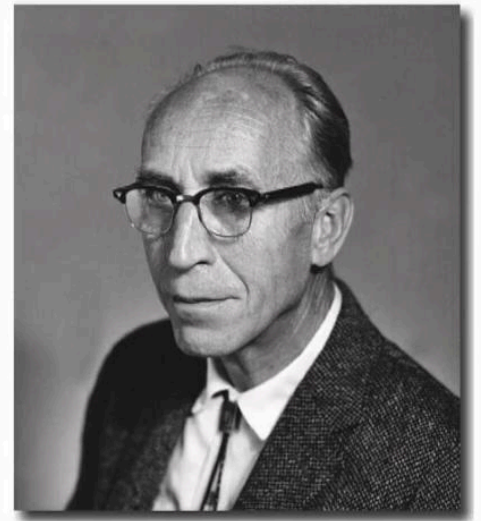


Thermodynamique

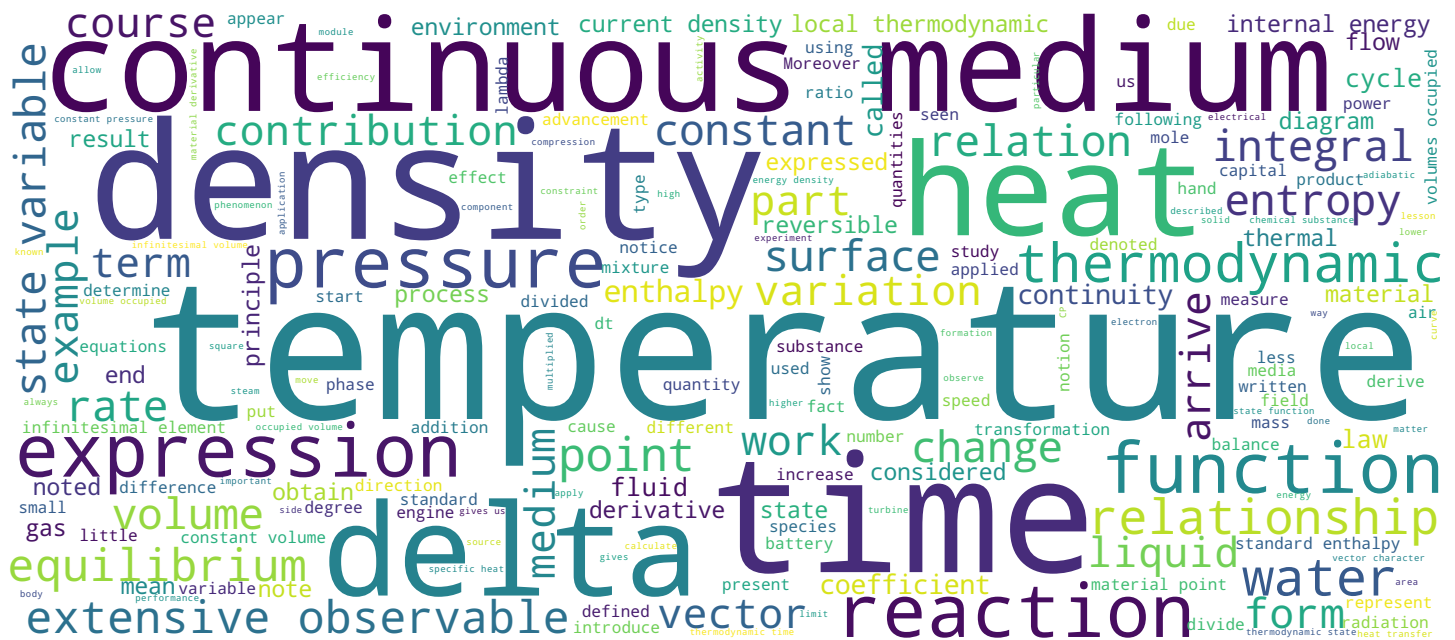
Introduction à la thermodynamique des milieux continus



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Video





- Hypothèses fondamentales
- Etat thermodynamique local
- Fonctions d'état extensives réduites - densités
- Equation de continuité pour une observable extensive

Thermodynamique

Hello and welcome to the thermodynamics classes. My name is Michael and I am the Pope of Alexandria and I am a professor at the Catholic University of Leuven. My fields of activity are fluid mechanics and more precisely Dysphasic flows as well as reactionary flows. In this part of the course, we will present the introduction to the thermodynamics of continuous medium, the structure of our presentation and the following. First, we will present the assumptions fundamental thermodynamics of the continuous medium. Then, we will study the notion of local thermodynamic time as well as that the notion of extensive state function is reduced. In other words, densities. Finally, we deliver the form general continuity equation for an extensive observable.

Notes

Summary



0m 05s



Thermodynamique

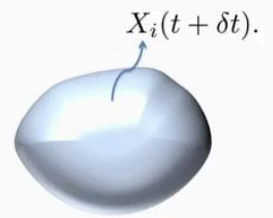
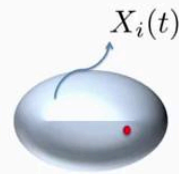
Give the advance. We are interested in the process of thermodynamic system, more precisely to the process. But balance is another expression so that this type of process is irreversible. During a process at equilibrium, the behavior of thermodynamic systems is no longer homogeneous. This means that the owner of the system are likely to extend in space and time. For this reason, it is convenient to treat a thermodynamic system as a continuous medium. We see in this figure the representation of a continuous medium. This is a classical macroscopic medium which consists of a very large number of elementary microscopic particles. In peacetime, it occupied the perfect rated volume, was the surface of the enclosure and noted by part, raised heads. During a process at equilibrium, the continuous medium can be moved and can be deformed. After a moment, Delph occupied another volume noted by Bethe. In addition, Delta, as soon as there is another surface of the enclosure noted by because it will be, was no longer delta. For a continuous environment, we define the state variable as we have already done for the variable state of a thermodynamic system in a state of equilibrium.

Notes

Summary



0m 52s



Thermodynamique

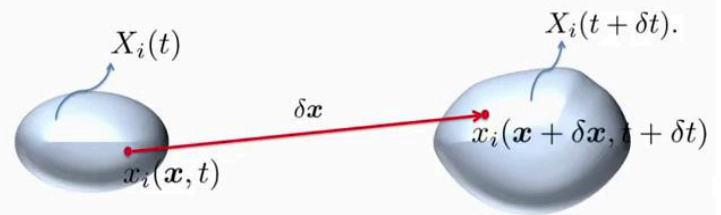
So for this system, we define for example the variable state x capital Y which is a function of time. We then write x capital i of t ait. After a moment δ of alphabet. The value of this variable. It did. It is denoted by a capital X . Was more δ pay. But not given that during a process in equilibrium, the behavior of a most homogeneous continuous medium. The study of this type of process can only be not be based essentially on state variables. Then the characteristic parameter of a continuous medium are necessarily 100, i.e. function of the position X and time. For this reason, it is convenient to introduce the notion of local thermodynamic system. We consider a point in the space occupied by the continuous medium attempted this point, as well as its immediate vicinity constitutes a local thermodynamic system. It is a local system and large enough to hold a large number of of elementary particles in order to be insensitive to statistical fluctuations. But it is small enough to be considered as infinitesimal. Then a local thermodynamic system can be described as a material point. According to this point of view, a continuous medium can be considered as the total weight in material it occupies.

Notes

Summary



2m 13s



$$X_i(t) = \int_{V(t)} x_i(\mathbf{x}, t) dV$$

$$\mathbf{v}(\mathbf{x}, t) \equiv \lim_{\delta t \rightarrow 0} \frac{\delta \mathbf{x}}{\delta t} = \frac{d\mathbf{x}}{dt}$$

Thermodynamique

For a local thermodynamic system, we define state variables by analogy of state variables of continuous medium. But now this new state variable, i.e. the state variables of a local system, are necessarily 100. For example, we define the analog of the state variable x capital Y and it is a then. And the field x tiny y dx dt . After a delta E ta pays seven material points. Seven Local thermodynamic system serves to be moved and was not the delta X . Moreover, after a moment, delta t ait the value of the lowercase X field will have no effect there. It is denoted by lowercase x y of x plus delta x and t + delta T . Since the continuous medium can be considered as the set of material weights it occupies then the value of the size variable X capital y is given as the integral over the volumes occupied by the so-called middle without lower case x y . We then say that the 100 x minuscule y is the density of the variable capital x . Moreover, for a thermodynamic system local, we can define kinematic state variables. For example, the speed of a material weight is defined as the delta limit and not zero. The delta x port material used by Delta Tet.

Notes

Summary



3m 48s

- Le premier principe de thermodynamique requiert l'existence d'une fonction d'état *quantité de mouvement* $P(t)$.
- Le second principe de thermodynamique requiert l'existence d'une fonction d'état *entropie* $S(t)$.
- L'état thermodynamique d'un système est complètement déterminé par :
 - i. la connaissance de $P(t)$, $S(t)$ et...
 - ii. par un ensemble de N variables d'état *extensives*: $X_1(t), \dots, X_N(t)$.
- Ces variables sont globales, i.e. indépendantes des coordonnées spatiales. Néanmoins, elles sont dépendantes du temps.

Thermodynamique

Now we will give the definition thermodynamic states at the global level. The first principle of thermodynamics requires the existence of a state function which we call momentum and which is noted by such besides. The second principle of thermodynamics requires the existence of state functions, which we will call entropy and which is noted by SDP. The thermodynamic time of a system is completely determined by the knowledge of the entropy quantity of motion and by a knowledge of the set of n extensive state variables. Upper case X to upper case X one. It is important to mention that these variables are global, i.e. independent of spatial coordinates. Nevertheless, they are time-dependent. The definition of the thermodynamic time of a system is realized in a completely analogous way.

Notes

Summary



5m 30s

- L'état thermodynamique *local* d'un système est caractérisé par $N + 2$ champs :
 - les densités d'entropie et de quantité de mouvement, $s(\mathbf{x}, t)$, $p(\mathbf{x}, t)$ et...
 - les densités des n autres variables d'état : $x_1(\mathbf{x}, t), \dots, x_N(\mathbf{x}, t)$.

$$S(t) = \int_{V(t)} s(\mathbf{x}, t) dV, \quad P(t) = \int_{V(t)} p(\mathbf{x}, t) dV, \quad (1)$$

$$X_i(t) = \int_{V(t)} x_i(\mathbf{x}, t) dV, \quad i = 1, 2 \dots N. \quad (2)$$

- Les densités sont des variables *extensives réduites*: elles sont dépendantes des coordonnées spatiales et du temps.

Thermodynamique

the local thermodynamic state of a system is characterized by a plus of 100. Its 100 are the entropy densities and momentum and the densities of the other n state variables. This is the relationship between entropy and its density. The entropy of a continuous medium is given as the integral over the volumes occupied by the media of the entropy density. Moreover, the momentum of continuous medium is given as the integral over the volumes occupied by the medium of the momentum density, and we have similar relations between the other size variables of the continuous medium. It is the density. It is important to mention that the densities are of reduced extensive variables, i.e. they are dependent on spatial and time coordinates. This density is also called 100 of thermodynamic size.

Notes

Summary



6m 28s

- Observable physique extensive : $F(t) \equiv F(S(t), P(t), X_1(t), X_2(t), \dots, X_N(t))$.
- Observable extensive réduite : $f(\mathbf{x}, t) \equiv f(s(\mathbf{x}, t), p(\mathbf{x}, t), x_1(\mathbf{x}, t), \dots, x_N(\mathbf{x}, t))$.
- Relation entre l'observable extensive et l'observable extensive réduite :

$$F(t) = \int_{V(t)} f(\mathbf{x}, t) dV. \quad (3)$$

- Objectif : dériver l'équation de continuité pour $F(t)$.
 - Considérons un milieu continu qui occupe un volume $V(t)$.
 - $\partial V(t)$: surface de l'enceinte du système.
 - $\mathbf{v}(\mathbf{x}, t)$: champs de vitesse de la matière en $V(t)$.
 - dS : élément infinitésimal de la surface de l'enceinte.

Thermodynamique

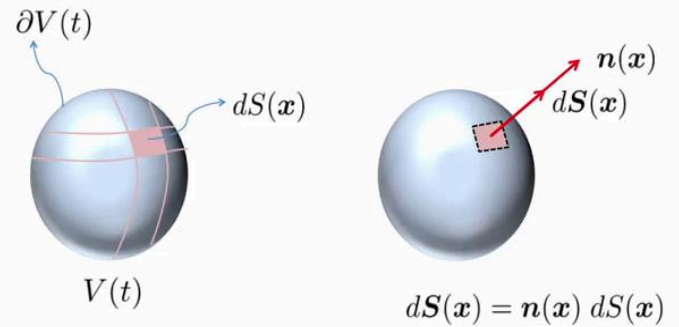
We now consider an extensive physical observable F . This extensive observable is a function of of John plus two size variables of the continuous medium at the local level. We have a similar situation. We consider a tiny reduced extensive observable f and this observable is a function of more density of state variables, i.e. the function of more than 100 of thermodynamics. And here is the relationship between the observables extensive observable extensive reduced. The expansive verb is given as the integral under the occupied volume by the middle of the object, extensive verb reduced. This means that the lovable object, extensive, reduces f lowercase and the density becomes barb extensive f uppercase. Our objective is to derive the equation of the continues t for the extensive f barb good of size. For this purpose, we consider an environment occupied by n volume of earthworms. Not that if LB was the surface of the system enclosure Bdx and was the speed center of the matter. The volume occupied by the medium is. DS represents the vector of an infinitesimal element of the enclosure surface.

Notes

Summary



7m 28s



Thermodynamique

First, we elaborate on the infinitesimal element vectors of S . We consider a continuous medium. It is the medium to a volume noted by V . The surface of the enclosure is noted by $\partial V(t)$. On this surface, we consider an infinitesimal element. The area of this infinitesimal element is ds . Moreover, we consider a vector unit normal to this infinitesimal element. This vector is denoted by \mathbf{n} . The vector $d\mathbf{S}$ is a vector that is parallel to the unit vector. Then it is a normal vector to the element infinitesimal and its amplitude is equal to the double infinitesimal area. So this relationship ends as the definition of the vector of an infinitesimal element.

Notes

Summary



8m 41s

- La variation d'une observable $F(t)$ est donnée par

$$\delta F(t) = \int_{V(t)} \underset{\downarrow}{\delta f(\mathbf{x}, t)} dV + \int_{\partial V(t)} f(\mathbf{x}, t) \underset{\downarrow}{\delta(dV)}.$$

variation temporelle de la densité au sein du volume

variation temporelle du volume infinitésimal local

- Mais $\delta(dV) = d\mathbf{S} \cdot \delta\mathbf{x}$, $\delta\mathbf{x}$: déplacement du volume infinitésimal.

$$\dot{F}(t) \equiv \frac{dF(t)}{dt} = \lim_{\delta t \rightarrow 0} \frac{\delta F(t)}{\delta t},$$

Thermodynamique

We consider the relationship between an extensive observable and its density. Purpose of this relationship. The variation of innumerable capital extenders is given by this relation. According to this relation, the variation Delta F Dean observable f capital letter is the sum of two contributions. The first contribution is due the temporal variation of the density to that of the volume. It is expressed by the integral on the volumes occupied by the medium of the delta f variation of the density. The second contribution is due the temporal variation of the local infinitesimal volume. It is expressed as one of the surfaces of the density set f multiplied by the time variation delta two, but. By the way, regarding the temporal variation delta was supposed to. It is given as the product between two vectors. The first vector is of s and the second vector is delta x. Delta x represents the displacement of the local infinitesimal volume. Now, we can obtain an expression for the observable rates of change. Capital F if we divide each term of this equation by Delta T and if we take the limit Delta head a zero, we notice that the rate of change of an extensive observable is given as the delta zero limit of the variation in delta F divided by Delta T.

Notes

Summary



9m 36s

- La variation d'une observable $F(t)$ est donnée par

$$\delta F(t) = \int_{V(t)} \underset{\downarrow}{\delta f(\mathbf{x}, t)} dV + \int_{\partial V(t)} f(\mathbf{x}, t) \underset{\downarrow}{\delta(dV)}.$$

variation temporelle de la densité au sein du volume

variation temporelle du volume infinitésimal local

- Mais $\delta(dV) = d\mathbf{S} \cdot \delta\mathbf{x}$, $\delta\mathbf{x}$: déplacement du volume infinitésimal.

$$\dot{F}(t) \equiv \frac{dF(t)}{dt} = \lim_{\delta t \rightarrow 0} \frac{\delta F(t)}{\delta t}, \quad \frac{\partial f(\mathbf{x}, t)}{\partial t} \equiv \lim_{\delta t \rightarrow 0} \frac{\delta f(\mathbf{x}, t)}{\delta t}, \quad \mathbf{v}(\mathbf{x}, t) \equiv \lim_{\delta t \rightarrow 0} \frac{\delta \mathbf{x}}{\delta t}.$$

- A l'aide des dernières définitions, nous arrivons à

$$\dot{F}(t) \equiv \frac{d}{dt} F(t) = \int_{V(t)} \frac{\partial f(\mathbf{x}, t)}{\partial t} dV + \int_{\partial V(t)} f(\mathbf{x}, t) \mathbf{v}(\mathbf{x}, t) \cdot d\mathbf{S}. \quad (4)$$

Thermodynamique

The rate of change of an extensive observable is denoted by \dot{F} of others. Moreover, the time derivative of the density with respect to time is given as the delta limit zero of the variation of the density of ΔF divided by ΔT . Finally, the velocity of a local infinitesimal volume is given as the limit of the head and not zero of the displacement of the infinitesimal volume divided by ΔT . Now we divide each term of this equation by ΔT and we take the shelf limit 1 to 0. Using the last definitions, we arrive at to this expression for the rates of change of an extensive observable. This result is known as the renown transport theorem. According to these results, the rate of change of an observable extensive f d'oc is the sum of two contributions. The first contribution is expressed by the integral on the occupied volumes by the thousands of the partial derivative of the density with respect to time. The second contribution is expressed by the integral below the surface of the whole product, between the tiny density f and the velocity center. These terms constitute a convective or observable current.

Notes

Summary



11m 05s

Causes physiques du taux de variation d'une observable $F(t)$

1. Interaction du milieu continu avec son extérieur; elle est décrite par le champs *densité du courant*, $\mathbf{j}_f(\mathbf{x}, t)$.
2. Production ou destruction de $F(t)$ au sein du milieu; elle est décrite par le champs *densité de source*, $\pi_f(\mathbf{x}, t)$.

$$\dot{F}(t) = - \int_{\partial V(t)} \mathbf{j}_f(\mathbf{x}, t) \cdot d\mathbf{S} + \int_{V(t)} \pi_f(\mathbf{x}, t) dV. \quad (5)$$

Thermodynamique

Now we consider the causes of the rate of change of an extensive observable F . There are physical causes. The first cause is the interaction of the continuous medium with its exterior. It is described by the direction of the current density noted by say. The second cause is the production or extensive observable destruction capital F within the environment. It is described by the no shock density. Even worse, the rate of change of objects extensive variables f is the sum of two contributions. The first contribution is the object of a degradation of the swing surface, that of the current density, and the second contribution is the integral over the volume occupied by the medium of the source density. We notice that the negative signs before this era introduced to respect the sign convention we have adopted. Now we have expressions equivalent for the rates of variation of innumerable extensives. This expression includes integrals on the surface of the enclosure. These integrals can be transformed into integrals under the occupied volume through the media using the divergence theorem.

Notes

Summary



12m 28s

- Théorème de la divergence pour une fonction vectorielle $\mathbf{g}(\mathbf{x}, t)$:

$$\int_{\partial V(t)} \mathbf{g}(\mathbf{x}, t) \cdot d\mathbf{S} = \int_{V(t)} \nabla \cdot (\mathbf{g}(\mathbf{x}, t)) dV,$$

- En utilisant ce théorème, les deux relations pour $\dot{F}(t)$ sont mises sous la forme :

$$\dot{F}(t) = \int_{V(t)} \frac{\partial f(\mathbf{x}, t)}{\partial t} dV + \int_{V(t)} \nabla \cdot (f(\mathbf{x}, t) \mathbf{v}(\mathbf{x}, t)) dV, \quad (6)$$

et

$$\dot{F}(t) = - \int_{V(t)} \nabla \cdot (\mathbf{j}_f(\mathbf{x}, t)) dV + \int_{V(t)} \pi_f(\mathbf{x}, t) dV. \quad (7)$$

Thermodynamique

And here is the divergence theorem for a vector function. According to this theorem, the integral on the launching surface of the vector function \mathbf{f} is equal to the integral over the occupied volumes by the middle of the divergence of the function \mathbf{G} . Using the seven theorems of, dividing then by using this theorem, the two relations for the rates of variation \dot{f} are put in this form as well as in this form. Now we can consider the equality of scale of the expressions.

Notes

Summary



13m 43s

Par identification des deux dernières relations, nous arrivons à

$$\int_{V(t)} \left(\frac{\partial f(\mathbf{x}, t)}{\partial t} + \nabla \cdot (f(\mathbf{x}, t) \mathbf{v}(\mathbf{x}, t)) \right) dV = \int_{V(t)} (-\nabla \cdot (\mathbf{j}_f(\mathbf{x}, t)) + \pi_f(\mathbf{x}, t)) dV.$$

Etant donné que $V(t)$ est arbitraire, la dernière équation est valable seulement si les fonctions qui sont intégrées sont égales, i.e.

$$\frac{\partial f(\mathbf{x}, t)}{\partial t} + \nabla \cdot (f(\mathbf{x}, t) \mathbf{v}(\mathbf{x}, t)) = -\nabla \cdot (\mathbf{j}_f(\mathbf{x}, t)) + \pi_f(\mathbf{x}, t). \quad (8)$$

Thermodynamique

By the notifications of the last two relationships. We arrive at this equation, but so far, we did not introduce any constraints on the volumes occupied by the media. This means that until now, the volume occupied by the media is quite arbitrary. But since the volume occupied by the media was arbitrary last year, the equation is only valid if the functions that are integrated are equal. We then arrive at this equation. We observe that the theme of the left side of this equation are the derivative partial density f tiny with respect to time, a theme of divergence. We can manipulate this time a bit.

Notes

Summary



14m 18s

- Soit un point $\mathbf{x} = (x, y, z)$ occupé par le système et $f(\mathbf{x}, t)$ une densité.

$$df = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$$

$$\xRightarrow{\frac{d}{dt}} \quad \frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial z} \frac{dz}{dt}$$

$$\Rightarrow \quad \frac{df}{dt} = \frac{\partial f}{\partial t} + \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right) \cdot \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$\Rightarrow \quad \dot{f}(\mathbf{x}, t) \equiv \frac{df(\mathbf{x}, t)}{dt} = \frac{\partial f(\mathbf{x}, t)}{\partial t} + \mathbf{v}(\mathbf{x}, t) \cdot \nabla f(\mathbf{x}, t).$$

↓
dérivée matérielle (ou Lagrangienne) d'une densité

Thermodynamique

We consider a material point X of spatial coordinates x, y and z occupied by the systems and f minuscule a density. Since the density is a function of of three spatial coordinates and time, the total differential of the density is given by this expression. Now, if we divide this expression by DT , we arrive at this equation and we notice that these three terms on the right side of this equation can be put in the form of an inner product between two vectors. Then we can rewrite this equation in this form. We note that that this bacterium is identified as the speed of the material point. In addition, this land is identified as the grunts of density f . We arrive then at this expression which rises as the definition of the material derivative or derivative, the ingredient of a density. The material derivative called density f is represented by \dot{f} of other and it represents the rate of change of the density measured by an observer in the reference frame of the material point.

Notes

Summary



15m 00s

- En introduisant la définition de la dérivée matérielle d'une densité $f(\mathbf{x}, t)$ et l'identité mathématique :

$$\nabla \cdot (f(\mathbf{x}, t) \mathbf{v}(\mathbf{x}, t)) = f(\mathbf{x}, t) \nabla \cdot (\mathbf{v}(\mathbf{x}, t)) + \mathbf{v}(\mathbf{x}, t) \cdot \nabla f(\mathbf{x}, t),$$

à la dernière équation, nous arrivons à l'équation de continuité pour $F(t)$:

$$\dot{f}(\mathbf{x}, t) + f(\mathbf{x}, t) \nabla \cdot \mathbf{v}(\mathbf{x}, t) = -\nabla \cdot (\mathbf{j}_f(\mathbf{x}, t)) + \pi_f(\mathbf{x}, t). \quad (9)$$

- Pour une observable extensive $F(t)$ du caractère vectorielle :

$$\dot{\mathbf{f}}(\mathbf{x}, t) + \mathbf{f}(\mathbf{x}, t) \nabla \cdot \mathbf{v}(\mathbf{x}, t) = -\nabla \cdot (\mathbf{j}_f(\mathbf{x}, t)) + \boldsymbol{\pi}_f(\mathbf{x}, t).$$

Thermodynamique

By introducing the definition of the material derivative of a density f is the following mathematical identity. Concerning the product divergence between a density and a vector to the last equation, we finally arrive at the equation of continuity for the extensive variable object f capital. This is the continuity equation for the capital extensive object. Until now, we have considered two extensive observables of scalar character. We can repeat these processes to derive the continuity equation for a vector extensive observable. For an extensive observable of vector character. The result is quite similar and we will arrive at this relationship which is the equation of continuity for the object of the vector character. We note that this quantity represents the density and this quantity is of tensor character. In this intense layer, it is considered as a matrix in dimension three of three, since this matrix is a representation of the subgroup of rotations of the Euclidean group. In addition, this amount $\mathbf{P}\mathbf{F}$ represents the density of sources for the extensive observer. It is of vector character. Now, using this equation, we can derive the equations of continuity for the state variable of a continuous medium. These equations imply a thermodynamic time center.

Notes

Summary



Champs d'état thermodynamique

1. Densité de quantité de mouvement, $p(\mathbf{x}, t)$.
2. Densité d'entropie, $s(\mathbf{x}, t)$.
3. Densités de r substances chimiques, $n_1(\mathbf{x}, t), n_2(\mathbf{x}, t), \dots, n_r(\mathbf{x}, t)$.
4. Densité de charge électrique, $q(\mathbf{x}, t)$.

Autres champs physiques d'intérêt particulier

1. Vitesse, $\mathbf{v}(p, s, \{n_A\}, q)$.
2. Densité de masse, $m(p, s, \{n_A\}, q)$.
3. Densité d'énergie totale, $e(p, s, \{n_A\}, q)$.
4. Densité d'énergie interne, $u(p, s, \{n_A\}, q)$.

Thermodynamique

This is the list of thermodynamic size centers of a continuous medium. We have the density of the quantities of motion \mathbf{p} . The entropy density s . The density of chemical substance n in n_2 . n_r and the electric charge density q . In addition, there are other physical fields of special interest. Here is a list of other physical fields of special interest. We have the speed \mathbf{V} , the mass density M , the total energy density and the internal energy density. They are a function of its thermodynamic state. We have also reached the end of this part of the course of thermodynamics in the next part of the course. We will use the general form of the continuity equation for an extensive observable, to derive the continuity equations of state variables of a continuous medium, as well as the continuity equations for other particular extensive Thales observables.

Notes

Summary



17m 57s