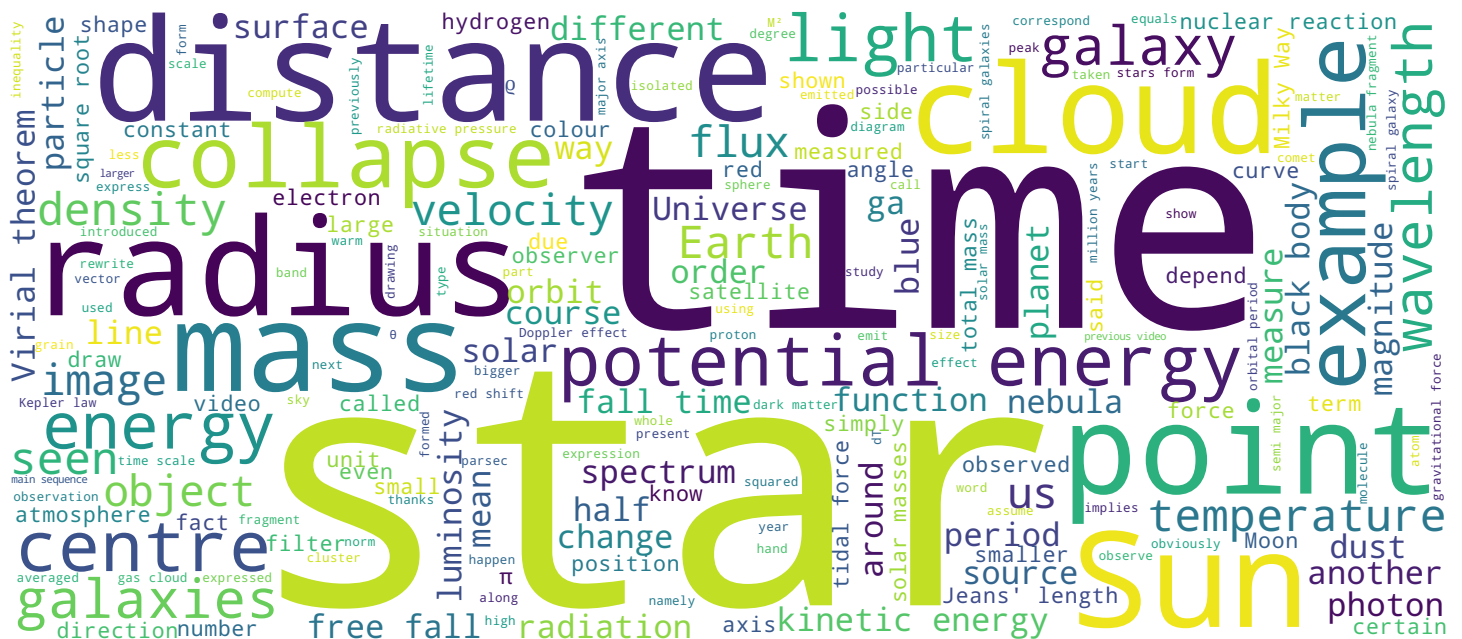


NGC 604: star-forming region

Image: NASA /ESA - HST

Frédéric Courbin





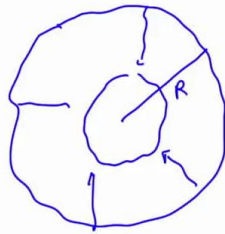
In the next three videos we will take interest in stellar formation and evolution. In this first video we will see under which conditions a cloud of gas and dust can start its collapse until eventually forming a star. As we see here, stars form in groups from a gas cloud which fragments due to local inhomogeneities and the resulting pieces if they are over a certain mass or over a critical radius, known respectively as Jeans' mass and Jeans' length. These pieces contract and warm up up to the point where nuclear reactions of hydrogen fusion start and give birth to a star.

Notes

Summary



0m 05s



ρ, n, N

Virial $\langle K \rangle = -\frac{1}{2} \langle U \rangle$

$$\langle K \rangle = \frac{3}{2} N k T$$

$$\langle U \rangle = -\frac{3}{5} \frac{G n^2}{R}$$

$$\frac{3}{5} \frac{G n^2}{R} = 3 N k T$$

Introduction to Astrophysics

As we said, stars form from fragments of a nebula. We draw here a nebula fragment. This fragment, if it is over a specific mass or a specific radius will collapse and as such become smaller and smaller, and denser. We will see that it implies that the fragment will warm up. Let us consider a nebula with a given radius R and as usual a given density, a total mass and a total number of particles composing the nebula : atoms and molecules. The nebula fragment is isolated and we can then use the Virial theorem which tells that, averaged in time, the kinetic energy is minus one half of the potential energy. We can express the kinetic energy as a function of temperature : $\frac{1}{2} k T$ by degree of freedom. A 3 dimensional space means 3 degrees of freedom and N particles leads to N degrees of freedom, times k the Boltzmann constant and T the temperature. If we now assume our nebula fragment to be more or less spherical in shape, we can compute the potential energy like we did in a previous video : $\frac{3}{5} G M^2$ (M being the total mass) over the radius of the nebula if it can be approximated to be spherical in shape. Both quantities are equal by the Virial theorem and we have then : $\frac{3}{5} G M^2 / R = 3 N k T$.

Notes

Summary



0m 42s

Jeans' Radius and Mass

Si $\langle K \rangle < -\frac{1}{2} \langle U \rangle \rightarrow$ effondrement

$$3 N k T < \frac{3}{5} \frac{G M^2}{R}$$

$$k T < \frac{G \bar{m}}{5 R} \cdot N \quad \text{avec} \quad \bar{m} = \frac{M}{N}$$

Par $M = \frac{4}{3} \pi R^3 \cdot \rho \quad \rho(r) = \rho$

$$R > \sqrt{\frac{15 k T}{4 \pi G \bar{m} \rho}} \quad \bar{m} = \mu m_H$$

Introduction to Astrophysics

As we have seen with the Virial theorem and in particular with the simulation shown in the video 1.4 on the Virial theorem, If there is a lack of kinetic energy relative to the potential energy, the gas cloud in this case will start to collapse. Let us take the aforementioned equation. We have $3 N k T < 3/5 G M^2/R$. By rearranging the terms, we have $k T$ smaller than $G \bar{m}$ averaged that we introduced in previous videos, over R times M . With \bar{m} averaged being the mean mass : M over the number of particles. If we now also assume that $M = 4/3 \pi R^3$ times the density, supposing that the density does not depend on the radius and that it is therefore constant everywhere in the cloud, we can then rewrite this inequality in another form. R goes from the denominator to the numerator since we invert the inequality $R > \text{square root of } (15 k T) / (4 \pi G \bar{m} \rho)$. The mean mass can be expressed as a function of the mean molecular mass we previously talked about times the atomic mass unit. If there is a lack of kinetic energy to support the shape of the cloud then it will collapse and the inequality we wrote here comes to defining a critical radius which is the minimum radius a cloud of a constant density can have before collapsing.

Notes

Summary



2m 31s

Jeans' Radius and Mass

$$R_{\text{Jeans}} = \sqrt{\frac{15 kT}{4\pi G \bar{m} \rho}} \quad \text{avec} \quad \bar{m} = \mu m_H$$

Rayon minimum pour que nuage s'effondre ($\rho = \text{cte}$)

si $R > R_{\text{Jeans}} \rightarrow \text{effondrement}$

M_{Jeans} : masse dans R_{Jeans}

\rightarrow Masse limite avant effondrement

Introduction to Astrophysics

The radius we just defined is called Jeans' length named for the astronomer who first introduced it. It is the square root of $(15 k T) / (4 \pi G m \rho)$. This length is the minimal radius from which the gas cloud collapses at a constant density. If R is bigger than Jeans' length, then we have a collapse. A star can potentially form from such a collapse. There is some mass contained within Jeans' length which allows us to define Jeans' mass, the mass inside Jeans' length. It is the limit mass before the collapse of a cloud of a given radius, Jeans' length.

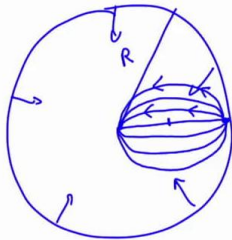
Notes

Summary



4m 43s

Free-fall Time



Introduction to Astrophysics

We have a collapsing cloud and we now seek to know the time taken for the cloud to collapse. This is called the free-fall time. How much time does it take for each point of a roughly spherical cloud to collapse into a single point, say at the centre of the cloud. We have a cloud with a radius R just like previously and we have a particle leaving from the edge of the cloud and going to the centre of the cloud. We can consider a straight line to actually be an ellipse of infinite eccentricity. I draw here another ellipse with a smaller eccentricity and again another one with another eccentricity. You recall Kepler's laws - we already see the drawing here - which said that the orbital period is independent from the eccentricity. All trajectories I am drawing here, if for example a particle leaves this point and follows this path until reaching the centre, it will take the same time as if it took this other path here. We thus consider that we have a particle here on an extremely eccentric orbit with a radius of $R/2$. The free-fall time is therefore the time taken to go from this point to the centre which corresponds to half the orbital period. The orbit has a radius $R/2$.

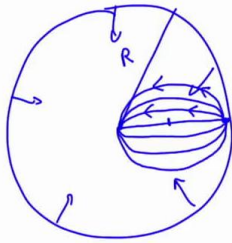
Notes

Summary



6m 06s

Free-fall Time



$$t_{cl} = \frac{P}{2}$$

$$P^2 = \frac{4\pi^2}{GM} \left(\frac{R}{2}\right)^3$$

$$P^2 = \frac{3\pi}{8 G \rho}$$

$$t_{cl} = \sqrt{\frac{3\pi}{32 G \rho}}$$

Ne dépend que de ρ

Introduction to Astrophysics

The free-fall time is P , the period of the orbits drawn here, - P being independent of the eccentricity which is what we're going to compute with Kepler's third law- divided by 2. This period - according to Kepler's law - is $4\pi/G M^2$ and here is the a^3 which is the cube of the semi-major axis, the orbital semi-major axis is $R/2$. We put the total mass of the cloud in a point here at the centre of the cloud. If we rewrite the equation, we get this. It is the period of the relevant orbit. The free-fall time is equal to the period / 2. Since the period is squared I have a square root here. The free-fall time is $3\pi/32 G \rho$. What we especially notice is that the free-fall time only depends on the density ρ .

Notes

Summary



7m 40s

Free-fall Time

	Density (kg.m^{-3})	t_{ff}
Universe	10^{-27}	10^{11} y
Galaxy	10^{-21}	10^8 y
Interstellar medium	$10^{-21} / 10^{-17}$	$10^5 / 10^8 \text{ y}$
Solar System	10^{-12}	1000 y
Sun	1400	1800 s

Introduction to Astrophysics

Here are a few examples of free-fall times, a few orders of magnitude. We have here the density of some objects including the Universe as a whole, the density of which being 10^{-27} kg/m^3 and we have the corresponding free-fall times. We see that if the whole Universe collapsed without any resistance it would take 10^{11} years. This is of course an order of magnitude. A galaxy with a density of roughly 10^{-21} kg/m^3 would collapse in roughly 100 million years. Clouds in the interstellar medium take tens of thousands of years to a few million years to collapse. The solar system would take 1000 years and a star such as the Sun with a density of roughly 1.4 tons/ m^3 would collapse in half an hour.

Notes

Summary



8m 58s

Gravitational Contraction

Energie totale

$$dE_{\text{tot}} = dU + dK = -L dt$$

Viriel

$$dU + 2dK = 0$$

$$\Rightarrow \begin{cases} dU = -2L dt \\ dK = +L dt \end{cases}$$

L fait perdre de l'énergie potentielle
La moitié de U perdue est convertie en énergie cinétique

Chauffage \rightarrow photon \rightarrow pression de radiation

Introduction to Astrophysics

Yet the Sun does not collapse in half an hour. To understand this fact, let us have a look at the energy balance of the Sun. If we look at the total energy, its change is equal to the change of potential energy plus the change of kinetic energy. This is equal to the energy loss rate, namely the Sun's luminosity times a time unit. For an isolated system we have the Virial theorem which tells that $dU + 2 d K = 0$. We now have a system of two equations with two unknowns which is easy to solve. We get that $dU = -2 L dT$ and $dK = + L dT$. This means that the star radiating implies a loss of potential energy. The luminosity of the star creates a loss of potential energy. We also notice that half of the lost potential energy is gained, converted into kinetic energy. If there is a gain of kinetic energy it means that the star warms up. If there is a warm up, there is photon emission by black body or ionization and if there is photon emission there is a radiative pressure exerted by the photons on the external layers of the collapsing stars. This radiative pressure counteracts the gravitation and prevents the star from collapsing in a time of the order of the free-fall time.

Notes

Summary



9m 49s

$$t_{\text{vie}} = \frac{U}{2 \cdot L} = \frac{3}{10} \frac{G M^2}{R \cdot L}$$

Temps de Kelvin-Helmholtz

Pour $1 M_{\odot}$ $t_{KH} \sim 10^7 \text{ ans} \sim 500 \text{ fois trop court}$

Introduction to Astrophysics

We can go some steps further and estimate the lifetime of a star by saying that this lifetime is the time needed to shine half the potential energy since this is what the star loses. We have then half the potential energy, $U / 2$ radiated at a rate corresponding to the luminosity of the star. In the case of a sphere, it is simply $3/5$ divided by 2, or $3/10$ times $G M^2$ the mass of the star over R , this is so far the potential energy of a sphere, divided by 2, and the L we have here. This is called the Kelvin-Helmholtz time scale. It is the lifetime of the star if it radiated half of the available potential energy. This is of course only an order of magnitude. If we make a numerical application for one solar mass, we find that the Kelvin-Helmholtz time scale is around 10 million years. The Sun is five billion years old so the time scale is roughly 500 times too small in the case of the Sun, a star of one solar mass. There is therefore another energy source that makes the stars shine.

Notes

Summary





We have seen in a simplified manner how the stars form and how they take their energy from the gravitational collapse on the one hand and the nuclear reactions on the other hand. As you have indeed understood, death and birth of stars are deeply linked. Generations of stars follow one after the other : young stars are formed from the material, the remains of dead stars. It is within diffuse nebulae like the one pictured, that stellar remains and rising stars live side by side. In the upcoming videos, and in particular in the very next one, we will see how stars evolve once they started up the nuclear reactions in their centre. We will see what are the observational methods to study the many steps of their life.

Notes

Summary

12m 55s

