ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

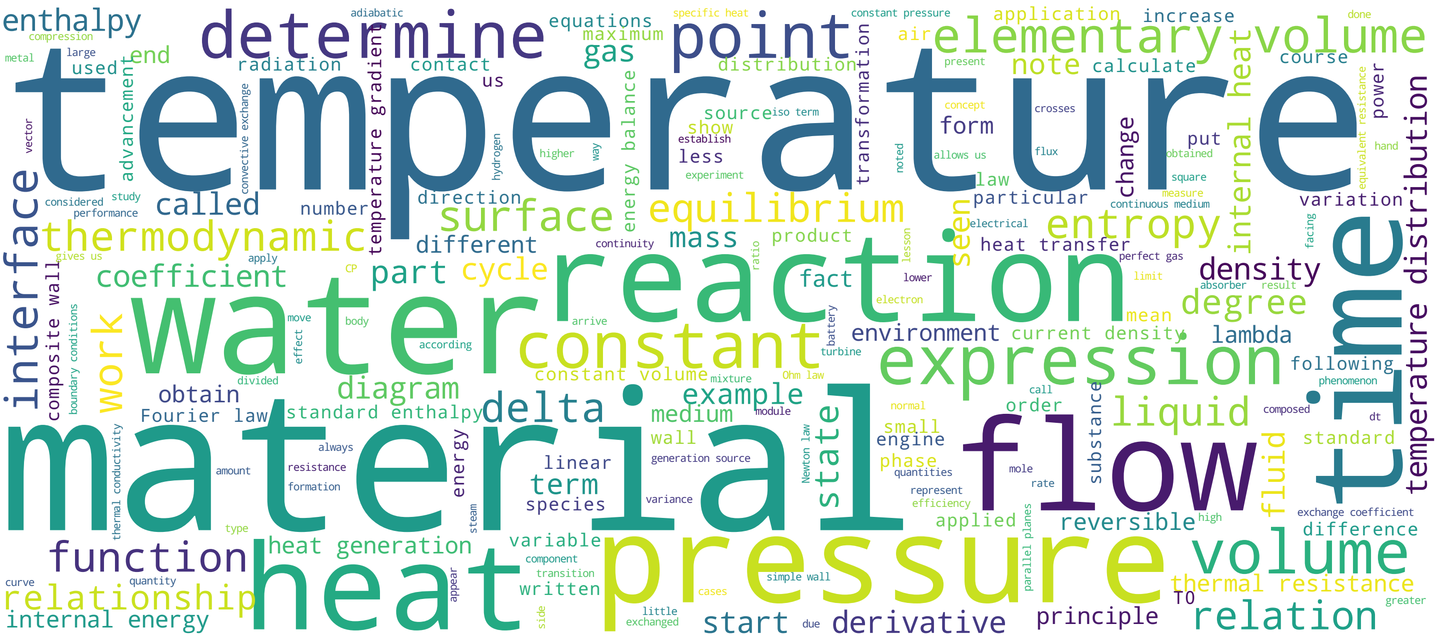
RESCIF
Réseau d'excellence des sciences de l'ingénieur de la Francophonie

Thermodynamique

Transfert conductif en régime stationnaire: Application

Jean-Baptiste Biot, 1774-1862

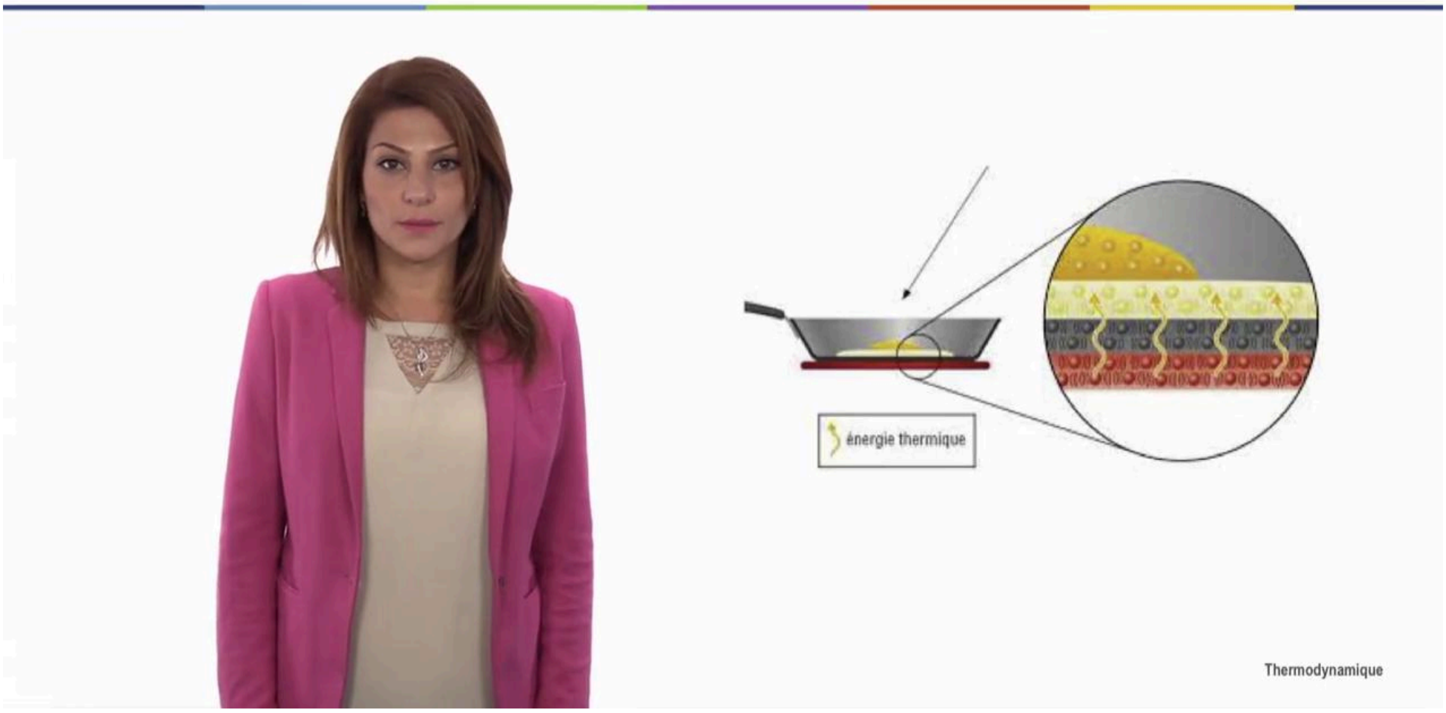
Dr. Chantal Maatouk



Video

EPFL

Introduction



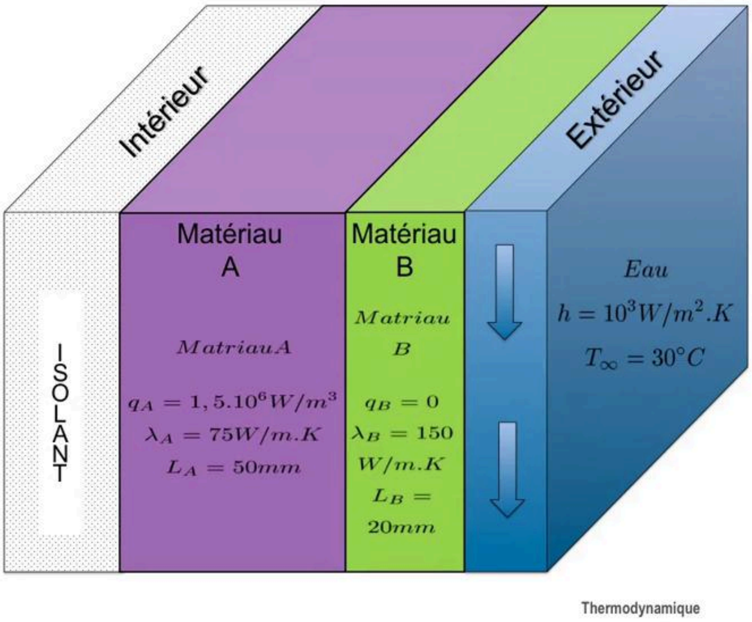
Welcome to the online heat transfer courses. My name is Chantal Mato and I am D. in energetics from the Ecole des Mines de Paris. Today, I am a teacher-researcher at Saint Joseph University, at the Higher School of Engineering in Beirut, Lebanon. In this session we will deal with the heat transfer in a wall composite very closely, mainly conduction in a composite wall. For this exercise, we will apply the heat equation, Fourier's law and Newton's law, Ohm's law and we will make the equivalence between the electric and the thermal.

Notes

Summary

0m 04s

Application: Etude d'un mur composite

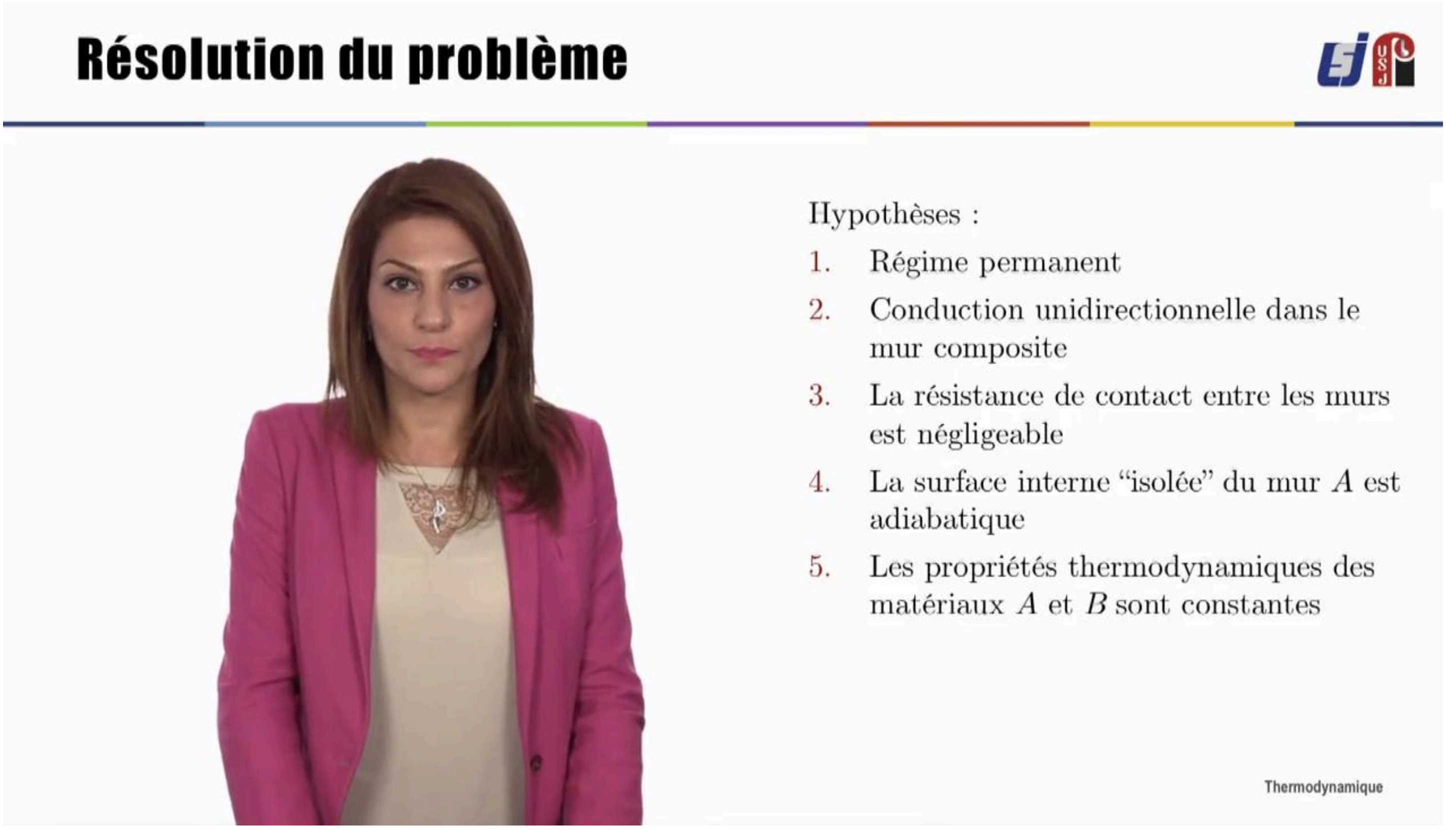


The application that we are going to treat consists in studying the thermal transfers in a composite wall, as you can see in the following figure. The wall is composed by material A. This material has a source of internal heat generation which is noted as A. The thermal conductivity of this material is lambda and it has a thickness to this material. It is joined by another material B whose thermal conductivity is different. It is lb thick and has no internal heat generation source. On the inside of the wall, we see that we have a thermal insulation and on the outside, material B is cooled by a water film at 30 degrees and is characterized by a convective heat transfer with an exchange coefficient convective H of 1000 watts per square meter per degree.

Notes

Summary

0m 44s



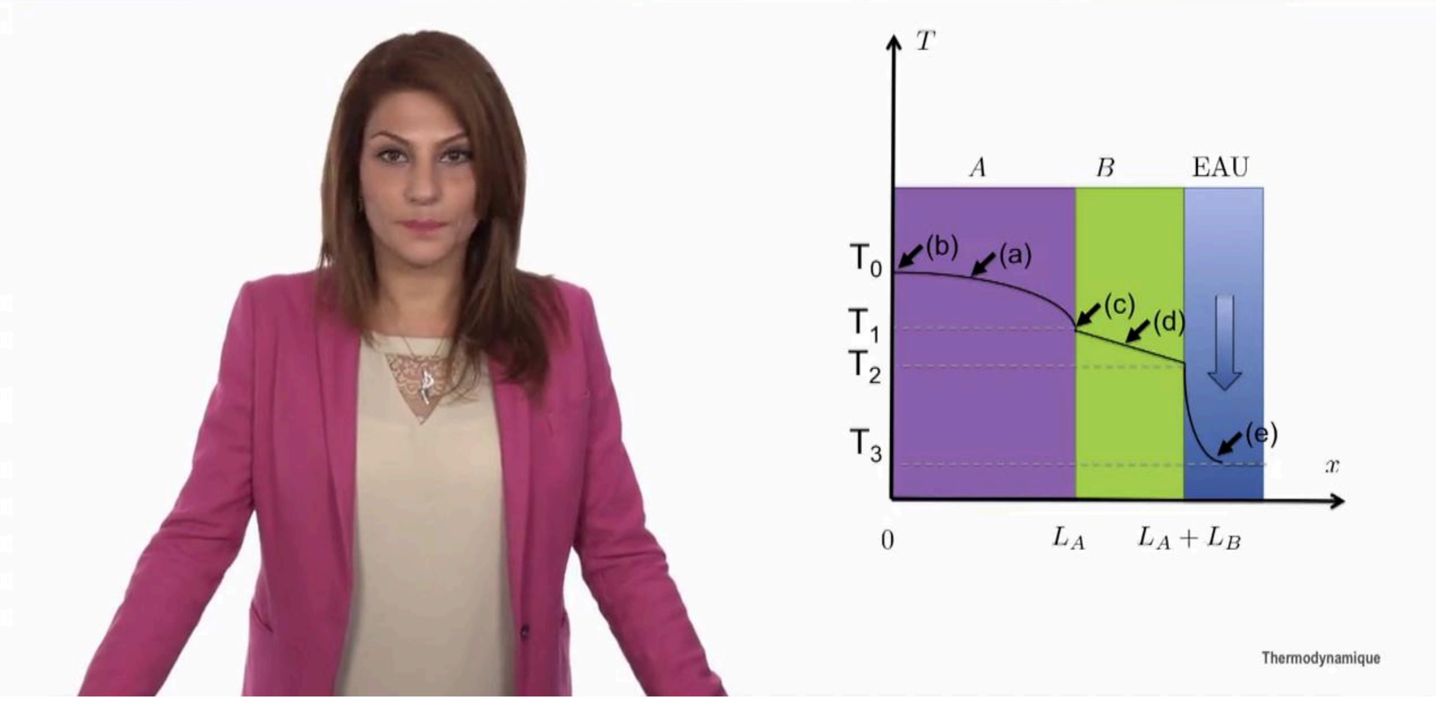
The objective of this work is to plot the temperature distribution in this composite wall and determine the temperature at the interfaces of this wall, i.e. on the insulated surface of material A and on the cooled interface of material B. In order to solve this problem, we will start by setting up the assumptions necessary to solve our problem. First, we will consider that the operating regime is permanent. Then, we consider that we are facing a problem of simple wall. So our wall is considered to be composed of parallel planes that are iso terms and thus the temperature gradient in this wall is well merged with the normal to these parallel planes. In particular, we will consider that the contact between the different materials that make up its plan, this wall is perfect. Thus, we will neglect the resistance of contact between these different walls on the side of the insulated surface. As you can see, we will consider that the surface is adiabatic or we are facing to a zero flow condition at this interface. And finally we will consider the properties of materials A and B as being independent of temperature and therefore constant.

Notes

Summary

1m 38s

Analyse du problème

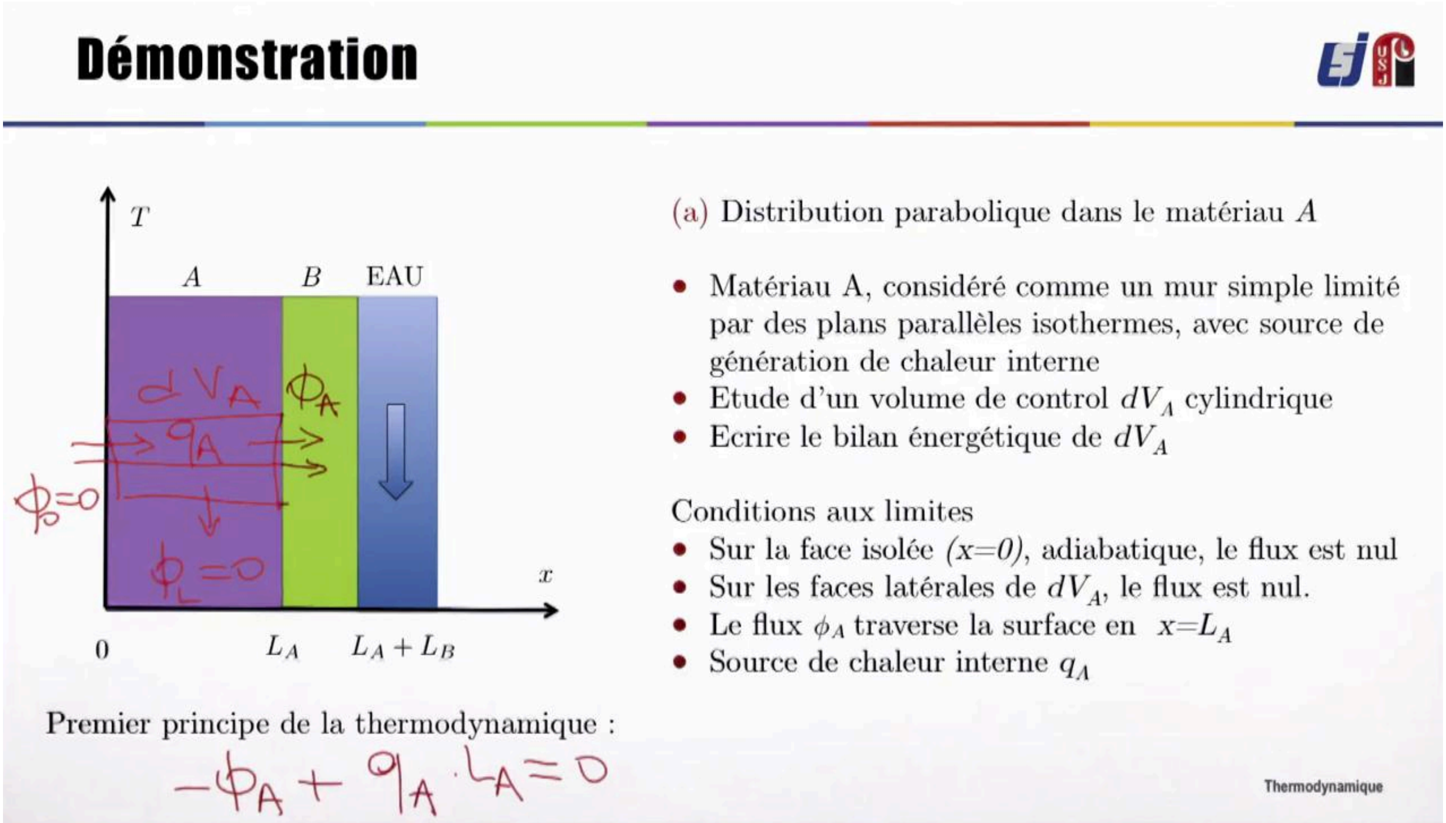


For this application, we will start by analyzing this problem from the physical conditions defined previously in the assumptions. We will therefore characterize the temperature distribution in this wall such as in material A, we will have a distribution of temperature which is rather parabolic with a maximum temperature at the isolated interface, i.e. an x equal to zero. On this interface, since the flux is zero, then we have a temperature which is constant at this interface at the point indicated, it is on the figure. We have a surface of contact between two materials as we have neglected the resistance of contact between these two materials, we can consider that the flow going out of material A is the same as the flow going into material B and that the temperature at this interface is the same in material B. In the absence of a generation source of internal heat, the temperature distribution will be linear and finally the interface between material B and water, or the temperature difference between material B and water will be due to the fact that we have a heat exchange convective conductor between these two parts.

Notes

Summary

3m 07s



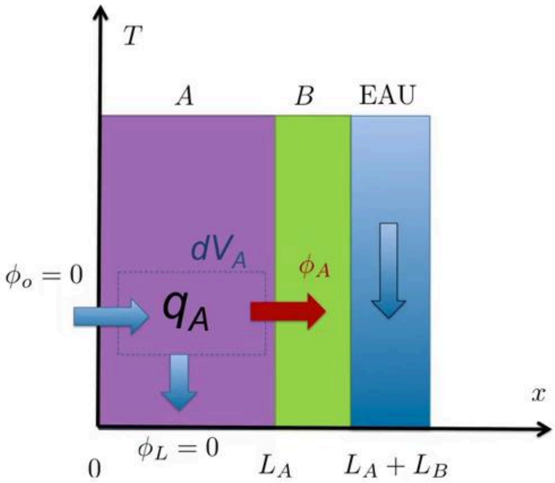
Let us prove that in material a the distribution is indeed parabolic. For this, we will consider the material A as a simple wall bounded by parallel ISO term planes with a source of internal heat generation. In this material, we will isolate an elementary volume. We will note VAT. This elementary volume has a cylindrical shape whose axis is coincident with the normal to the ISO plane term an x equal to zero. The flow into this control volume is zero. We will note if zero on one the lateral surfaces since we is facing planes which are ISO at term the flow is lateral, it is also zero. In X, we have a Fiat flow that crosses the boundary of the elementary volume and in the elementary volume itself, we have a power density Q1 which will be considered as a source of internal heat generation. By writing the energy balance applied on this elementary volume, we can write that if a is the outgoing flux of this elementary volume plus q a, it has. The internal power generated in the volume is equal to zero.

Notes

Summary

4m 32s

Démonstration



- Système d'équation: $\begin{cases} \dfrac{d^2T}{dx^2} + \dfrac{q_A}{\lambda_A} = 0 \\ T(0) = T_0 \\ T(L_A) = T_1 \end{cases}$
- Première intégration

$$\frac{dT}{dx} + \frac{q_A}{\lambda_A} x = C_1$$

- Deuxième intégration

$$T(x) = -\frac{q_A}{2.\lambda_A} x^2 + C_1.x + C_2$$

$$T(x) = -\frac{q_A}{2.\lambda_A} x^2 + [\frac{T_1 - T_0}{L_A} + \frac{q_A.L_A}{2.\lambda_A}]x + T_0$$

Distribution parabolique

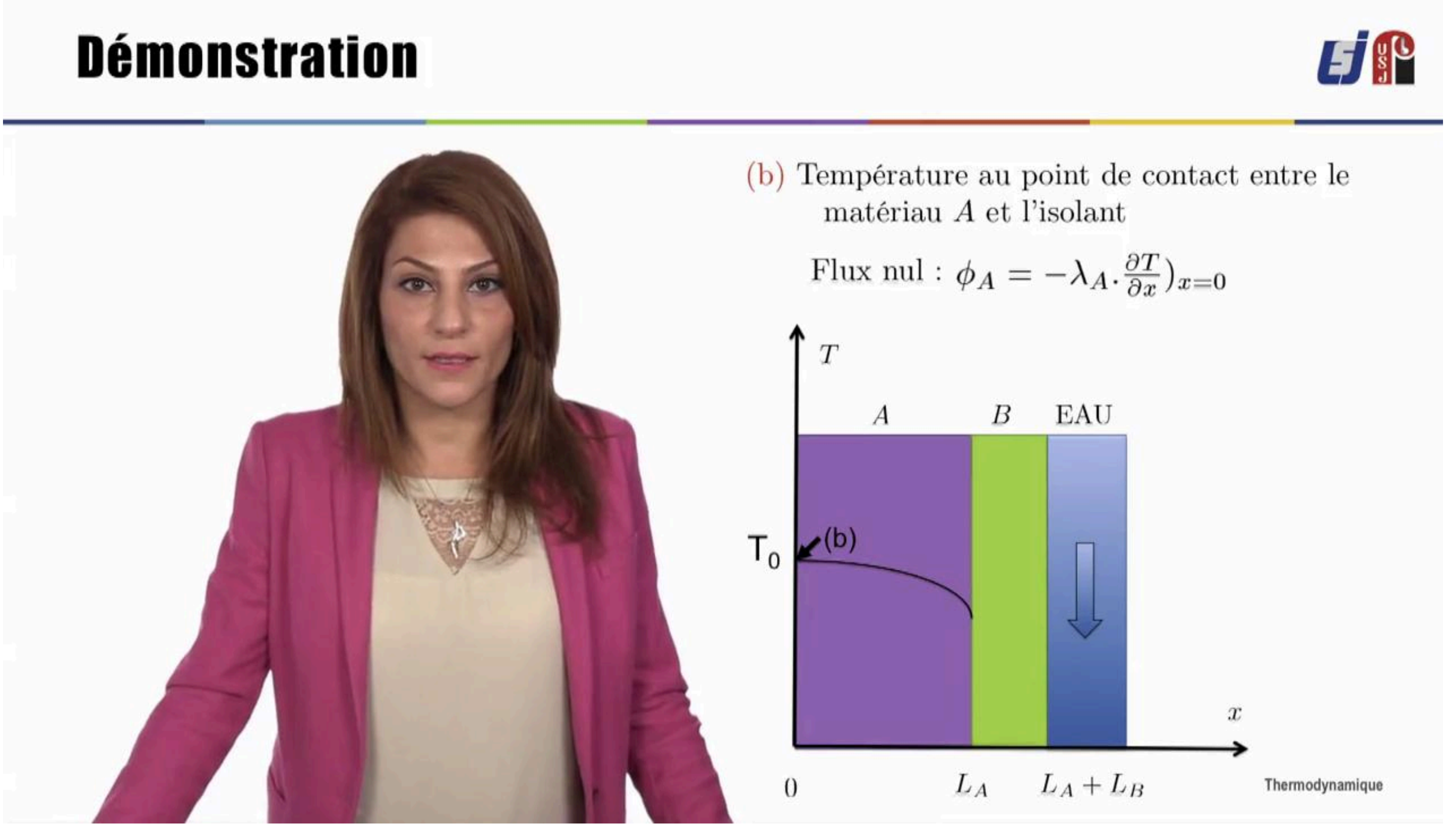
Thermodynamique

By expanding the resulting expression. We can then write the heat equation applied to the volumes of rubber tree control, as well as boundary conditions, as seen in the following system of equations. So the boundary conditions are an x equal to zero and x equal to l at. These are the boundaries of this material has. And this expression corresponds to the heat equation that we have that can be developed in this control volume or in the material to. The double integration of this expression allows us to determine the temperature distribution in the material A. As you can see, the resulting polynomial is a polynomial of the Second Age and thus we can see that the distribution is indeed parabolic.

Notes

Summary

5m 57s

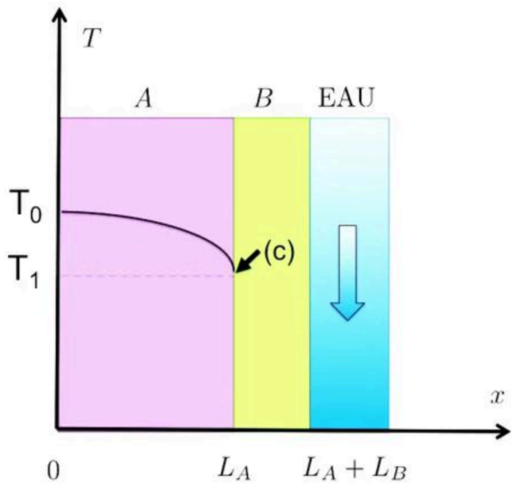


It was then shown that the distribution of the temperature in the material A is well parabolic. Now we will see that an X is equal to zero. The temperature of the material is the highest and largest of the material A. Since we are facing a zero flow, so the heat flux expressed according to Fourier's law gives us a derivative of the temperature with respect to X1X equal to zero which is zero, which means that the temperature in this surface is indeed maximum and we has a zero temperature variation at this interface.

Notes

Summary

6m 45s



(c) Température au point de contact entre les matériaux *A* et *B*

$$\phi_A(x = L_A) = \phi_B(x = L_A) \Rightarrow \frac{\lambda_B}{\lambda_A} = 2$$

$$T_A(x = L_A) = T_B(x = L_A) = T_1$$

Thermodynamique

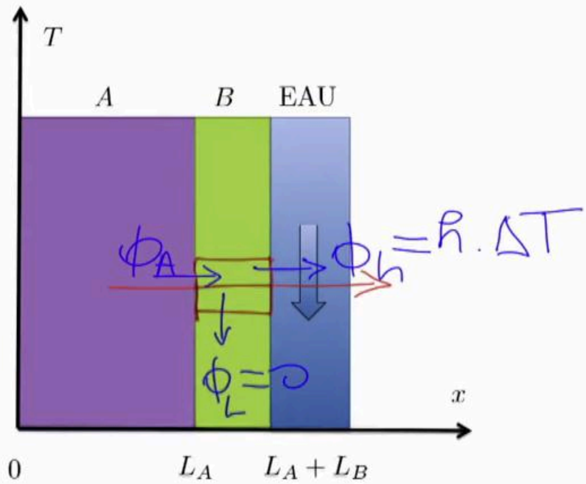
We are at the interface between the two materials A and B. On this surface, the temperature of the material in X is equal to l A is equal to the temperature of the material B in x equal to. It also has. The outflow from material A is equal to the inflow into material B. At this abscissa, and therefore the change of slope of the temperature distribution, it is equal to the ratio thermal conductivity of material B and material A.

Notes

Summary

7m 22s

Démonstration



Premier Principe de la thermodynamique:

$\phi_A - \phi_h = 0$

(d) Distribution linéaire dans le matériau B

- Matériau B, considéré comme un mur simple limité par des plans parallèles isothermes, sans source de génération de chaleur interne
- Etude d'un volume de control dV_B cylindrique
- Ecrire le bilan énergétique de dV_B

Conditions aux limites

- Le flux ϕ_A traverse la surface en $x = L_A$
- Sur les faces latérales de dV_B, le flux est nul
- Le flux ϕ_h traverse la surface en $x = L_A + L_B$

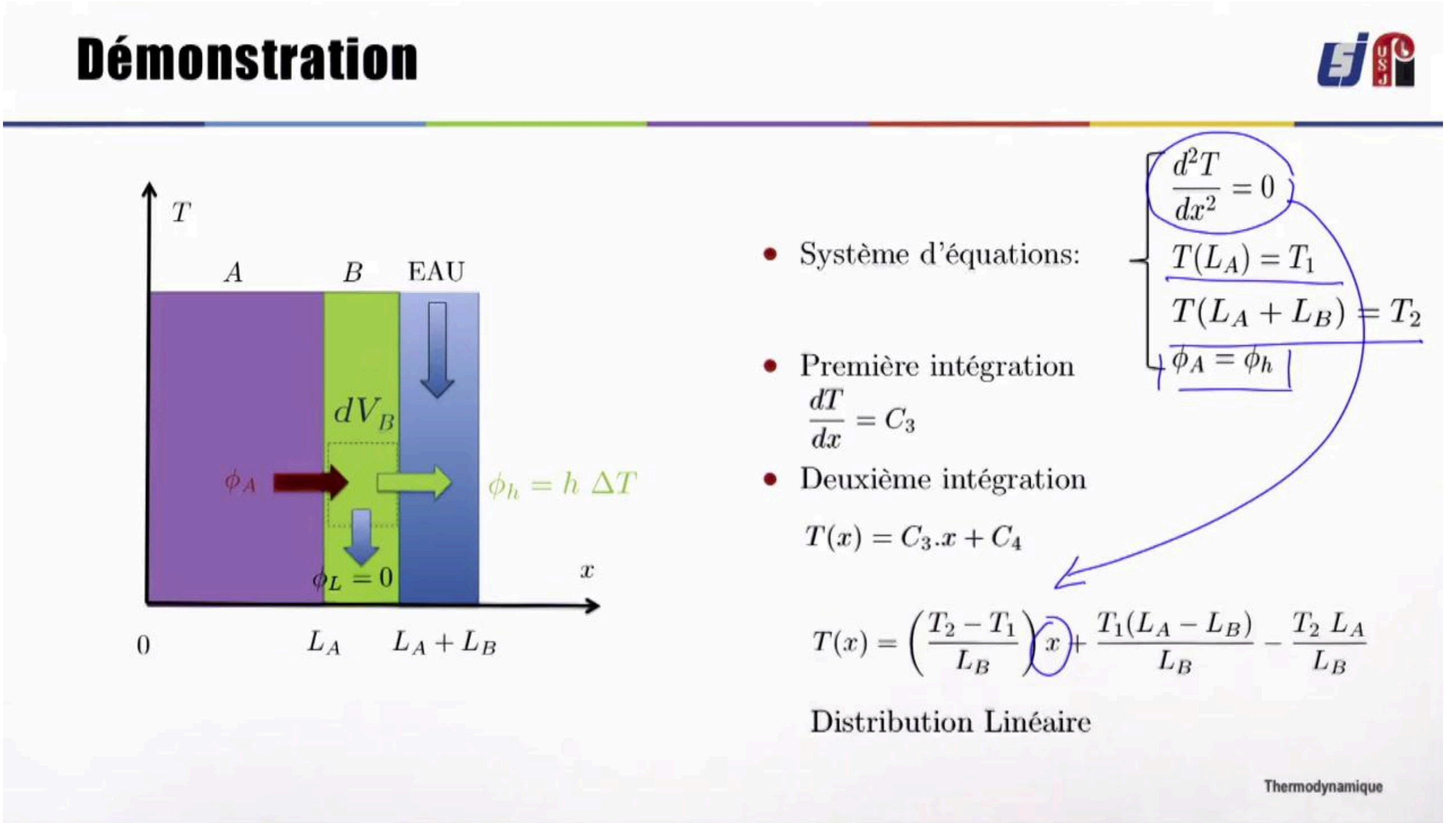
Thermodynamique

Now we will determine the temperature variation in material B. If we also consider the material B as a simple wall bounded by ISO parallel planes, term without internal heat generation source. In this material, we will isolate a volume DVB and we will establish the energy balance on this elementary volume. Then the elementary volume. It has a cylindrical shape with an axis confused with the normal to its parallel solids iso term in x equal to l a. We have a Fiat flow that crosses the boundary of this elementary volume. On the side surfaces we have a flux if l which is null since the parallel surfaces are in term. And in x lb we have a flow so h. Coming out of this elementary volume. This flow is exchanged between the material BEI and the cooling fluid which is water by conductors. Convection and therefore H is the amount of heat exchanged by convection. It is governed by Newton's law as son H. It is equal to H the convective exchange coefficient times the difference temperature difference between material B and the water surface. In steady state, the sum of all the flows entering this elementary volume is equal to zero. Thus, we can write the first principle of thermodynamics applied to this elementary volume, such that fi a. Flash equal to zero.

Notes

Summary

7m 54s



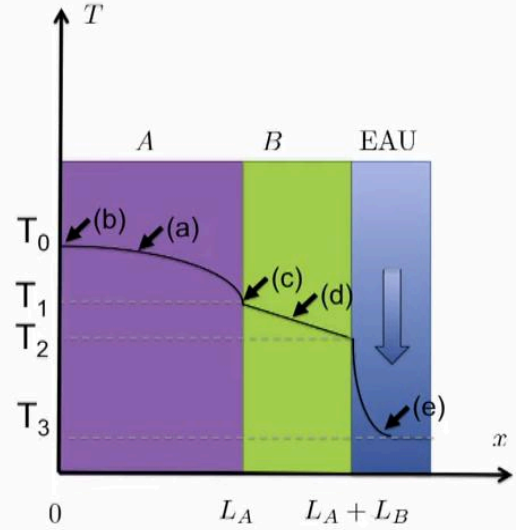
Similarly, we can write the equation of the heat applied to the elementary volume DVB, as well as the conditions at the limits, as you can see in this system of equations. So the elementary volume is bounded in X equal to l a and x equal to lb. Boundary conditions. From the first principle, the equation to equals h and from the equation of heat, we can, by integrating twice this expression, determine the temperature distribution in material B. We can see that it is a polynomial of the first order and thus the distribution in material B is linear.

Notes

Summary

9m 46s

Détermination des températures T_0 et T_2



Thermodynamique

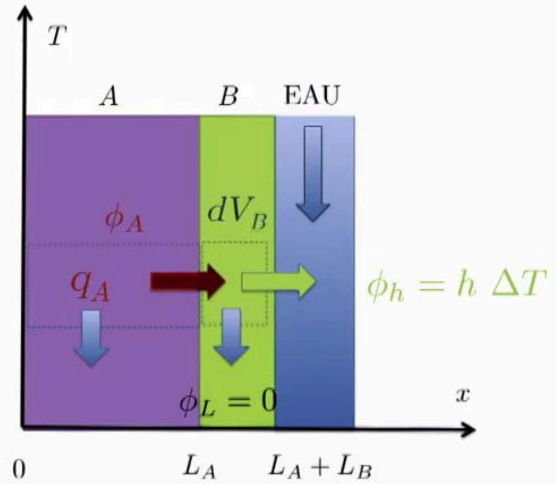
Let's now determine the temperatures T0 and T2 at the ends of this composite wall.

Notes

Summary

10m 31s

Détermination des températures T_0 et T_2



- Détermination de T_2

$$\begin{cases} \phi_A = \phi_h \\ \phi_A - q_A.L_A = 0 \\ \phi_h = h.(T_2 - T_\infty) \end{cases}$$

$$T_2 = T_\infty + q_A \frac{L_A}{h}$$

- Application numérique : $T_2 = 105\ ^oC$

Thermodynamique

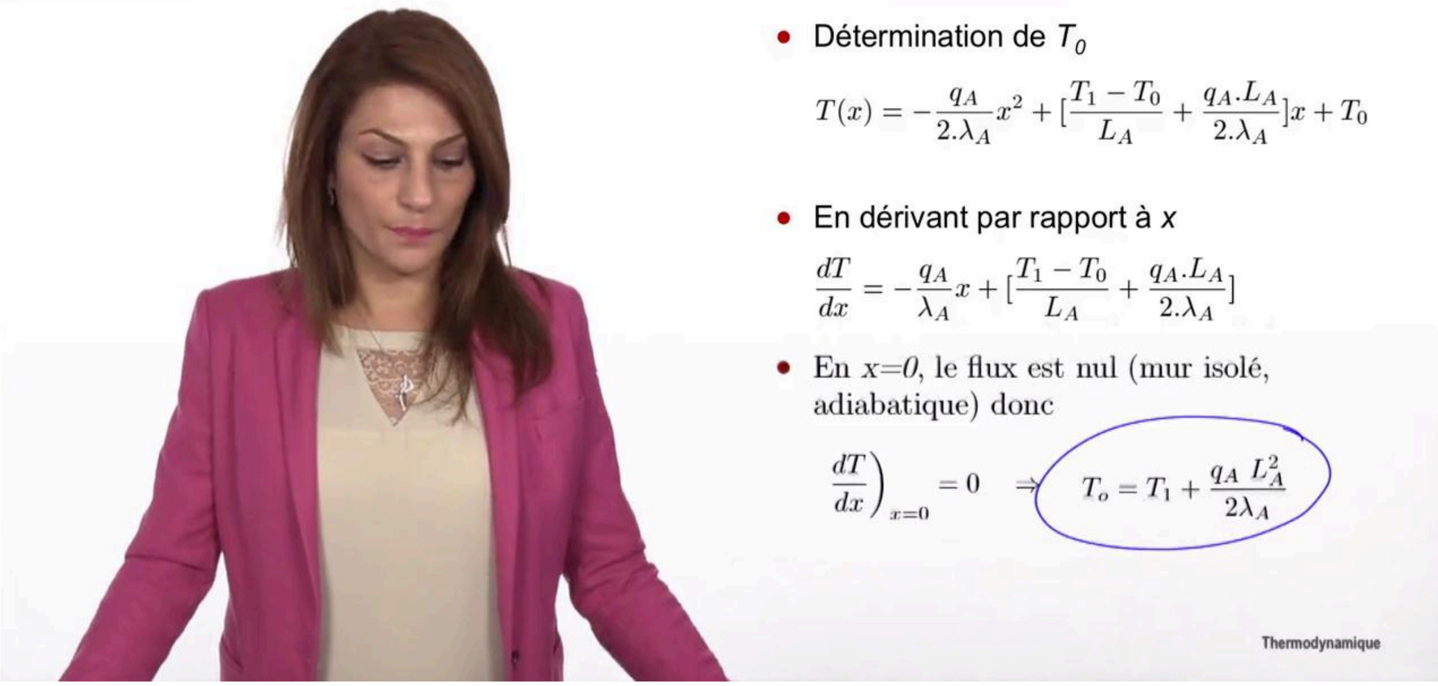
We now determine at the temperature two from the energy balance established in material B. It was determined that if a the flow of heat coming from material A, it is equal to the exchanged heat flow by conduction convection on the interface between material B and water. From the energy balance established in material A. We wrote fi a. Depending on the power density generated in the material A and according to Newton's law, we could calculate fi h from these three expressions. We can then determine the equation of T2 which allows us to calculate this temperature according to of temperature, water and the various characteristics in the material A and the convective exchange coefficient h. Thus, we can calculate a temperature T2 which is equal to 105 degrees C.

Notes

Summary

10m 38s

Détermination des températures T_0 et T_2



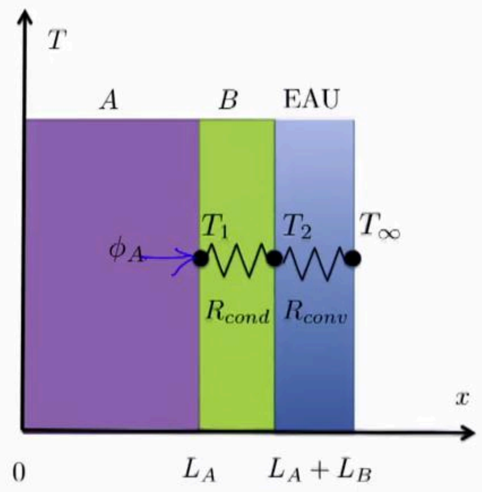
Let's now determine the temperature of T0. As we have seen at the beginning, we had put the hypothesis that one X is equal to zero. We have a zero flow since the surface is isolated. So by applying Fourier's law, we have that the flux fi a which crosses this interface is equal to zero. Thus, we can say that the derivative of the temperature on x1 x is equal to zero and null. The temperature distribution in the material A by deriving this expression and replacing X by zero. We can therefore determine the temperature or the expression of the temperature in x equal to zero. As you can see, this temperature depends on T1 and other known parameters. Therefore, in order to calculate T0, it is necessary to determine the value to determine the temperature T1.

Notes

Summary

11m 39s

Détermination des températures T_0 et T_2



$A.N.: T_1 = 115°C$
$T_2 = 140°C$

- Détermination de T_1
- Application de la loi d'Ohm entre $x = L_A$ et l'eau

R_{cond} : Résistance thermique conductive

$$R_{cond} = \frac{L_B}{\lambda_B} = \frac{0{,}02}{150} = 1{,}33.10^{-4}\left(\frac{m^2.°C}{W}\right)$$

R_{conv} : Résistance thermique convective

$$R_{conv} = \frac{1}{h} = \frac{1}{1000} = 10^{-3}\left(\frac{m^2.°C}{W}\right)$$

R_{eq} : Résistance équivalente

$$R_{eq} = R_{cond} + R_{conv} = 11{,}33.10^{-4}\left(\frac{m^2.°C}{W}\right)$$

$$\Rightarrow \boxed{\phi_A = \frac{(T_1 - T_\infty)}{R_{eq}}} = \phi_h = h.(T_2 - T_\infty)(W)$$

$$\Rightarrow T_1 = T_\infty + R_{eq}.\phi_h \ , \ T_0 = T_1 + \frac{q_A.L_A^2}{2.\lambda_A}$$

Thermodynamique

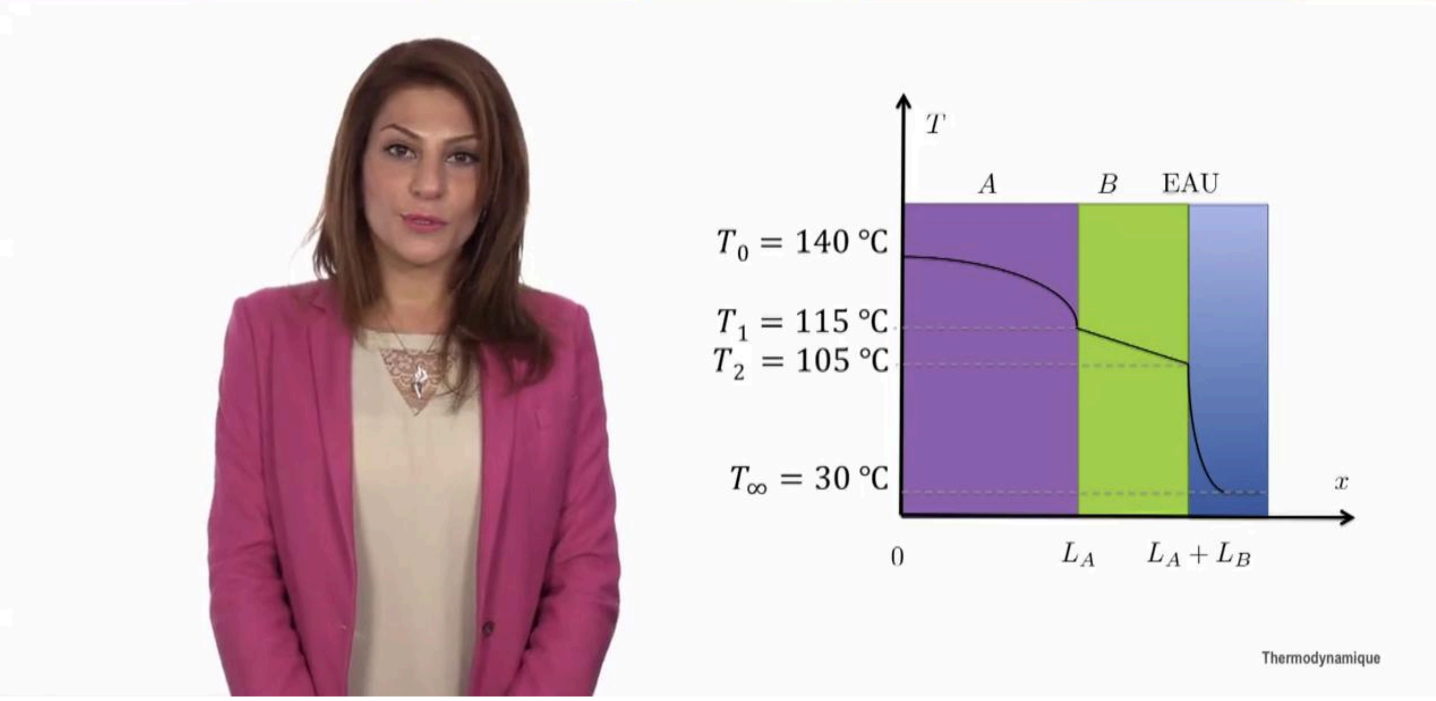
We will apply Ohm's law between X equal to L, A and water. We start by defining a conductive thermal resistance R which is equal to LB over lambda B and therefore dependent on the conductivity of the material as well as its thickness. We also define a resistance convective thermal r conf which is equal to one on h h. This is the convective exchange coefficient. As you can see on the diagram, the two resistors are in series, the equivalent resistance is equal to the sum of the two resistances in series. The flow A is unknown. This flow at. It crosses this equivalent resistance by entering through point A t1. By applying Ohm's law, we can write. If A is equal to half infinite field divided by the equivalent resistance, and since we know Fiat which is equal to H, we can determine. Then the expression of t1 t1 calculated is equal to 115 degrees and therefore determine the value of T0 which is equal to 140 degrees. You can see that the temperature gradient in the material P is lower than the temperature gradient we have between the interface of material B in contact with water and the temperature of the water. This is directly related to the fact that the convective thermal resistance is greater than the conductive thermal resistance.

Notes

Summary

12m 35s

Conclusions



In conclusion, during this session we have dealt with the problem of transfer thermal conductive in a composite wall. The heat equation, Fourier's law, has been applied, Newton's law, Ohm's law, and thermal and electrical equivalence. We have seen by solving this problem than in a single wall with an internal heat generation source, the temperature distribution is rather parabolic. In a single wall, without internal heat generation, the temperature distribution is linear. We have also seen that on an isolated interface, we are facing a condition. Of zero flow. In this case, the temperature of the material at this interface is maximum. It was also done that the temperature gradient in a given material depends mainly on the thermal resistance of the material. When you have a material with low thermal resistance, the temperature gradient in this material is rather low, whereas it is much higher when it is a high thermal resistance. Thank you for your attention and see you next time.

Notes

Summary

14m 08s