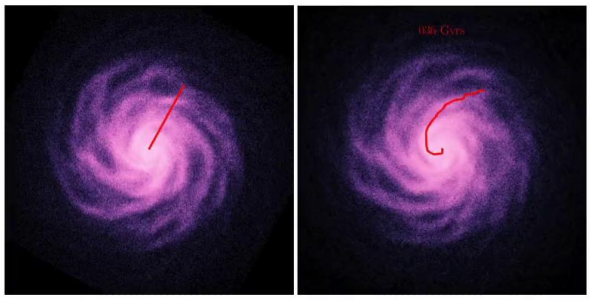
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE



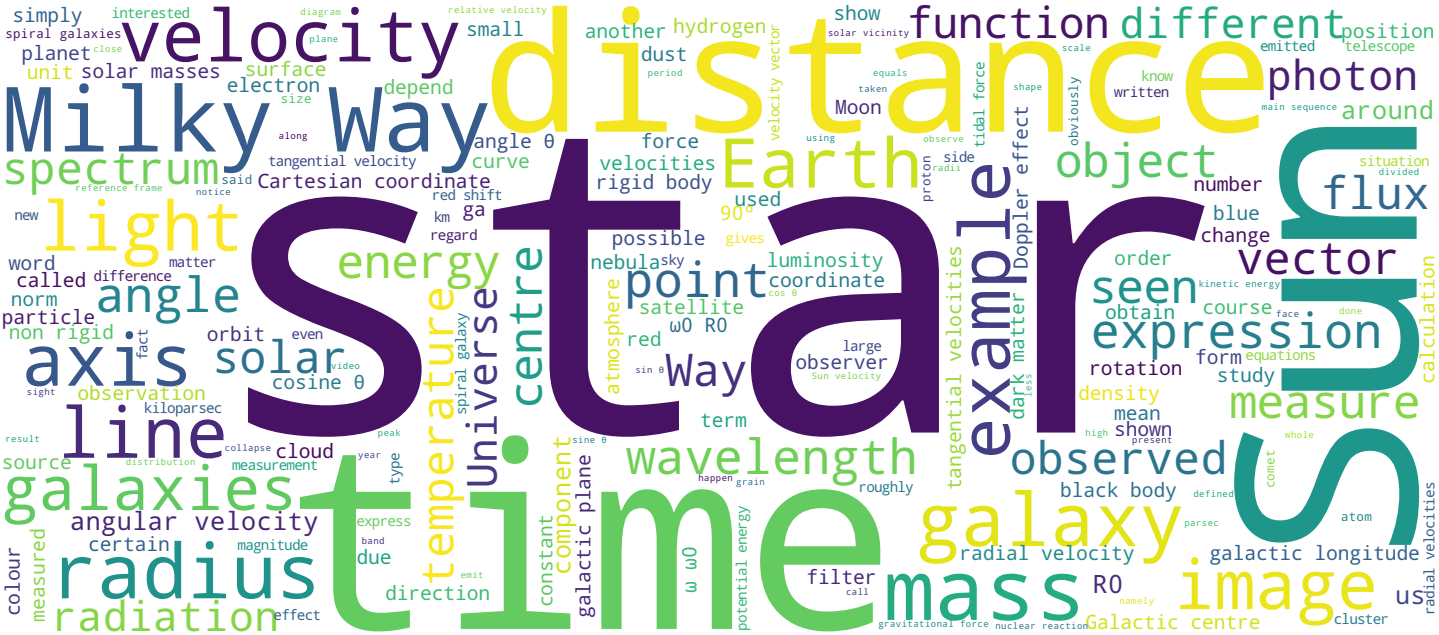
Non-rigid rotation of galaxies

Images: Yves Revaz (EPFL)

Rotation of the Milky-Way and Oort Constants

Introduction to Astrophysics; Chapter 6.3

Frédéric Courbin



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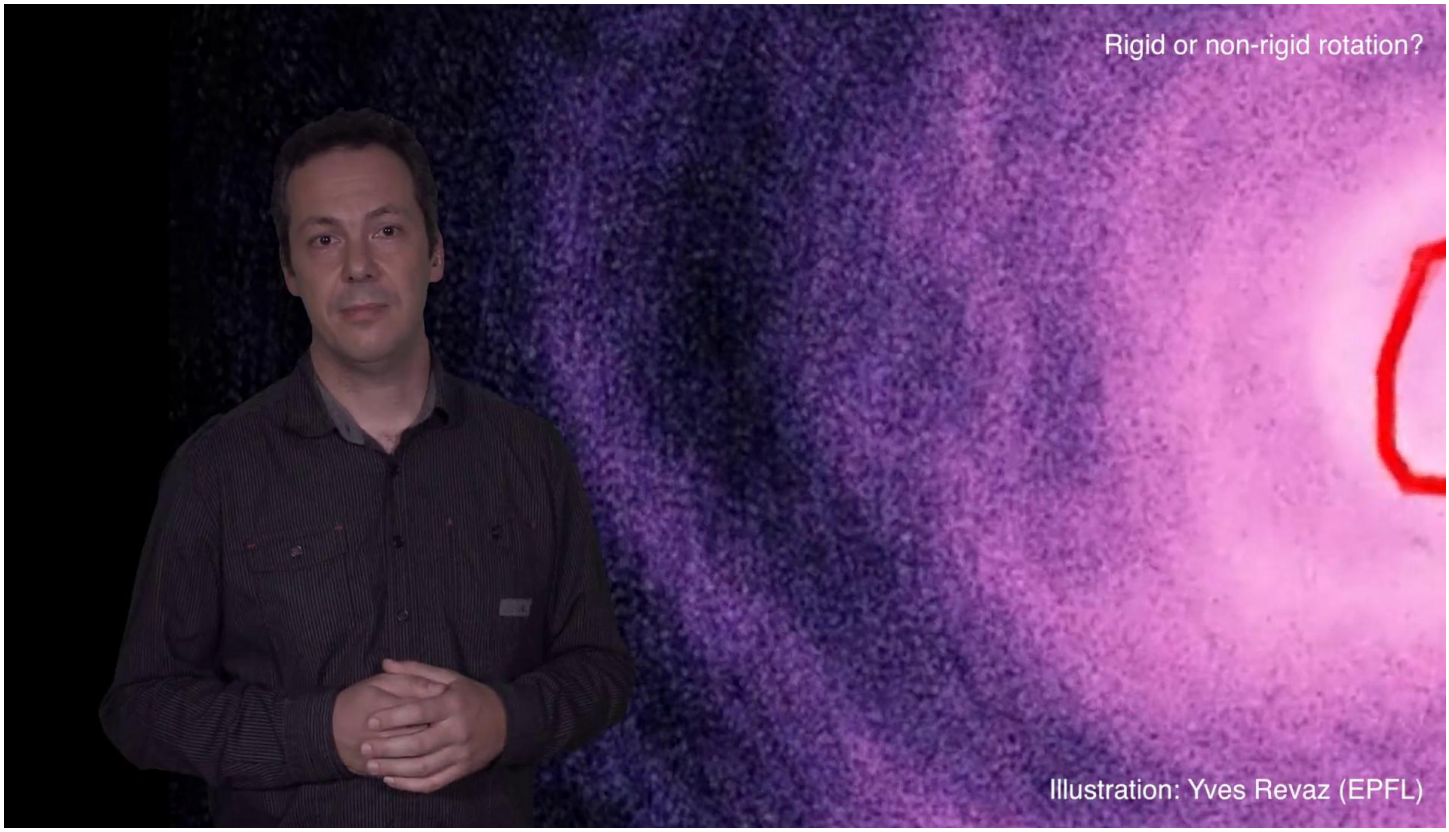


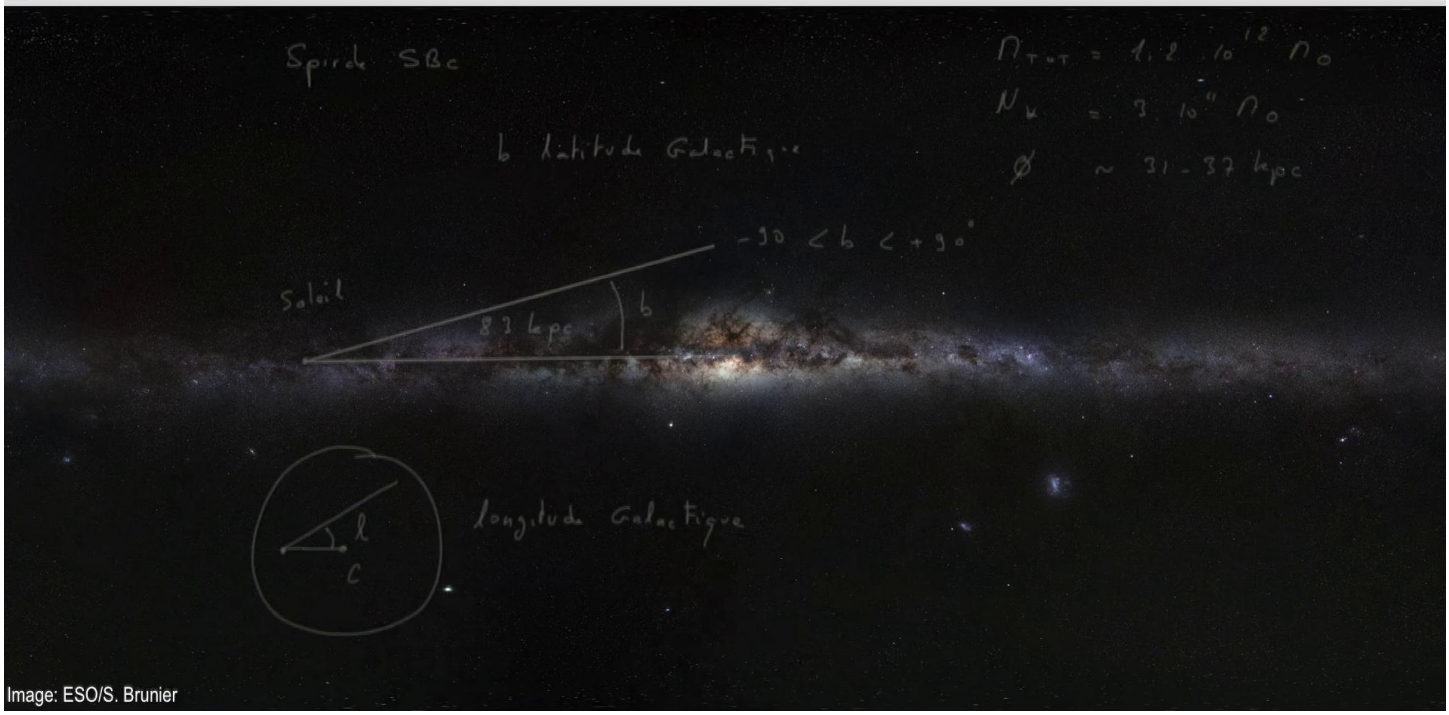
The Milky Way is a barred spiral galaxy like the one showed here face-on. It is quite simple to visualize a galaxy that is seen from the outside, just like in the image produced by a telescope. However, it is much more difficult to picture a galaxy while living in it. This is our case on Earth, around the Sun, in the Milky Way. How to describe the Sun's motion in the Milky Way? What are the observables we can use? This is what will see in this lecture taking the reasoning and the calculations made by the Dutch astronomer Jan Oort in 1927. The same Jan Oort who proposed the idea of a comet cloud around our solar system from which comets originate diving toward the Sun from time to time. It is natural to ask the question whether our Galaxy rotates as a rigid body as shown here, where all the points placed along an axis joining the galactic centre to the outter parts rotate with the same angular velocity. The situation is represented with a the red line rotating around the galaxy. Or, conversely, our Galaxy may be in a non-rigid rotation as shown here where all the points on the red line have an angular velocity that depends on the radius. We will show that the study of the movements of the stars in the vicinity of the Sun allows us to answer this question.

Notes

Summary

0m 07s

The Milky-Way



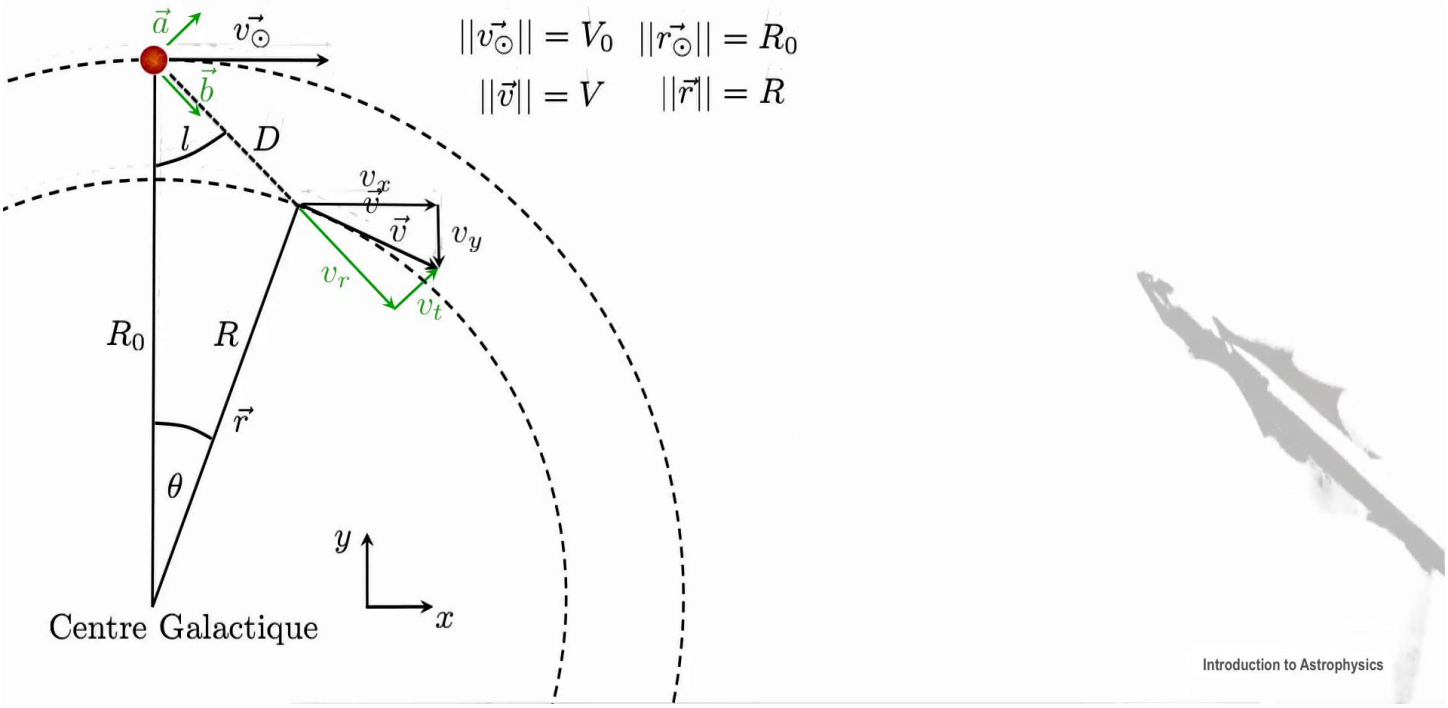
First, a few informations about the Milky Way. We are in a spiral barred galaxy SBc that we can only see edge on as we are inside. The total mass of the Milky Way, so the mass of stars, gas, dust and dark matter is equal to 1.2* 10^12 solar masses. The number of stars in the Milky Way is roughly 1*10^11 How do we find a way in the Milky Way? The Sun is about 8 kiloparsecs from the Galactic centre, more precisely 8.3 kpc. The diameter of the Galaxy is about 31 to 37 kpc depending on the different estimates available to date. How do we find a way in the Milky Way? We use a coordinate system taking into account the Sun's position in the Milky Way. The first coordinate is b - the galactic latitude, it is the angle between the galactic plane and the direction toward observer. b varies from -90º to +90º, the galactic poles being at +90º et -90º, b = 0 indicates the galactic equator. The galactic longitude can be measured if have a face on view in the Milky Way with the centre here and the Sun at 8.3 kps. The angle between the direction Sun - Galactic centre and the direction of observation is the galactic longitude.

Notes

Summary

1m 35s

Motion of the Sun in the Milky-Way



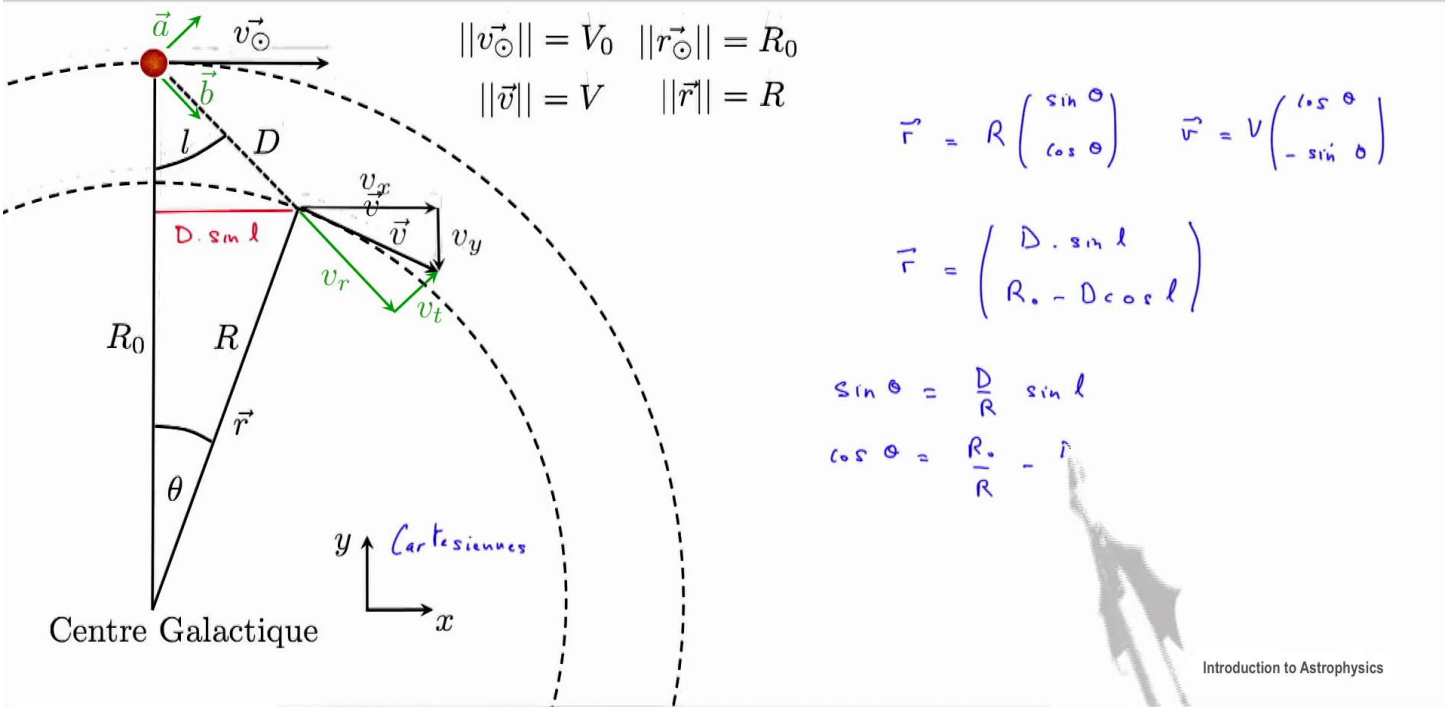
What we will try to describe now using this coordinate system with the galactic latitude and longitude, as we are placed in the plane of The Milky Way, is the relative movement of the stars with respect to the Sun. Each star has a velocity v, this is the Galactic plane, the Galactic centre in here, stars are at certain distance R. The norm of the vector r is R, The Sun is at the distance R0 from the Galactic centre. θ is the angle between R0 and R, and the galactic longitude l is the angle between the line Sun - Galactic centre and the line joining the Sun and the observed stars. The stars as well as the Sun have a given velocity, and what we will study now is the relative motion of the stars with respect to the Sun. This is the observable that enables us the study of the Sun's motion in the Milky Way. If the Galactic disk is in rotation, we have that V0 = ω0 R0, for the stars : V = ω R. If the rotation is that of a rigid body, then ω(R) = cste and equal to ω0, as all the angular velocities in the Galaxy are equal to the angular velocity of the Sun. The next step is to see whether the stars' angular velocity is constant or not, in other words, does the disk rotate like a rigid body or not.

Notes

Summary

3m 45s

Motion of the Sun in the Milky-Way



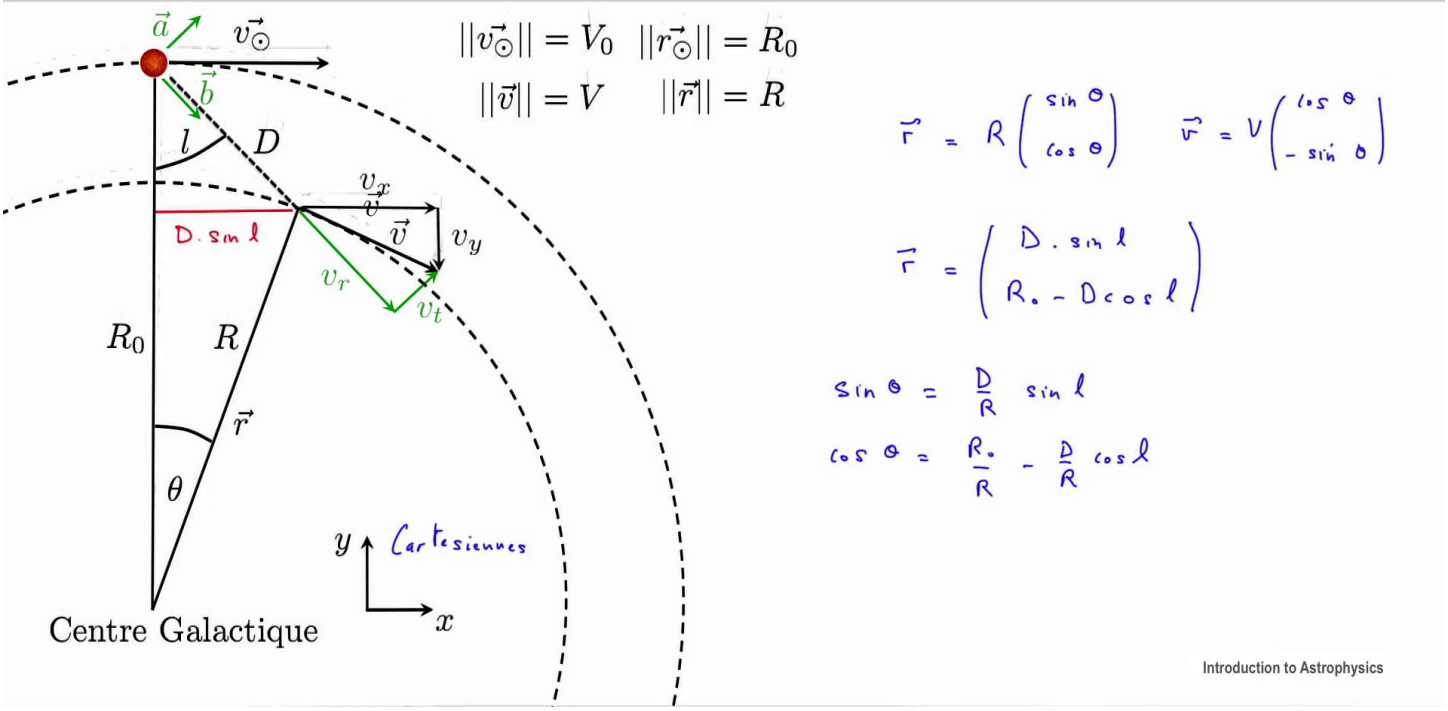
Different things must be defined, let's set a Cartesian coordinate system with x, y. Then, the velocities and radii will be decomposed along the x and y axis, so Vx, Vy, Rx, Ry, and so on. In this coordinate system, r is equal to the norm R times sine θ for the x-axis, cosine θ for the y-axis. r is just projected onto the x or y axis. The same is true for the velocity vector, it is equal to v, looking at the image one can see that there is cosine θ along the x axis and minus sine θ along the y-axis. This is the projection of v and r onto the x and y axes using the angle θ. This angle θ is not observed, the angle that is measurable from the Earth is the angle l - the galactic longitude. One can notice that this distance is equal to D times sine l, and so r can be written for the x axis as: D sine l, and for the y axis, as the vector r in the y direction it is this distance, here, R0 minus that term, so one has R0 minus D cosine l. Now we have two expressions for r and it is possible to identify the x and y components of the vector r in order to obtain the expressions for sine θ and cosine θ: by identifying the 2 components of the vector r: sinus θ = D/R sin l and cosine θ is equal to R0/R - D/R cos l.

Notes

Summary

5m 42s

Motion of the Sun in the Milky-Way



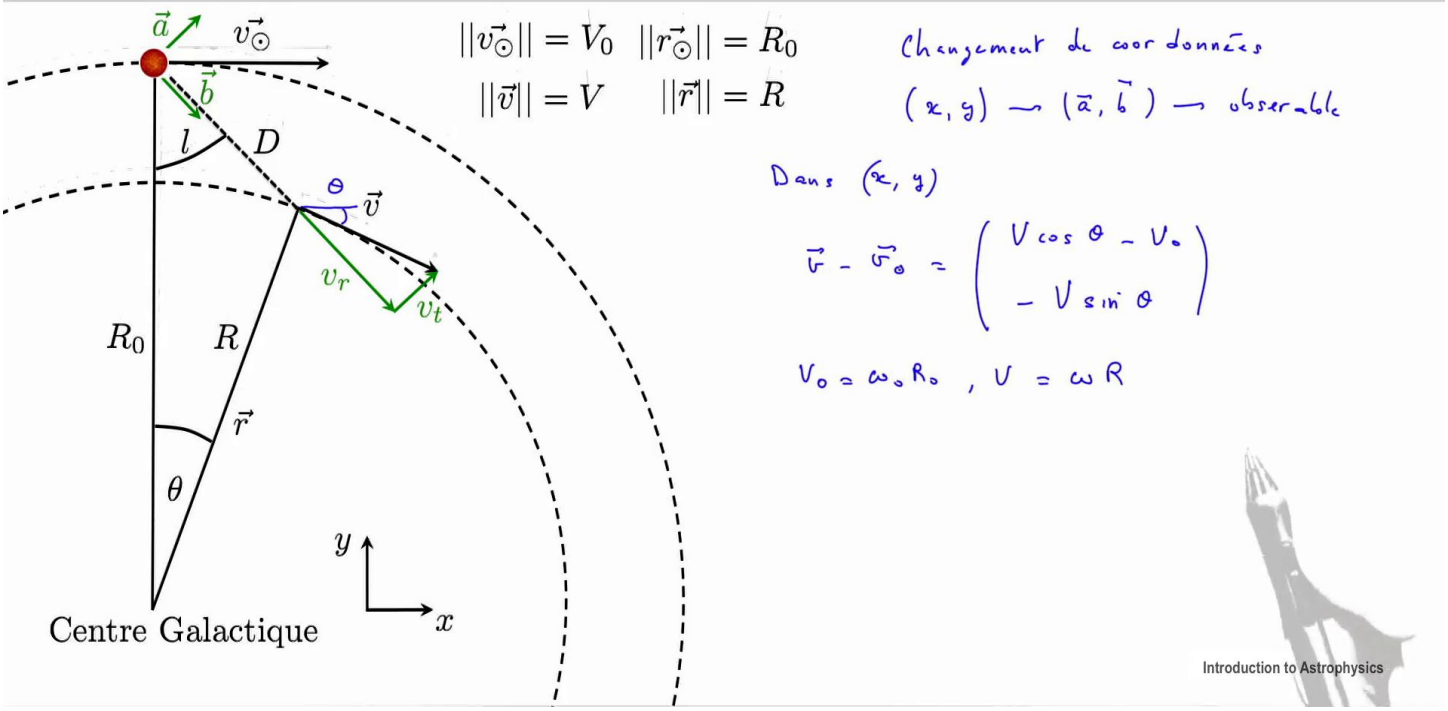
sin θ is equal to this expression, cos θ is equal to this one. And there is a factor R that should not be forgotten. Thanks to this calculation it was possible to eliminate the angle θ that cannot be observed and to replace it by the angle l, which is more relevant because it is the one that can be observed.

Notes

Summary

7m 57s

Motion of the Sun in the Milky-Way



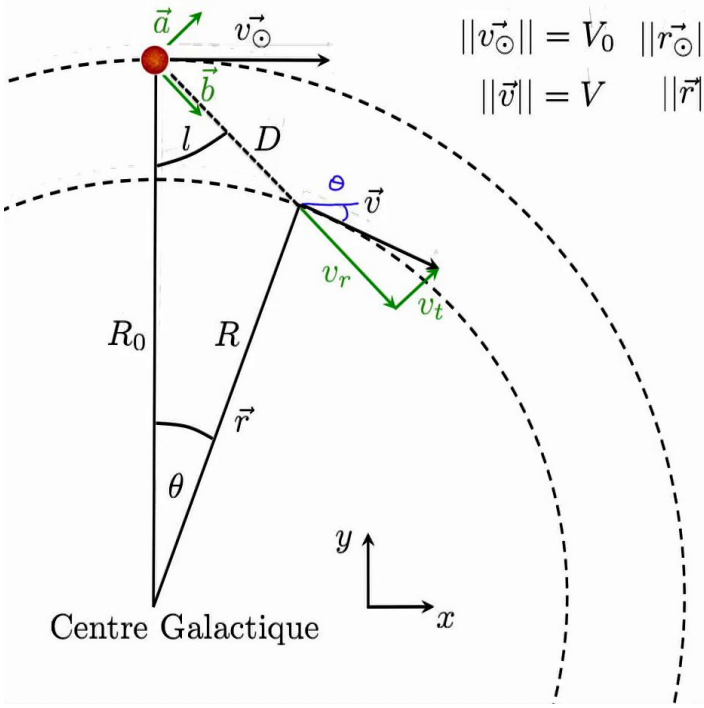
Up to now, the calculations were made using the Cartesian coordinate system with the velocity vector for the x and y axis, which is nice, since we are used to work in this frame. Unfortunately, these axes are inconvenient to measure the velocities that we are interested in: the radial velocities - velocities along the line of sight measured using the Doppler effect, and the tangential velocities that are the projections onto the celestial vault measured by observing the stellar motion over intervals of several years. Because of that, we change the coordinate system from the Cartesian system (x, y) to the system a and b, where a represents tangential velocities and b radial velocities. This is what will be observed. In Cartesian coordinates, in the (x,y) reference frame, one can express the velocities with regard to the Sun's velocity: v - v of the Sun is equal to, as the angle θ is here, v cos θ - V0 for the x axis and minus V sin θ for the y axis. One can see that V0 is equal to ω0 R0, V is equal to ωR. Then, we have also equations calculated before that allowed us to replace cos θ and sin θ by sin l and cos l - the observed values.

Notes

Summary

8m 19s

Motion of the Sun in the Milky-Way



$||\vec{v_\odot}|| = V_0 \quad ||\vec{r_\odot}|| = R_0$

$||\vec{v}|| = V \quad ||\vec{r}|| = R$

Changement de coordonnées

$(x, y) \rightarrow (\vec{a}, \vec{b}) \rightarrow$ observable

Dans (x, y)

$$\vec{v} - \vec{v}_\odot = \begin{pmatrix} V\cos\theta - V_0 \\ -V\sin\theta \end{pmatrix}$$

$V_0 = \omega_0 R_0 \quad , \quad V = \omega R$

$$\vec{v} - \vec{v}_\odot = \begin{pmatrix} R_0(\omega - \omega_0) - \omega D\cos l \\ -D\omega\sin l \end{pmatrix}$$

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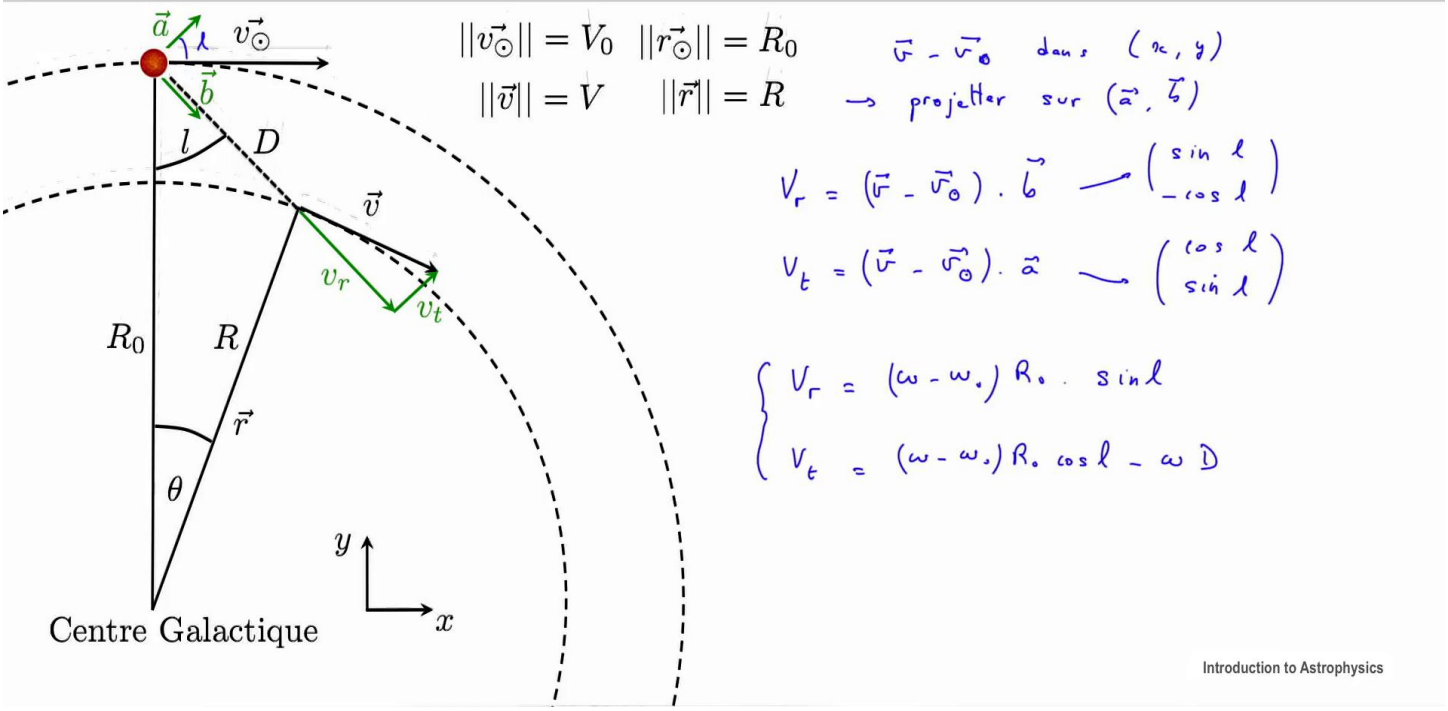
If one uses all these equations it is possible to write the relative velocity of the stars with regard to the Sun's velocity in the following way. R (ω - ω0) minus ω D cos l for the first component, - D ω sin l for the second component. In this expression V and V0 were replaced using ω0 R0 and ω R, and θ was replaced by l using the equations calculated before.

Notes

Summary

10m 10s

Motion of the Sun in the Milky-Way



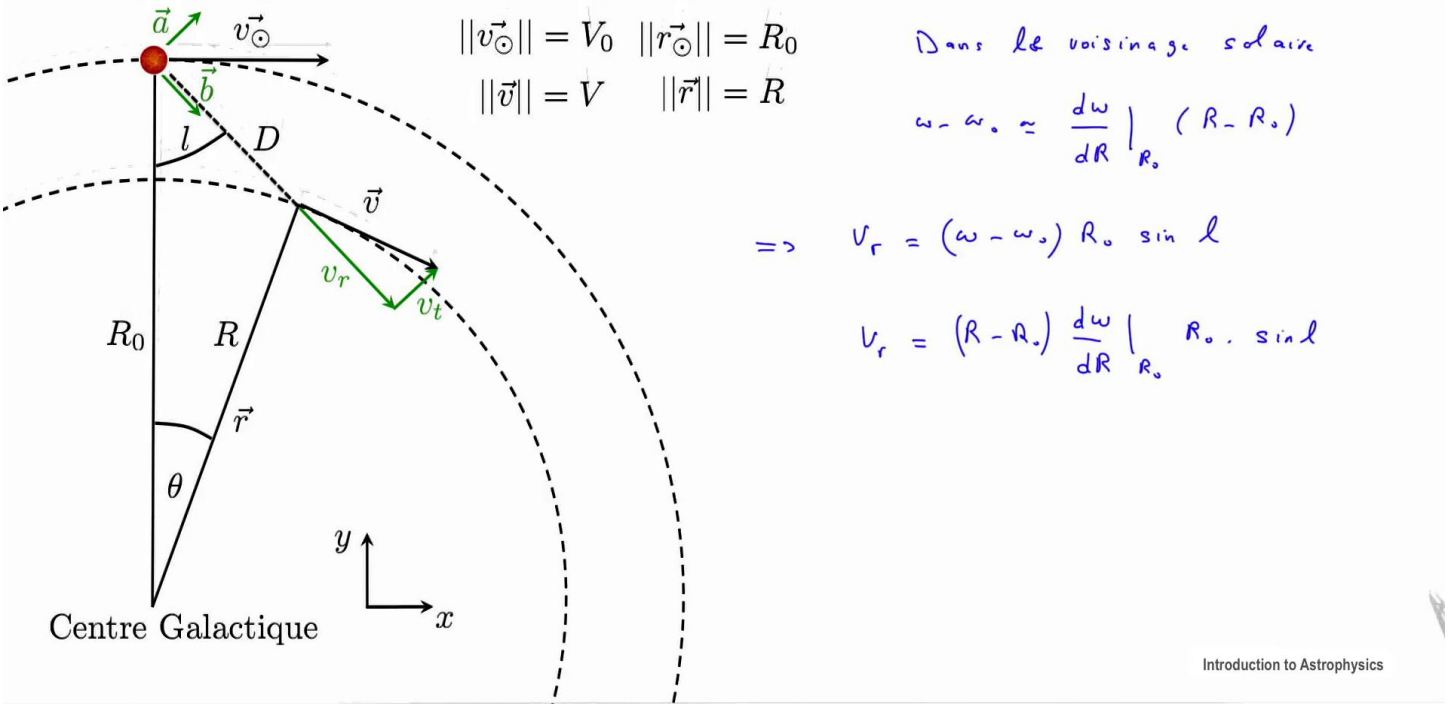
The equation for v - v Sun in the (x,y) reference frame will be projected onto the (a,b) reference frame. We use the scalar product so that Vr - radial velocity is equal to v - v Sun in Cartesian coordinates dot b vector, along the line of sight. The same for the tangential velocity, v - v Sun in Cartesian coordinates dot a in Cartesian coordinates. The b vector in Cartesian coordinates has one component equal to sine l, here and minus cosine l here. This is the vector b. For the vector a one finds the angle l here, and so the components are equal to cosine l and sine l. Calculating the scalar product one finds Vr = (ω - ω0) R0 sin l for the radial velocity, and for the tangential velocity Vt = (ω - ω0) R0 cos l - ωD. Now, we have at disposal equations for the radial and tangential velocities, so the velocity of the stars relative to the Sun in the coordinate system that we are interested in a for tangential velocities, b for radial velocities. Both are functions of an observed angle, the galactic longitude, that is equal to zero when observing toward the Milky Way's centre. In general it takes values from 0 to 360º, or from -180 to 180º scanning in this way the lines of sight running on from the galactic plane.

Notes

Summary

10m 45s

Solar Neighborhood and Oort Constants



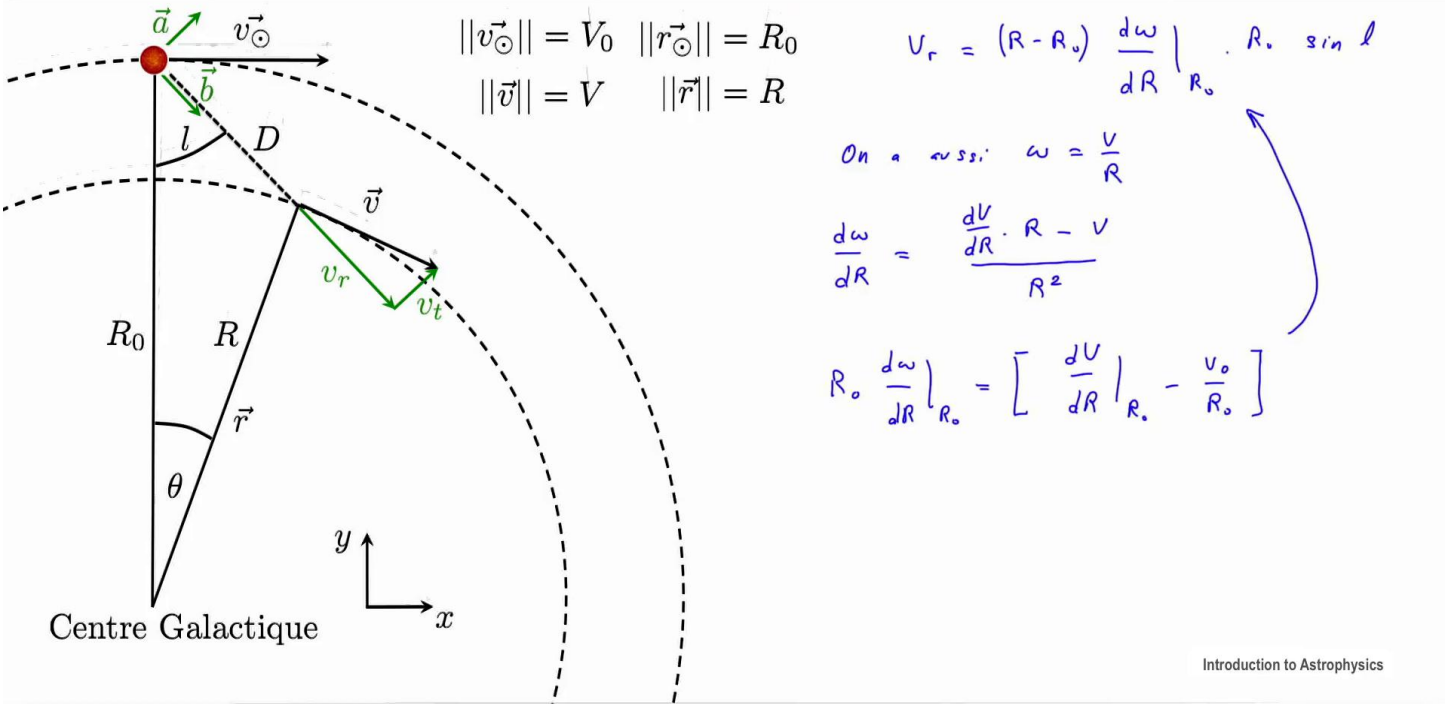
It is not possible to see stars in every part of the Milky Way, in any case, better measurements are obtained for stars close to us. In the solar vicinity, ω - ω0 can be expressed using a limited Taylor series giving dω/dR at R0 times (R - R0). This is the first-order Taylor expansion around the point R0, so for R close to R0, in other words when the stars are relatively near the Sun. Replacing this into the expression for the radial velocity (ω - ω0) R0 sin l, leads to Vr equals this expression. This is a new expression for a relative velocity of stars, more precisely, the radial component of this velocity in the solar vicinity.

Notes

Summary

12m 33s

Solar Neighborhood and Oort Constants



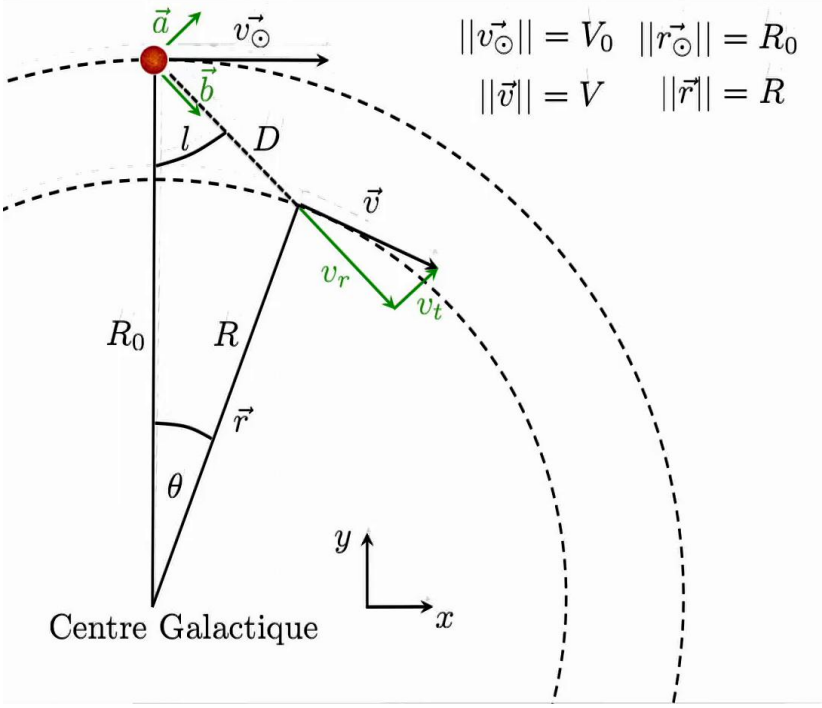
Rewriting this expression once again. We have that the angular velocity is equal to V/R, so deriving this equation gives the following expression. Now, we take the derivative around the point R0, so this expression at R0 times R0, gives this formula. It will be replaced in the expression above in order to obtain a new equation for the radial velocity.

Notes

Summary

13m 41s

Solar Neighborhood and Oort Constants



$||\vec{v_\odot}|| = V_0 \quad ||\vec{r_\odot}|| = R_0$

$||\vec{v}|| = V \quad ||\vec{r}|| = R$

$$V_r = \left[\left.\frac{dV}{dR}\right|_{R_0} - \frac{V_0}{R_0} \right] (R - R_0) \sin l$$

$$V_t = \left[\left.\frac{dV}{dR}\right|_{R_0} - \frac{V_0}{R_0} \right] (R - R_0) \cos l - \omega D$$

On a aussi $R_0 - R \sim D \cos l$

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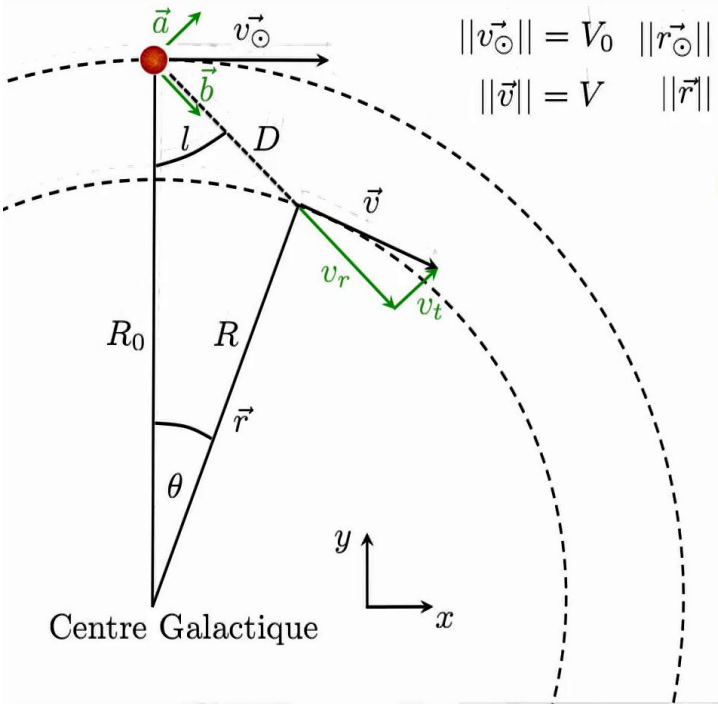
This new equation for Vr is as follows. A similar calculation can be made for the tangential velocity. These equations are not the final formulae. We have also that R0 - R is small, so the difference between R0 and R, this length here, is roughly equal to D times cosine l. This is valid for small angles l, or for R close to R0. This can be used to replace (R - R0) into the equations above.

Notes

Summary

14m 26s

Solar Neighborhood and Oort Constants



$||\vec{v_\odot}|| = V_0 \quad ||\vec{r_\odot}|| = R_0$

$||\vec{v}|| = V \quad ||\vec{r}|| = R$

$V_r = A \cdot D \sin 2l$

$V_t = A \cdot D \cos 2l + B \cdot D$

A et B : Constantes de Oort

$$A = -\frac{1}{2}\left[\left.\frac{dV}{dR}\right|_{R_0} - \frac{V_0}{R_0}\right]$$

$$B = -\frac{1}{2}\left[\left.\frac{dV}{dR}\right|_{R_0} + \frac{V_0}{R_0}\right]$$

$V = \omega R \quad A \neq 0 \rightarrow$ rotation non-rigide

car $\omega = \omega(R)$

Introduction to Astrophysics

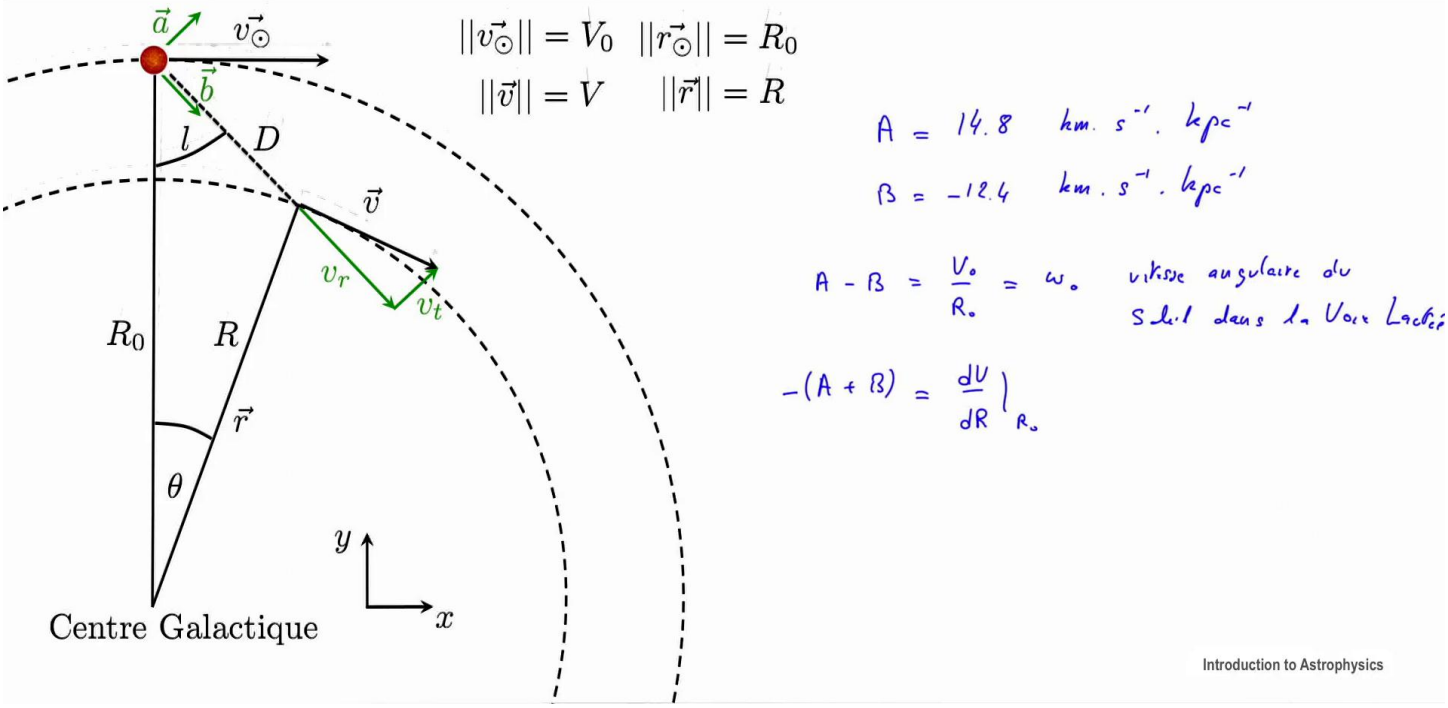
If this is done one can show that the radial velocity is equal to a constant times D *sine 2l and the tangential velocity is equal to A D cos 2l + a constant times D. Those are not just some constants. A and B are the Oort's constants. They can be written in the following way. We know that V=ωR. One can see that for the rotation of a non-rigid body, meaning that ω depends on R, the term within brackets cannot be equal to zero. In other words, if A is not equal to zero the rotation is that of a non-rigid body, and ω=ω(R).

Notes

Summary

15m 13s

Solar Neighborhood and Oort Constants

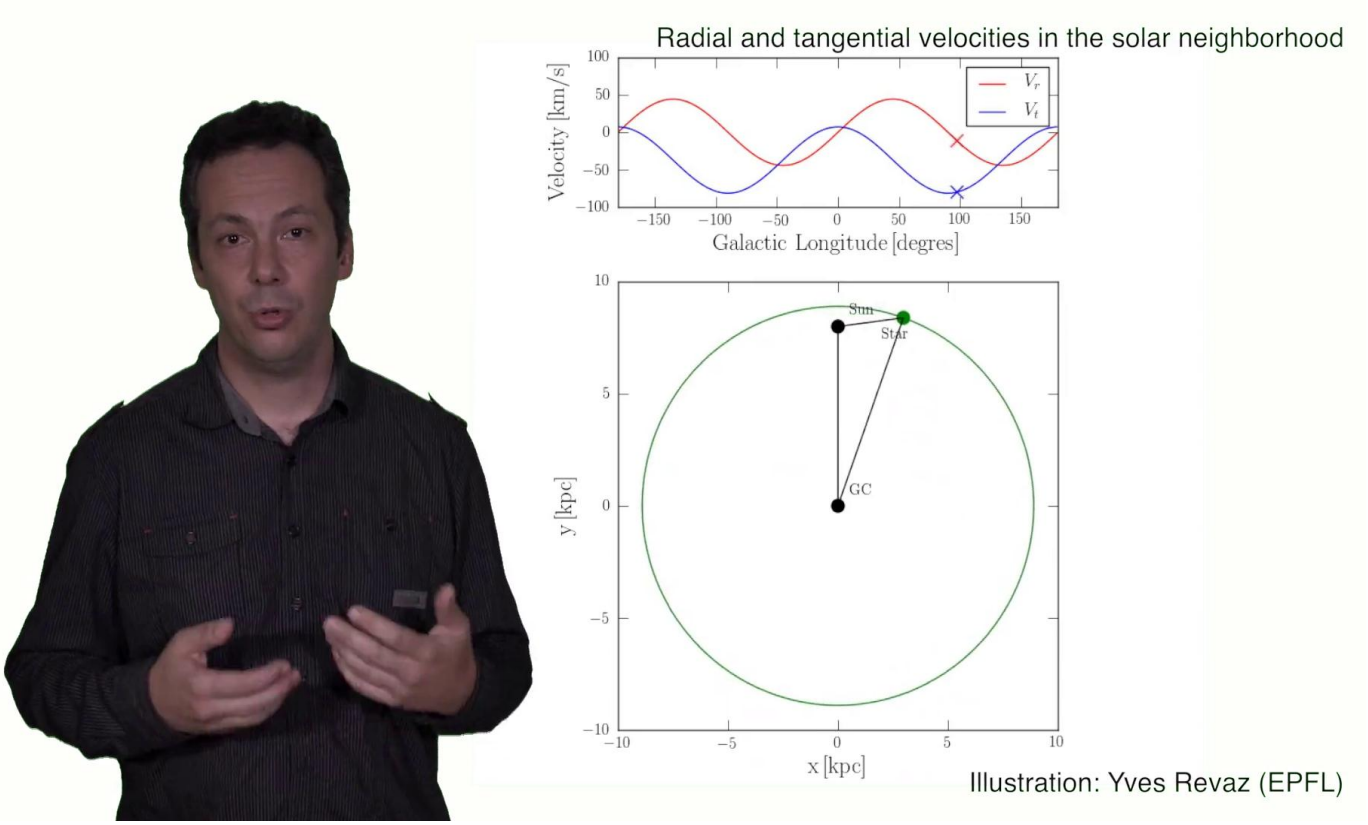


What are the measured values for A and B? Measurering radial and tangential velocities as a function of the galactic longitude, can be done by measuring the velocities on the line of sight using the Doppler effect and the tangential motion of stars, one finds A equal to roughly 14.8 km per second and per kiloparsec. one can see that the unit of A is that of an angular velocity. Also form the measurements one obtains B equal to -12,4 km per second and per kiloparsec. What can be also seen taking both expressions for A and B is that A - B = V0/R0 which is equal to ω0 - the solar angular velocity in the Milky Way. One can also notice that (-A + B) = dV/dR for R0. In other words, a measure of the Oort's constants, valid in the vicinity of the Solar System, gives the angular velocity of the Sun in the Milky Way and the derivative of the velocity around R0.

Notes

Summary

16m 28s



This image presents the results of the calculations. In this animation one can see at the bottom a schematization of the Milky Way, the galactic disk face on, with the Sun placed at a distance R0 = 8 kpc from the Galactic centre. A star is situated at a distance D = 3 kpc from the Sun, so in the solar vicinity. At the top of the image one can see the relative velocity of that star with regard to the Sun's velocity. Both radial and tangential velocities are predicted by the Oort's curves and the Oort's velocities. The radial and tangential velocities are measured for stars at all the galactic longitudes, so the stars situated at different galactic longitudes in the Galactic plane. The results completely confirm Oort's predictions that the rotation of the Milky Way is that of a non-rigid body.

Notes

Summary

17m 56s