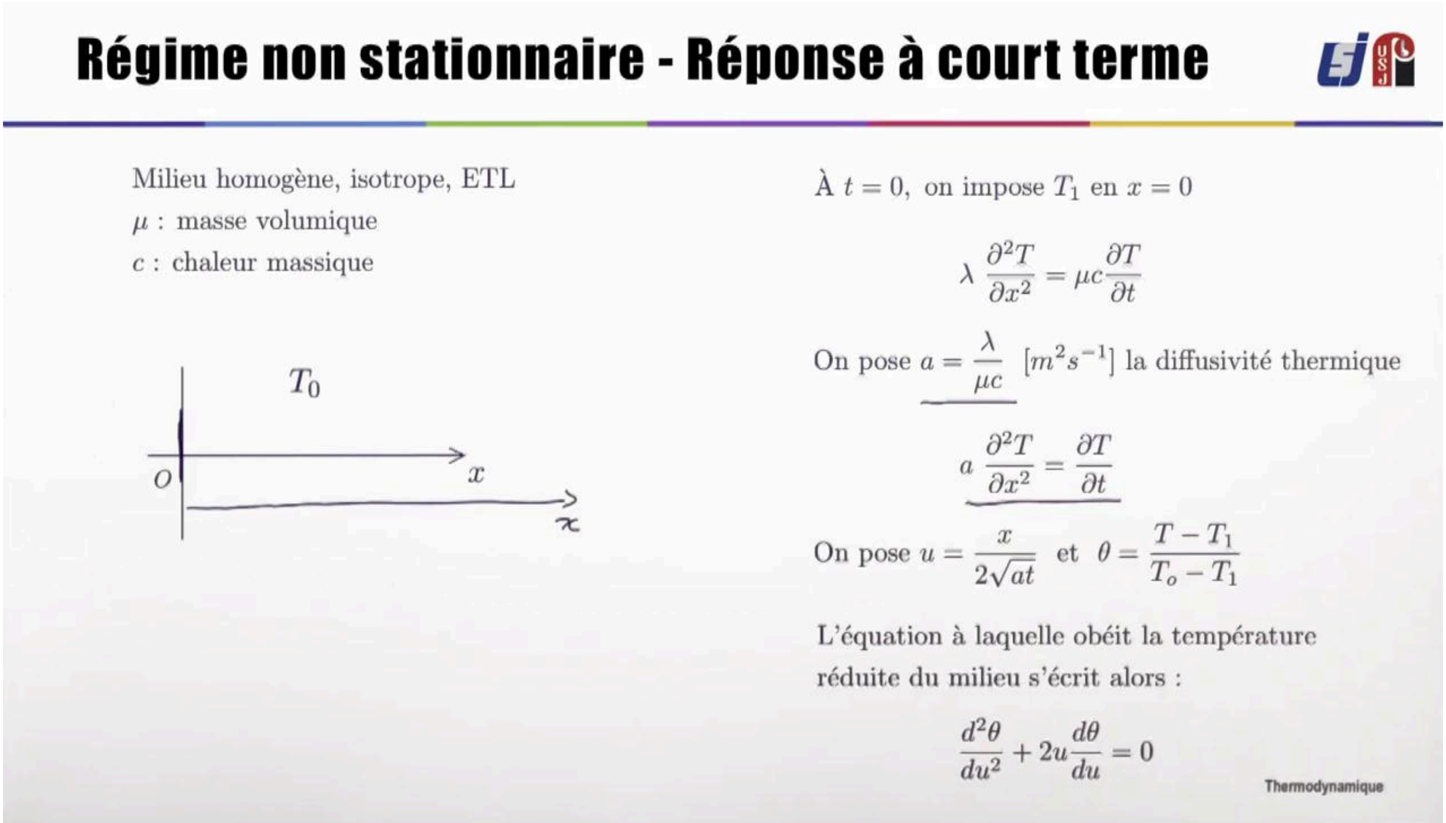
Thermodynamique

Transfert conductif en régime non stationnaire

Prof. Marwan Brouche

Jean-Baptiste Biot, 1774-1862

Video

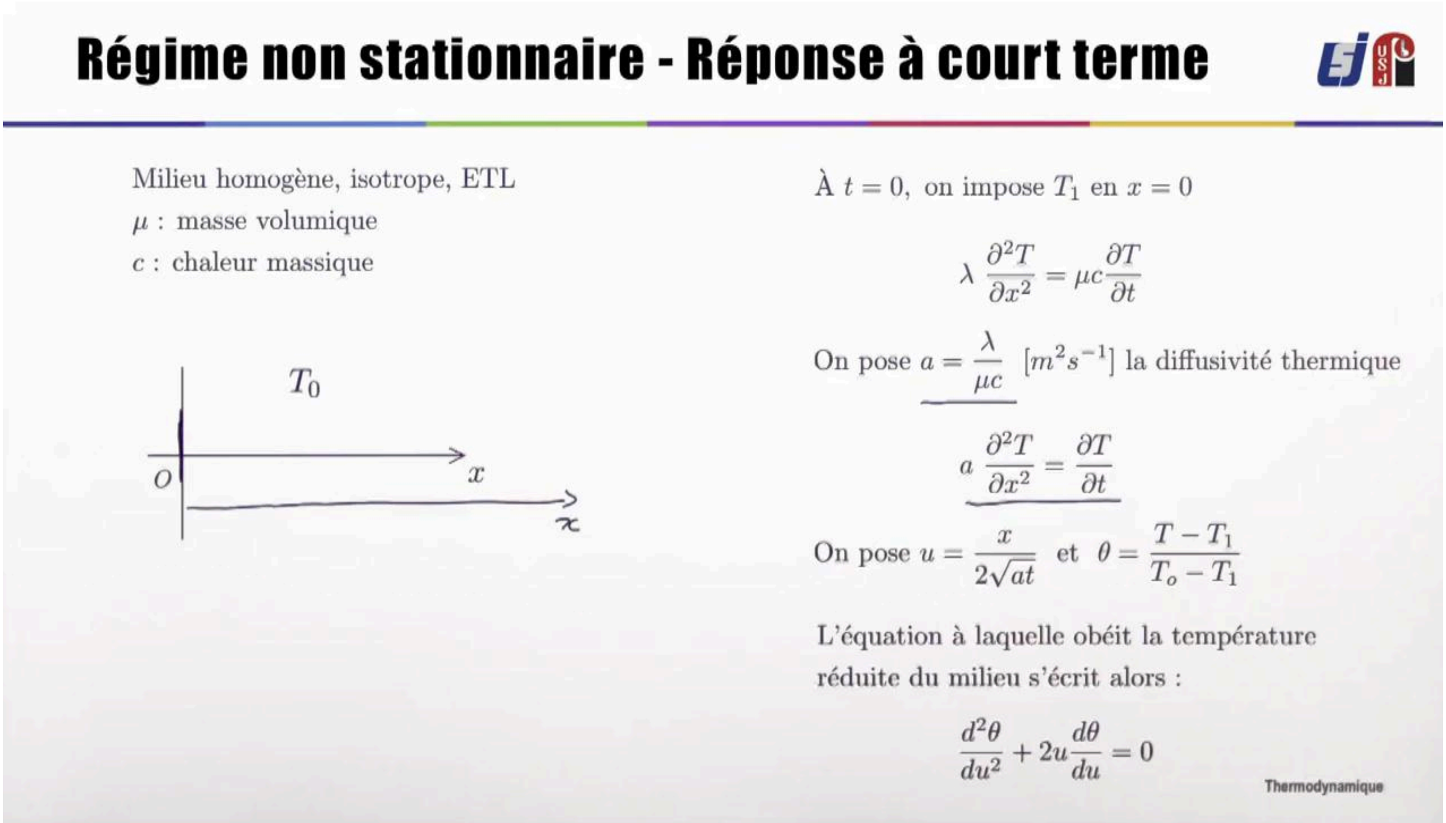


We will now deal with the second part of our course, namely conduction in non-steady state, i.e. when the temperature depends on space and time. To deal with this part, we will make two examples. The response is a medium to some excitement, but its short-term response and the response of a medium to a sinusoidal excitation. Along this part here, we will always assume that our conduction is unidirectional. We consider a homogeneous medium and isotropic and in local thermodynamic equilibrium. This medium is always characterized by its density mu and its mass heat. It is. This medium will expand two x equal to zero to infinity and it is at the temperature. Homogenous t0 at time T equal to zero. It can be imposed in x equal to zero, i.e. on this whole section. The temperature T1. The temperature of the environment will lead to the heat equation that we had established in the first part of our course. We will introduce a thermal parameter which is defined by the ratio of lambda firmus that we will call the thermal diffusivity. To solve such an equation. We pass by a change of variable. We pose u is equal to x on two root dt and a is equal to witness t1 on t0 t1.

Notes

Summary

0m 04s



We can notice that those of the variables are dimensionless and we can obtain the equation to which the reduced temperature of the medium obeys and which can be written in the form of the second derivative with respect to the square plus twice u. The derivative of head with respect to u will be equal to zero.

Notes

Summary

1m 33s

Régime non stationnaire - Réponse à court terme

Fonction d'erreur

u	erf(u)	u	erf(u)	u	erf(u)	u	erf(u)
0	0	0,3	0,328627	0,8	0,742101	1,8	0,989091
0,05	0,056372	0,4	0,428392	1	0,842701	2	0.995322
0,1	0,112463	0,5	0,520500	1,2	0,910314	2,5	0,999593
0,15	0,167996	0,6	0,603856	1,4	0,952285	3	0,999978
0,2	0,222703	0,7	0,677801	1,6	0,976348	∞	1

La solution de l'équation précédente s'écrit:

$$\theta(u) = C\int_0^u e^{-v^2}\,dv$$

Pour calculer C, on utilise les conditions initiales:

à $t = 0$, $u = \frac{x}{2\sqrt{at}} \to \infty$ et $\theta = \frac{T - T_1}{T_0 - T_1} = 1$

D'où $\theta(\infty) = C\int_0^\infty e^{-v^2}\,dv = 1$

Or, $\int_0^\infty e^{-v^2}\,dv = \frac{\sqrt{\pi}}{2}$ d'où $C = \frac{2}{\sqrt{\pi}}$

Finalement $\theta(u) = \frac{2}{\sqrt{\pi}}\int_0^u e^{-v^2}\,dv = erf(u)$

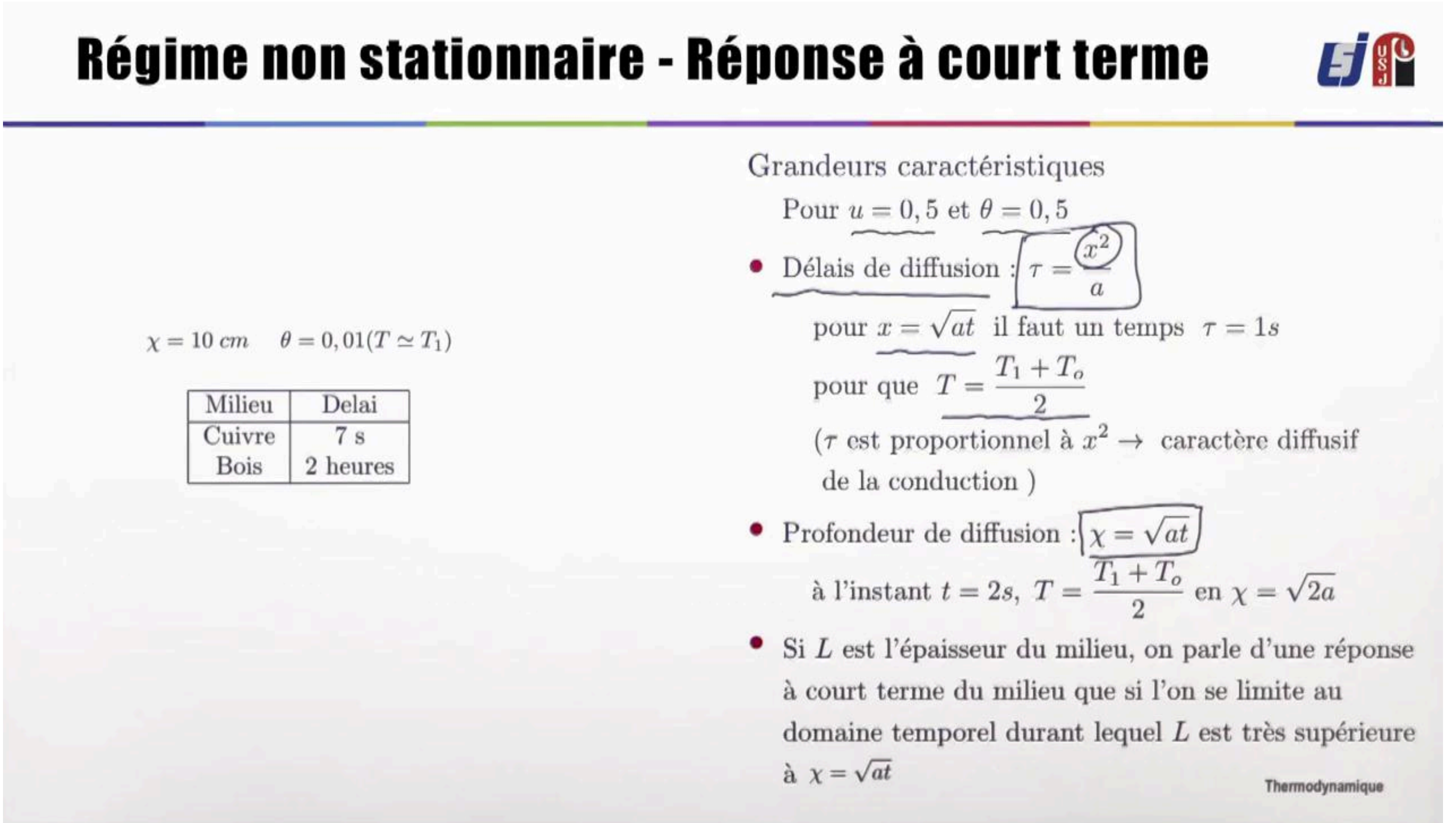
Thermodynamique

The solution of the equation just obtained is written as u. Equal to the integral between zero and u of the exponential of less squares dv. This solution is obtained by integrating the previous equation twice with respect to U. It remains to calculate the constant C and for that we use the initial conditions, that is to say. When the time T tends to zero, the variable U will tend to infinity and the reduced temperature theta will tend to one. So the theta function of infinity will can be written as c the integral between zero and infinity of the exponential of v square dv and will tend to one. And so we can obtain the value of the constant C which is written in the form of these roots. Since then, finally, the expression of the reduced temperature will be written in the form of two roots of Pi by the integral between zero and u exponential of less vector and dv. This function is known as the error function. In the table below, we have the tabulation of the error function and in this table, there is a particular value that interests us. This is the value for which U is equal to to zero five and for this value of U is equal to zero five. We have the error function which will be equal to zero five. That is to say, the reduced temperature will be equal to one half.

Notes

Summary

1m 53s



From these particular values for U is equal to zero five and is equal to zero five. Characteristic quantities can be defined. These quantities are. The first one is the delay of the diffusion which is defined by rate equal to x squared on one. To understand the physical meaning of this quantity, it is enough to take. The value of x is equal to the root of A. And to calculate the time, i.e. what will be equal to one second for the temperature to be in. This value of X becomes equal to T1 plus zero over two. We can notice that in the formula of time, the time is proportional to the spatial variable squared, which gives the decisive character of the contradiction. The second quantity that can be defined, is the decision depth which is defined by. If is equal to the root of a by the time also, we will take an example with an order of magnitude at time t is equal to two seconds. The temperature is equal to T1 zero over two years which is equal to root twice. So now to justify the name of the short-term response. If we take it which is the thickness of the middle, we talk about a short-term response of the environment.

Notes

Summary

3m 25s

Régime non stationnaire - Réponse à court terme

$\chi = 10\ cm \quad \theta = 0,01 (T \simeq T_1)$

Milieu	Delai
Cuivre	7 s
Bois	2 heures

Grandeurs caractéristiques

Pour $u = 0,5$ et $\theta = 0,5$

- Délais de diffusion : $\boxed{\tau = \frac{x^2}{a}}$

  pour $x = \sqrt{at}$ il faut un temps $\tau = 1s$

  pour que $T = \frac{T_1 + T_o}{2}$

  (τ est proportionnel à $x^2 \rightarrow$ caractère diffusif de la conduction)

- Profondeur de diffusion : $\boxed{\chi = \sqrt{at}}$

  à l'instant $t = 2s,\ T = \frac{T_1 + T_o}{2}$ en $\chi = \sqrt{2a}$

- Si L est l'épaisseur du milieu, on parle d'une réponse à court terme du milieu que si l'on se limite au domaine temporel durant lequel L est très supérieure à $\chi = \sqrt{at}$

Thermodynamique

If we limit ourselves to the temporal domain during which the thickness is much greater than the depth of fusion which is equal to the root of a carried. In the tables below, we have taken orders of magnitude. Proxi is equal to ten cm and we calculated the time that it takes for this place, which is ten cm, the reduced temperature becomes equal to 0.01, i.e. the temperature becomes almost equal to T1 and this for two media. The first medium is copper. And this delay will be seven seconds. And the second medium, which has a much less conductive medium thermal than copper, i.e. wood. And in these cases, the delay is 2 hours. This picture there. And this reasoning can explain the fact that sometimes, when we put our hand on a copper bar that is At 70 degrees, for example, there is an immediate burning sensation. Whereas if we put our hand on wood that is worn at the same temperature, we have rather a pleasant sensation of heat. But of course, this is only valid for if we limit ourselves to a time frame that is not very large. What does that mean? It is always assumed that the thickness of the medium, is much higher than the depth of diffusion.

Notes

Summary

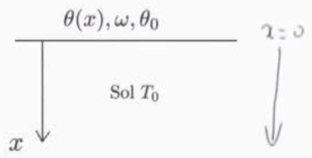
4m 48s

Régime non stationnaire

Milieu homogène, isotrope, ETL

μ : masse volumique

c : chaleur massique



$\underline{T}(x,t) = T_0 + \underline{\theta}(x)e^{j\omega t}$

- Régime périodique dans un demi-espace

Equation de la chaleur :

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{a}\frac{\partial T}{\partial t}$$

$$\frac{d^2\underline{\theta}(x)}{dx^2} = \frac{1}{a}j\omega\underline{\theta}(x)$$

Equation caractéristique: $r^2 = \frac{j\omega}{a}$

$$r_1 = (1+j)\sqrt{\frac{\omega}{2a}}; \quad r_2 = -(1+j)\sqrt{\frac{\omega}{2a}}$$

On pose $\delta = \sqrt{\frac{2a}{\omega}}$

Thermodynamique

The second example is dealt with under the parking regime. It will be the periodic response of a medium to a periodic excitation. We know that the air on the surface of the Earth, the temperature can vary along a day, but it can also vary along the year. We will assume that these variations are sinusoidal and omega pulses. We will model the soil by a semi-infinite medium. Who goes x is equal to zero down. In X equal to zero, we are on the surface of the Earth and then we goes towards the positive X's, then we sink into the ground. The amplitude of the temperature variation at the Earth's surface is assumed to be equal to zero and the depth x this state of x, and it is assumed that the temporal average soil temperature was zero. To solve such a problem. We use complex notations and write the temperature function in complex notation as the sum of T0. More than x exponentials of j omega T. Starting from the heat equation, we can arrive at the equation that governs the variation of the amplitude of the variation in depth which is written as the derivative of ratio with this square is equal to one over zero total omega of x. The Cartesian equation of the equation just established is r square equals with j omega on one which has two solutions R1 and R2 and we pose Delta is equal to the square root of two times on omega.

Notes

Summary

6m 17s

- La solution de l'équation de la chaleur s'écrit alors

$$\underline{\theta}(x) = \underline{\theta}_1 e^{\frac{-(1+j)}{\delta}x} + \underline{\theta}_2 e^{\frac{(1+j)}{\delta}x}$$

$$\underline{\theta}_2 = 0 \ \text{ sinon } \underline{\theta}(\infty) \rightarrow \infty$$

D'où $\underline{\theta}(x) = \theta_0 e^{\frac{-(1+j)}{\delta}x}$

et $\underline{T}(x,t) = T_0 + \theta_0 e^{\frac{-x}{\delta}} e^{j(\omega t - \frac{x}{\delta})}$

$$T(x,t) = T_0 + \theta_0 e^{\frac{-x}{\delta}} cos\ (\omega t - \tfrac{x}{\delta})$$

- Phénomène de diffusion $\rightarrow$ phénomène de propagation atténué.
Avec $\delta = \sqrt{\frac{2a}{\omega}}$ la profondeur de pénétration.

- Solution: $a = 7,5.10^{-7}\ m^2 s^{-1}$
Variations journalières: $\omega = 7,3.10^{-5}\ rads^{-1}$ $\delta = 14\ cm$
Variations annuelles: $\omega = 2,7.10^{-7}\ rads^{-1}$ $\delta = 2,7\ m$

Thermodynamique

The solution of the equation just established will be written in the form of the sum of two terms in exponential plus j on delta pa x plus an exponential term of one plus j on delta pa x. Of course, there are state constants. One was with two that we go to seek. To find the values. If we take the particular case where it tends to infinity. In these cases, theta of X will tend to infinity, which is physically impossible. So necessarily the theta constant. Two. Will be equal to zero. It remains for us to calculate. The constant was one. For this, we take the boundary conditions, i.e. X equal to zero. The amplitude of the variation was at zero points. Which was finally made in the form of the sum from t0 smaller to zero exponential of delta mold multiplied by exponential from J to megabyte X on Delta. Of course, at actual temperature, this will be the real part of this complex function. So finally, the spatial and temporal distribution of the temperature in the soil as a function of of the sinusoidal excitation imposed at X equal to zero will be written as t0 plus zero exponential of minus x on delta by cosine omega control and on delta.

Notes

Summary

7m 54s

Régime non stationnaire

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et $\underline{T}(x,t) = T_0 + \theta_0 e^{\frac{-x}{\delta}} e^{j(\omega t - \frac{x}{\delta})}$

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Thermodynamique

It is immediately obvious that in this particular case of diffusion, we tend rather towards a propagation attenuated by the exponential term of at least x. On delta, there is a particular value for which the amplitude of the variation will be practically zero. This is for X is equal to Delta. Now, we can take a particular example to give an order of magnitude, we take an is equal to 7.5 and calculate the penetration depth. In the first case or the variation is counted on one day and in these cases Delta is equal to about fourteen cm and we take the second case, i.e. a variation of the temperature along the year and in these cases, we find a delta equal to 2.7 meters. From there, we can understand why, to keep 120, we make cases that are always under at a depth of three meters from the surface of the Earth. So, we have just seen two examples which deal with thermal conduction in a non-steady state. The first is the short-term response of a medium which undergoes a thermal excitation. And the second is the periodic response of a medium which is also excited, but this time by a periodic signal. I invite you all to go and see the video in which the application is treated What we just did and in an engineering application which is the cooling of components in electronics.

Notes

Summary

9m 16s