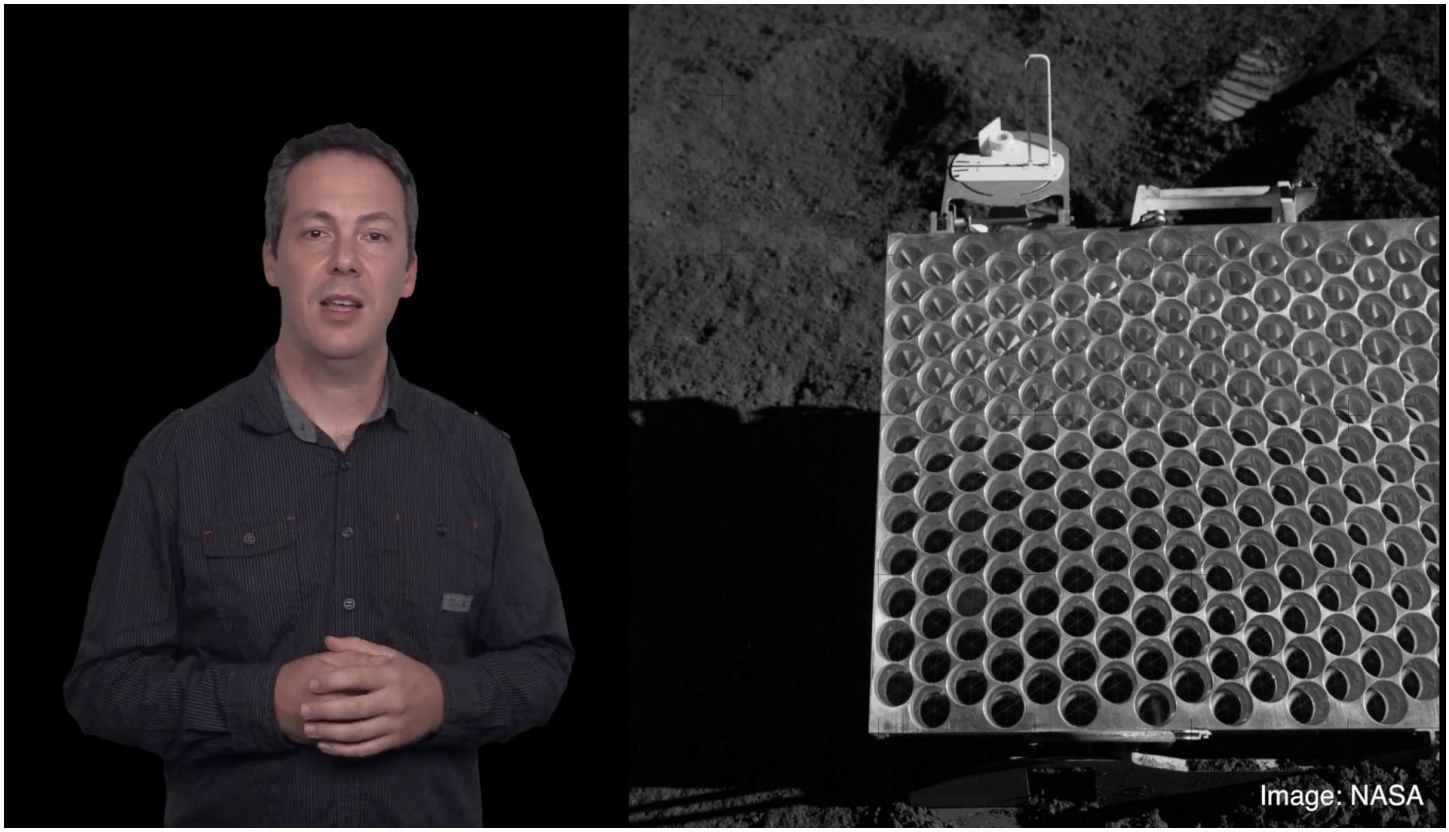


Let's write the angular momentum of the Moon, its mass, angular velocity, the square of its orbital radius. It was said that L dot is positive. Let's take the time derivative of L. Now, using Kepler's laws, particularly the third law, one can write that $\omega^2 r^3$ is equal to a constant. Computing the time derivative which is zero, and factorizing this expression leads to an ωr term here and finally one gets 2ω dot $r^2 + 3\omega \dot{r} r = 0$. Obviously, neither ωr nor r is equal to zero, it is the whole expression that equals zero, implying ω dot $r^2 = -3/2\omega \dot{r} r$. This equation will be used to replace ω dot r^2 in this formula. This leads to L dot equals m times $-3/2\omega \dot{r} r + 2\omega \dot{r} r$. Finally, L dot = $1/2 m \omega r \dot{r}$. It was said that L dot is positive, so looking at this expression one sees that if m, w and r are positive $\dot{r}$ also has to be positive. If $\dot{r}$ is positive it implies that r increases and so the Moon moves away from The Earth.

Notes

Summary

14m 13s

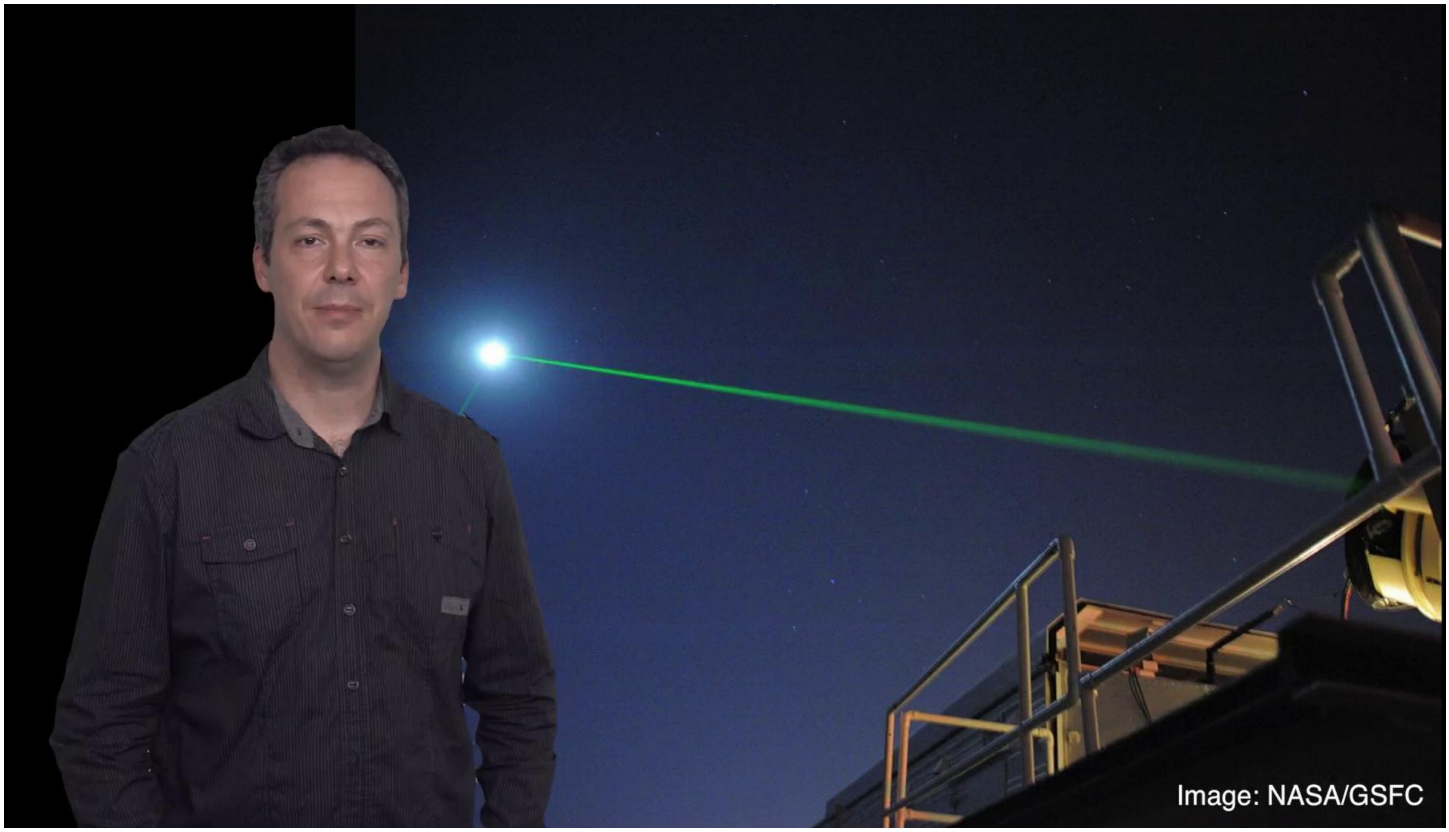


This distance increase can be measured thanks to the retroreflectors placed on the Moon by the astronauts during the Apollo 11 mission in 1969. The property of those retroreflectors is that they reflect the light exactly in the incidence direction. If one aims at them with the beam of a laser from the Earth it is possible to measure the Earth - Moon distance very precisely by simply measuring the time it takes for photons to make their return journey.

Notes

Summary

16m 32s



In practice, the beam of the laser light is sent from a telescope, photons are reflected and then collected by the same telescope after around two seconds. Thanks to this measurement it is possible to deduce that the change of the Earth - Moon distace is equal to about 3,8 cm per year.

Notes

Summary

16m 58s

In the Solar System, Io, one of the four satellites of Jupiter, show an exuberant volcanic activity due to the tidal forces of Jupiter. Its surface is strewn with volcanoes.

Notes

Summary

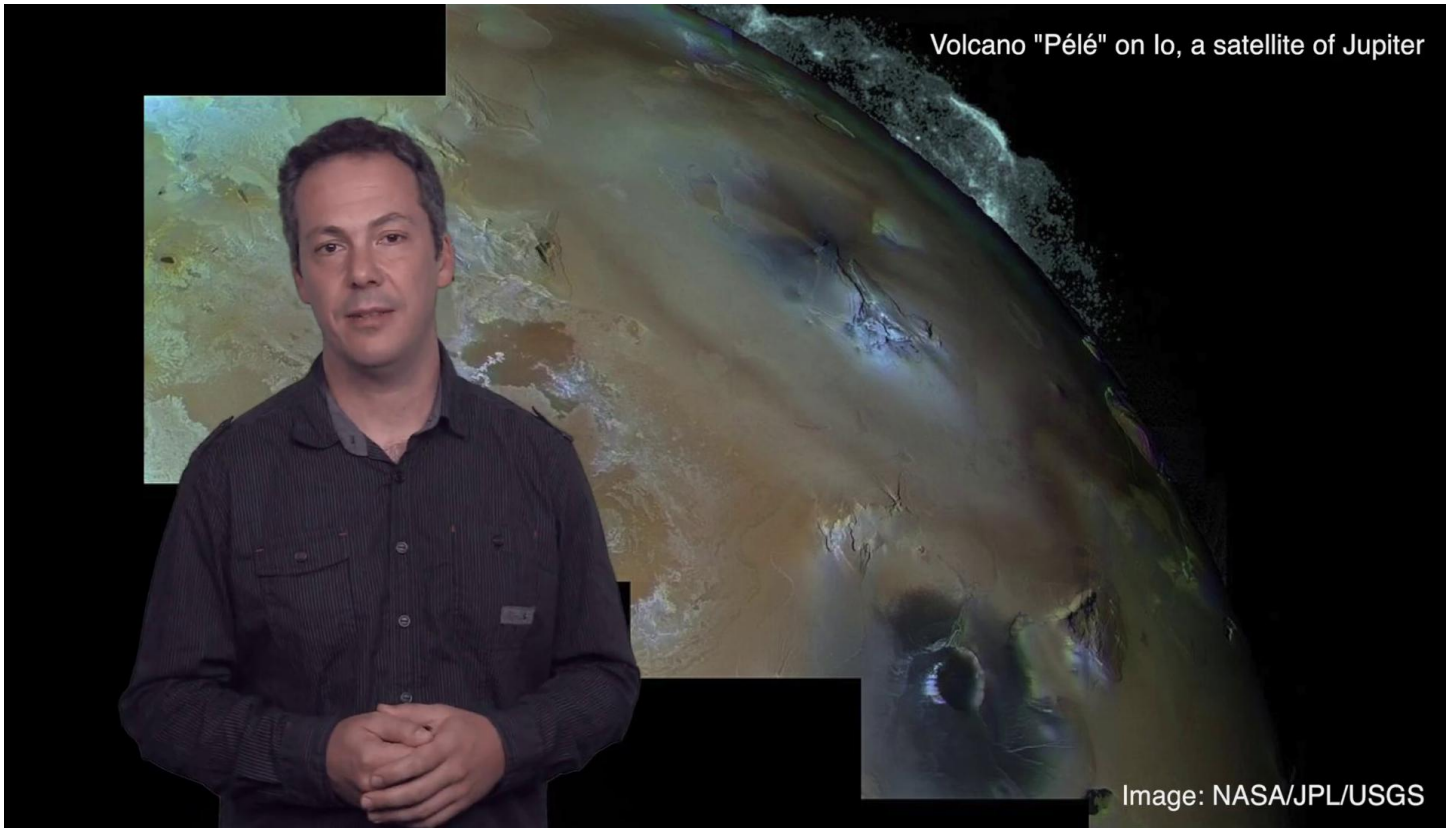
17m 16s

Of the four galilean satellites, Io is the closest moon to Jupiter and also the one that has the most eccentric orbit. The tidal forces regularly compress and dilate the rocky mantle, causing it to melt.

Notes

Summary

17m 26s



The result is a collection of volcanoes on the satellite's surface, and some of them, like the volcano Pele shown here, eject matter fast enough that it overcomes the satellite's gravitational field and escape. In the next lecture, we will have the opportunity to see that the tidal forces can not only cause the rocky mantle to melt, but may also cause the dislocation of the satellite, or even dislocate a celestial body in general.

Notes

Summary

17m 43s