Thermodynamique

Equations de continuité thermodynamique

UCL Université catholique de Louvain

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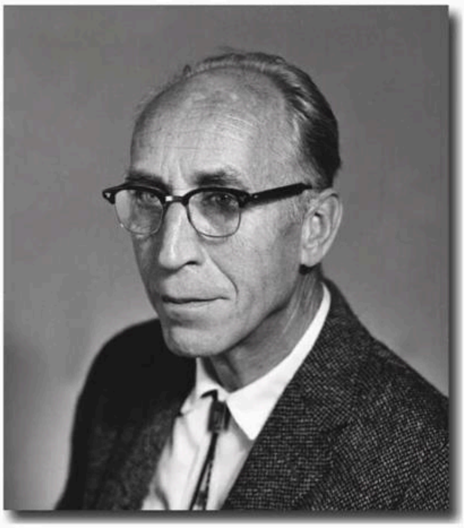

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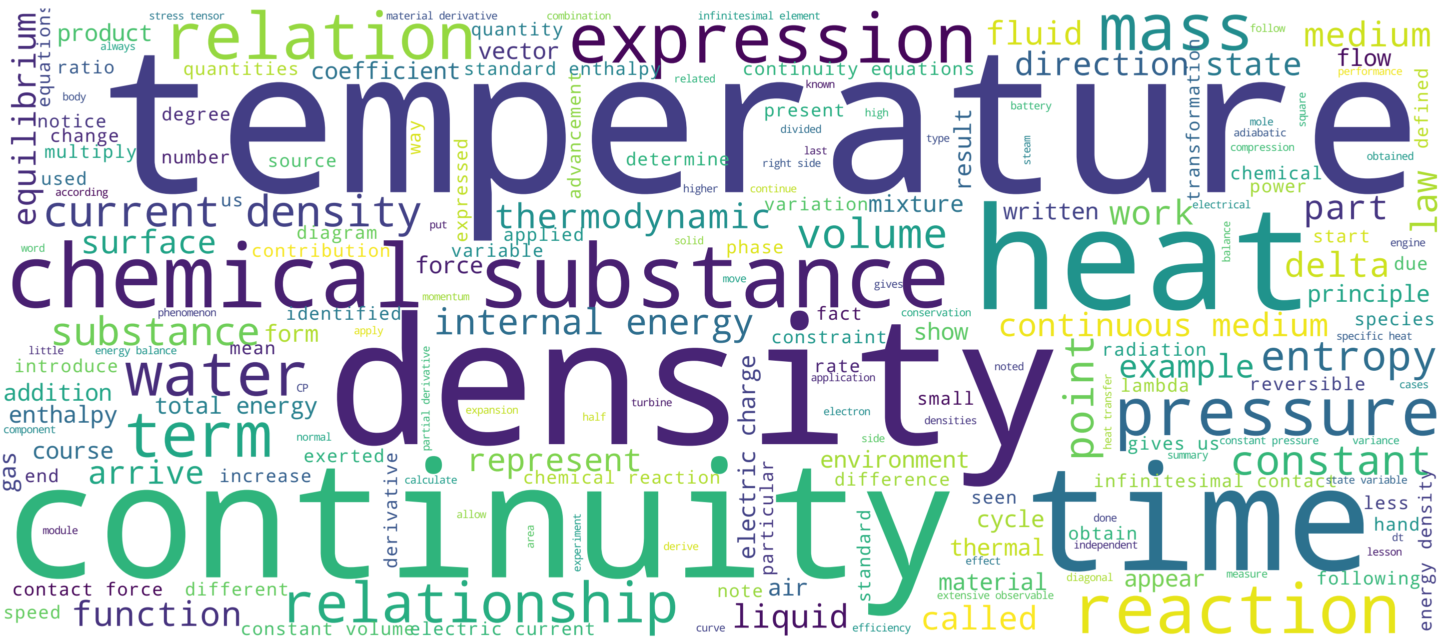
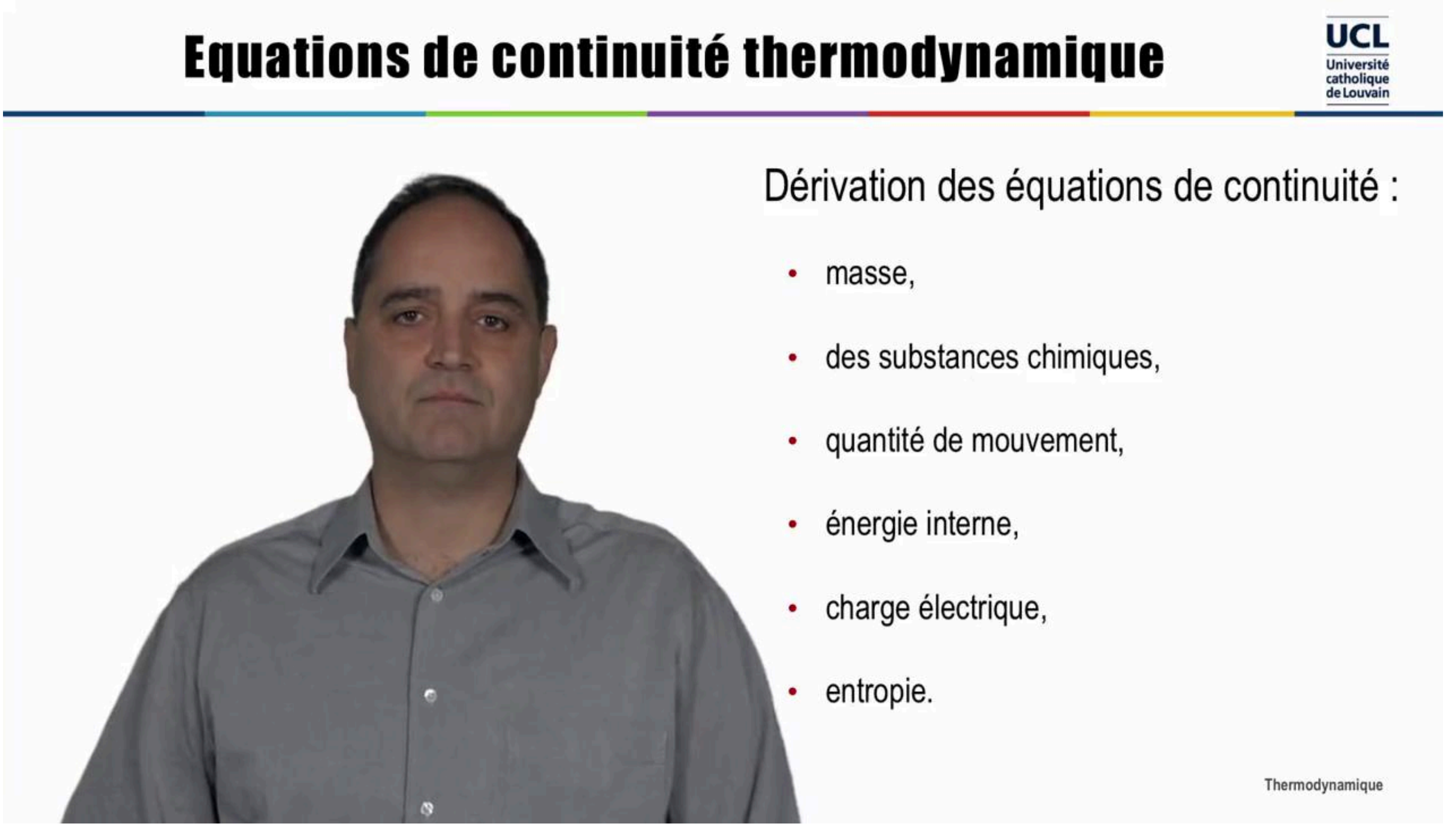


Video



Hello and welcome to the thermodynamics classes. In the previous presentation, we obtained the general form of the continuity equation for an extensive observable of a continuous medium in this part of the course. We will use this general form to derive the continuity equations for the state variables of a medium continuous, as well as the continuity equations for other extensive observables of particular interest. More precisely, we will present the derivation continuity equations for mass for chemical substances. For the quantity of movement. For internal energy. For the electric charge and finally for the entropy.

Notes

Summary

0m 05s

Equations de continuité thermodynamique

- Rappel :

$$\dot{f}(\boldsymbol{x},t) + f(\boldsymbol{x},t)\,\nabla\cdot\boldsymbol{v}(\boldsymbol{x},t) = -\nabla\cdot\left(\boldsymbol{j}_f(\boldsymbol{x},t)\right) + \pi_f(\boldsymbol{x},t)\,. \qquad (9)$$

- Nous considérons un mélange de r substances chimiques, en présence de n réactions chimiques.
- La vitesse angulaire de son centre de masse est supposée nulle : $\boldsymbol{\omega}(t) = 0$.
- Le principe de *conservation de la masse* requiert qu'il n'y ait pas de densité de source ni de courant de masse.
- Alors, l'équation de continuité, appliquée à la densité de la masse, résulte en

Thermodynamique

We recall the general form of the continuity equation for a capital F extensive observable. In this equation, lowercase f represents the density of objects ab extensives f in the present of the material derivative of the density, while if f represents the density of current for f and f represents the density of things for. We consider a mixture of substances in the presence of chemical reactions. This is the continuous environment that we propose to the ethics of this environment. The angular velocity of its center of mass is assumed to be zero. We then write that the leading omega is equal to zero. We start with the continuity equation for the mass. The principle of conservation of mass requires that there is no source density or mass current. Then the continuity equation applied to the density of the mass results directly in this equation.

Notes

Summary

0m 48s

Equations de continuité thermodynamique

- Rappel :

$$\dot{f}(\boldsymbol{x},t) + f(\boldsymbol{x},t)\,\nabla\cdot\boldsymbol{v}(\boldsymbol{x},t) = -\nabla\cdot\left(\boldsymbol{j}_f(\boldsymbol{x},t)\right) + \pi_f(\boldsymbol{x},t)\,. \qquad (9)$$

- Nous considérons un mélange de r substances chimiques, en présence de n réactions chimiques.
- La vitesse angulaire de son centre de masse est supposée nulle : $\boldsymbol{\omega}(t) = 0$.
- Le principe de *conservation de la masse* requiert qu'il n'y ait pas de densité de source ni de courant de masse.
- Alors, l'équation de continuité, appliquée à la densité de la masse, résulte en

$$\dot{m} + m\,\nabla\cdot\boldsymbol{v} = 0. \qquad (10)$$

Thermodynamique

In this equation, M represents the density of the mixture mass.

Notes

Summary

1m 48s

Equations de continuité thermodynamique

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- Pour les substances chimiques, l'équation de continuité donne :

$$\dot{n}_A + n_A \nabla \cdot \boldsymbol{v} = -\nabla \cdot \boldsymbol{j}_A + \pi_A \,, \quad A = 1, \ldots, r \,.$$

- densité de source de la substance A : $\pi_A = \sum_{a=1}^{n} \omega_a \, \nu_{aA}$

Coefficients stoechiométriques

taux de la réaction a : $\frac{d\xi_a}{dt}$

- densité de courant de la substance A : $\boldsymbol{j}_A$

$$\dot{n}_A + n_A \nabla \cdot \boldsymbol{v} = -\nabla \cdot \boldsymbol{j}_A + \sum_{a=1}^{n} \omega_a \, \nu_{aA} . \qquad (11)$$

Thermodynamique

We continue with the equations of continuity for the chemical substances in the mixture. For chemical substances, the equation of continuity of which this relation we have is relation of the seven types of relationship for each chemical substance. In this relationship, n a represents the density of the chemical substance A. In addition, pH represents the density of substance A strains. It is due to chemical action. It is given by this relation. In this relation we mega a and the rate of the chemical reaction, it is given as the differential time of the chemical reaction. If we then write that omega equals to Dexia on lands, moreover it is this coefficient that has are the ethico-metric coefficient related to the chemical substance at. Finally, Guy A. Represents the current density of the chemical substance at si. We use the expression for the sulfur density of the substance. In this equation, we arrive at the continuity equation for the chemical substance a. We then have r equation of this type for the chemicals in the mixture.

Notes

Summary

1m 53s

Equations de continuité thermodynamique

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- La densité de masse de mélange est liée aux densités des r substances par

$$m = \sum_{A=1}^{r} m_A n_A \,. \tag{12}$$

- m_A: densité d'une unité de A (*masse molaire*), $m_A = \frac{\partial m}{\partial n_A} = cte.$

- Des deux dernières équations, nous arrivons à

$$\dot{m} + m \nabla \cdot \boldsymbol{v} = -\boldsymbol{\nabla} \cdot \left(\sum_{A=1}^{r} m_A \boldsymbol{j}_A \right) + \sum_{a=1}^{n} \omega_a \left(\sum_{A=1}^{r} m_A \nu_{aA} \right) = 0 \,. \tag{13}$$

densité de courant de masse

densité de source de masse

Thermodynamique

Moreover, we notice that the density of mass of mixture helium density of air, substances by this relation. Here M A represents the dynamic density of the chemical substance. In other words, it is the molar mass of the chemical substance A. It is defined as the partial derivative of the density and mass of mixture m with respect to the density of the chemical substance n at, the molar mass of a chemical substance is constant. Now we consider the continuity equations of chemical substances. More precisely, we consider the continuity equation for the chemical substance a and we multiply each term of this equation by the molar mass of the chemical substance a. We do this for all the continuity equations and then we take the sum of them. This result can be combined with this equation. In this way, we can derive the continuity equation for the mass. To summarize the last two equations, we arrive at this equation which is nothing else than the equation of continuity for the mass, and then it must be equal to zero. Based on the structure of this equation, i.e. in brackets and identify as the current density of March. Moreover, it is it if it is to define as the density of source of mass.

Notes

Summary

3m 11s

Equations de continuité thermodynamique

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- Selon le principe de *conservation de la masse,* il n'y a pas de densité de source ni de courant de masse :

Loi de Lavoisier

$$\sum_{A=1}^{r} m_A \boldsymbol{j}_A = 0\,, \qquad (14) \qquad \text{et} \qquad \sum_{A=1}^{r} m_A \nu_{aA} = 0\,, \quad a = 1, \ldots, n\,. \qquad (15)$$

- Introduisons la vitesse $\boldsymbol{v}_A$ de la substance A qui satisfait la relation :

$$\boldsymbol{v}_A n_A = \boldsymbol{v} n_A + \boldsymbol{j}_A\,. \qquad (16)$$

- A l'aide des dernières relations, la vitesse du milieu est donnée par :

$$\boldsymbol{v} = \frac{\sum_{A=1}^{r} m_A n_A \boldsymbol{v}_A}{\sum_{A=1}^{r} m_A n_A} \quad \Longrightarrow \quad \boldsymbol{v} = \frac{\sum_{A=1}^{r} m_A n_A \boldsymbol{v}_A}{m}\,. \qquad (17)$$

Thermodynamique

But according to the principles of conservation of the mass, there is no source density or mass current. This means that all land in parentheses on the right side of the previous equation must be zero. We then arrive at this result. This inflation implies that the densities of chemical substances are not all independent. On the other hand, the ex weighted by the molar masses of the substances is equal to zero. Moreover, we arrive at this equation. This equation implies that the geometric coefficients related to to a chemical reaction are not all independent. On the other hand, the extreme weighted by the molar masses of the substances that participate in this reaction is equal to zero. This result is known as Lavoisier's law. Now we use the speed of the chemical substance that satisfies the following relationship. In other words, the current density of the chemical substance is given as the difference between the velocity v, a and towards multiplied by the density of substance a. We can now introduce this inflation in this expression to get an inflation for the mixing speed using Daniel Relations. The speed of the medium is given by this expression and we notice that this air in the dominator right side of this equation is equal to the density of the mass of the mixture. This is who provides us with this relationship for the speed of the medium, but.

Notes

Summary

4m 40s

Equations de continuité thermodynamique

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- La relation constitutive de la mécanique s'écrit $\boldsymbol{p} = m\,\boldsymbol{v}$, i.e.

$$\boldsymbol{p} = \sum_{A=1}^{r} m_A\, n_A\, \boldsymbol{v}_A\,. \tag{18}$$

- La densité de source de la quantité de mouvement est due aux forces de champs extérieurs : $\boldsymbol{\pi_p} = \boldsymbol{f}^{\,\mathrm{ext}}$.
- La densité de courant est l'opposé du *tenseur des contraintes*: $\mathrm{j}_{\boldsymbol{p}} = -\,\boldsymbol{\tau}$.
- L'équation de continuité pour la quantité de mouvement s'écrit comme :

$$\dot{\boldsymbol{p}} + \boldsymbol{p}\,\nabla\cdot\boldsymbol{v} = \nabla\cdot\boldsymbol{\tau} + \boldsymbol{f}^{\,\mathrm{ext}}\;. \tag{19}$$

Thermodynamique

We now turn to the continuity equation for momentum. The constitutive relation of the clipper mechanics is equal to m vp being the density of the momentum using the previous version. For the speed of the middle. The momentum density in peace can be written in this form. Concerning the continuity equation for the momentum, the density of breath of the momentum of motion is due to the forces of the external field. We then write. Pipette is equal to f outside. An example of an external blood force is the force of gravitation. Moreover, the current density is the opposite of the dry time of the constraints. We then write that if p is equal to to less than time, extend the stress tensor. The main constraints are related to to the forces of the contacts which are exerted in continuous medium, i.e. to the forces which are exerted on the surface occupied by the continuous medium. The negative signs that appear in front of the stress tensor is introduced to respect the sign convention we have adopted. The continuity equation then that the momentum is written in this form. Now, we can introduce in this relation the constitutive relation of the mechanics and then apply the rules of the skit for the pedantic material derivative of the density of motion candidates.

Notes

Summary

6m 24s

Equations de continuité thermodynamique

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- La relation constitutive de la mécanique s'écrit $\boldsymbol{p} = m\,\boldsymbol{v}$, i.e.

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- L'équation de continuité pour la quantité de mouvement s'écrit comme :

$$\dot{\boldsymbol{p}} + \boldsymbol{p}\,\nabla\cdot\boldsymbol{v} = \nabla\cdot\boldsymbol{\tau} + \boldsymbol{f}^{\,\mathrm{ext}}\ . \tag{19}$$

- A l'aide d'équation de continuité pour la masse, la dernière équation résulte en la 2ème loi de Newton :

$$m\,\dot{\boldsymbol{v}} = \nabla\cdot\boldsymbol{\tau} + \boldsymbol{f}^{\,\mathrm{ext}}. \tag{20}$$

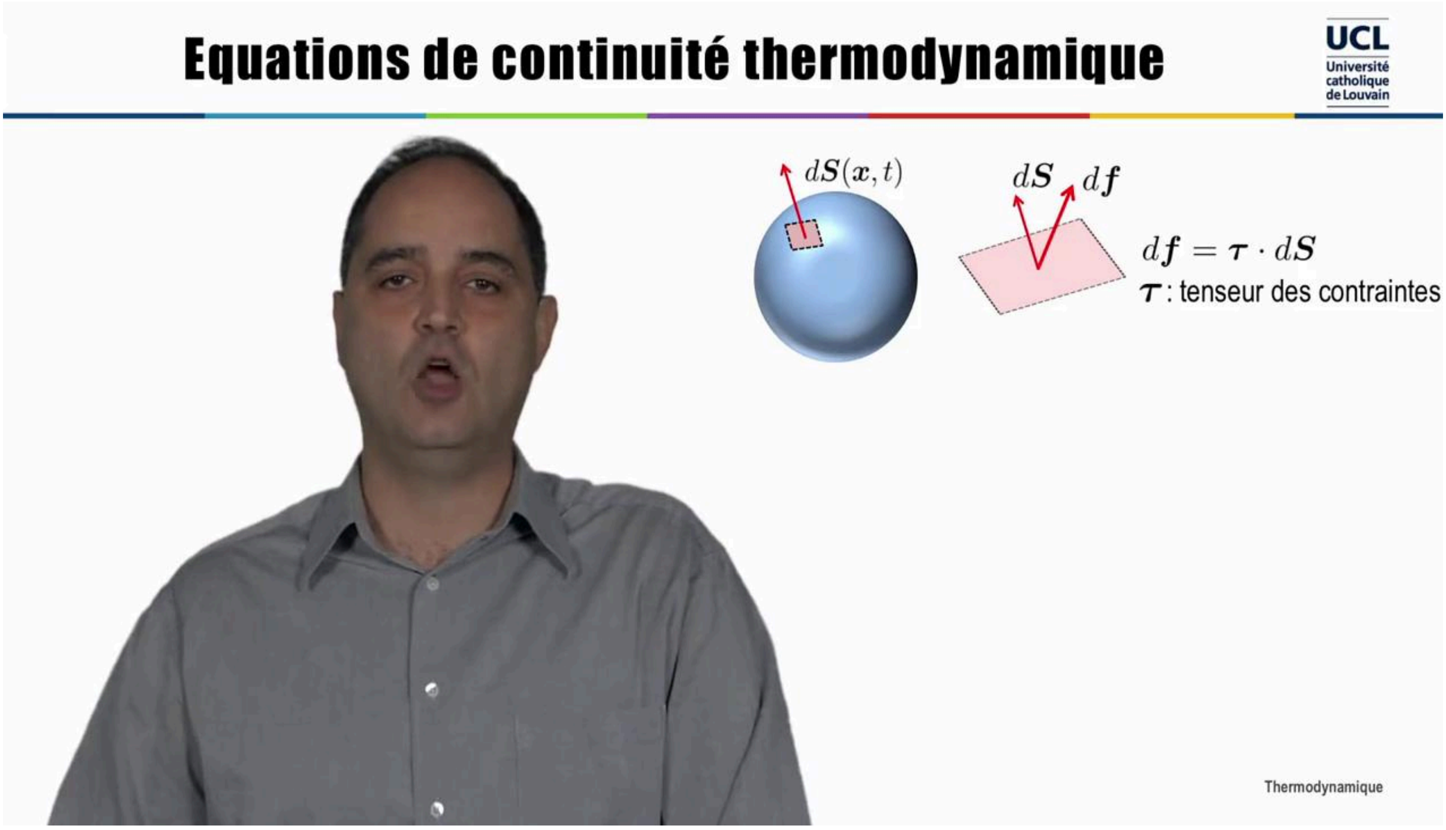
Thermodynamique

Then we can introduce the continuity equation for the mass. In summary, using the continuity equation for the mass, the last equation results in Newton's second law. This is the second law of sheep.

Notes

Summary

7m 58s

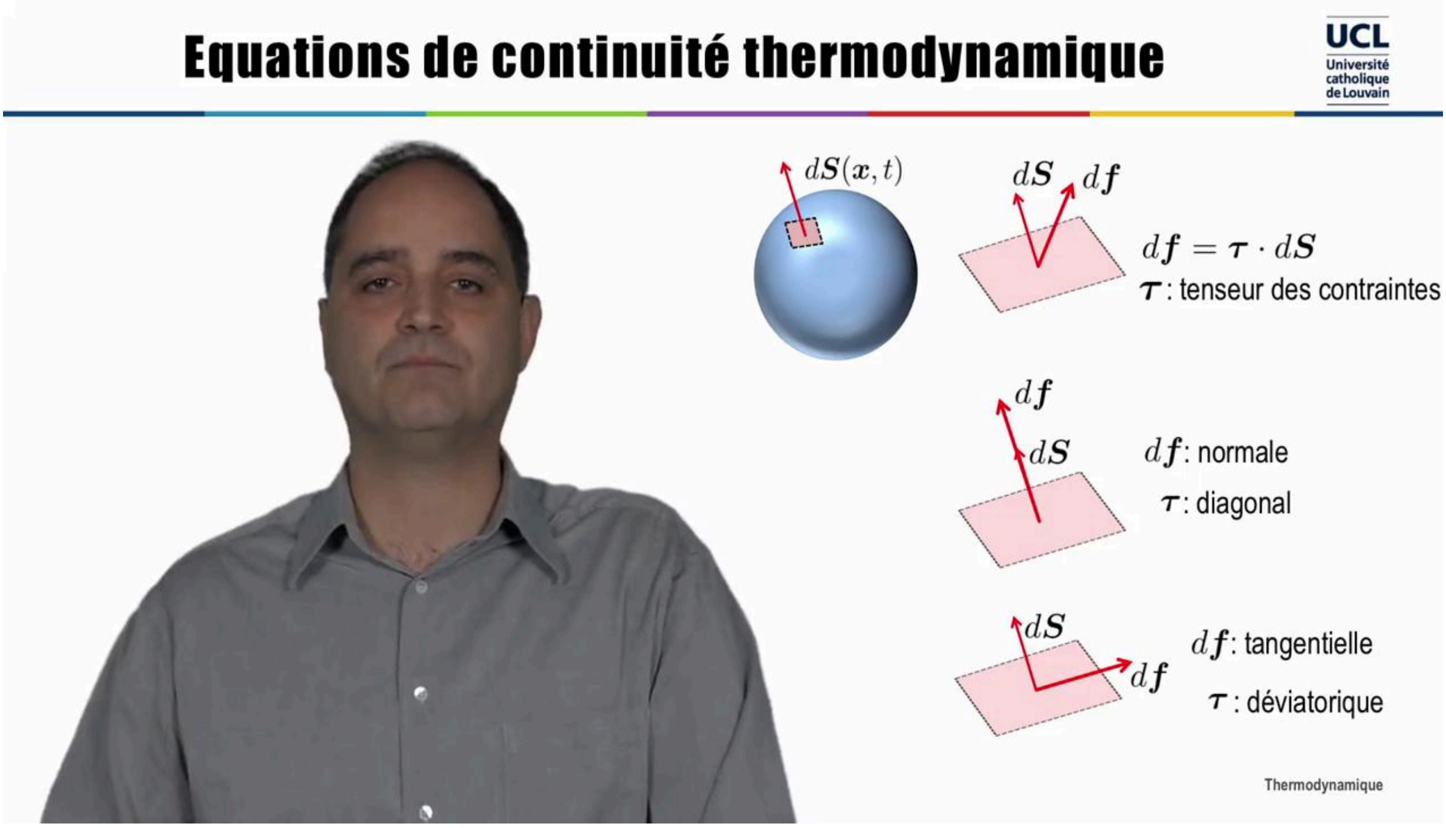


Now we elaborate a bit more on the spark of constraints. We consider a surface occupied by the continuous medium and under this surface, we take an infinitesimal element. The vector of this infinitesimal element is noted by de s. There will be a contact force on this infinitesimal element. This infinitesimal contact force is denoted by DF. In general, the direction of this force of infinitesimal contact is different from the direction of the SDS vector, but it is convenient to have a relationship between these two vectors. The most general relationship between two vectors is carried out with the help of a greenhouse time. We then write that as soon as f, the infinitesimal contact force is equal to the product of the time tensor multiplied by the vector of S. Time is identified as the spark of inert constraints, expressions for that ancestor is in the circle of tensions. For a continuous medium whose elementary particles have no rotation intrinsic, we can show, with the help of the principle of conservation of the moment, of the quantities of motion that the stress spark is symmetrical. In addition, there are special cases concerning the direction of the force of infinitesimal contacts of F.

Notes

Summary

8m 14s



The first special case is when the force of F has the same direction as the direction of the vector of S. In these cases, the force of an infinitesimal contact of F is normal to the infinitesimal element where it is exerted. In seven cases, the time, that of the constraints in diagonal is in greenhouse is diagonal. When the term off diagonal are touched guazzelli. The fact of a normal contact force to the surface is accepted and the change of the volume of the medium, i.e. the compression or expansion of the medium. The second particular case is when the infinitesimal contact force of F is normal with RDS. In these cases, DF is tangential to the surface where it is exerted, and we can show that the stress tensor is catholic intense R and V catholic when its trace is equal to zero. The trace of intense r is the sum of diagonal terms of times R. The fact of a contact force tangential to the surface where it is exerted is the deformation of the medium. In these cases, we say that this contact force is a shearing force.

Notes

Summary

9m 42s

Equations de continuité thermodynamique

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- L'énergie totale est la somme de l'énergie interne et de l'énergie cinétique :

$$e = u + \frac{1}{2} m \, \boldsymbol{v} \cdot \boldsymbol{v}. \tag{21}$$

- L'équation de continuité pour l'énergie totale s'écrit sous la forme :

$$\dot{e} + e \nabla \cdot \boldsymbol{v} = -\nabla \cdot \boldsymbol{j}_e + \pi_e \, . \tag{22}$$

- $\pi_e = \boldsymbol{v} \cdot \boldsymbol{f}^{\,\mathrm{ext}}$: travail des forces extérieures.
- $\boldsymbol{j}_e = -(\boldsymbol{\tau} \cdot \boldsymbol{v}) + \boldsymbol{j}_u,$

  courant d'énergie interne, e.g. chaleur.

  travail des contraintes

- La combinaison des trois dernières équations donne :

$$\dot{e} + e \nabla \cdot \boldsymbol{v} = \nabla \cdot (\boldsymbol{\tau} \cdot \boldsymbol{v}) \; - \; \nabla \cdot \boldsymbol{j}_u \; + \; \boldsymbol{v} \cdot \boldsymbol{f}^{\,\mathrm{ext}} \, .$$

Thermodynamique

We now continue with the continuity equation for the total energy. the total energy is the sum of the internal energy and. the Kinetic Energy is the same also for the densities. Then the total energy density is the sum of the internal energy density U, plus the density of the kinetic energy which is represented by this term. The continuity equation for the total energy was created in the following form. Here Pillet and the density of sources for the total energy. It is equal to the work of external forces. We then write that worse is the inner product between the speed and the external force. In addition, the current density is for the total energy and the sum of two contributions. The first contribution is expressed by this area which is the work of constraint. The second contribution is expressed by this quantity which is identified as the current density. For internal energy, a current of internal energy, no doubt, a current of heat for example. Now we can introduce in the continuity equation for the total energy. These two expressions. The combination of the three becomes the equation in this relationship.

Notes

Summary

11m 09s

Equations de continuité thermodynamique

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$$\left.\begin{aligned} &\dot{e} + e\,\nabla\cdot\boldsymbol{v} = \nabla\cdot(\boldsymbol{\tau}\cdot\boldsymbol{v}) - \nabla\cdot\boldsymbol{j}_u + \boldsymbol{v}\cdot\boldsymbol{f}^{\,\mathrm{ext}} \\ &e = u + \frac{1}{2} m\,\boldsymbol{v}\cdot\boldsymbol{v} \end{aligned}\right\}\Longrightarrow$$

$$\left(\dot{u} + \frac{1}{2}\dot{m}\,\boldsymbol{v}\cdot\boldsymbol{v} + m\,\boldsymbol{v}\cdot\dot{\boldsymbol{v}}\right) + \left(u + \frac{1}{2} m\,\boldsymbol{v}\cdot\boldsymbol{v}\right)\nabla\cdot\boldsymbol{v} \;=\; \nabla\cdot(\boldsymbol{\tau}\cdot\boldsymbol{v}) - \nabla\cdot\boldsymbol{j}_u + \boldsymbol{v}\cdot\boldsymbol{f}^{\,\mathrm{ext}}$$

$$\Longrightarrow (\dot{u} + u\,\nabla\cdot\boldsymbol{v}) + \frac{1}{2}(\boldsymbol{v}\cdot\boldsymbol{v})\,\overbrace{(\dot{m} + m\,\nabla\cdot\boldsymbol{v})}^{0} + \boldsymbol{v}\cdot m\,\dot{\boldsymbol{v}} \;=\; \nabla\cdot(\boldsymbol{\tau}\cdot\boldsymbol{v}) - \nabla\cdot\boldsymbol{j}_u + \boldsymbol{v}\cdot\boldsymbol{f}^{\,\mathrm{ext}}$$

$$(\dot{u} + u\,\nabla\cdot\boldsymbol{v}) + \boldsymbol{v}\cdot m\,\dot{\boldsymbol{v}} = \nabla\cdot(\boldsymbol{\tau}\cdot\boldsymbol{v}) - \nabla\cdot\boldsymbol{j}_u + \boldsymbol{v}\cdot\boldsymbol{f}^{\,\mathrm{ext}}$$

$$m\,\dot{\boldsymbol{v}} = \nabla\cdot\boldsymbol{\tau} + \boldsymbol{f}^{\,\mathrm{ext}}$$

Thermodynamique

We then have this equation plus the expression for the total energy density of a continuous medium. We can introduce this expression in this equation and use the rules of the chain for the material derivative of the total energy density. This will provide us with this equation. Now we can group together the terms on the left sides of this equation. To arrive at this relationship. We notice that these terms here on the left side of this equation is zero. In view of the continuity equation for the mass. This gives us this equation and now we notice that this embedded product also appears in Newton's second law. This is the second law of time. Now we can multiply each term of the second law of sheep by V. Enter the results into this equation.

Notes

Summary

12m 37s

Equations de continuité thermodynamique

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$$\dot{u} + u\,\nabla\cdot\boldsymbol{v} = -\boldsymbol{v}\cdot(\nabla\cdot\boldsymbol{\tau}) + \nabla\cdot(\boldsymbol{\tau}\cdot\boldsymbol{v}) - \nabla\cdot\boldsymbol{j}_u\,,$$

- Identité mathématique :

$$\nabla\cdot(\boldsymbol{\tau}\cdot\boldsymbol{v}) = \boldsymbol{v}\cdot(\nabla\cdot\boldsymbol{\tau}) + \boldsymbol{\tau}:\tilde{\nabla}\boldsymbol{v}\,.$$

$$\tilde{\nabla}\boldsymbol{v} \equiv \frac{1}{2}\left(\nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^{\mathrm{T}}\right) = \frac{1}{2}\left(\frac{\partial v^i}{\partial x_j} + \frac{\partial v^j}{\partial x_i}\right):$$ tenseur des déformations

- A l'aide de cette identité mathématique, nous arrivons à l'équation de continuité pour l'énergie interne :

$$\dot{u} + u\,\nabla\cdot\boldsymbol{v} = -\nabla\cdot\boldsymbol{j}_u + \boldsymbol{\tau}:\tilde{\nabla}\boldsymbol{v}\,. \qquad (23)$$

Thermodynamique

We then arrive at this equation. Now we can combine these two terms that appear on the right side of this equation. This is done using the following mathematical identity. This quantity that appears here is a differential operator under the velocity centers. It is then intense R and it is given by this relation. It is defined as half the sum of of the velocity gradient of the speed keeper transpose. This advance is called two-formation thinking. In addition, this operation represents the double scalar product between two cancers. The operation is as follows. We multiply each term of the first cancer with the equivalent term of the second cancer and then we take the sum of these products. This gives us the double scalar product between two tensors using this identity is mathematical. We arrive at the continuity equation for the internal energy. This is the continuity equation for the internal energy.

Notes

Summary

13m 39s

Equations de continuité thermodynamique

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- L'équation de continuité appliquée à la charge électrique donne :

$$\dot{q} + q\,\nabla\cdot\boldsymbol{v} = -\nabla\cdot\boldsymbol{j}_q\,. \tag{24}$$

pas de densité de source de charge électrique!

- $\boldsymbol{j}_q$: densité de courant électrique *conductif*,
- $q\boldsymbol{v}$: densité de courant électrique *convectif*,
- $\boldsymbol{j} = \boldsymbol{j}_q + q\boldsymbol{v}$: densité de courant électrique.
- La densité de la charge électrique q est liée aux densités des r substances :

$$q = \sum_{A=1}^{r} q_A\, n_A\,. \tag{25}$$

- q_A: charge électrique d'une unité de la substance A: $q_A = \dfrac{\partial q}{\partial n_A} = cte.$

Thermodynamique

We continue with the continuity equation for the electric charge. The discontinuity equation applied to the electric charge. In this expression, we notice the absence of density and source of electric charge. In addition, the quantity x that appear in this equation is the conductive electric current density. By the way, the product of the density of the electric charge and the velocity is identified as the convective electric current density. The sum of these two densities is equal to the electric current density which is denoted by G. In addition, we note that the density of electric charging is related to the density of air substances by this relation. Here, PIO represents the electrical charge of one unit of substance A. It is defined as the partial derivative of the charge density electrical or in relation to the density of the chemical substance to. Everything to the electrical recharge of Unitaid and the substance A is constant. Now we consider the continuity equation for the chemical substance to and we multiply each term of this equation by q or a. We do this for all continuity equations of chemical substances and we take the sum then this sum using this equation. We will again give the continuity equation for the electric charge.

Notes

Summary

14m 56s

Equations de continuité thermodynamique

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- La combinaison de la dernière relation avec les équations de continuité pour les substances chimiques donne :

$$\dot{q} + q\,\nabla\cdot\boldsymbol{v} \;=\; -\nabla\cdot\left(\sum_{A=1}^{r} q_A\,\boldsymbol{j}_A\right) + \sum_{a=1}^{n}\omega_a\left(\sum_{A=1}^{r} q_A\,\nu_{aA}\right).$$

- L'identification de cette relation avec l'équation de continuité pour la charge électrique donne :

$$\boldsymbol{j}_q = \sum_{A=1}^{r} q_A\,\boldsymbol{j}_A\,, \tag{27}$$

et

$$\sum_{A=1}^{r} q_A\,\nu_{aA} = 0\,, \quad a = 1, ..., n\,. \tag{26}$$

Thermodynamique

In summary, the combination of the last relation with the continuity equations for the chemicals in this equation. This equation is nothing more than that the continuity equation for the electric charge then. This air in brackets is identified as the productive electric current density, while this term is identified as the heating density or the characteristic that is equal to zero. In summary, the identification of this relationship with the continuity equation for electric charging. We in this equation as well as this equation.

Notes

Summary

16m 33s

Equations de continuité thermodynamique

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- L'équation de continuité pour l'entropie s'écrit sous la forme :

$$\dot{s} + s\,\nabla \cdot \boldsymbol{v} = -\nabla \cdot \boldsymbol{j}_s + \pi_s \,. \tag{28}$$

- $\boldsymbol{j}_s$: densité de courant d'entropie (interaction du milieu avec son exterieur).
- π_s: densité de source d'entropie au sein du milieu.
- Le deuxième principe de thermodynamique requiert que :

$$\pi_s \geq 0 \,.$$

Thermodynamique

And we will conclude this presentation with the continuity equation for entropy. The continuity equation for entropy is written in the following form. In this relation, JS represents the current density of entropy and the null interaction of the medium with external field. Moreover, PS represents the entropy shock density within the medium and the double of non-equilibrium processes within the environment. The second principle of thermodynamics requires that the entropy sulfur density be in negative. We have also reached the end of this part of the thermodynamics course. In the next part, we will use the equations of continuity for the state variables that we have derived in this part of the course to realize the local energy balance of continuous medium.

Notes

Summary

17m 18s