

Previously we learned all about the characteristics of blackbody radiation which is an important form of continuum radiation. In astronomy, stars and planets are usually considered to emit approximately like blackbodies. In addition to blackbody radiation, there are two other processes which commonly give rise to continuum emission at radio wavelengths; free-free emission and synchrotron emission. At lower frequencies below say a few tens of gigahertz, the contribution of these two types of emission tend to be much stronger than blackbody emission and account for large fraction of all the radio emission in the sky. Today we will focus on these two other types of continuum emission which are both produced by accelerated charged particles. In the case of free-free emission, the acceleration is produced by electrostatic forces while for synchrotron emission, the acceleration is due to a magnetic field. As charged particles are an essential ingredient to these two types of emission, it's important to note they cannot be produced by an entirely neutral medium but that the emitting region must contain some charged particles.

Notes

Summary

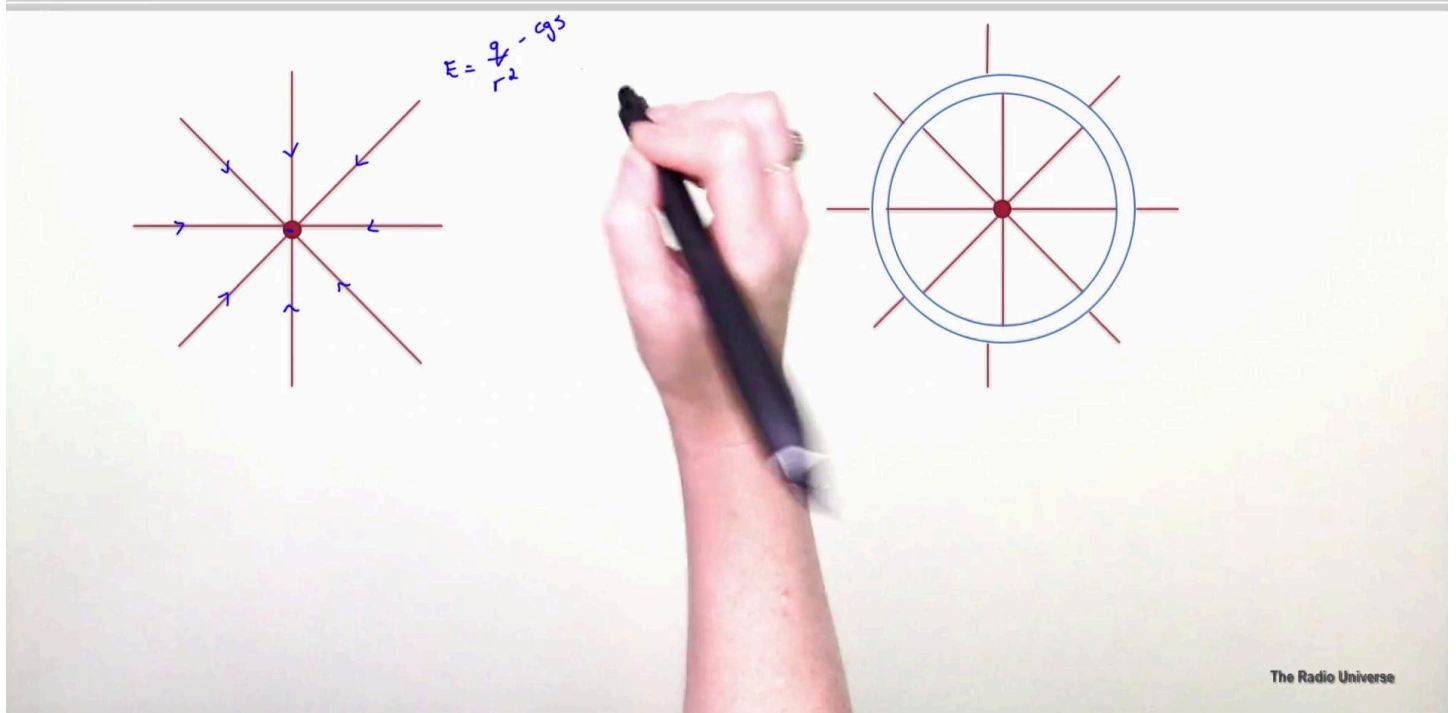


0m 05s





# Emission from an accelerated charge



From Maxwell's equations, we can say that an accelerating charged particle creates a change in electric field and thus the change in magnetic field and electromagnetic radiation is just a change in electric magnetic field. So accelerating charged particle generates electromagnetic radiation. In this lecture, we'll derive the intensity and the angular distribution of this emission from an accelerating charged particle. If you say you have a stationary charged particle, for example, if this was a negatively charged particle then you know that this particle would be surrounded by an electric field represented here by these radials electric field lines and these this electric field would drop off as the distance squared so that the electric field is given by this equation. It's equal to the charge over the distance squared. In SI units, you would replace this expression with you add a constant to this expression so you'd say 'e' is equal to 'kq' over 'r' squared. But in CGS units the constant is one and so it's safe enough to just drop this from the equation.

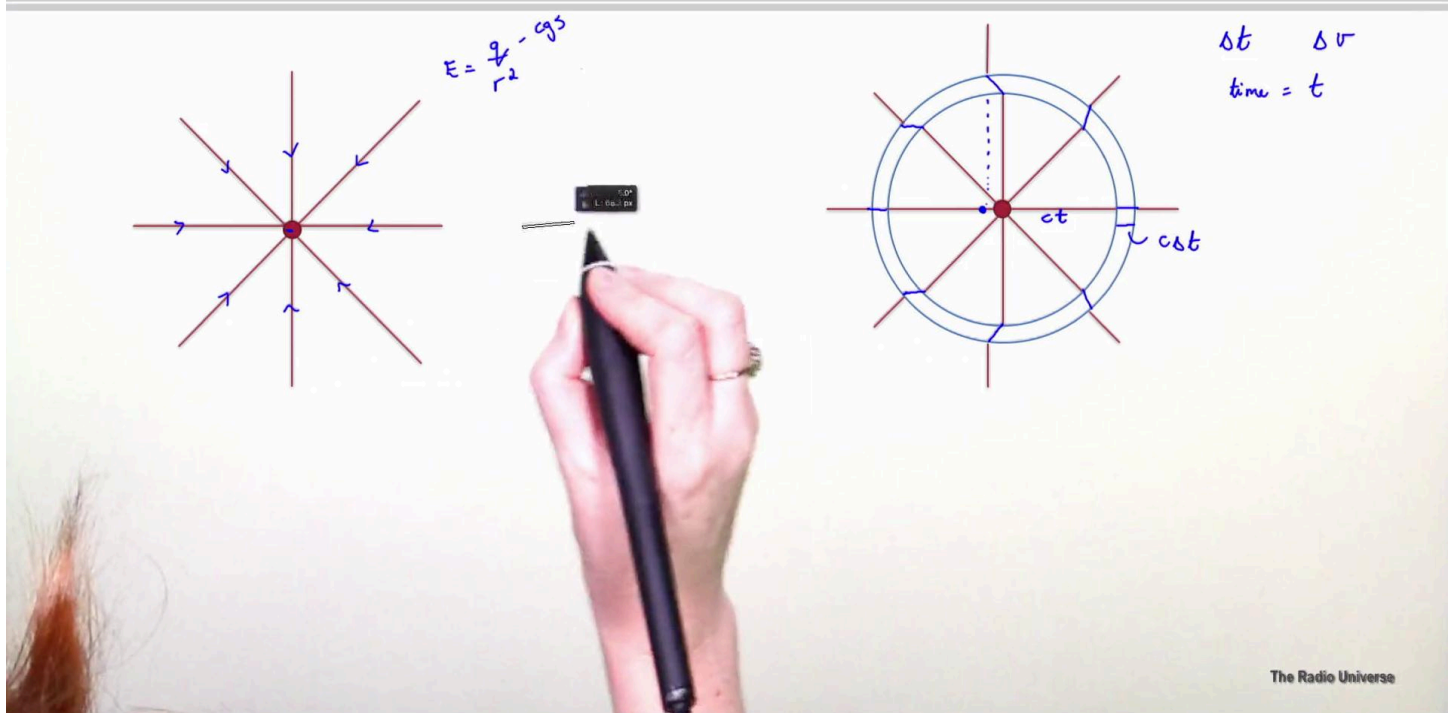
Notes

Summary



1m 45s

# Emission from an accelerated charge



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If you now imagine that you accelerate this charged particle for some very short time  $\Delta t$  so that it has a change in its velocity of  $\Delta v$  and then you re-observe the system at some time later, time  $t$ , what you would see is that the information about the acceleration of this charged particle can only travel as fast as the speed of light. So information about the fact that this charged particle has had a change in its velocity has only propagated some finite distance which is given by the speed of light times the time since the event. So inside this radius, the radial field lines will now be centered on the new position of your charge particle but outside of this radius the field doesn't yet know about the change in this charged particle's velocity and so the field lines here will still be centered on the old position of your charged particle and so there will be some very narrow region here given by  $c \times \Delta t$ . There will be a small region  $c \times \Delta t$  where there will be a distortion between the inside and outside field lines. If we now zoom in on one of these field lines then what you see would look something like this.

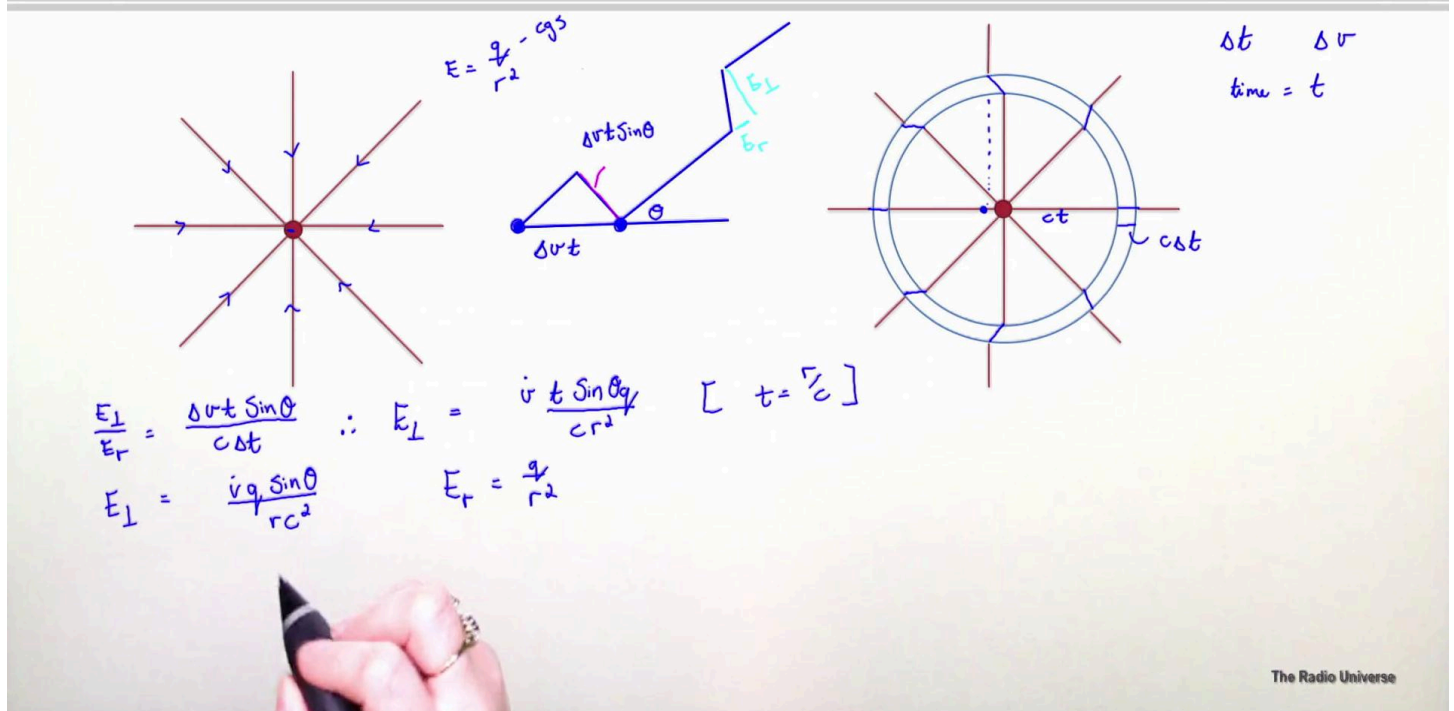
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3m 03s

# Emission from an accelerated charge



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If this is the original position of your charged particle then it has moved some distance which is given by the change in its velocity times time. If this is an angle theta then this component here is given by the expression delta 'v' times time times the sine of theta. And what this tells you is that there's now a perpendicular component to the electric field as well as a radial component to the electric field and that there is a fixed relationship between the perpendicular component and the radial component which is given by this expression. But we already know what the radial component of the electric field equals. It's given by this expression here so that means that the perpendicular component is given by this expression. Delta 'v' over delta 't' is just the acceleration so we can replace this by 'v' dot which represents the acceleration of the charge and into this equation we can use the substitute the relation that the time is equal to the distance over the speed and so from this we derive that the perpendicular electric field component is equal to this. If we now compare the perpendicular electric field component to the radial electric field component, we can see that while the radial component falls off as one over 'r' squared, the perpendicular component only falls off as one over 'r'.

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Summary



4m 35s

# Emission from an accelerated charge

$E = \frac{q}{r^2} - \text{cgs}$   
 $\frac{E_{\perp}}{E_r} = \frac{\Delta v t \sin \theta}{c \Delta t} \therefore E_{\perp} = \frac{q \sin \theta}{rc^2} [t = \frac{r}{c}]$   
 $E_{\perp} = \frac{q \sin \theta}{rc^2}$   
 $E_r = \frac{q}{r^2}$   
 Poynting flux  $\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{B}$   $|\vec{E}| = |\vec{B}|$   
 $|\vec{S}| = \frac{c}{4\pi} E^2$

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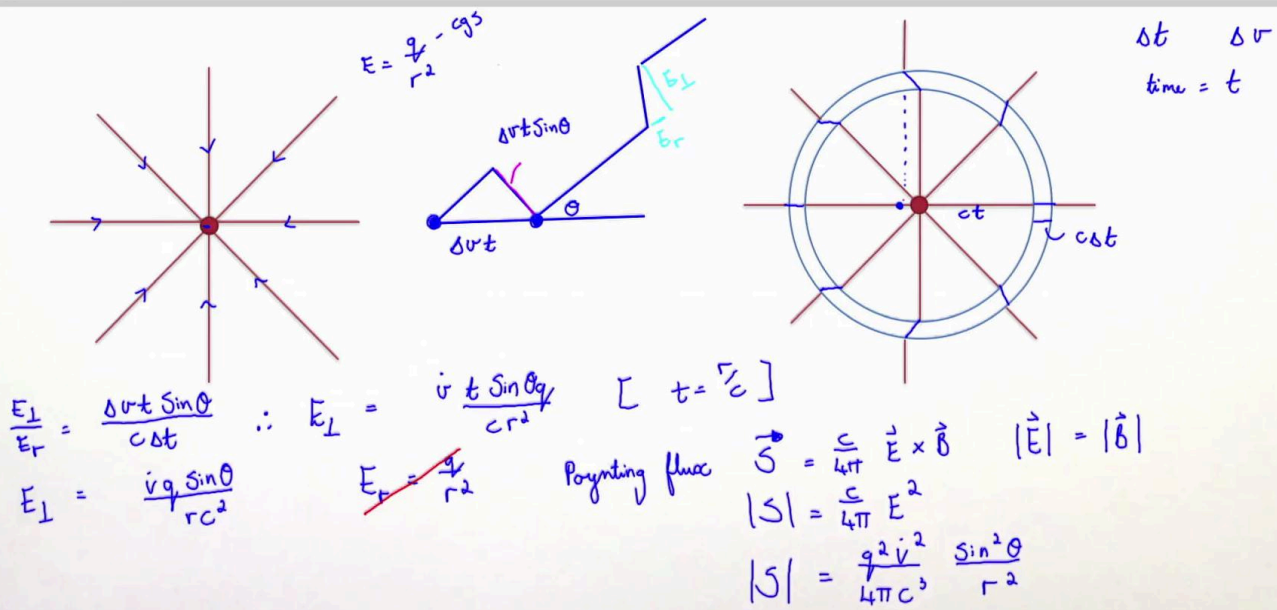
So from a very far away distance the radial component would be dominated by the perpendicular component and can be mostly ignored. This expression for the perpendicular component tells us that the electric field changes directly as the particle accelerates. So if you have a particle that's accelerating and it's changing its velocity as a sinusoid then you will have an electric field which also changes as a sinusoid. So the electric field changes exactly in step with the changing of the velocity of the charged particle. We can calculate what the radiated power would be for a charged particle that's emitting electromagnetic radiation using the equation for the Poynting flux. This is given by this expression where the Poynting flux is equal to 'c' over four Pi and the cross product of the electric and magnetic field vectors. But in CGS units the magnitude of the electric field is equal to the magnitude of the magnetic field and so we can rewrite this as being 'c' over four Pi that the magnitude of the flux is equal to 'c' over four Pi times the electric field squared where the electric field is just given by the perpendicular component of the electric fields as this becomes negligible at large distances.

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Summary



# Emission from an accelerated charge



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So this gives us the expression that it gives us the expression that an accelerated charge particle radiates with a dipole power pattern and that the power is proportional to the acceleration squared.

Notes

Summary





# Emission from an accelerated charge

$$|S| = \frac{q^2 \dot{v}^2}{4\pi c^3} \frac{\sin^2 \theta}{r^2}$$

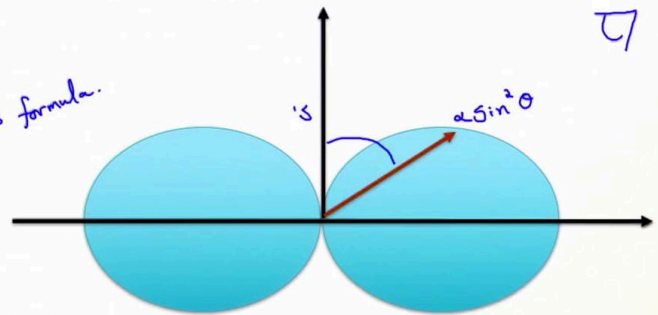
Total Power

$\int_{\text{sphere}}$

$$|S| dA = \frac{2}{3} \frac{q^2 \dot{v}^2}{c^3}$$

Larmor's formula.

$$\propto \dot{v}^2$$



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So we've just learned that an accelerating charge produces a power per unit area that's equal to this equation and so what that tells us is that an accelerating charge has a dipole pattern where the the outside of this dipole pattern is given by the expression. It's proportional to sine squared theta where theta is the angle between the viewer and the acceleration of the charge so if this is the acceleration vector then this is the dipole and this dipole looks roughly like a doughnut so these two lobes would come out of the page and join in a doughnut shape approximately. From this it's possible to work out the total power radiated over all angles by integrating this expression over all angles which gives us this equation. This expression is called Larmor's formula. Confusing me sometimes this other form of the equation is also called Larmor's formula. The important thing to take away from this is the shape of the radiated emission and the fact that it's proportional to the acceleration squared. The largest accelerations in the universe are produced by electrostatic forces. Remember the gravity of the comparatively weak force compared to the electrostatic force.

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8m 20s

# Emission from an accelerated charge

$$|S| = \frac{q^2 \dot{v}^2}{4\pi c^3} \frac{\sin^2 \theta}{r^2}$$

Total Power

$$\int_{\text{sphere}} |S| dA$$

Larmor's formula.

$$= \frac{2}{3} \frac{q^2 \dot{v}^2}{c^3}$$

$$\propto \dot{v}^2$$

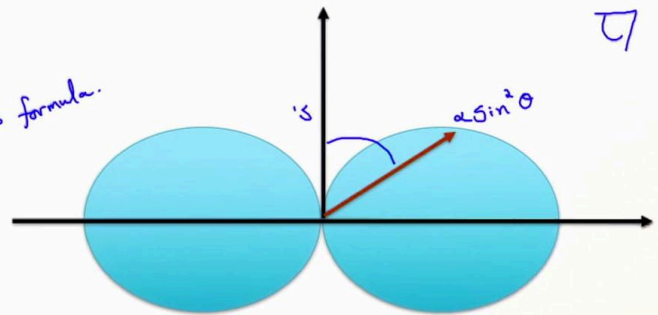
Electrostatic Force

charge/mass ratio

$$F = ma$$

	$q$	
e	1	$9.11 \times 10^{-28} \text{ g}$
p	1	$1.66 \times 10^{-24} \text{ g}$

$$\left(\frac{m_p}{m_e}\right)^2 = 4 \times 10^6$$



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And so what determines the acceleration of a charged particle is essentially its charge to mass ratio. Because remember that force equals mass times acceleration and the force is only dependent on the charge of the particle. So for the same charge if you have a greater mass, you'll have a smaller acceleration and vice versa. Now, for example, an electron and a proton have roughly equal charge. However, the mass of an electron is significantly less than the mass of a proton. An electron has a typical mass of around 9.11 times ten to the minus 28 grams while a proton has a much larger mass. And so if we compare the mass of a proton over the mass of the electron and square it then we get something like 4 times ten to the 6. And what that tells you is that the acceleration of the electron will be significantly more than the proton for the same force and that because of this larger charge to mass ratio, that there will be roughly 4 times ten to the 6 more emission associated with an electron compared to a proton. And so that tells you that most of the radiation in the universe is generated by electrons and the comparative amount of radiation generated by a proton is significantly less.

Notes

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9m 48s