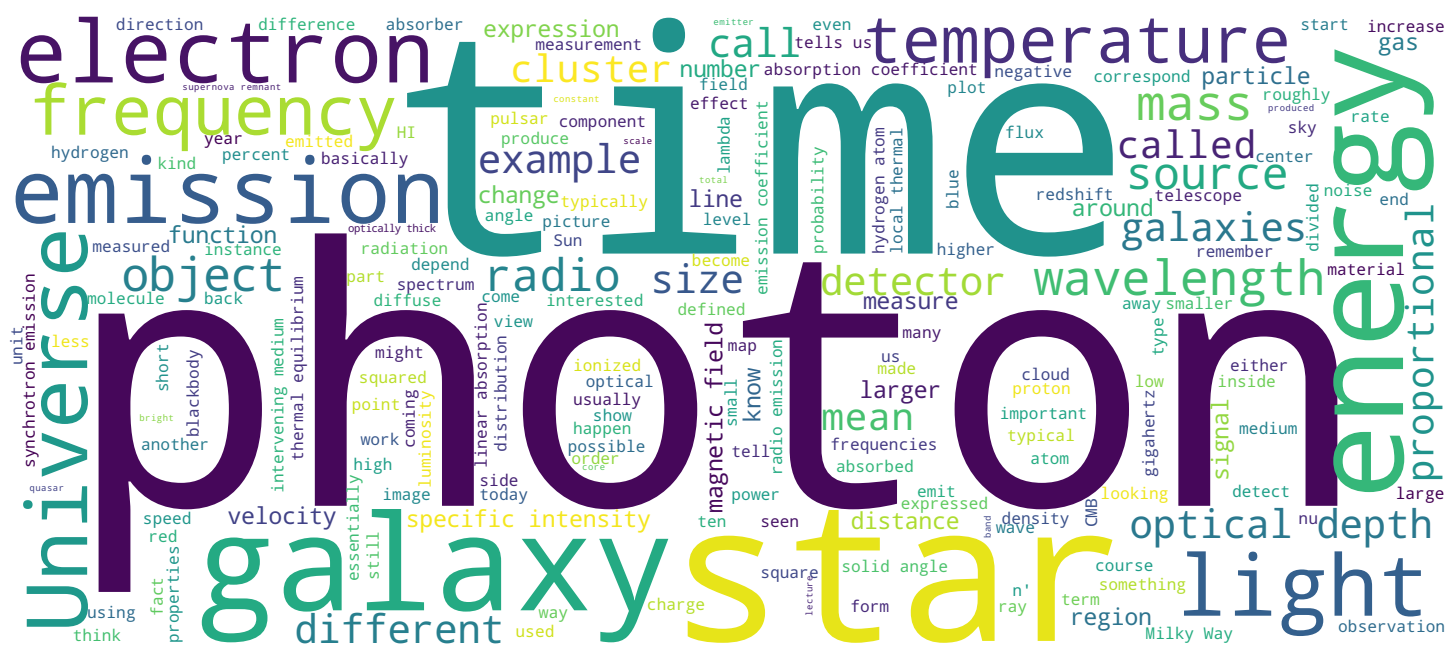
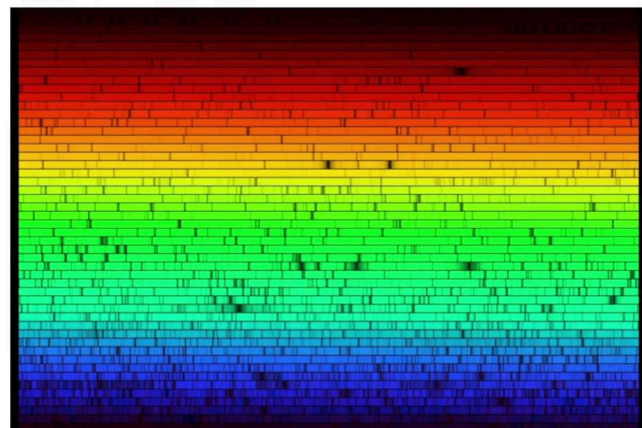


Kim McAlpine

NOAO/AURA/NSF



## Search MOOC



## Video





Once a photon is generated in Astrophysical object, it may have to interact with a number of intervening particles on its way to our telescope before we detect it. During this journey it could be absorbed, it could be scattered or in fact, the intervening particles could emit their own radiation distorting the radiation from the original source that we were interested in. So as astronomers we need to be able to characterize all these processes that happen to the photon along its journey towards us and we do that with the radiative transfer equation which we will explore now in this lecture.

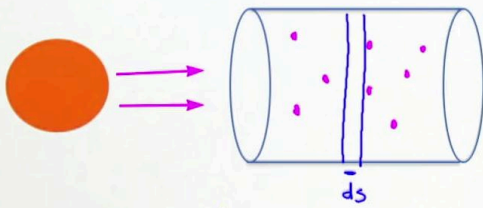
Notes

Summary



0m 05s

# Transport Equation



$$dP = \kappa ds$$

↳ linear absorption coefficient

$$\frac{dI_\nu}{I_\nu} = -\kappa ds$$

The Radio Universe

If you have photons from your source that you're interested in, they might be traveling through some intervening medium that exists between the source and your detector. So you need to understand what the effect of this intervening source is on the photons that you're interested in studying or perhaps you're even interested in studying the intervening medium. So you can say that the probability of a photon being absorbed in some infinitesimally small slab with size 'ds' is proportional to the size of the slab so the probability is proportional to the size of the slab with some probability proportionality constant which is called the linear absorption coefficient. It's important that you consider only this very small slab because if you were to consider this whole absorber at once, you'd realize that some of the photons will be absorbed at the beginning of the of this absorber and as you penetrate deeper and deeper there'll be fewer and fewer photons to absorb so this linear approximation won't hold anymore. From this we can say that the change in specific intensity is proportional to the negative of this is equal to sorry, equal to the negative of the absorption coefficient times 'ds'.

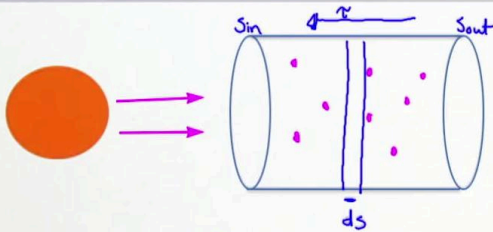
Notes

Summary



0m 40s

# Transport Equation



$$dP = K ds$$

linear absorption coefficient

$$\int_{S_{in}}^{S_{out}} \frac{dI_\nu}{I_\nu} = - \int_{S_{in}}^{S_{out}} K ds$$

$$\ln(I_\nu) \Big|_{S_{in}}^{S_{out}} = - \int_{S_{in}}^{S_{out}} K ds$$

$$\frac{I_\nu(S_{out})}{I_\nu(S_{in})} = e^{-\int_{S_{in}}^{S_{out}} K ds} = e^{-\tau}$$

$$\tau = - \int_{S_{out}}^{S_{in}} K(s) ds$$

$\tau$



The Radio Universe

This is a macroscopic description of how these photons are being absorbed. Doesn't really depend on the properties of this material that's between you and the source. It could be anything inside this box of intervening material and it's only valid if the sizes or the separations between the particles are very much larger than the particles themselves. From this we can integrate this differential equation across the whole range of the intervening material. And so from this what we get is that this integral is the log of the specific intensity and if we take the exponential on both sides here then what we'll end up with is that we can define this our expression here has a special name. This is called or defined as the optical depth. So optical depth is defined as being and it's defined so that optical depth increases as you look from the detector into the absorber so you'll see that the sign here has changed around so that this ends up equaling 'e' to the minus tau, 'e' to the power of minus optical depth. So it's important to remember that then optical depth here increases as you look deeper into the absorber. Optical depth is basically a measure of how opaque your object is and we can define two regimes.

Notes

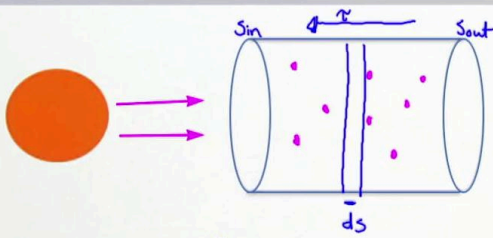
Summary



2m 16s



# Transport Equation



$$dP = K ds$$

linear absorption coefficient

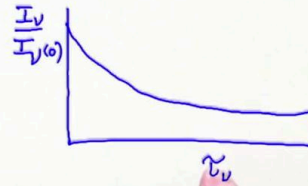
$$\int_{S_{in}}^{S_{out}} \frac{dI_\nu}{I_\nu} = - \int_{S_{in}}^{S_{out}} K ds$$

$$\ln(I_\nu) \Big|_{S_{in}}^{S_{out}} = - \int_{S_{in}}^{S_{out}} K ds$$

$$\frac{I_\nu(S_{out})}{I_\nu(S_{in})} = e^{-\int_{S_{in}}^{S_{out}} K ds} = e^{-\tau}$$

$$\tau = - \int_{S_{out}}^{S_{in}} K(s) ds$$

$\tau \ll 1$  optically thin  
 $\tau \gg 1$  optically thick



The Radio Universe

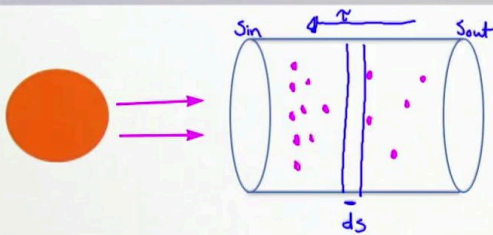
We define when optical depth is less than one, we say that an object is optically thin and this means that the object is approximately semi-transparent so you could see through it. An example of an optically thin medium would be something like a thin or light smog where you could still see your hand or something through the smog but the but there is some intervening medium or there is an optically thick source and that is when tau is very much greater than one and an optically thick source is essentially an opaque source. An example of an optically thick source would be something like a very dense cloud where you could only really see the surface of the cloud and you couldn't actually see deeply into the cloud. What this expression tells us is that as you as you penetrate deeper into the absorber, the specific intensity of your incident source is declining as an exponential function so if you are to measure the inputs, the outputs specific intensity over the input specific intensity then it would be declining as a function of optical depth. You can also think of optical depth here which is, in general, frequency dependent as being a measure of how much photon energy remains after the photon has penetrated some depth into the absorbing medium.

Notes

Summary



# Transport Equation



$$dP = K ds$$

linear absorption coefficient

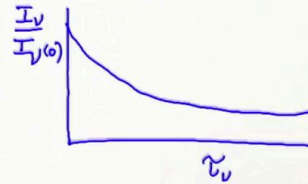
$$\int_{S_{in}}^{S_{out}} \frac{dI_\nu}{I_\nu} = - \int_{S_{in}}^{S_{out}} K ds$$

$$\ln(I_\nu) \Big|_{S_{in}}^{S_{out}} = - \int_{S_{in}}^{S_{out}} K ds$$

$$\frac{I_\nu(S_{out})}{I_\nu(S_{in})} = e^{-\int_{S_{in}}^{S_{out}} K ds} = e^{-\tau}$$

$$\tau = - \int_{S_{out}}^{S_{in}} K(s) ds$$

$\tau \ll 1$  optically thin  
 $\tau \gg 1$  optically thick



$\frac{1}{K(s)} \sim$  geometric mean free path

The Radio Universe

So remember that optical depth depends on the size of the medium so the larger the size of the medium, the larger the optical depth will be. You can also think of this linear absorption coefficient as measuring the mean geometric path of a photon inside this inside this absorber so how far does a photon travel before it's absorbed? And if this was a homogeneous medium so if everywhere 'ks' was the same then it would be a mean one for the entire absorber but if, for instance, you have a very homogeneous medium where it's very much denser in one region and very much sparser in another then you would have a higher linear absorption coefficient here and a shorter mean free path and vice versa and you'd have a lower linear absorption coefficient here and a longer mean free path. So you can also think of it as being a local geometric mean free path.

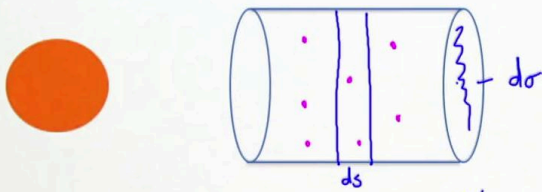
Notes

Summary



5m 42s

# Transport Equation



$$dP = ds d\sigma d\Omega = \frac{W}{dV \cdot d\Omega}$$

$$j_{em} = \frac{dI_\nu}{ds}$$

$$\frac{dI_\nu}{ds} = -k I_\nu + j_{em} \quad \text{RTE}$$

The Radio Universe

If you consider once again our intervening medium but now we consider the possibility that this intervening medium might be an emitter, we can say that the probability of a photon being emitted in some small volume inside this region, so imagine we have a small cylinder here which has size 'ds' here and surface area given by 'd' sigma then we can say that the probability of emission is proportional to the volume and it's also proportional to the solid angle with which you're trying to detect this emission so photons will be emitted in different directions and the larger the region of solid angle you consider, the more likely it is that a photon will be emitted into that solid angle. We define here the something called the emission coefficient which is the change in the specific intensity for a given distance in the absorber for 'ds' and we can see from this expression here that the units of this should be watts per unit change in volume per unit solid angle. From this we can now combine that with our previous expression for the absorption that we would get from the same intervening material and we can say that the total change in specific intensity is going to be equal to the absorption coefficient times the specific intensity plus the emission coefficient and we call this equation the full radiative transfer equation or the RTE for short.

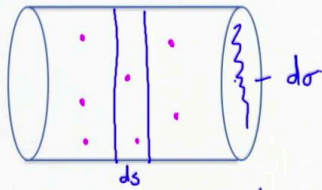
Notes

Summary



6m 50s

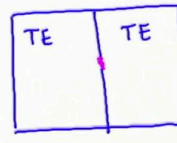
# Transport Equation



$$dP = ds d\sigma d\Omega = \frac{W}{dV \cdot d\Omega}$$

$$j_{em} = \frac{dI_\nu}{ds}$$

$$\boxed{\frac{dI_\nu}{ds} = -k I_\nu + j_{em}} \quad RTE$$



$$\nu + d\nu$$

$$\frac{dI_\nu}{ds} = 0 \quad I_\nu = B_\nu(\nu, T)$$

The Radio Universe

Now it might seem given the way that we've derived this expression that there is an arbitrary relationship between the absorption coefficient and the emission coefficient but in fact, this isn't true and this was first shown by Kirchhoff by considering two blackbody emitters which are in thermal equilibrium connected by a filter. So in Kirchhoff's thought experiments he considers that he has two blackbody emitters like this. They are connected by a small gap here and inside this gap there is a small filter which only allows a narrow range of frequencies to pass through to pass through the filter. Remember that the  $K$  and  $j$  will both be functions of frequency. Here you have a blackbody that's in thermal equilibrium here and another one at thermal equilibrium here at the same temperature and in this situation it's not possible for there to be any net power transferred across this filter because if that were the case then you would be able to drive some kind of heat engine here and that violates the second law of thermodynamics. So what that tells you here is that in this situation  $dI_\nu$  will in fact, be zero and because this is a blackbody, you also know that  $I_\nu$  will be equal to the Planck function at the temperature of the blackbody.

Notes

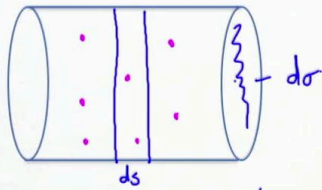
Summary



8m 45s



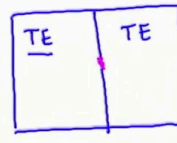
# Transport Equation



$$dP = ds d\sigma d\Omega = \frac{W}{dV \cdot d\Omega}$$

$$j_{em} = \frac{dI_\nu}{ds}$$

$$\boxed{\frac{dI_\nu}{ds} = -k I_\nu + j_{em}} \quad \text{RTE}$$



$$\nu + d\nu \quad \frac{dI_\nu}{ds} = 0 \quad I_\nu = B_\nu(T)$$

$$\frac{dI_\nu}{ds} = 0 = -k B_\nu(T) + j_{em}$$

$$\frac{j_\nu(T)}{k_\nu(T)} = B_\nu(T) \quad \text{Kirchhoff's Law}$$

LTE

→ good absorber is a good emitter!

The Radio Universe

So from this we can substitute that back into this equation. We will say then that and so if we rearrange this equation what that tells us is that at a given temperature and frequency there is a fixed relationship between the absorption coefficient and the emission coefficient and that fixed relationship is that they are equal to the Planck function and this is known as Kirchhoff's law. Although this expression has been derived specifically for the case of thermal equilibrium only, it also holds quite often in the case of local thermal equilibrium. So you can safely use this expression for local thermal equilibrium as well and this is very useful because it provides a way in a local thermal equilibrium system to obtain information about the absorption from the emission and vice versa and what this also tells you is that an object that's a good absorber is also a good emitter and that is why, for instance, in the whole blackbody scenario, we would say that blackbody is a perfect emitter but it's also a perfect absorber.

Notes

Summary

