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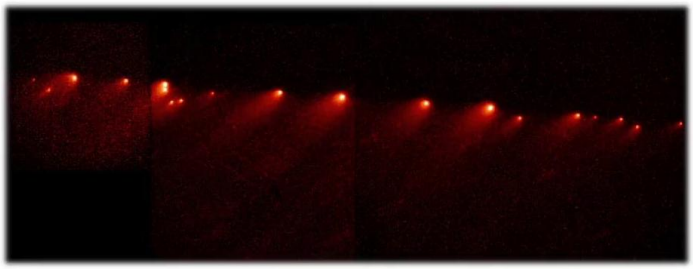


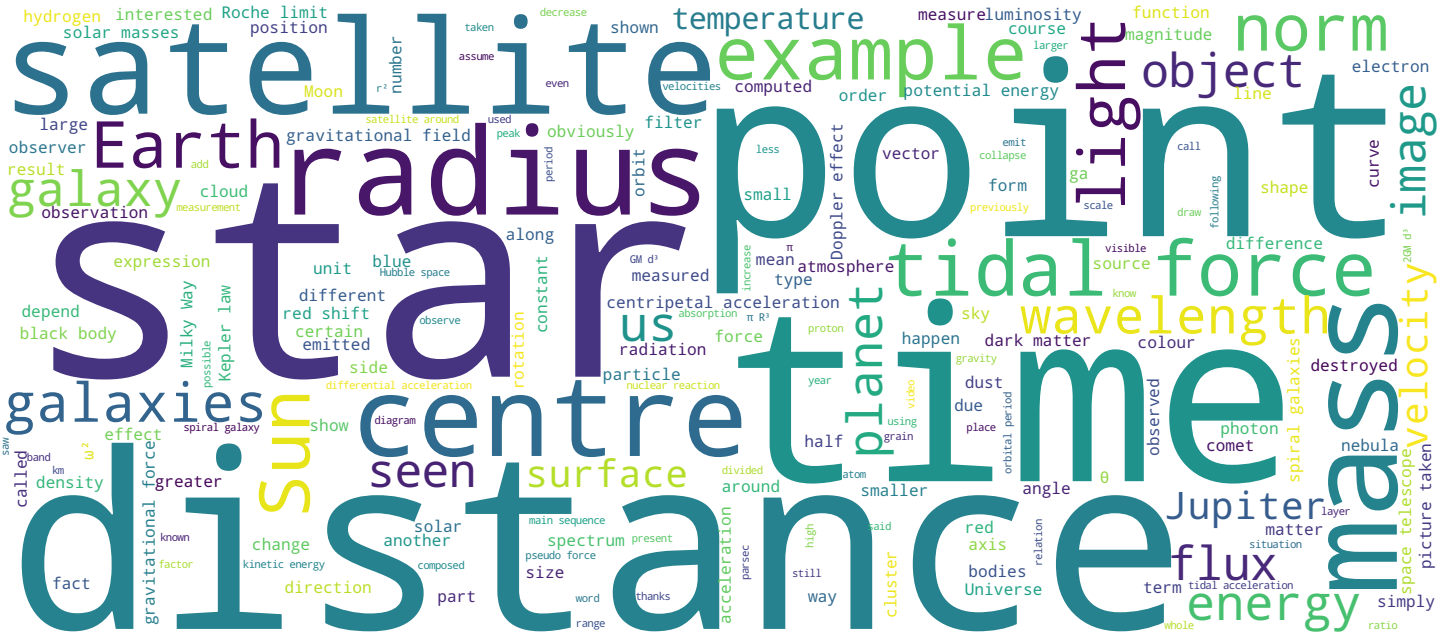
Tidal disruption of Comet Shoemaker-Lévy 9, in 1994

Image: NASA/HST

Roche limit

Introduction to Astrophysics; Chapter 4.1

Frédéric Courbin



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Video

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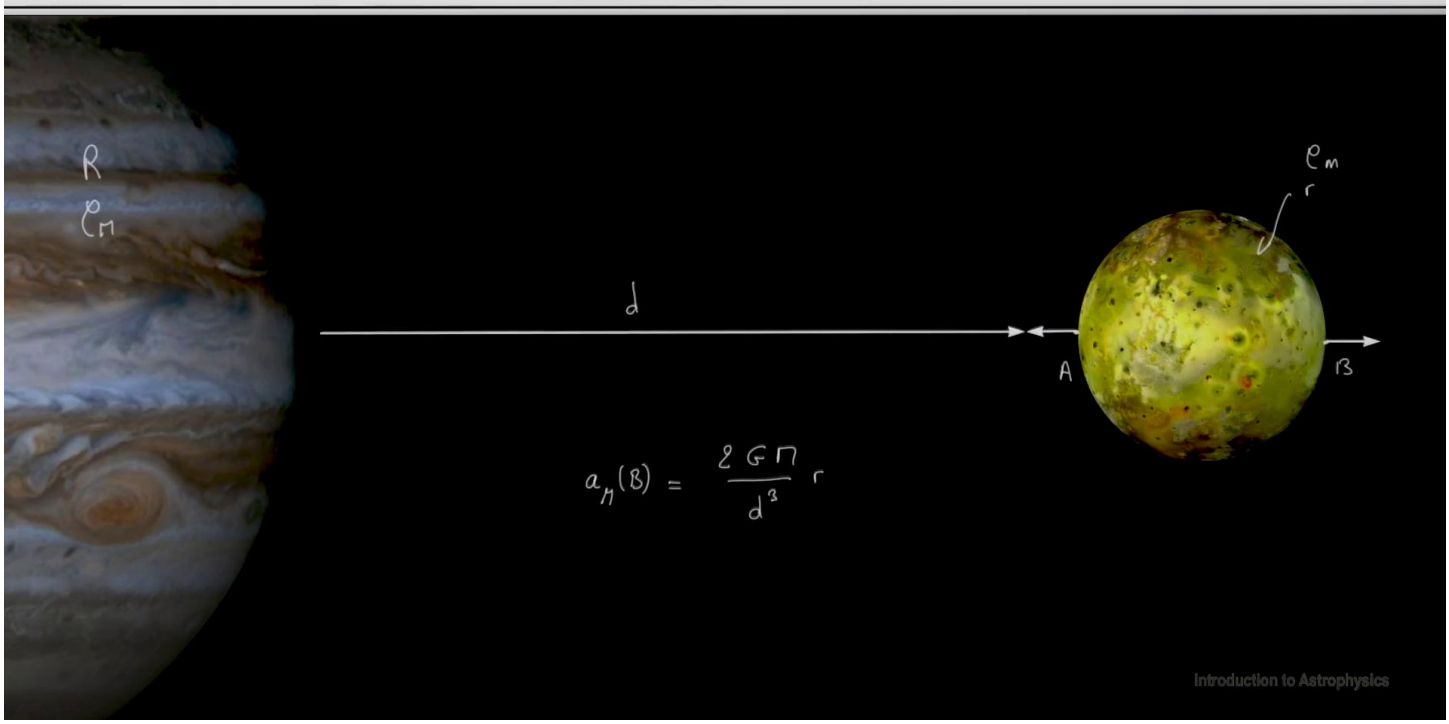
In extreme cases the tidal forces can distort the shape of celestial bodies up to the point of breaking them. Some planetary rings such as Saturn's, seen here by NASA's Cassini probe, may be the result of the destruction of a satellite under the effect of tidal forces. We will see here that there is a limit in the distance between two bodies before one of them breaks under the effect of the tidal forces from the other. This distance is known as the « Roche limit » from the French astronomer Edouard Roche who first performed the calculation in the 19th century.

Notes

Summary

0m 04s

Roche Limit



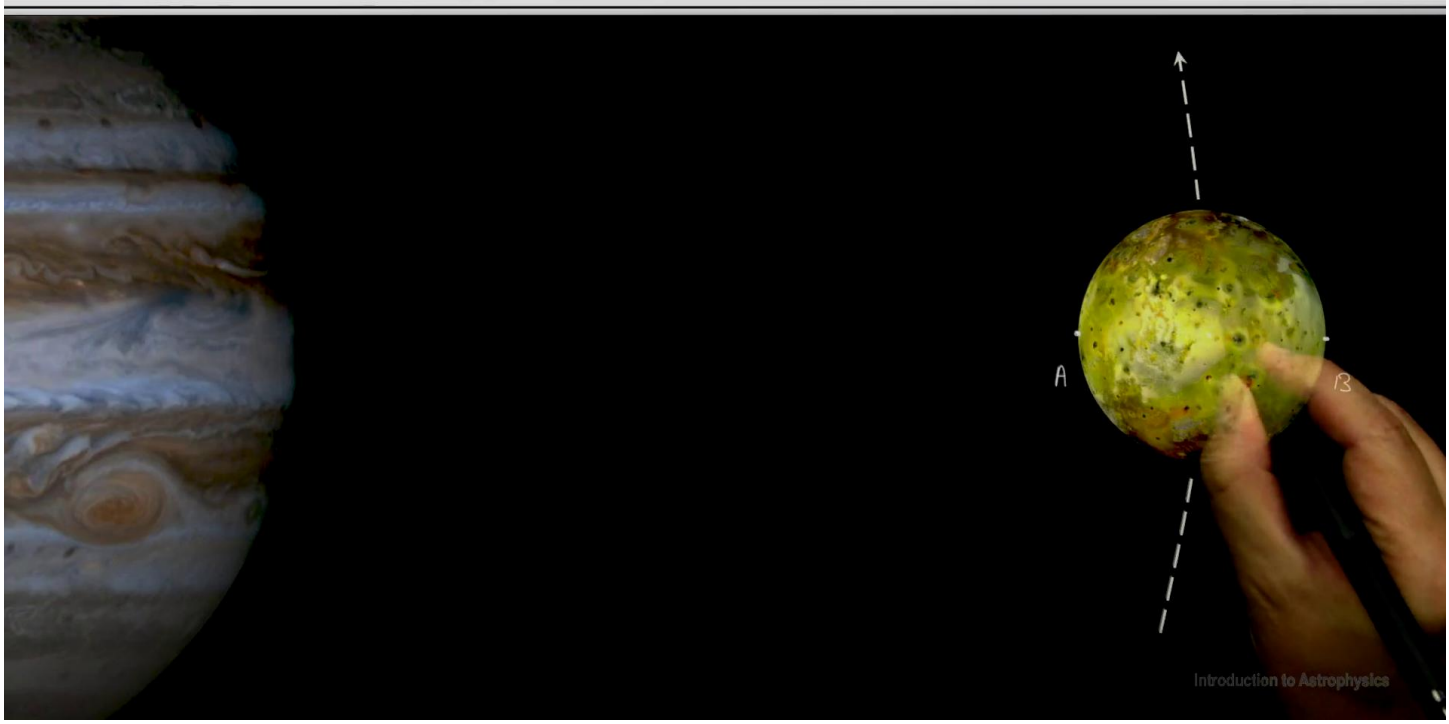
Let us compute the Roche limit, the distance at which a satellite is destroyed under the effect of the tidal forces exerted by a planet. Let us consider a planet of a given radius R. Here is Jupiter as an example. We assume it has a constant density ρ_M. We also consider a satellite, "Io" for example, or any other satellite which has a given density ρ_m and a given radius r. The two bodies are at a distance d from each other. We saw in the last chapter that we have tidal forces exerted on the points a and b here. These are differential forces due to the gradient of the gravitational field of Jupiter through the satellite, namely Io in this example. The tidal forces are equal with opposed signs. We have a force here on B and another one here on A. Both forces are stretching the satellite. We previously found that the tidal force, when neglecting the rotation of the satellite around Jupiter, exerted either on B or on A, has the same value, is equal to 2GM / d³, with d the distance between the two bodies, time the radius of the smallest object. This is the expression of the tidal forces when neglecting the rotation of the satellite around the planet.

Notes

Summary

0m 35s

Roche Limit



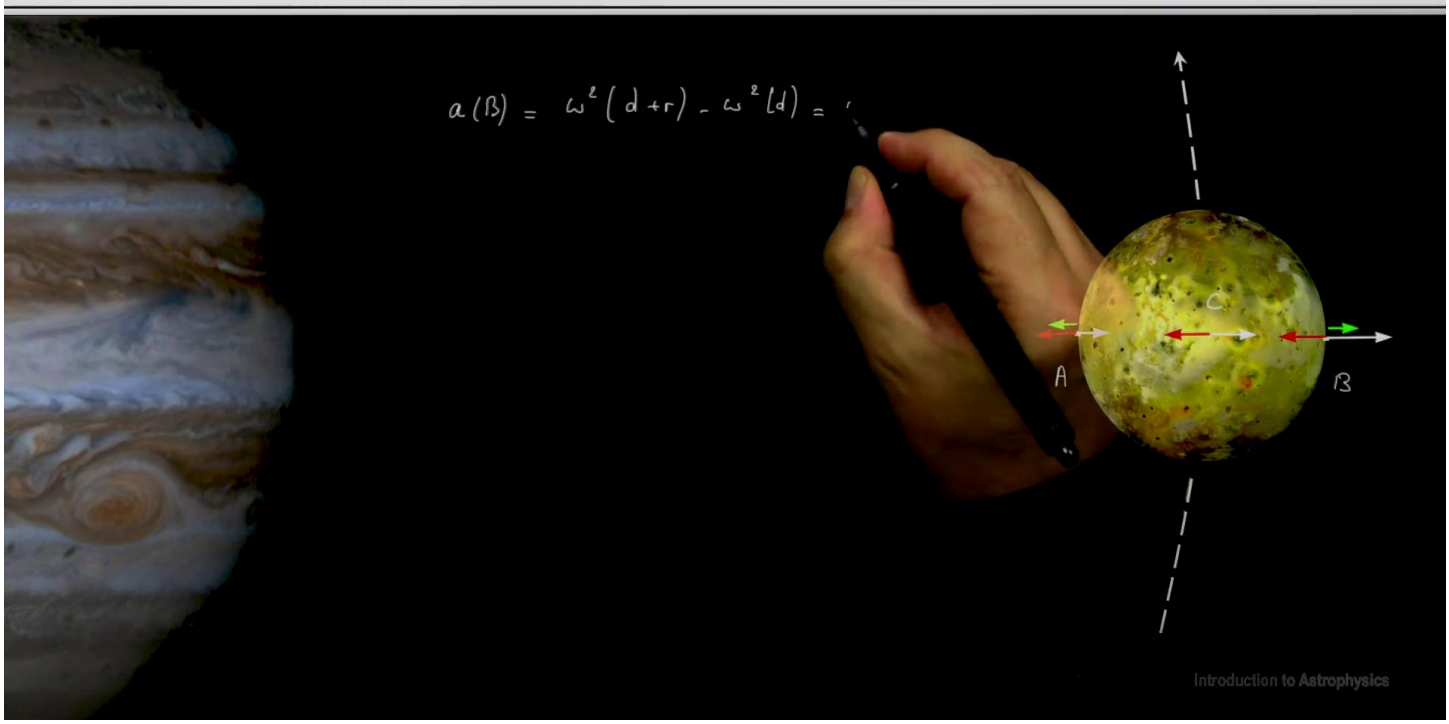
What happens when we do not neglect the orbital motion of the satellite any more? The angular velocity is given by 2 π / P, where the orbital period P is given by Kepler's laws. We have a centripetal acceleration at the three points here at the centre and at the edges of the satellite. This centripetal acceleration is a = ω ²r where r is the distance between the centre of Jupiter and each point we are considering. We had the points A and B there as well as the centre C of the satellite. Let us consider the different forces. There is first the centripetal acceleration. On the closest point to Jupiter, A, it is smaller than on the other points, such as C here or B there. The acceleration on A is ω² (d - r) At the centre we have an acceleration ω² d since the centre is at a distance d from the centre of Jupiter and on B the centripetal acceleration is ω² (d+r) We have thus a centripetal acceleration to the centre of Jupiter We have thus a centripetal acceleration to the centre of Jupiter At each point on the surface of the satellite is felt a pseudo force in particular at the points A, B and C the centre of the satellite. So we have like previously A, B and C.

Notes

Summary

2m 08s

Roche Limit



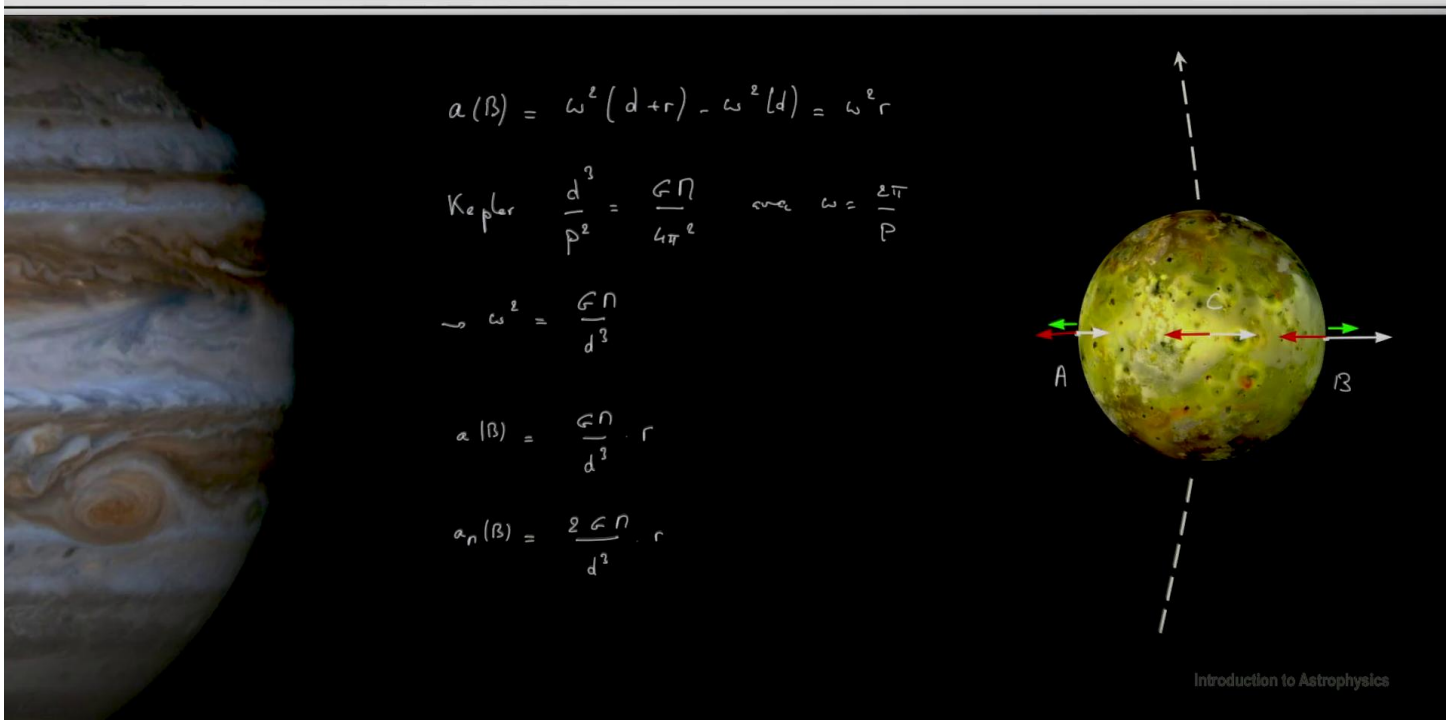
The pseudo force is in the opposite direction with a norm equal to the norm of the centripetal force. We have thus a small vector here on A, a bigger vector here on C and an even bigger one on B such that the norm of the different accelerations are the ones we wrote previously. We now want to compute the differential acceleration at the points A and B relative to the acceleration at point C. To this purpose we subtract, at A, B and C, the centripetal acceleration, or the pseudo force felt at C. We add a vector with the same norm as this one at C but with opposed direction. We have then a first arrow at C here and another one at B there such that they have the same norm. and a last one at A there with again the same norm. We add the two vectors at each point and we get the resulting force. Here at B we have a force roughly like that, at the centre we have obviously zero and at A the resulting force is in this direction. We are now interested in computing the norm of these two green arrows. Let us first write the norm of the green arrow at point B. The acceleration at point B is ω^2 (d+r)- ω^2 d which is simply the difference between the white and the red arrows, so finally = ω^2 r.

Notes

Summary

4m 10s

Roche Limit



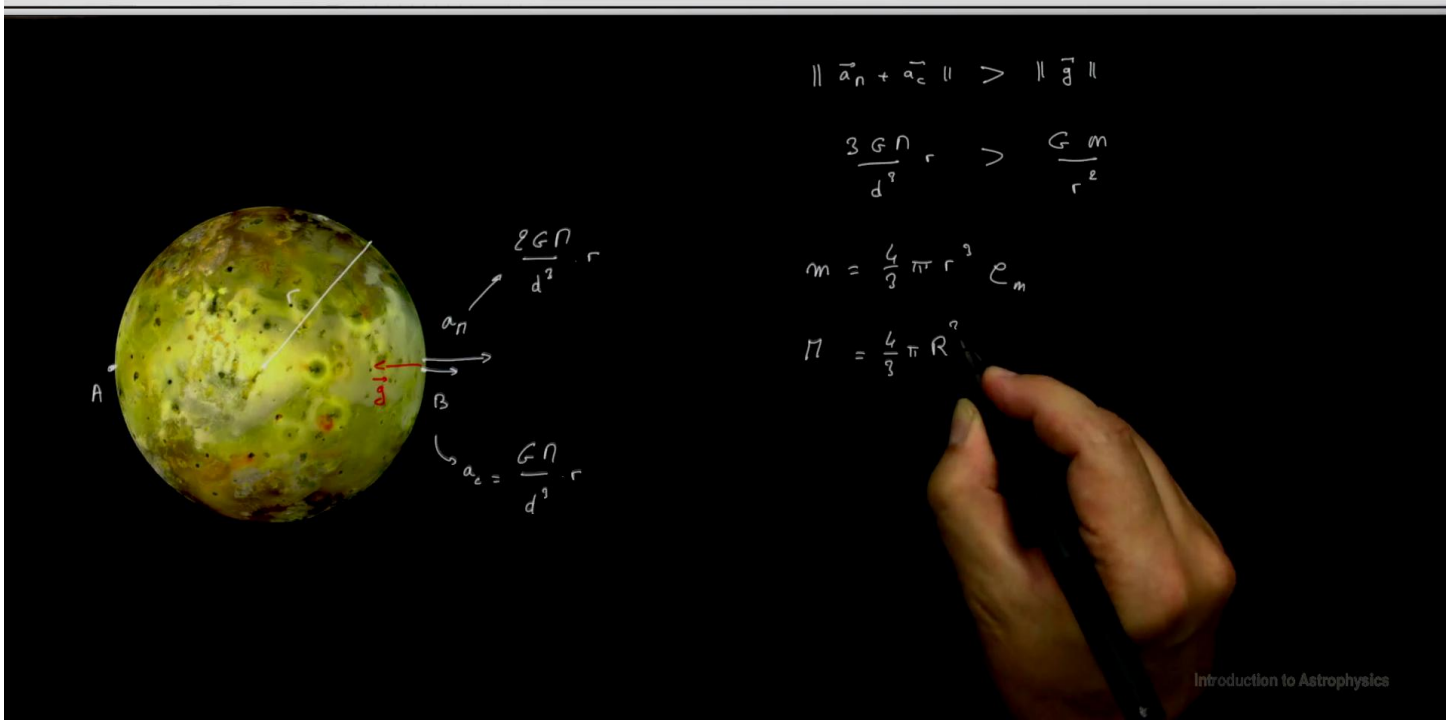
The third Kepler law tells us that $d^3 / P^2 = G M$ where M is the mass of the most massive body, over $4\pi^2$, with $\omega = 2\pi / P$, P being the orbital period. From there we deduce that $\omega^2 = GM / d^3$ that we inject here in order to get the acceleration at point B. It is equal to $(G M / d^3) * r$ As we've seen before, the tidal acceleration at point b, caused by the gradient of the gravitational force through the satellite, equals $(2GM / d^3) * r$. We thus have an effect of differential rotation. The differential acceleration caused by the rotation of the satellite around the planet which is equal to half the tidal force and as such is by no means insignificant.

Notes

Summary

5m 54s

Roche Limit



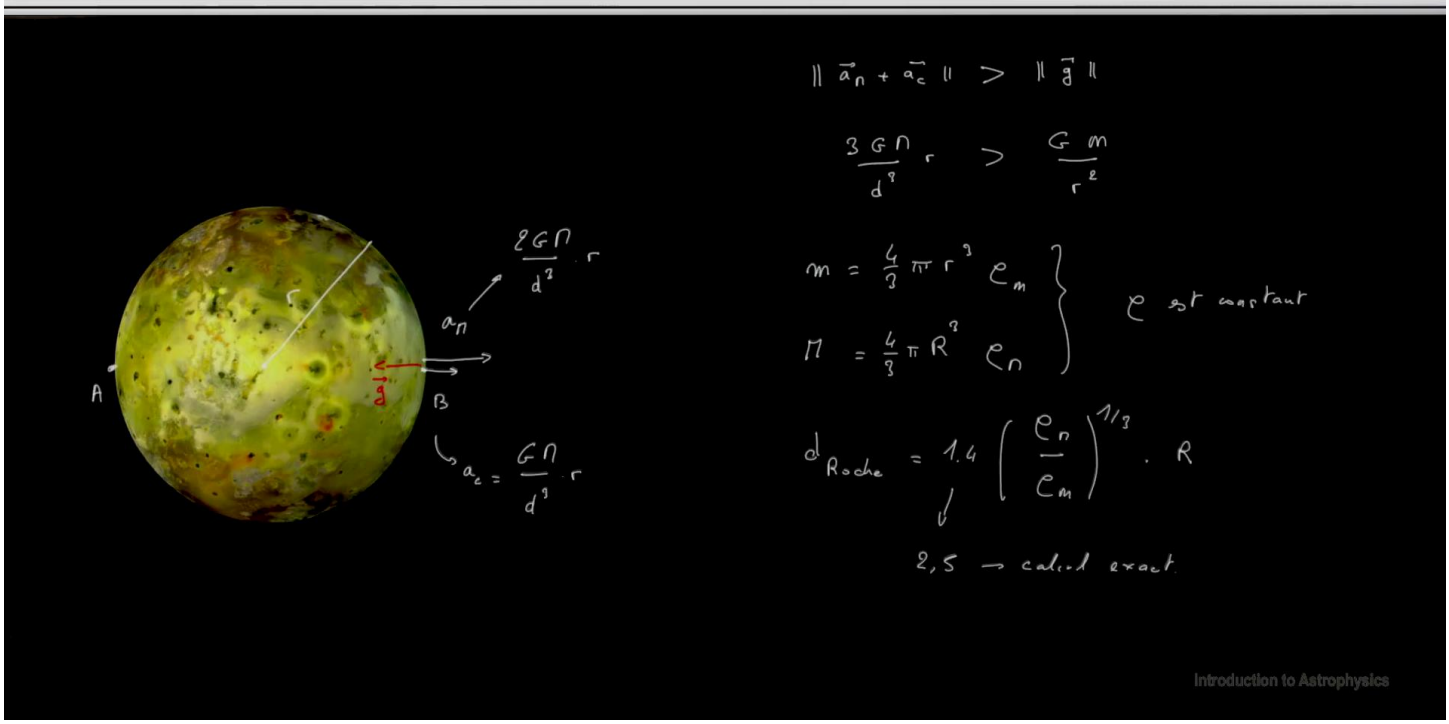
Now let us summarize what we computed. We still have our two points A and B. We will look in greater details what happens at point B. We have our satellite with a radius r and we have several forces. We have first the tidal force that we computed in the previous video, the differential acceleration which is half of this force there and in the other direction we have the g of the satellite. So here is the tidal acceleration which is equal to (2GM / d³) * r. We have the centripetal acceleration we just computed, which is equal to (GM / d³) * r. The satellite is torn apart if the norm of the sum of a_M + the tidal acceleration, so the sum of the two forces, is greater than the norm of the gravitational field which holds the satellite together. If the sum of these two accelerations is greater than g then the satellite is destroyed. Let us write it as equations : (3 GM / d³) * r > Gm where m is the mass of the satellite, over r², its radius. This term is the gravitational field at the surface of the satellite. To simplify we will assume that m = 4/3 π r³ * ρ_m, ro_m being the satellite density. With the same assumption, the mass of the planet, Jupiter here, is : M = 4/3 π R³ * ρ_M.

Notes

Summary

7m 18s

Roche Limit



We consider ρ to be constant. If we plug m and M in this equation here we will find the critical distance we are interested in : d_Roche = 1.4 times the ratio of the densities of the two bodies, to the power 1/3, times the radius of the larger body, Jupiter in this case. This is an approximation that does not take into account the internal cohesive forces If we make an exact calculation, we would find here a factor 2.5. This distance, d_Roche, is the one we have to inject in this equation here to have an equality. If the distance is smaller, the satellite is torn apart by the combined effect of tidal forces and centripetal acceleration caused by the planet.

Notes

Summary

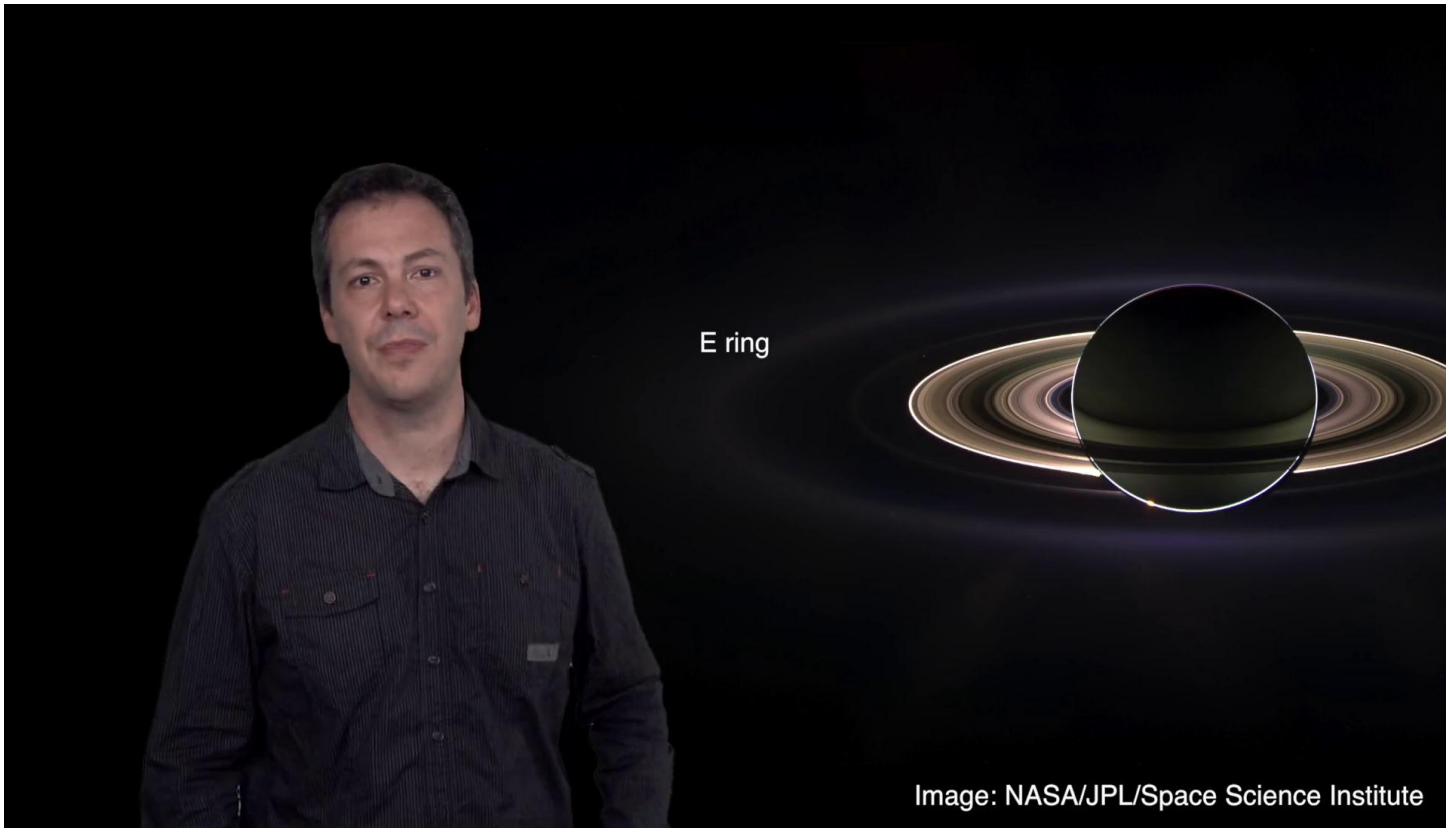
9m 35s

Let us come back to the planet Saturn. The rings you see here are the result of the break-up of a body that crossed the Roche limit and broke under the effect of the tidal forces from Saturn.

Notes

Summary

10m 33s

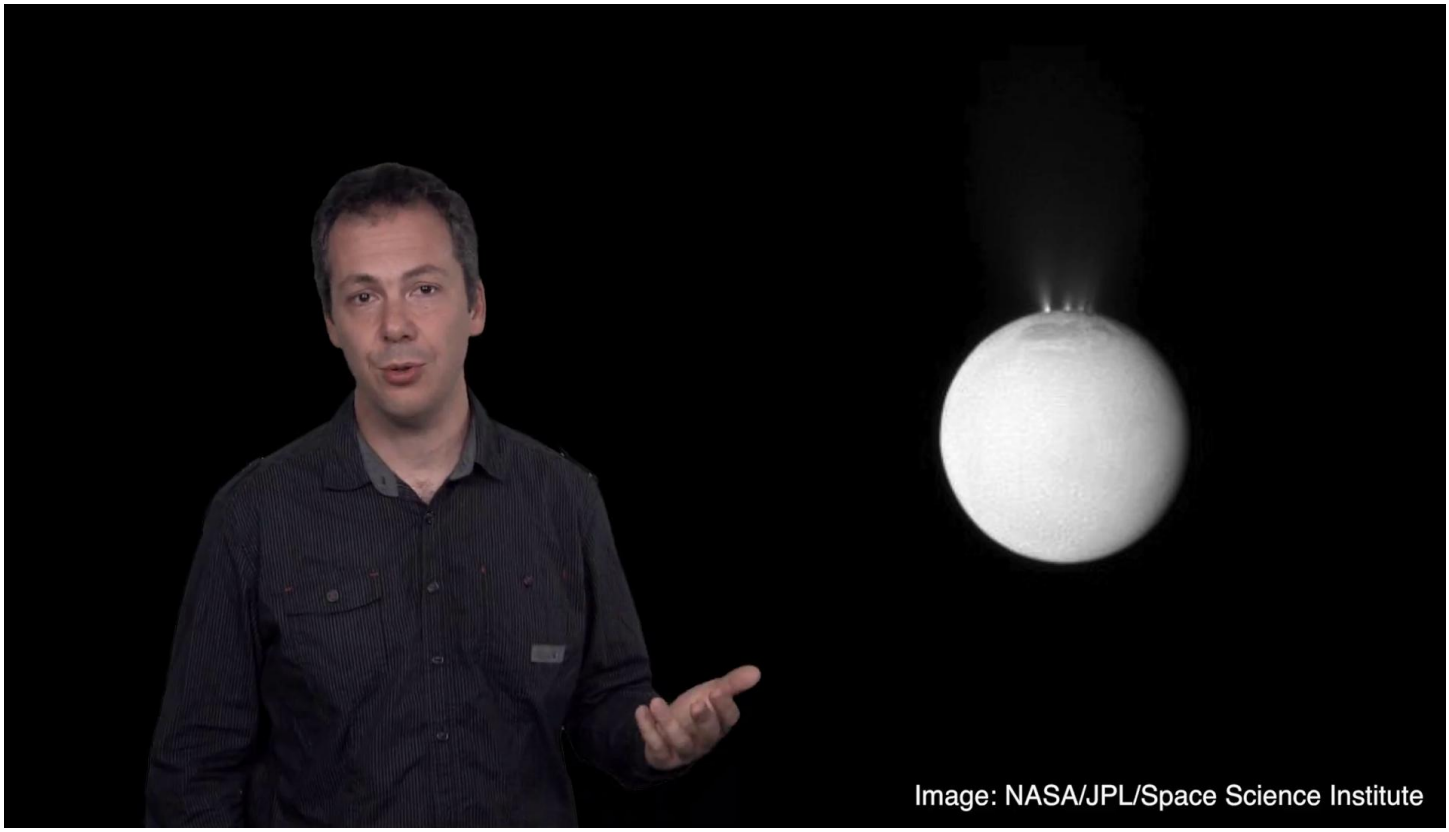


But all the rings don't have the same origin. The external ring, the E ring of Saturn, slightly blurred on this picture taken by the Cassini space probe is actually created by the extreme volcanic activity of a satellite of Saturn, Enceladus.

Notes

Summary

10m 43s

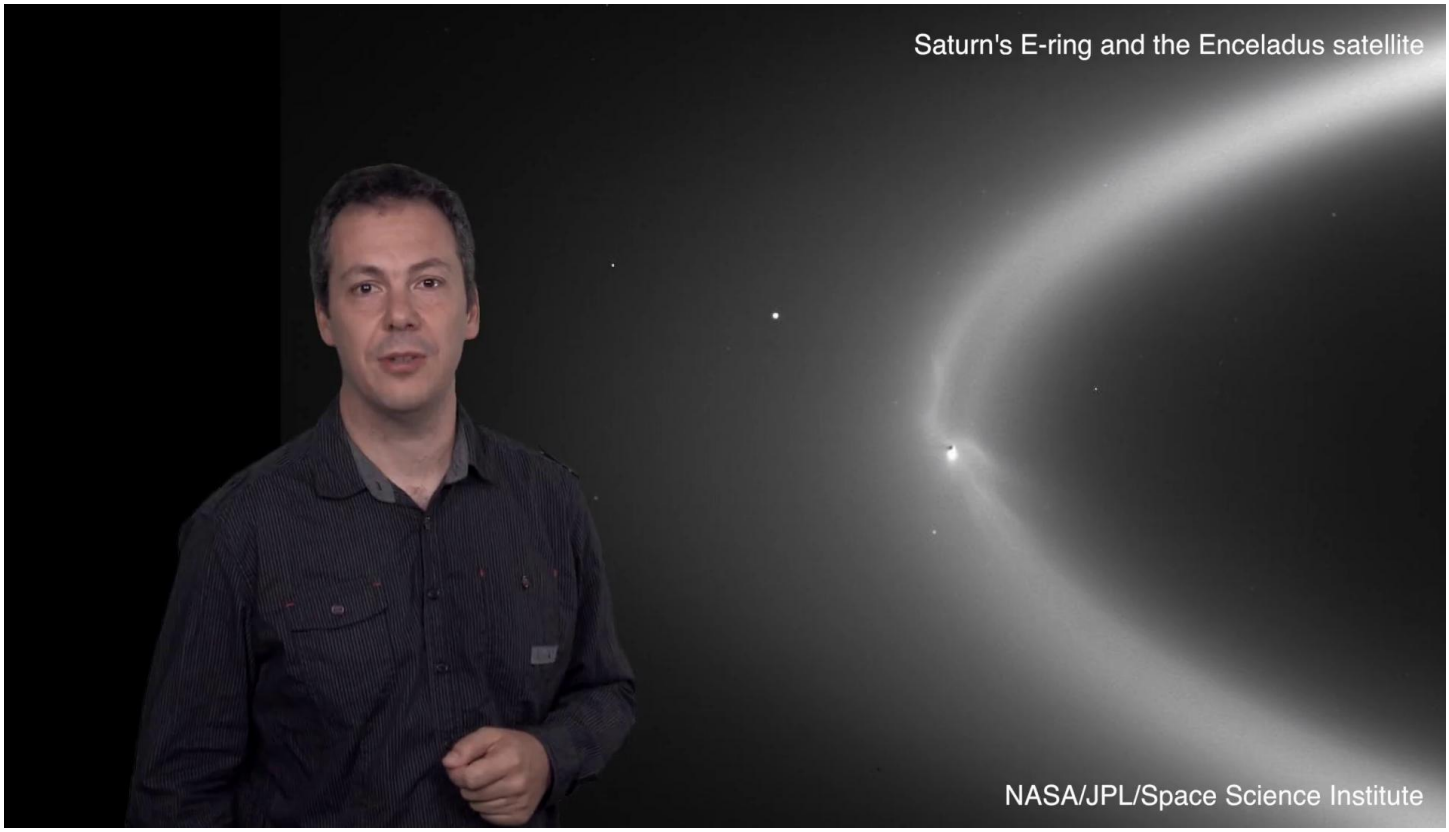


The plumes that you see going out of the satellite are composed of water vapour heated partly by the friction of matter in the internal mantle of the satellite under the effect of tidal forces from Saturn.

Notes

Summary

10m 57s



Part of the water vapour is ejected to the orbit of the satellite with some dust. The process creates a wide ring materializing the orbit of Enceladus around Saturn.

Notes

Summary

11m 09s

In the solar system, the bodies most affected by the Roche limit are probably the comets. They travel at high speed through the whole solar system and therefore get often quite close to strong gravitational fields such as Jupiter's or the Sun's. In both cases the comets can be destroyed if they cross the Roche limit. This is what happened to the comet Shoemaker-Levy 9 getting too close to Jupiter in may 1994. The comet broke up in 21 pieces clearly visible on this picture taken by the Hubble space telescope clearly visible on this picture taken by the Hubble space telescope where we see debris spreading over a distance of more or less 1 million kilometres.

Notes

Summary

11m 21s

Three months later these debris came back to Jupiter and eventually crashed on the surface, on 16th July 1994. Several impacts were visible on the planet's surface for several days. Some impact marks even reached the size of the Earth.

Notes

Summary

12m 02s