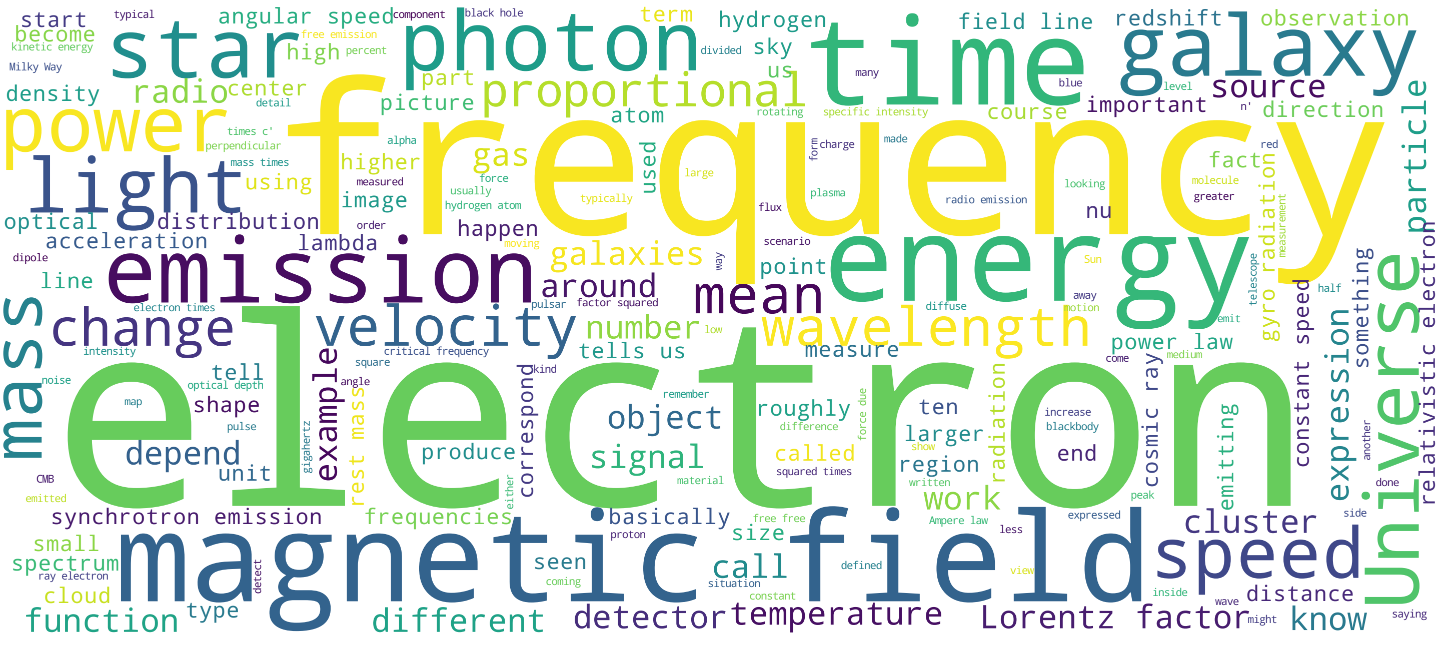
Continuum Emission (part 3)

Kim McAlpine



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Video

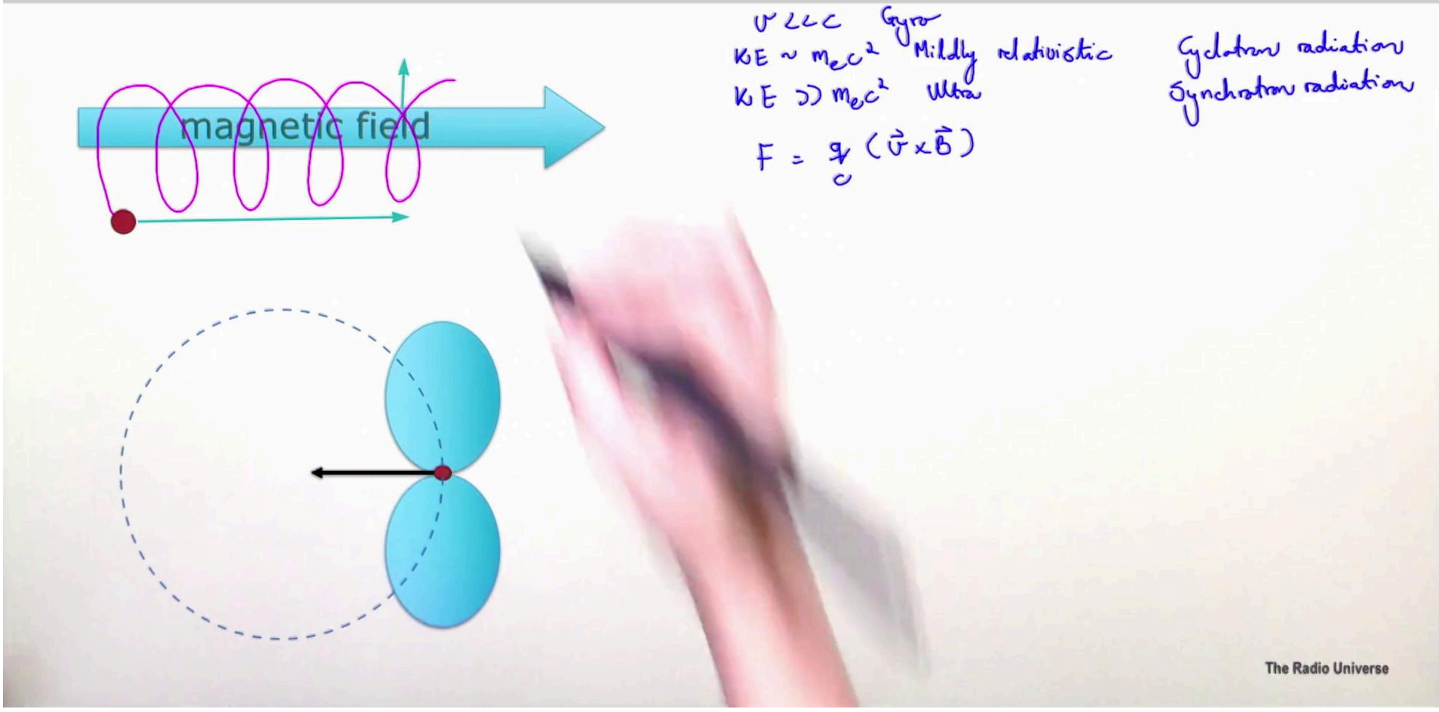
Today's lecture focuses on synchrotron emission. Synchrotron emission occurs when electrons accelerated to very high near relativistic speeds interact with a magnetic field line. In this case, the electrons will spiral around the magnetic field line and as they're accelerating they will emit radiation. In today's lecture, we'll learn all about how we can use synchrotron emission to learn something about the cosmic ray electrons which are producing this emission.

Notes

Summary

0m 05s

Magnetobremsstrahlung emission

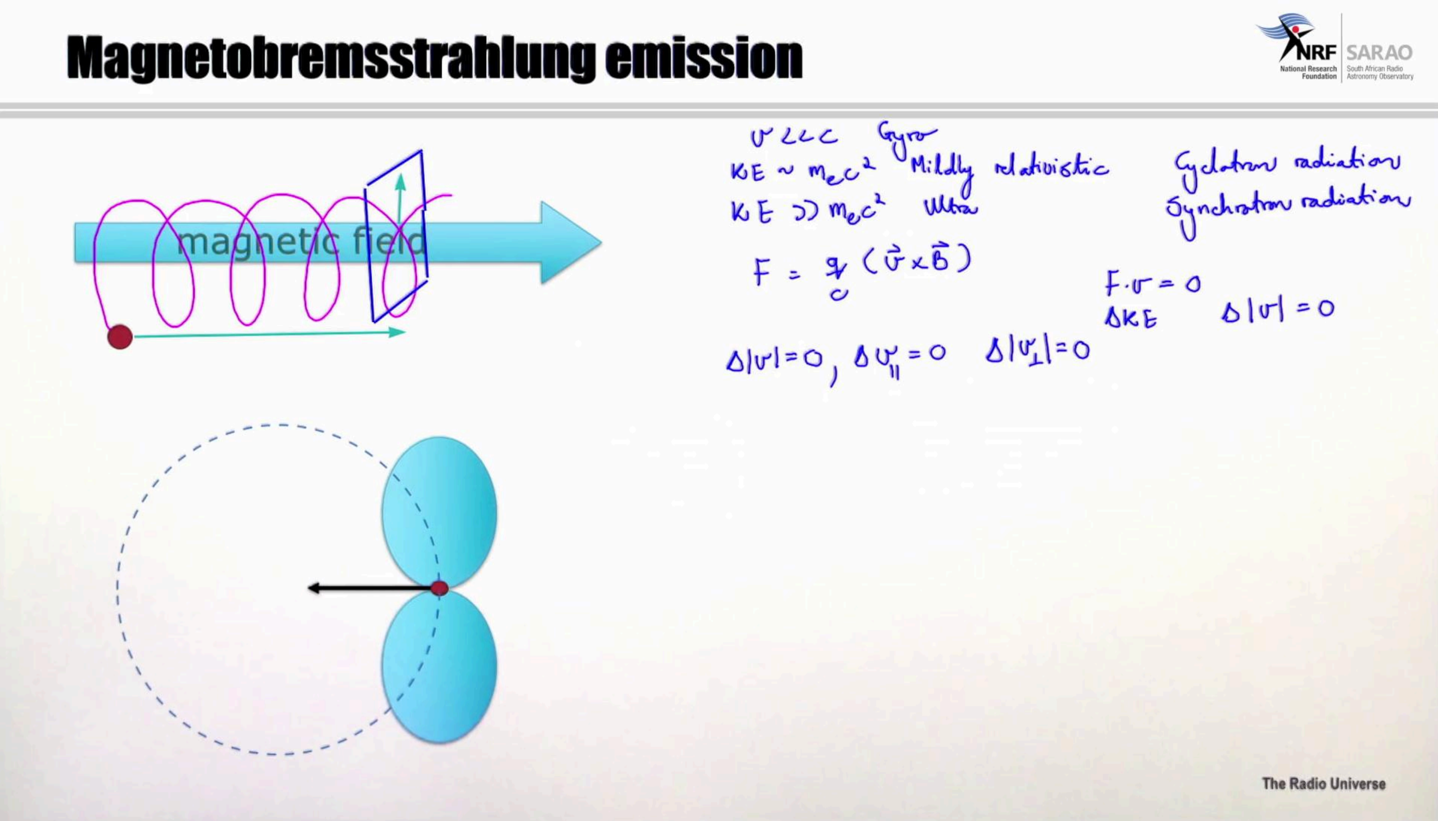


Magnetobremsstrahlung emission occurs when an electron is spiraling around a magnetic field line. The property of the emission and the name of the emission in this case depends on the speed of the electron. For very low speeds, where 'v' is very much less than the speed of light, we get gyro radiation. If the speed of the electron is such that its kinetic energy is approaching the rest mass of the electron times 'c' squared then we say that the particle is mildly relativistic and we have cyclotron radiation. If the kinetic energy is very much greater than the rest mass of the electron times 'c' squared then we say that it is ultra-relativistic and we have synchrotron radiation. When you have an electron and a magnetic field interacting with each other, the electron will experience a force according to Ampere's law which is given by the expression that the force is equal to the charge divided by 'c' times the cross product of the velocity of the electron and the magnetic field. This electron will have a velocity component in the direction that's parallel to the magnetic field and also a velocity component in the plane that's perpendicular to the magnetic field.

Notes

Summary

0m 34s

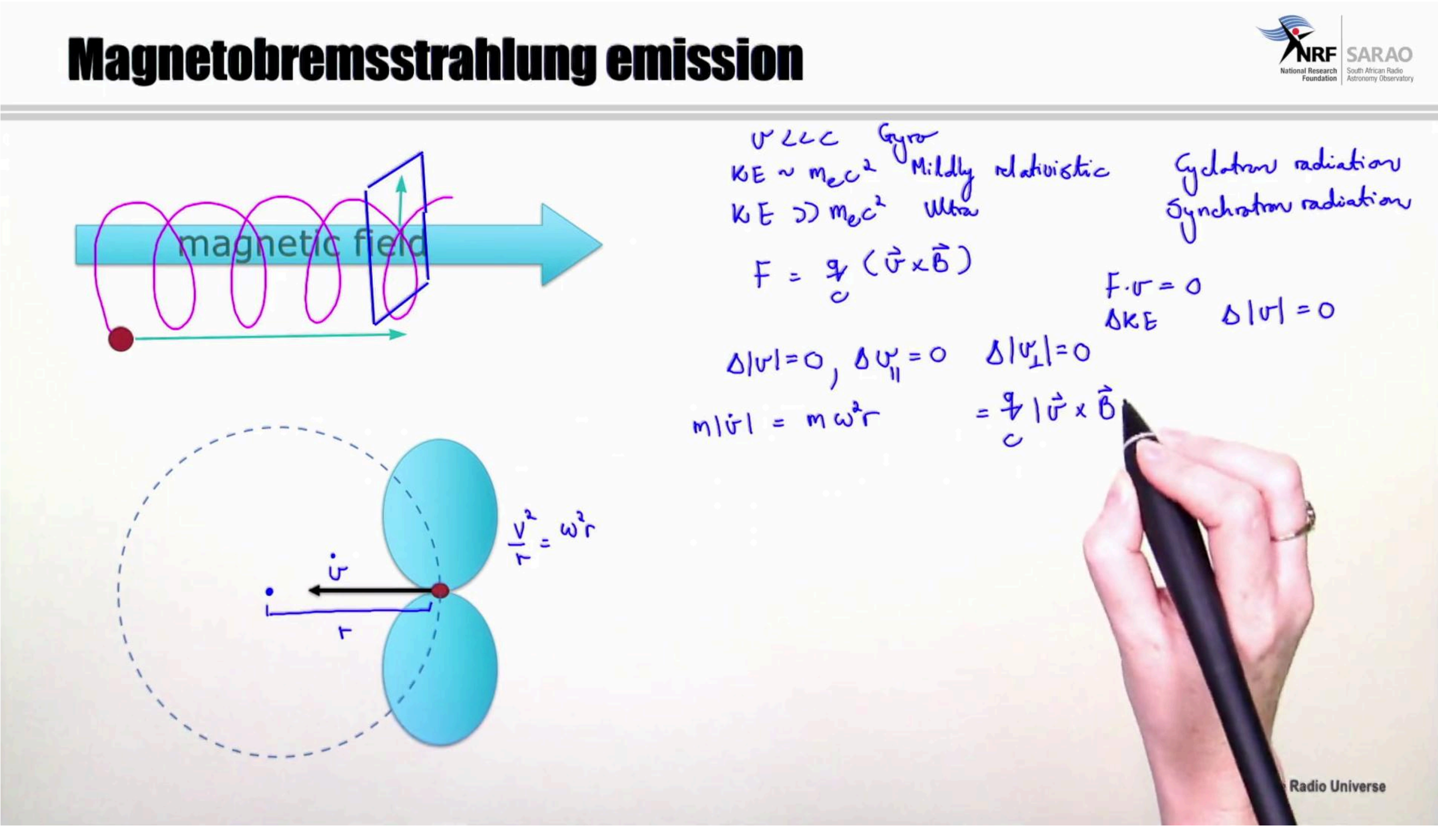


In the situation that we're considering, we can say that the force due to the magnetic field is everywhere perpendicular to the velocity of the electron and so that means that the dot product of the force times the velocity is zero and that no work is done on the electron by the magnetic field and if no work is done there can be no change its kinetic energy and so, therefore, there's no change in the speed of the electron. If we can say that there's no change in the speed of the electron then what we can also see is that in the direction that is parallel to the magnetic field, the electron doesn't change directions either and so, therefore, the velocity parallels that magnetic field doesn't change. And so because of that we can also say that the speed in the in the plane that is perpendicular to the magnetic field doesn't change so there's no change in the magnitude of the velocity in the plane perpendicular to the magnetic field. In this situation we can analyze the problem in the inertial frame that moves at constant speed with the electron in the direction parallel to the magnetic field and so what we have then is circular motion at a constant speed around the magnetic field line which is much simpler problem to analyze than considering this three-dimensional situation.

Notes

Summary

1m 55s

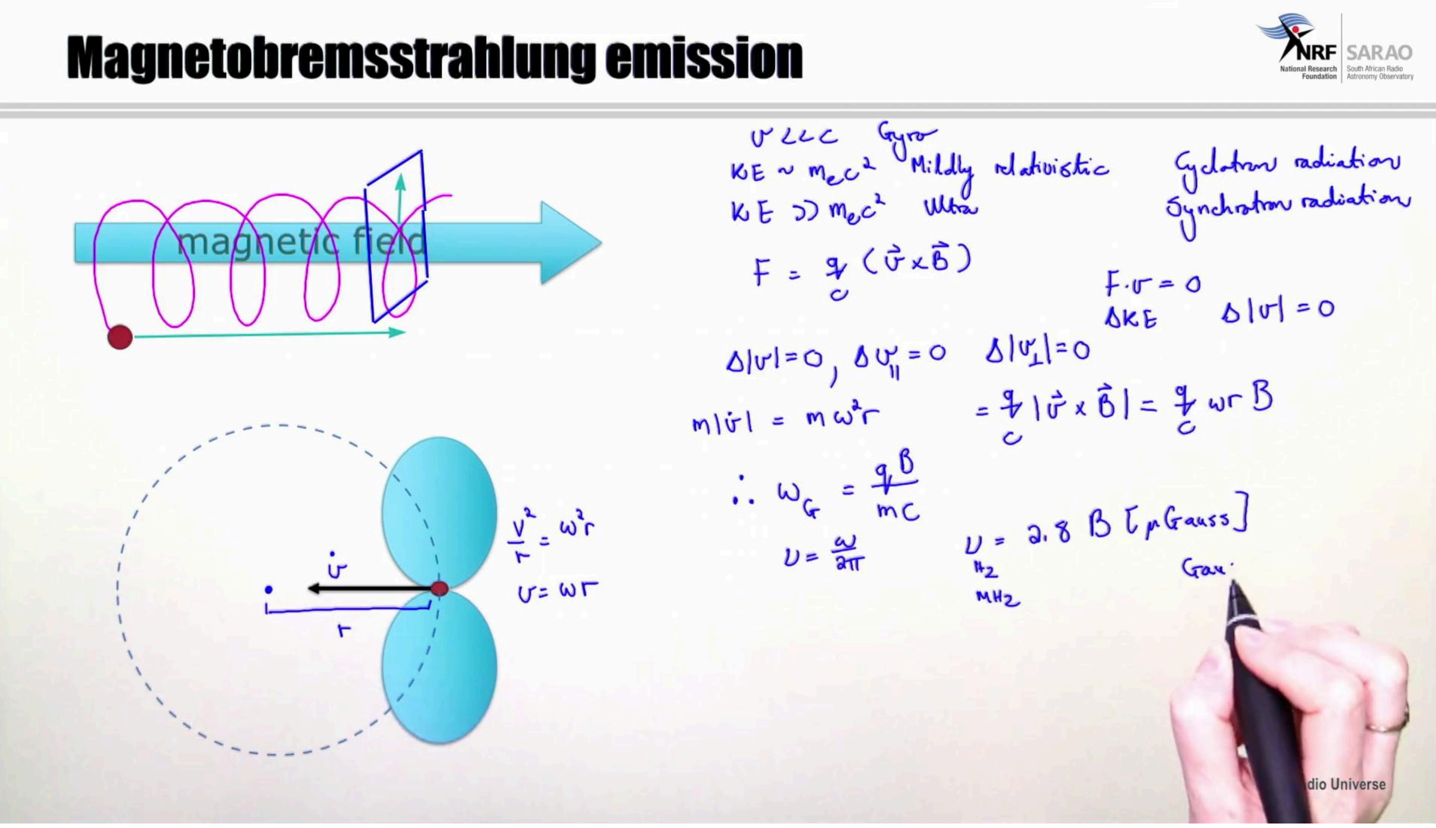


So in this scenario what you have is the magnetic field line which is coming directly out of the page. You have the electron which is experiencing a force due to Ampere's law and it is rotating in circular motion with constant speed because there's an acceleration in the direction of the magnetic field line here. You have an acceleration vector here and so that means the electron is radiating as a dipole in the direction perpendicular to the acceleration vector as we learned about in the earlier lectures as Larmor's formula. So what we can do in this scenario is we can use our usual trick of saying that the mass times the acceleration of the particle must be equal to the force on the particle and so if we say that then we have the mass times the centripetal acceleration here then that must be equal to the force due to Ampere's law. The centripetal acceleration is given by the speed squared over ‘r’ where ‘r’ is the separation here. But that can also be written as the angular speed Omega squared times ‘r’ so the mass times the acceleration which is the centripetal acceleration is equal to the force due to Ampere's law.

Notes

Summary

3m 27s

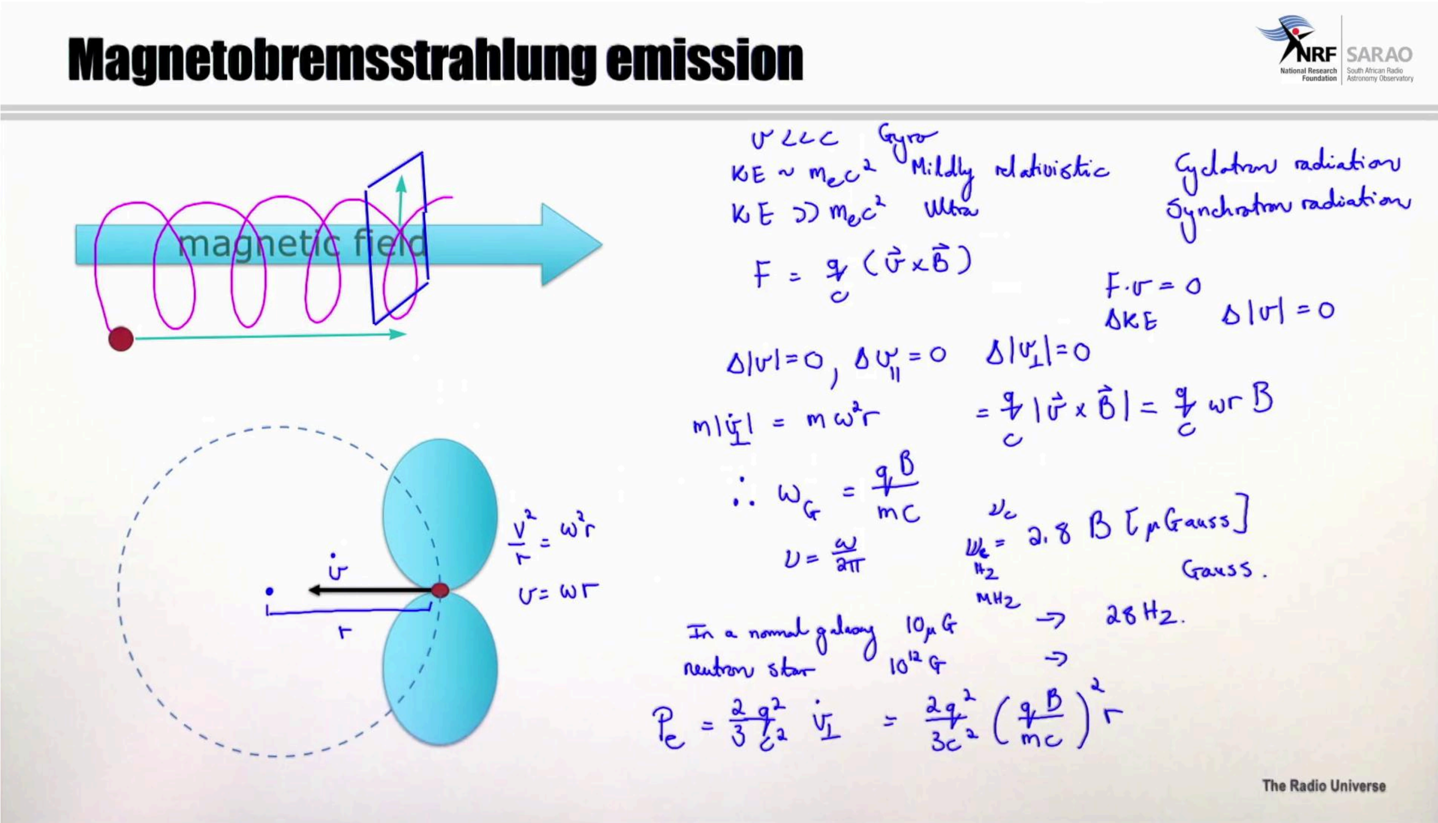


Now the magnetic field line is perpendicular to the velocity of the electron and so the cross product here is just equal to the speed times the magnitude of the magnetic field where the speed here can also be written as angular speed times 'r'. So we can write this as... If we rearrange this equation then we can calculate what the angular speed is of the motion of the electron in this circle where it has a constant speed and the angular speed will be equal to this expression where we can see that this expression does not depend on the velocity of the electron. So for non-relativistic particles they rotate around a magnetic field with a constant speed in the parallel direction. Also with a constant speed in the perpendicular plane and that constant speed is independent of the velocity of the electron. From this orbital angular speed, we can work out what the frequency of this electron is where the frequency is given by the angular speed over two Pi and if we plug all the right numbers in here then what that tells us is that the frequency here is equal to roughly 2.8 times the magnetic field where the frequency here is given in Hertz and the magnetic field here is given in microgauss or equivalently we can give the frequency in megahertz and give the field here in in units of gauss.

Notes

Summary

4m 49s

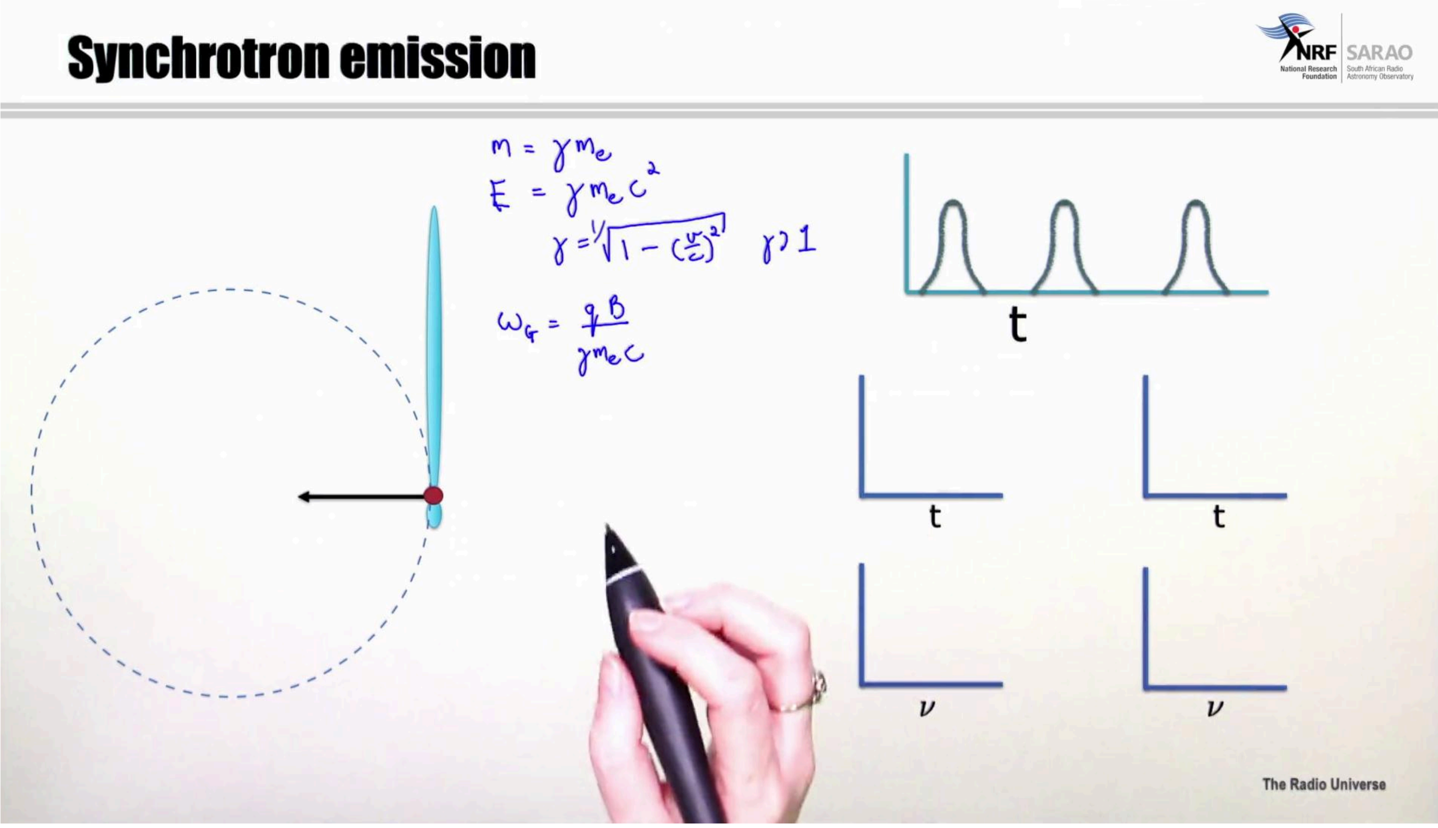


We call this frequency, the frequency here the critical frequency so you can call that nu 'c' and this critical frequency is the frequency of the emission that we'll see from the electron. So in a normal galaxy, you might have a typical interstellar magnetic field of around 10 microgauss which implies that the gyro radiation frequency would be around 28 Hertz. This is below the frequency where waves can propagate in a plasma in ICM. So you will never see this type of gyro radiation from a galaxy. However, the situation is different where you have extreme magnetic fields, for instance, in a neutron star where you can have fields as strong as 10 to the 12 gauss and so in that case you will have a much higher frequency emission for this gyro emission mechanism and so you will see some emission. We can now work out the power per electron using Larmor's formula for this scenario where the power per electron is given by the usual formula times the acceleration of the electron where the acceleration is just the acceleration in the plane perpendicular to the magnetic field because there's no acceleration in the parallel direction here. And this is given by our expression here so that's simply equal to...

Notes

Summary

6m 32s

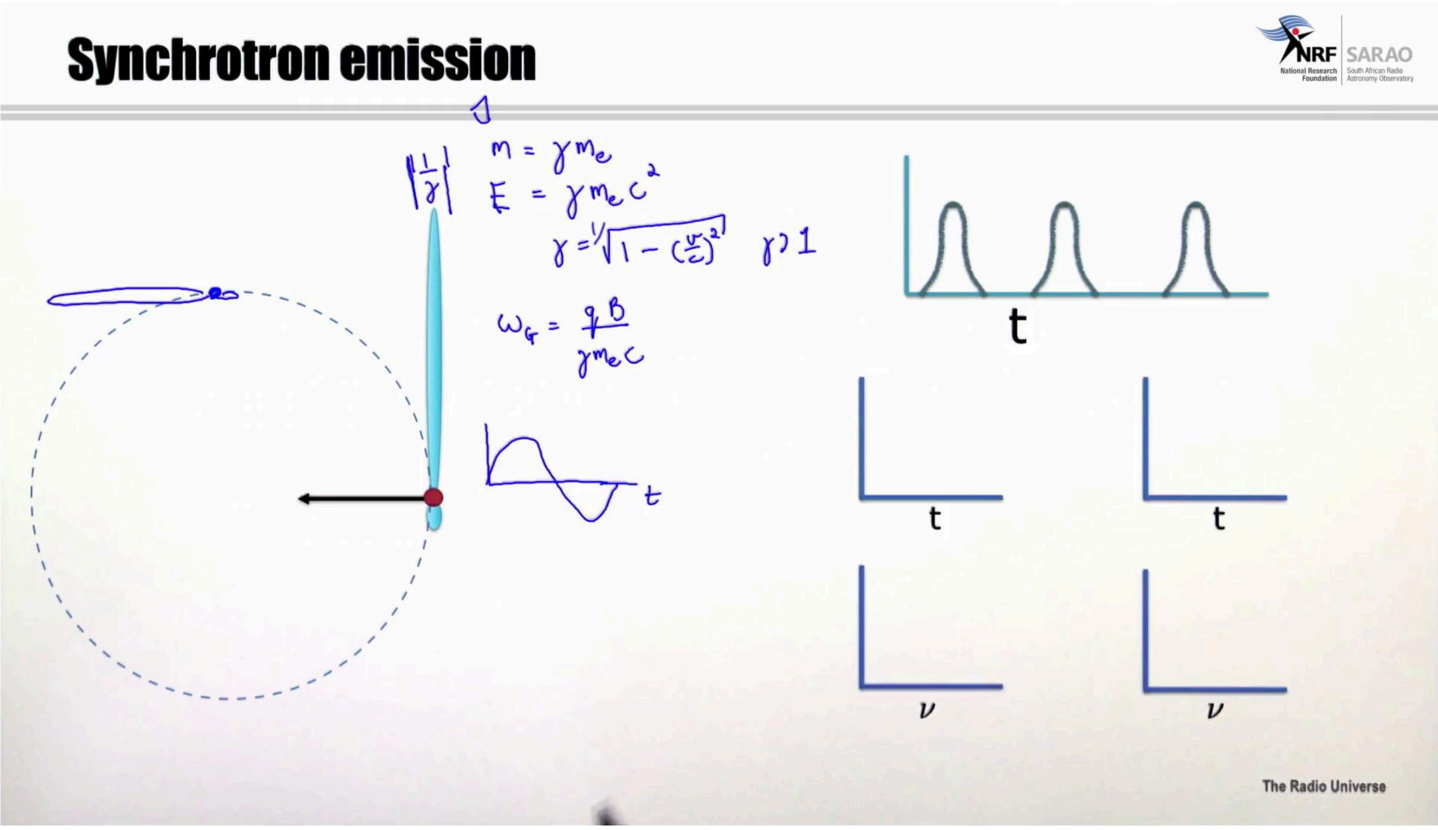


We've already established that you don't see radio emission from non-relativistic electrons except in the case of really extreme magnetic fields but you do see emission from relativistic electrons and then it's called synchrotron emission. In this case, we have to make a number of relativistic corrections to our formulas that we've been using. We aren't going to a lot of detail here but we'll just say that in the relativistic case the mass of the electron now becomes its relativistic mass which is equal to the Lorentz factor times the rest mass of the electron and the energy of this electron given by the usual formula that 'E' equals 'mc' squared is now equal to the Lorentz factor times the rest mass of the electron times 'c' squared where the Lorentz factor is equal to this where this is the speed of the electron. In this case we'll modify the speed of the angular rotation of the electron because we have a different mass so we'll modify our previous expression for the angular speed of the electron by saying that the mass is not equal to the relativistic mass. And so what this tells us is that the angular speed of rotation here will decrease a lot when the Lorentz factor is greater than one which is the case for ultra-relativistic particles.

Notes

Summary

8m 09s

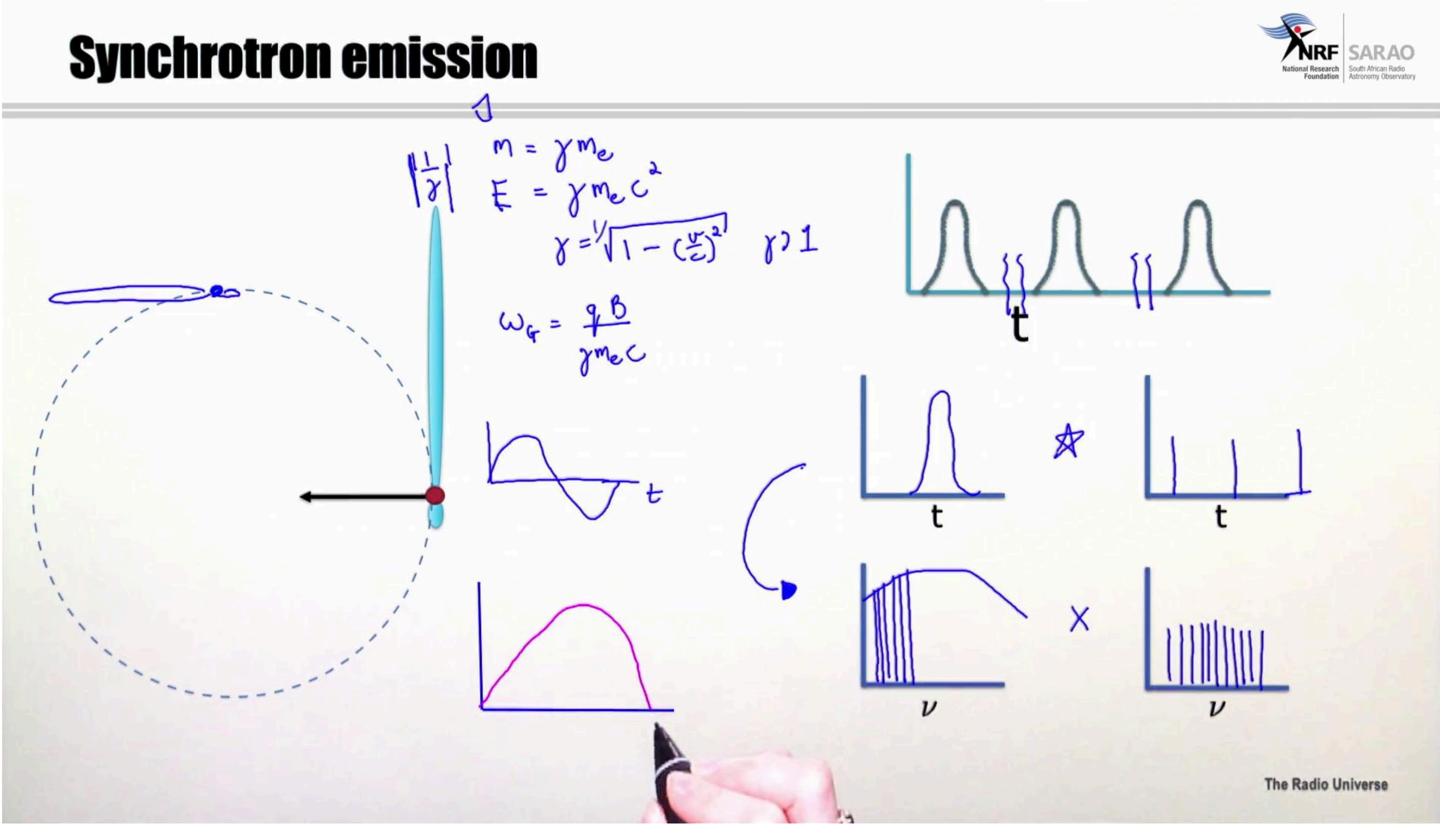


So what that tells us now is that the frequency of the rotation here will depend not only on the magnetic field but also on the speed and energy of the electron that we're dealing with. Other changes that happen in the relativistic case is that the normal dipole that we have from Larmor's formula becomes the very strongly beamed and squashed in the direction of motion of the electron. So what happens is that your dipole gets much stronger in the direction it's moving and it gets very much narrower so that the width here is roughly equal to one over the Lorentz factor. So if you're an observer looking at this radiation from some distance away before when we had the dipole, what you would have seen would have been a fairly regular sinusoidal change in the power as this electron rotated around the magnetic field line. But now because you have this much narrower pattern and this much slower rotation, if you're an observer looking from far away, you'll see a very brief pulse of emission and then for a long time the electron will be rotating and not beaming any so this if you're the observer here looking from this angle then you'll see a pulse as you travel here but as you as the electron travels around the circle then its emission will be primarily pointed away from you and so you won't see anything at all.

Notes

Summary

9m 44s



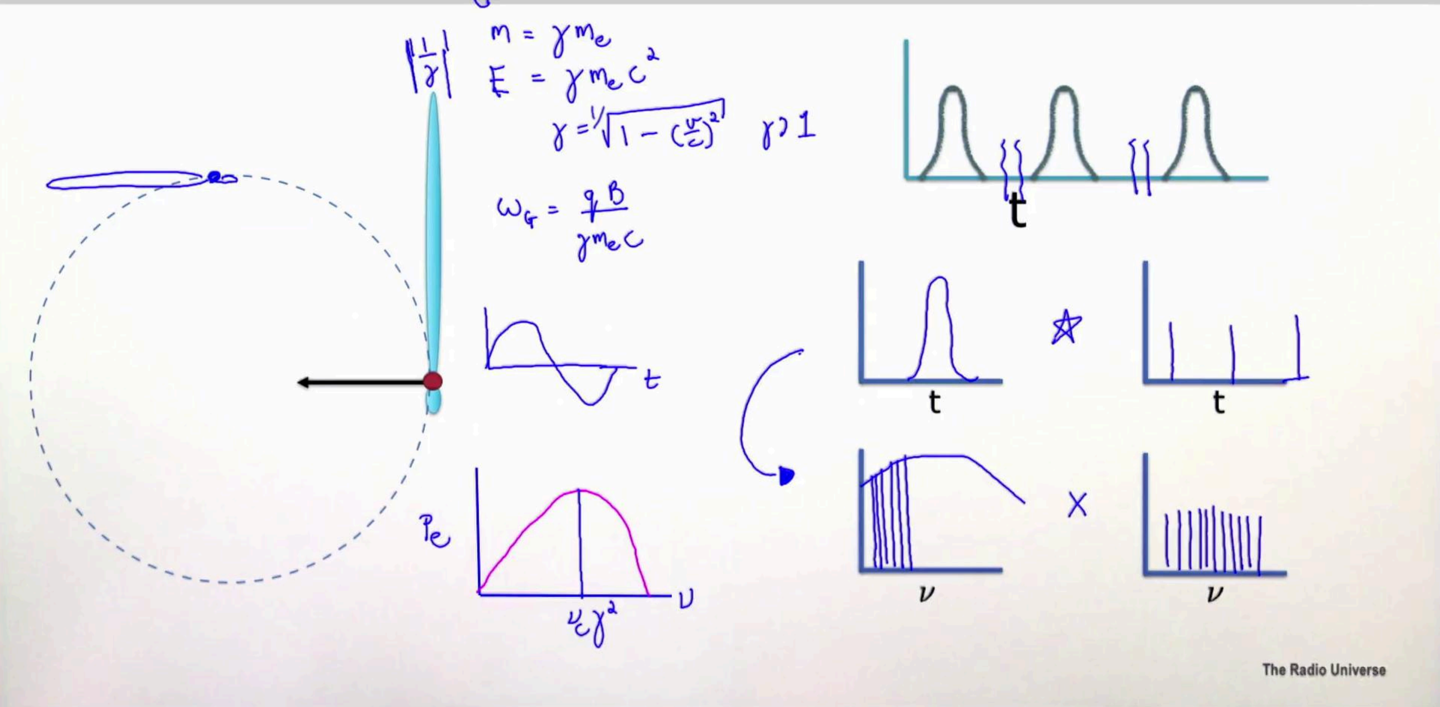
And so from that what you'll see as a function of time is a series of pulses that are separated by quite long times and we can represent this pulse distribution as a pulse as a function of time as being a convolution of a narrow pulse times some sampling function where the time here depends on this Omega 'g' and is longer for higher Lorentz factors. Now we won't go into too much detail about Fourier transforms because they'll be dealt with more thoroughly later in the course but just to say that we can convert the signal that we see in time to a signal that we see in frequency using a Fourier transform and in that scenario what tends to happen is that narrow features in time become wide features in frequency and so this very narrow pulse profile that you see in time will become a very wide frequency pulse and these very widely spaced samples in time will become very narrowly spaced samples in frequency and a convolution in time will now become a multiplication in frequency so if we multiply this wide distribution by this very narrow sampling frequency, sampling function in frequency then what you end up with is something that looks approximately like a continuous distribution and so we get continuous emission and the shape of this continuous emission looks roughly like this.

Notes

Summary

11m 23s

Synchrotron emission

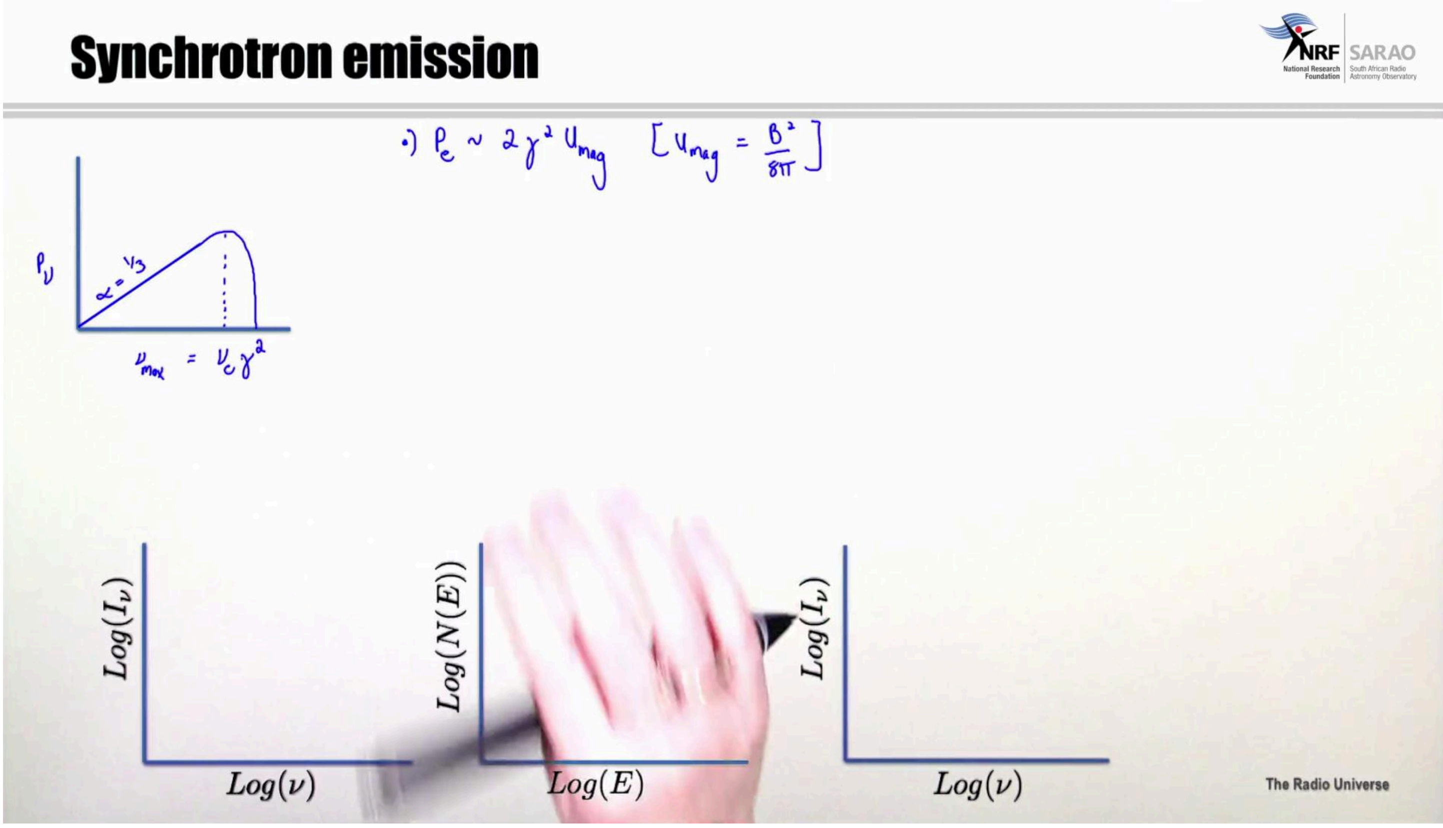


It spans a range in frequencies but it peaks at a well-defined frequency which is equal to our critical frequency from the previous slide times the Lorentz factor squared.

Notes

Summary

13m 04s

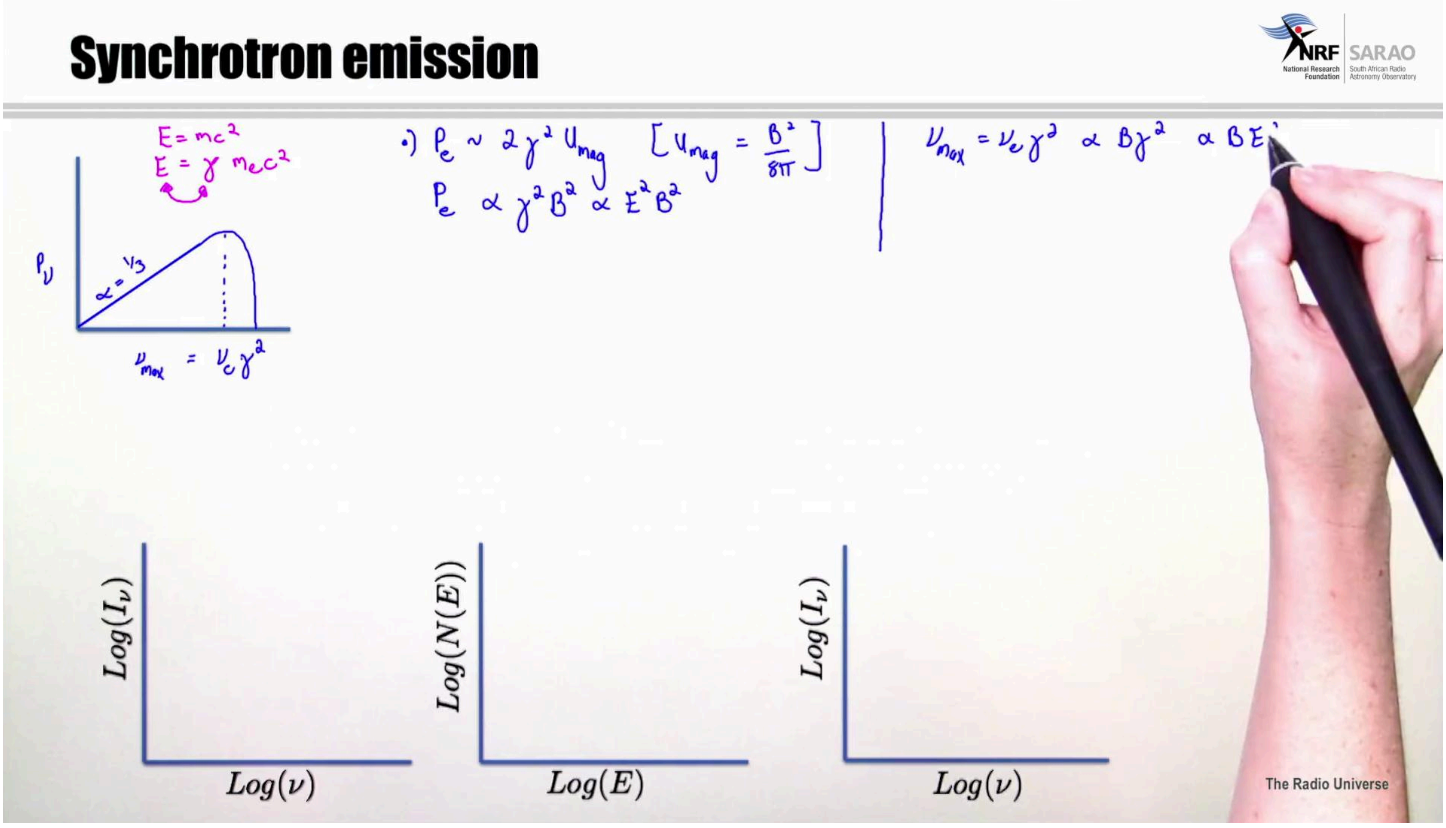


We learned in the previous slide that a relativistic electron emits radiation with a characteristic shape as a function of frequency. It looks roughly like this where the bottom part of this profile has a spectral index of roughly a third and it peaks at a very specific frequency which is equal to the critical frequency times the Lorentz factor squared. The interesting thing about this is that previously with gyro radiation the emission was always at this critical frequency or this gyro frequency regardless of the speed of the electron but now if you have a higher speed electron then you have a higher Lorentz factor and then you have a higher frequency for your emission. In order to work out the power for a given electron using Larmor's equation, we need to use a number of relativistic corrections and so we'll skip a lot of those details and just say that the power for a given electron is roughly equal to two times the Lorentz factor squared times the magnetic energy density. It's good to note here that the power for a relativistic electron is very much boosted compared to the power for a non-relativistic electron with gyro radiation. So the power here is much larger by almost a factor of the Lorentz the Lorentz number squared.

Notes

Summary

13m 29s

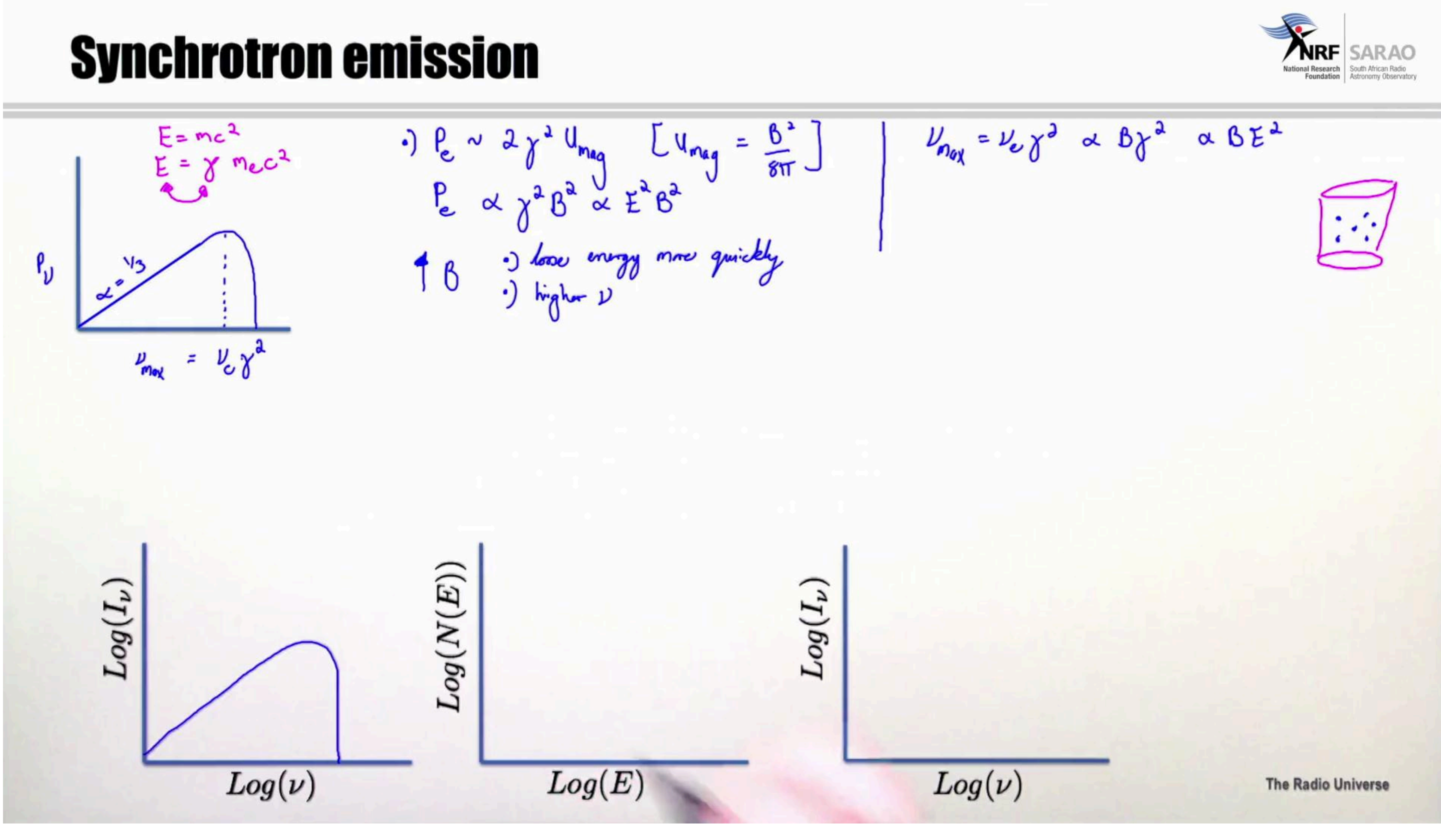


So that is why synchrotron emission is very much more powerful than a free-free emission, the gyro radiation would almost always be less powerful than a free-free emission in a typical ICM situation. So what we can see from this equation is that the power for a given electron is proportional to the Lorentz factor squared times the magnetic field squared. But the energy for a relativistic particle, remember by 'E' equals 'mc' squared, is equal to the Lorentz factor times the rest mass times 'c'. So what this is, is that the energy of the electron is roughly proportional to the Lorentz factor and so from that we can say that the power for given electron is also proportional to the energy of the electron times the magnetic field squared so the energy squared times magnetic field squared. We can also say that they're critical the frequency where the maximum in this radiation happens is equal to nu 'e' times gamma squared but nu 'e' you'll recall from our previous equation was proportional to the magnetic field so this is then proportional to the magnetic field times gamma squared which is then also proportional to the magnetic field times 'E' squared.

Notes

Summary

14m 50s

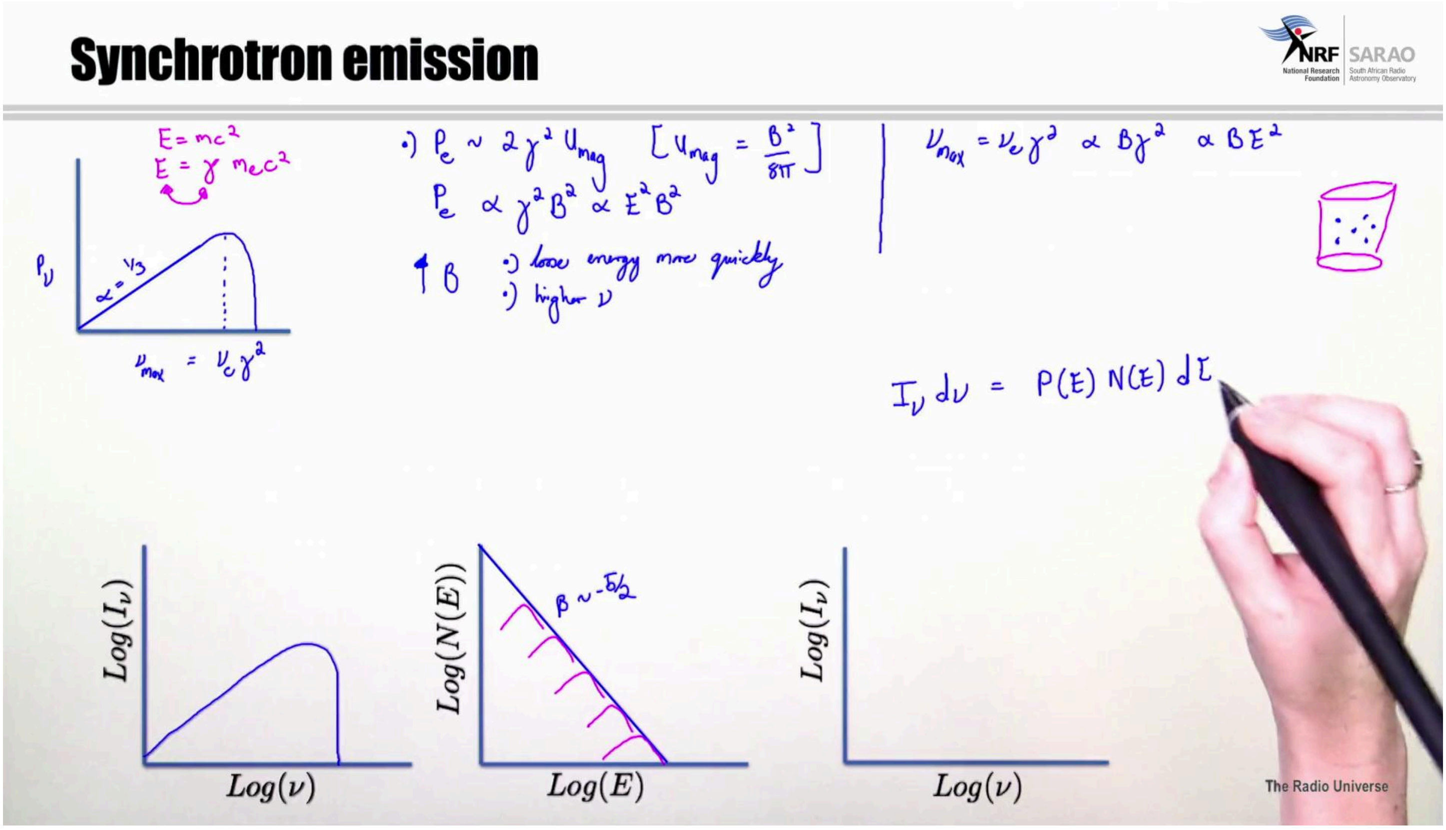


So a consequence of these two equations is that if you have a higher magnetic field for a given galaxy, 'B' is higher then you have a higher power so you radiate energy away faster and you also emit at higher frequencies. So what about the situation for an ensemble of electrons? Instead of just having one electron what happens if we have a whole collection of electrons? Well the interesting thing about this distribution is that the electrons are emitting in a region of frequency space that's relatively narrow and it depends on the energy of the electron so the shape of your emission for a collection of electrons depends on the shape of the energy distribution of the electrons because the energy determines the frequency of the emission. So if you have a given shape for your emission then if you want to know what the shape will look like as a function of frequency for a collection of electrons what you need to know is what does the energy profile of those electrons look like? We have relativistic electrons in our galaxy. They’re called cosmic rays and they have a power law distribution in energies.

Notes

Summary

16m 24s

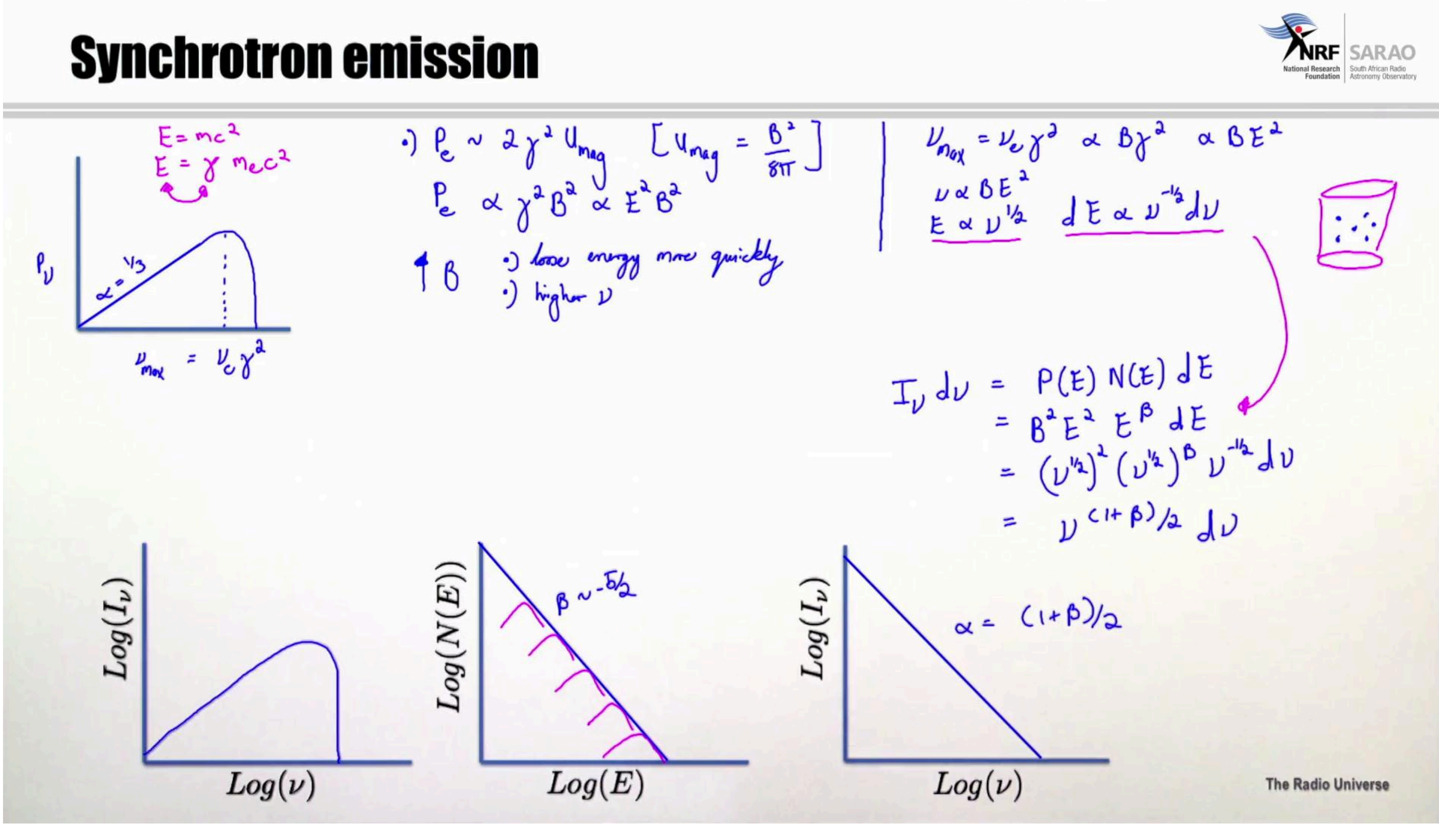


So where we can say that the number of electrons with a given energy is proportional to the energy to some power and so the power law here has a power law index of typically around minus five over two for cosmic-ray electrons in our galaxy. So what we can say is that for any energy here, if we take all the electrons that are emitting at that energy then they'll be emitting this shape radiation at these frequencies. So we can take a whole we can take all of them and add them up over the profile of all of the energies that we have in their galaxy. So from this what we can say is that the intensity of emission at a given frequency and a frequency interval depends on the power for a given energy so the power per electron for a given energy times the number of electrons at that energy times the energy interval.

Notes

Summary

17m 45s

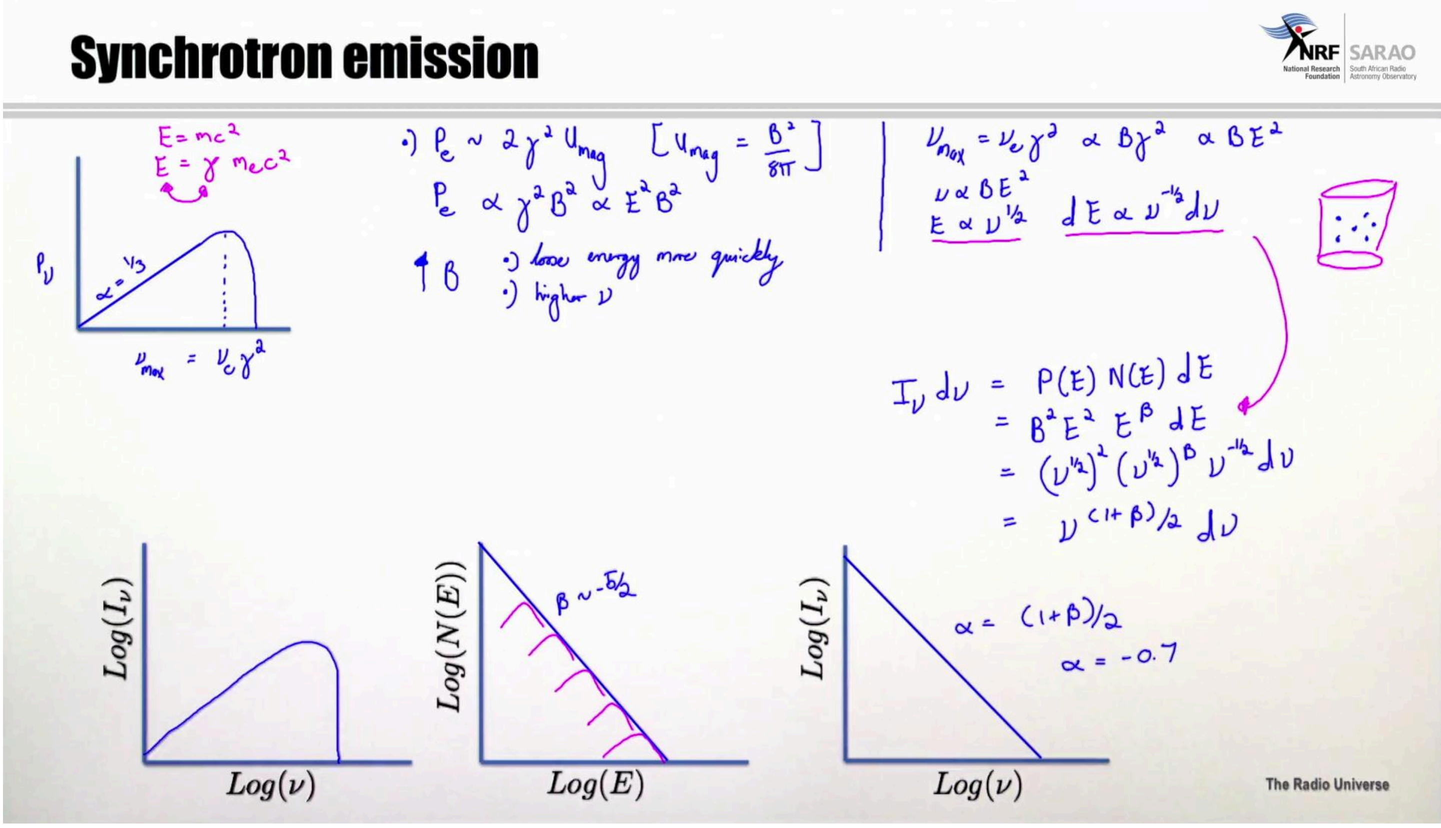


So from here we can substitute that the the power for a given energy is given by this expression and the number of electrons for a given energy is given by a power law so if we want to write this expression in terms of frequencies then what we can use is the fact that we can approximate this entire distribution by saying that nearly all of the emission is emitted near the peak of the distribution so all of the emission is given at this frequency and so then we can say that the frequency of the emission is proportional to 'B' times 'E' squared which means that energy is proportional to frequency to the power of a half which means that 'dE' is proportional to frequencies the power of minus the half 'd' nu. So if we substitute these two expressions into this expression then what we end up with is something like this. So what that tells us is that for a power law distribution in our cosmic ray electrons, we'll have a power law distribution in the specific intensity where the power law index here, alpha is equal to the power law index of the one plus beta over two. So this tells you that the the index for your synchrotron emission tells you something about the electron distributions of your emitting electrons.

Notes

Summary

19m 00s



And for typical galaxies and black holes in the universe, we usually measure a spectral index here of around minus 0.7 and so you can use this value to work out what the typical value of beta would be and the fact that we see such a universal distribution of alpha suggests that the electron energy distribution for cosmic rays is very similar in many situations.

Notes

Summary

20m 38s