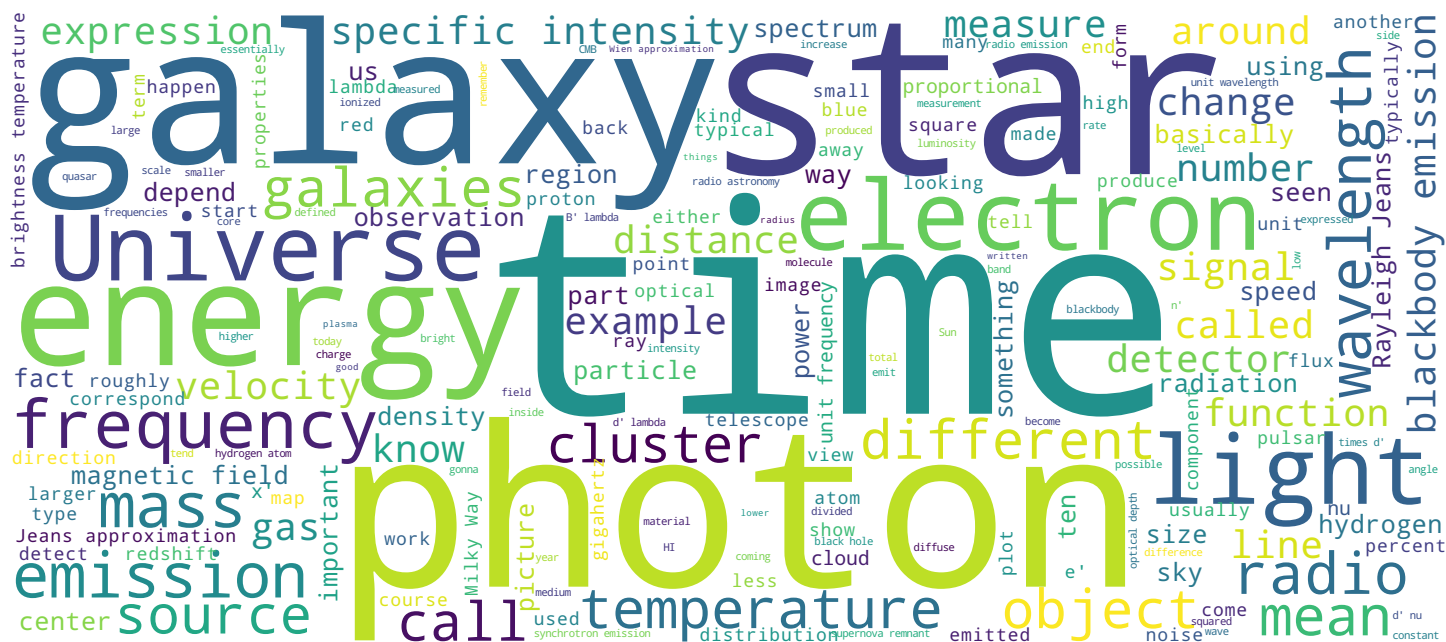
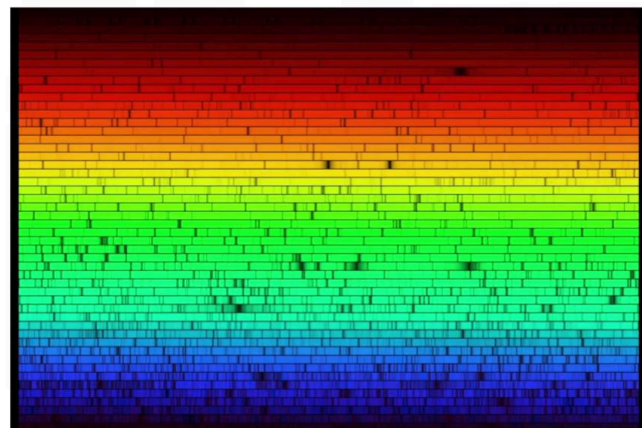


Kim McAlpine

NOAO/AURA/NSF



## Search MOOC



## Video

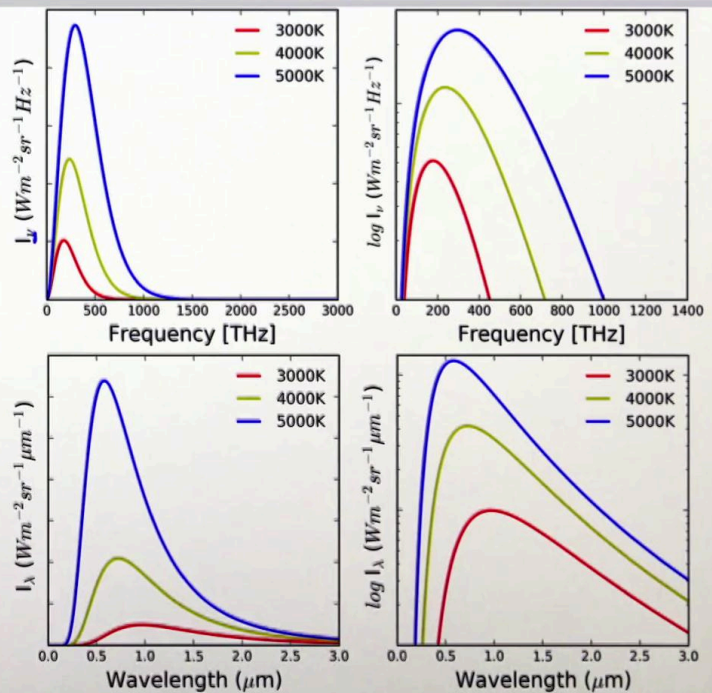


# Blackbody radiation

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

$$B_\lambda \quad (c = \lambda\nu)$$

$$\frac{I}{d\nu} = B_\nu \quad B_\lambda = \frac{I}{d\lambda}$$



This lecture continues our exploration of the properties of blackbody emission and particularly focuses on blackbody radiation as a function of frequency and its properties of the radio wavelength regime. In our previous plot we looked at how the blackbody emission changes as a function of wavelength where we measured blackbody emission in units of specific intensity per unit wavelength. But as I mentioned, you can also measure specific intensity per unit frequency. So here I'm showing you what blackbody emission looks like if you measure it in spectral and specific intensity per unit frequency as a function of frequency. It's important to note that if you take the blackbody emission and measure it in spectral specific intensity per unit frequency that you cannot just derive this expression by using the expression that we had before for 'B' lambda and substituting in the usual equation that 'c' equals lambda nu and that's because here what we're looking at is the change in intensity per unit frequency whereas before what we were looking at was the change in intensity per unit wavelength and the unit wavelength and unit frequencies will not necessarily be interchangeable with each other.

Notes

Summary



# Blackbody radiation

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

$$B_\lambda \quad (c = \lambda\nu)$$

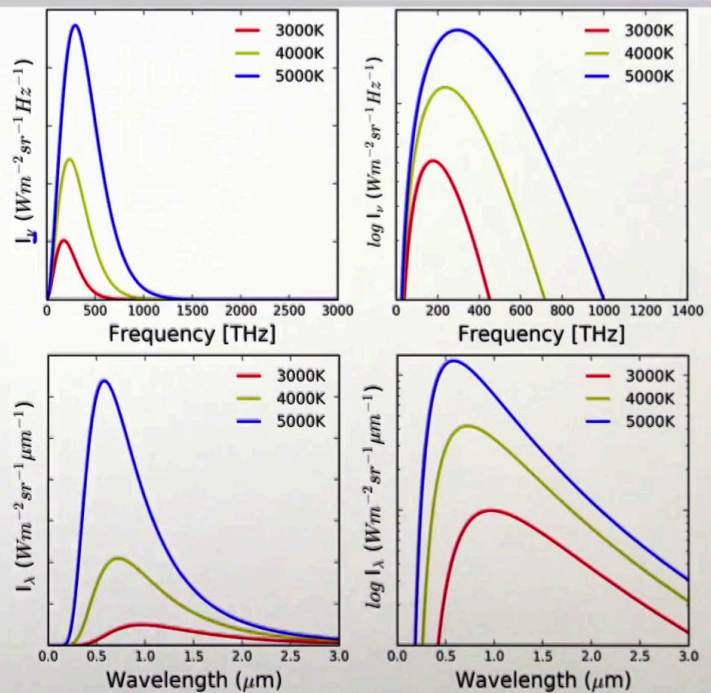
$$\frac{I}{d\nu} = B_\nu \quad B_\lambda = \frac{I}{d\lambda}$$

$$B_\nu d\nu = I = B_\lambda d\lambda$$

$$B_\nu = B_\lambda(T) \frac{d\lambda}{d\nu}$$

$$\frac{d\lambda}{d\nu} = \frac{d}{d\nu} \left( \frac{c}{\nu} \right) = -\frac{c}{\nu^2}$$

$$\therefore B_\nu(\nu, T) = B_\lambda\left(\frac{c}{\nu}, T\right) \times \frac{c}{\nu^2}$$



So the way to derive your blackbody equation as a function of specific intensity per unit frequency is to use the relationship that 'I' will be equal to 'B' lambda times 'd' lambda and it will also be equal to 'B' nu times 'd' nu and so, therefore, we can say that 'B' nu is equal to 'B' lambda times 'd' lambda over 'd' nu and 'd' lambda over 'd' nu is given by this and so we can then calculate this relation per unit frequency. You can verify for yourself that this expression here is the same as this expression here where 'B' lambda was given on the previous slide. It's also good to notice that people frequently plot blackbody emission in log-log per units so it just show you what the blackbody emission looks like if you plot it on a log scale and also notice here that as we go to increasing temperature, we have the shortening wavelengths which is then represented by the increasing frequency on this plot.

Notes

Summary



1m 36s

# Blackbody radiation - Rayleigh Jeans

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

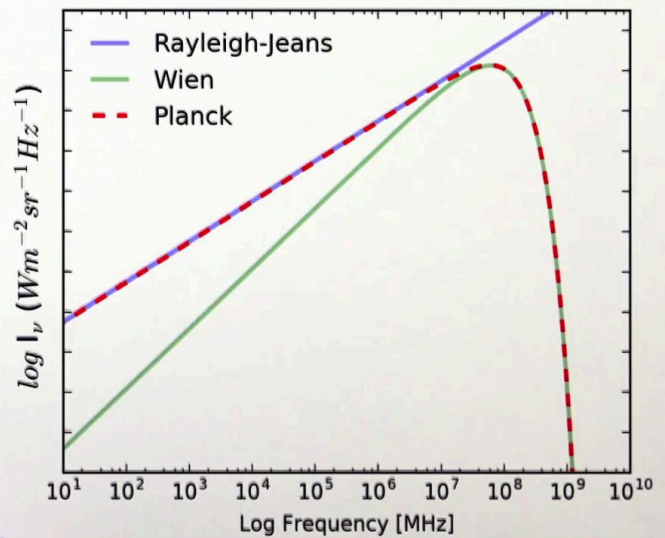
$\frac{h\nu}{kT} \ll 1$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$h\nu/kT \ll 1 \rightarrow B_\nu(T) \approx \frac{2\nu^2 kT}{c^2} \quad \checkmark$$

Wien approx

$$h\nu/kT \gg 1 \rightarrow B_\nu(T) \approx \frac{2h\nu^3}{c^2} e^{-\frac{h\nu}{kT}} \quad \because e^x - 1 \approx e^x$$



The Radio Universe

Because the full Planck equation is kind of a mouthful, most of the time radio astronomers use a much more convenient approximation for blackbody emission called the Rayleigh Jeans approximation. This approximation is valid for the lower frequencies that we tend to use in radio astronomy. So the approximation is valid for ' $h\nu$  over ' $kT$ ' very much less than one. So in this case what we can do is we can use a Taylor expansion which says that ' $e$ ' to the ' $x$ ' is equal to one plus ' $x$ ' plus ' $x$ ' squared over two factorial cubed over three factorial. And here if ' $x$ ' is very much less than one then what we're able to do is neglect these higher order terms, in which case we can replace this whole expression with the expression one plus ' $h\nu$  over ' $kT$ '. If you substitute this value into this equation then you can verify that in fact, what you end up with is this equation which is the Rayleigh Jeans approximation for the blackbody emission at low frequencies. There's also a high frequency approximation called the Wien approximation and in that case what you exploit is that if ' $x$ ' is very much greater than one then ' $e$ ' to the ' $x$ ' minus one will be approximately equal to ' $e$ ' to the ' $x$ ' and so you can replace this whole expression with the expression ' $e$ ' to the ' $h\nu$  over ' $kT$ ' and if you do that then you'll find that you get this expression which is the Wien approximation.

Notes

Summary



2m 57s



# Blackbody radiation - Rayleigh Jeans

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

$$\frac{h\nu}{kT} \ll 1 \quad \left(1 + \frac{h\nu}{kT}\right)$$

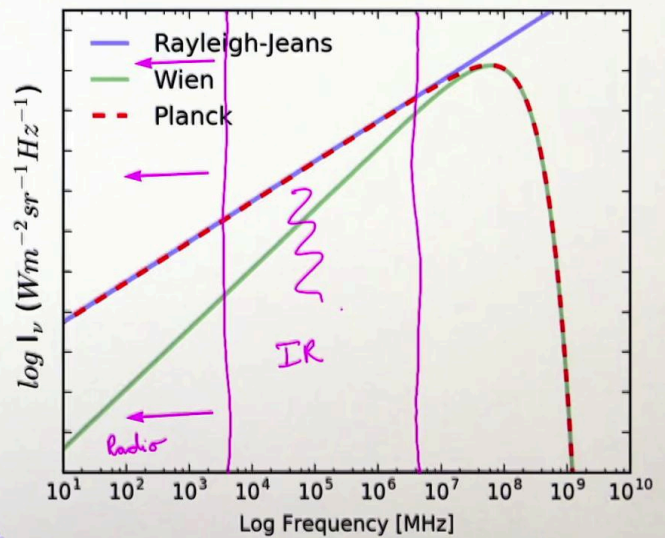
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

$$T_b \Leftrightarrow I_\nu$$

$$h\nu/kT \ll 1 \rightarrow B_\nu(T) \approx \frac{2\nu^2 kT}{c^2} \quad \checkmark$$

Wien approx

$$h\nu/kT \gg 1 \rightarrow B_\nu(T) \approx \frac{2h\nu^3}{c^2} e^{-\frac{h\nu}{kT}} \quad \because e^x - 1 \approx e^x$$



The Radio Universe

To show you how a good approximation the Rayleigh Jeans approximation is at low frequencies, I've plotted here the full Planck expression in red, the Rayleigh Jeans approximation in blue and the Wien approximation in green and you can now see that indeed both approximations work in the opposite frequency regime. The Rayleigh Jeans approximation works at all radio frequencies because the radio part of the spectrum is essentially this part of the plot, everything lower than this line. So in this regime, it's clear that the two that the approximation works very well and in fact, the approximation even works up to infrared frequencies which lives approximately in this part of the plot. The Rayleigh Jeans approximation is so ubiquitous in radio astronomy that we use it to define a measurement unit called the brightness temperature so we use it to define a way of measuring a specific intensity by specifying temperature. So what that means is that if you measure a brightness temperature that you can always convert it back to a specific intensity by using this Rayleigh Jeans formula so you take your brightness temperature, you stick it into this equation and then you get back a specific intensity.

Notes

Summary



# Blackbody radiation - Rayleigh Jeans

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

$$\frac{h\nu}{kT} \ll 1$$

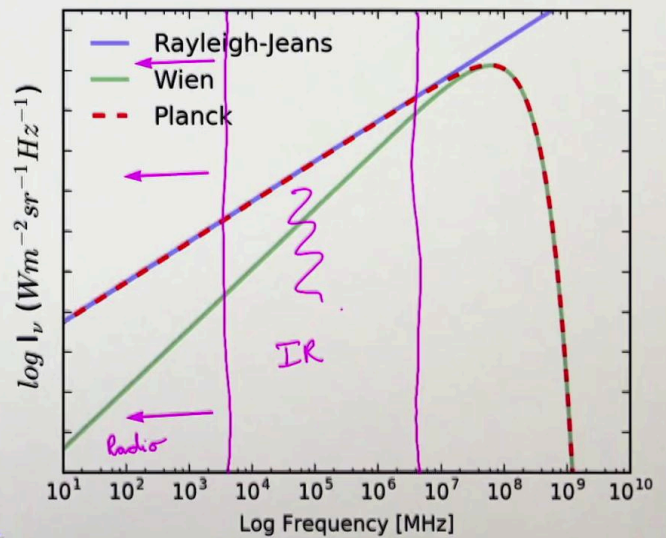
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

$$T_b \Leftrightarrow I_\nu$$

$$h\nu/kT \ll 1 \rightarrow B_\nu(T) \approx \frac{2\nu^2 kT}{c^2} \checkmark$$

Wien approx

$$h\nu/kT \gg 1 \rightarrow B_\nu(T) \approx \frac{2h\nu^3}{c^2} e^{-\frac{h\nu}{kT}} \quad \therefore e^x - 1 \approx e^x$$



The Radio Universe

Remember though that the brightness temperature isn't necessarily a physical temperature. It will only be a physical temperature if you have something that approximately emits like a blackbody and is opaque, but in all instances it is a measure of the specific intensity.

Notes

Summary

