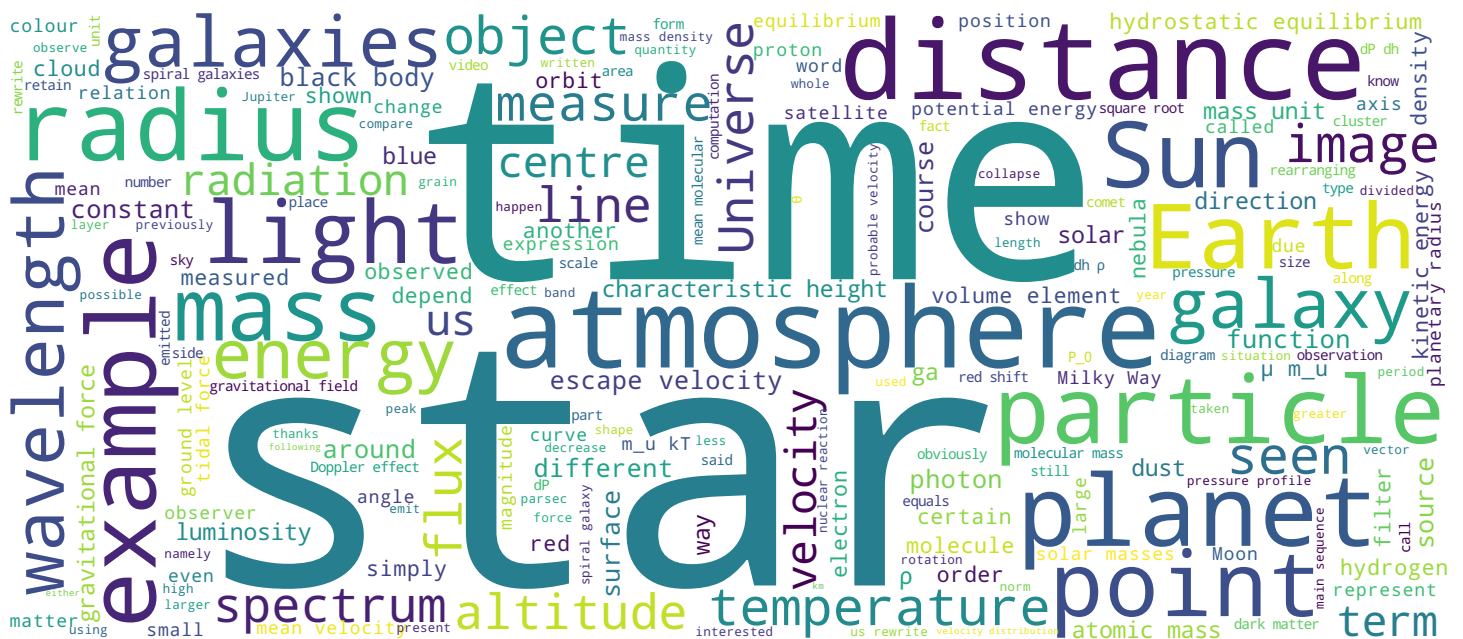


Thin atmospheric layer seen from the International Space Station

Image: NASA



Video



Thin Martian atmosphere, as seen by the Mars Global Surveyor



Image: NASA/JPL

How to characterize planetary atmospheres ? They can be either very thin such as the one of Mars shown here or extremely dense, which the case of gaseous planets such as Jupiter or Saturn. Why can some planets retain their atmosphere while others cannot ? In this video we will explore some answers to these questions.

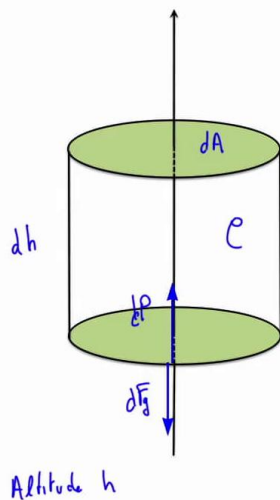
Notes

Summary

0m 05s



Hydrostatic Equilibrium Equation



$$dP \cdot dA = -dF_g$$

=

Introduction to Astrophysics

Among the tools we have access to to characterize planetary atmospheres is the hydrostatic equilibrium equation. As the name suggests, it expresses an equilibrium between the gas pressure acting in this direction and the gravitational force acting in the opposite direction. We represent these 2 forces acting on a volume element, and we will see how to use this equation, how to establish the equation of hydrostatic equilibrium which measures the equilibrium between an infinitesimal pressure and an infinitesimal gravitational force. We have drawn here a volume element with an area dA and a height dh . The mass density is ρ and this small volume element is at a given altitude h . h is the altitude in what follows. There is here an infinitesimal pressure, the force of which is $dP \cdot dA$. If we are interested in the pressure exerted on our infinitesimal volume, we write it as $dP \cdot dA$. Since we seek the equation of hydrostatic equilibrium, this must be equal to $-dF_g$ which is the gravitational force exerted on this infinitesimal volume.

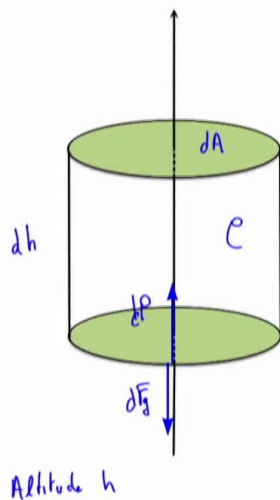
Notes

Summary



0m 25s

Hydrostatic Equilibrium Equation



$$\begin{aligned} dP \cdot dA &= -dF_g \\ &= -\rho(h) \cdot g(h) \cdot dA \, dh \end{aligned}$$

$$\frac{dP}{dh} = -\rho(h) \cdot g(h)$$

$$\left. \begin{aligned} \rho(h) &\sim \rho \\ g(h) &\sim g \end{aligned} \right\} \rho, g \text{ constants}$$

Introduction to Astrophysics

The gravitational force is of course the mass of the volume element, that is the density, which depends on the altitude, times the gravitational field which can also depend on the altitude, times the volume of our small cylinder : $dA \cdot dh$. We see that dA vanishes and we can then rewrite this equation in this form : $dP/dh = -\rho(h) \cdot g(h)$. Depending on the conditions in which we use this equation, we can very often assume that $\rho(h)$ is a constant, ρ . Moreover, if we have a big planet with a relatively thin atmosphere g is almost independent of the altitude. We can therefore assume $g(h)$ to be a constant g . Thus, ρ , g , or both of them can be constants.

Notes

Summary



1m 53s

Hydrostatic Equilibrium Equation

$$\frac{dP}{dh} = -\rho \cdot g$$

Expression pour $\rho = \frac{P \cdot \bar{\mu} m_u}{kT}$

$$\frac{dP}{P} = - \underbrace{\frac{g \bar{\mu} m_u}{kT}}_{\text{Dimension de l'inverse d'une longueur}} \cdot dh$$

Dimension de l'inverse d'une longueur

Eq. des gaz parfaits $P = nkT$

avec $n = \frac{\rho}{\bar{\mu} \cdot m_u}$

$\bar{\mu}$: masse moléculaire moyenne

m_u : unité de masse atomique

$$m_u = 1,66 \cdot 10^{-27} \text{ kg}$$

Introduction to Astrophysics

Let us rewrite the equation of hydrostatic equilibrium that we have just seen. We had : $dP / dh = - \rho g$. We had : $dP / dh = - \rho g$. Let us solve this equation. We recall the ideal gas law which tells us that $P = nkT$ with T the temperature, k the Boltzmann constant and n the numerical density of particles. This numerical density is the mass density over the mean mass of a particle. We have thus here μ , the mean molecular mass, the computation of which we will show later, and m_u is the atomic mass unit. Thanks to the ideal gas law, we get an expression for ρ . By rearranging the terms, we have that $\rho = P \mu m_u / kT$. By rearranging the terms, we have that $\rho = P \mu m_u / kT$ which I plug in the equation of hydrostatic equilibrium to get that dP , not any more over dh but over P , is equal to, by rearranging the terms $- g \mu m_u / kT$ times dh - $g \mu m_u / kT$ times dh . We have on the left hand side a dimensionless quantity so the right hand side must also be dimensionless. Therefore the dimension of this term is the inverse of a length.

Notes

Summary



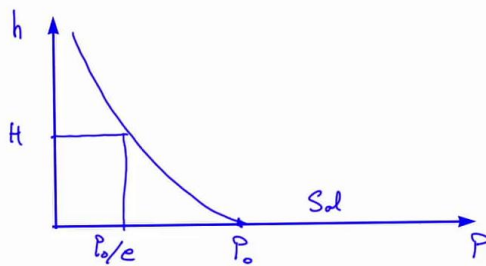
2m 55s

Pressure Profile

$$\frac{dP}{P} = -\frac{1}{H} \cdot dh \quad \text{avec} \quad H = \frac{kT}{g \bar{\mu} \cdot m_u}$$

$$P = K e^{-h/H} \quad H: \text{hauteur caractéristique}$$

$$P = P_0 e^{-h/H}$$



Introduction to Astrophysics

Let us rewrite once again the equation of hydrostatic equilibrium in a slightly different manner. We still have dP / P and the term dh we had previously. We had a term which was the inverse of a length here, so we write $1 / H$ with $H = kT / g \mu m_u$. This equation is now very easy to solve. The pressure is a constant times the exponential of $-h / H$. H is the characteristic height of the atmosphere. At ground level, when $h = 0$, P is the ground pressure. We have thus simply that P is equal to P_0 , the ground pressure, times the exponential of $-h / H$. Let us draw this relation on a figure. We can plot either the altitude as a function of the pressure, or the other way around. We have here the altitude h and here is P . We are thus drawing the pressure profile. The pressure profile is an exponential curve which looks roughly like this. Here is the zero altitude so the pressure at this point is P_0 . At this point, where $h = H$, the pressure is P_0 / e . Again, we are here at ground level and here at high altitude. This of course assumes that we have solved this equation in an extremely simple case.

Notes

Summary



5m 05s

Mean Molecular Mass

Calcul de la masse moléculaire moyenne : $\bar{\mu}$

Pour la Terre

78% azote moléculaire : $N_2 \rightarrow 14 \text{ nucléons}$ $N_2 \rightarrow 28 \text{ uma}$

22% oxygène moléculaire : $O_2 \rightarrow 16 \text{ "}$ $O_2 \rightarrow 32 \text{ uma}$

$$\bar{\mu} = \frac{(0.78 \times 28) + (0.22 \times 32)}{0.78 + 0.22} = 29 \text{ uma}$$

Introduction to Astrophysics

A word on the computation of the mean molecular mass, which we noted as a mean μ . Let us make the computation with the Earth as an example. It represents the mean number of particles in the atmosphere. We simplify the Earth's atmosphere by assuming that there is 78% of molecular nitrogen and 22% of molecular oxygen. What we want to do is to count the mean number of particles in the atmosphere. The molecular nitrogen is N_2 , the molecular oxygen is O_2 . The nitrogen atom has 7 protons and 7 neutrons therefore 14 nucleons. 14 nucleons for 1 nitrogen atom means that for the molecule N_2 we have 28 atomic mass units. The oxygen atom has 16 nucleons and thus for O_2 we have 32 atomic mass units. What is the mean molecular mass? It is a weighted mean mass. The weights are simply the proportion of each element in the atmosphere. We have 78% times 28 atomic mass unit plus 22% times 32 atomic mass unit which we normalize to $0.78 + 0.22$ and we find 29 atomic mass unit.

Notes

Summary



Condition to Retain Atmosphere

Vitesse d'éjection des molécules $>$ vitesse la plus probable
 $v_e > v_m$

$$\frac{1}{2} m v_e^2 = m g R_p$$

$$v_e = \sqrt{2 g R_p}$$



Introduction to Astrophysics

Once we have g , ρ and H , how can we use this quantity to see for example if a planet is able to retain its atmosphere or not? We can consider the escape velocity of a molecule. We can compare the escape velocity of a molecule in the atmosphere with its most probable velocity. If a particle has a very large ejection speed, it can break away from the gravitational attraction of the planet and as such the planet does not retain its atmosphere. Therefore we want to compare the escape velocity of the molecules with the most probable velocity of the molecules in the atmosphere. We write the escape velocity v_e and it is greater than the mean velocity of the molecules. v_e is the escape velocity which is obtained by computing the kinetic energy of a particle with a mass m and the escape velocity and equalling this kinetic energy to the potential energy of the particle in a gravitational field g where R is the planetary radius since we consider the altitude zero. This relation yields immediately the escape velocity, i.e., the square root of $2 g$ times the radius of the planet. We have on the one hand the escape velocity and on the other hand the most probable velocity. We have a gas with thermal agitation.

Notes

Summary



8m 56s

Condition to Retain Atmosphere

Vitesse d'éjection des molécules $>$ vitesse la plus probable
 $v_e > \bar{v}_m$

$$\frac{1}{2} m v_e^2 = m g R_p$$

$$v_e = \sqrt{2 g R_p}$$

Boltzmann

$$\bar{v}_m = \sqrt{\frac{2 k T}{\bar{\mu} \cdot m_u}} = \sqrt{2 g H}$$

$$v_e > \bar{v}_m$$

$\Rightarrow R_p > H$ planète peut retenir son atmosphère.

Introduction to Astrophysics

As such, the velocity distribution is a Boltzmann distribution just like in thermodynamics and the most probable mean velocity is the Gaussian mean of the velocity distribution, which is equal to the squared root of $2 k T$, where k is still the Boltzmann constant, over $\bar{\mu} m_u$. Here is where the H we previously saw appears. We can rewrite this mean velocity as the square root of $2 g H$, where H is the characteristic height we previously computed. The relation we wrote, so, $V_e >$ and the mean V_m , can be finally written as a comparison between the planetary radius and the characteristic height of the atmosphere. In such a case, if $R_p > H$ the planet can keep its atmosphere. We have thus established a very simple relation from the characteristic height of a planet that tells, by simply comparing the planetary radius with the characteristic height of the atmosphere, if a planet can retain its atmosphere or not.

Notes

Summary



Planetary Atmospheres



	Vénus	Terre	Mars
Solar Constant (kW.m^{-2})	2,6	1,4	0,6
Albedo	0,7	0,3	0,2
Black Body Temp. (K)	230	255	216
Ground Temp. (K)	735	288	210
P_0 : Ground Pressure (bar = atm.)	93	1	6×10^{-3}
H: Height Scale (km)	14	8	11
R: Planet Radius (km)	6052	6371	3390

Introduction to Astrophysics

Here is the data from the previous video for Venus, the Earth and Mars with the addition of the new quantities we just introduced : the pressure at ground level, the characteristic height H and the radius of the planets. We first notice that in the 3 cases the planet retains its atmosphere since H is negligible in regards to the planetary radius. De facto, the 3 planets, the Earth, Venus and Mars have indeed an atmosphere. In the second place we notice extreme differences between the surface pressures although the 3 planets are otherwise very similar and they moreover have comparable black body temperatures. This shows once again that the composition of the atmospheres and their ability to interact with radiation completely determines the physics at the surface of a planet as well as the complete temperature and pressure profiles of the atmospheres with the altitude.

Notes

Summary



12m 09s