Teaching Problem Solving II

Foundations for Teaching in Higher Education

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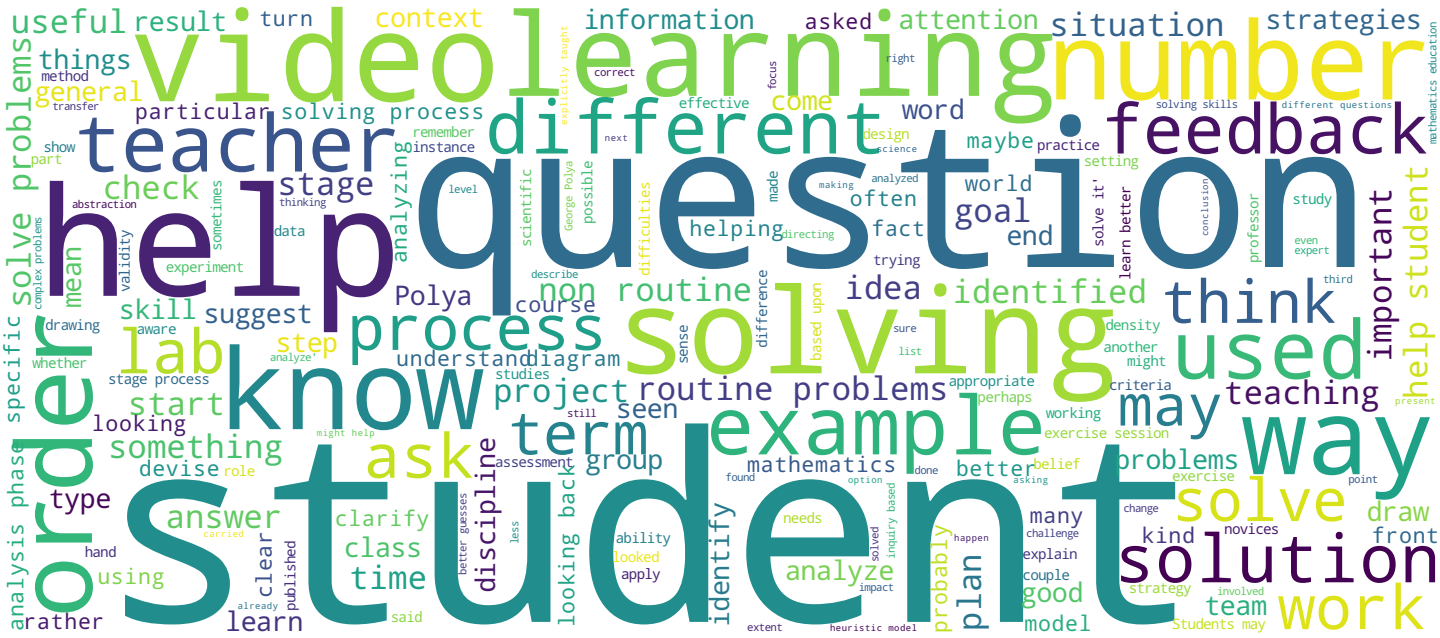

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Video

EPFL

Goals of this video

You should be able to:

- Describe Polya's **4-stage problem-solving process**
- Identify a number of **heuristics** that can play a role in solving problems
- Identify you can use this to help students learn **transferable problem solving skills**

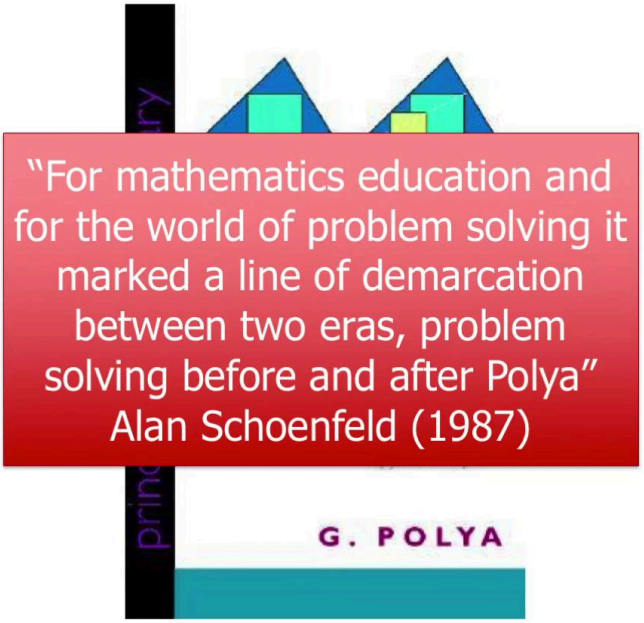
In the first video on problem solving, we looked at the idea that helping students abstract from the specific problem to a more general set of rules can be very useful in terms of helping them to solve non-routine problems. In this video we look at a particular model of abstraction or a particular version of abstraction which can be quite useful to you as a teacher and to the students themselves. So, the goals of this video are number one that you should be able to describe a particular abstract model of the problem-solving process that is the one which was identified first by George Polya back in the 1940s. Secondly that arising from that, that you should be aware of a number of different strategies what Polya called heuristics which can be used to help students solve problems. And thirdly based upon this that you can help students to learn transferable problem-solving skills using Polya's approach.

Notes

Summary

0m 04s

Polya – Problem Solving Heuristics



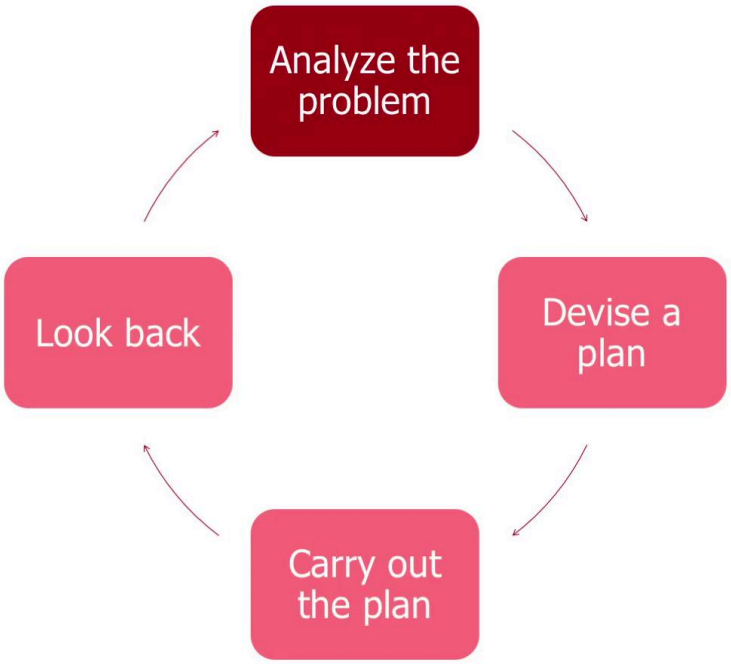
So who is George Polya and where do his ideas come from. First of all, Polya was a mathematician a practicing mathematician who was a professor of mathematics in ETH Zurich from 1914 to 1940. And after that he was a professor in Stanford. In addition to his work on mathematics he was also interested in the process through which people came to solve problems. Polya had two key insights here. First that people learn to solve problems by solving lots of problems but that these problems needed to be structured in such a way that the strategies involved in solving them became explicit to the student. Secondly, he identified that very often people solve problems complex problems by guessing and then testing if their guess worked. His goal was to help people develop better guesses as to where to start. In order to do this he published a number of different books the first of which 'How to solve it' was published in 1945. 'How to solve it' is identified as a key moment in mathematics and mathematics education. Alan Schoenfeld whose work featured heavily in another video here said, for mathematics education and for the world of problem solving it marked a line of demarcation between two eras, problem solving before and after Polya. That's what this book meant in the world of problem solved.

Notes

Summary

1m 05s

Polya's four-stage problem-solving process



In the book 'How to solve it' Polya described solving of non-routine problems as happening in a four-stage process. The first stage of this process is analyzing the problem. Polya actually used the word 'understand' rather than 'analyze' the problem. We've switched that to 'analyze' here because we find that students sometimes take 'understand' the problem to be quite a passive thing whereas 'analyze' the problem emphasizes that this is an active process for the student.

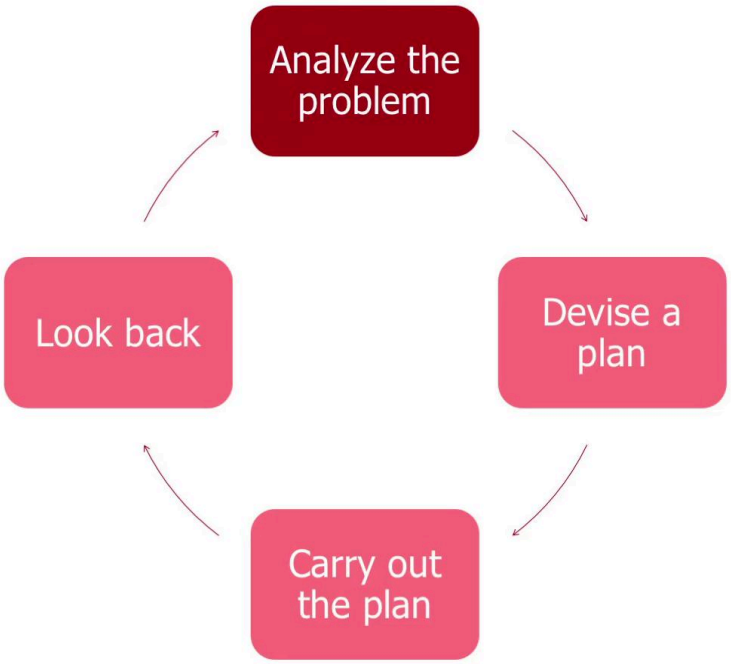
Notes

Summary

2m 39s

Polya's four-stage problem-solving process

- Are there words that you need to define?
- What do you need to find or show?
- Can you restate the problem in your own words?
- Is there a drawing that might help to clarify the problem?



Now within each of these four stages, Polya has identified a number of different heuristics or strategies, rules of thumb, which can be used to help students make better guesses about how to solve the problem. These strategies can often be phrased as questions and it's these questions which you as a teacher can use in order to prompt students in the right direction when solving problems. So for example under the heading of analyzing the problem he proposes a number of different questions which can be used. First of all are there words that you need to define in the problem? Secondly what do you need to find or show in the problem? Can you restate the problem in your words? Is there a drawing that might help to clarify the problem? So, these are questions which can be used to help students to analyze a problem. Some of these questions are quite powerful. So, for example there is research evidence that training in diagramming has some of the biggest impact on learning how to solve problems. Therefore this question as to whether there is a drawing that might help to clarify the problem may well be a very very powerful. But it's not one that's gonna be useful in all settings.

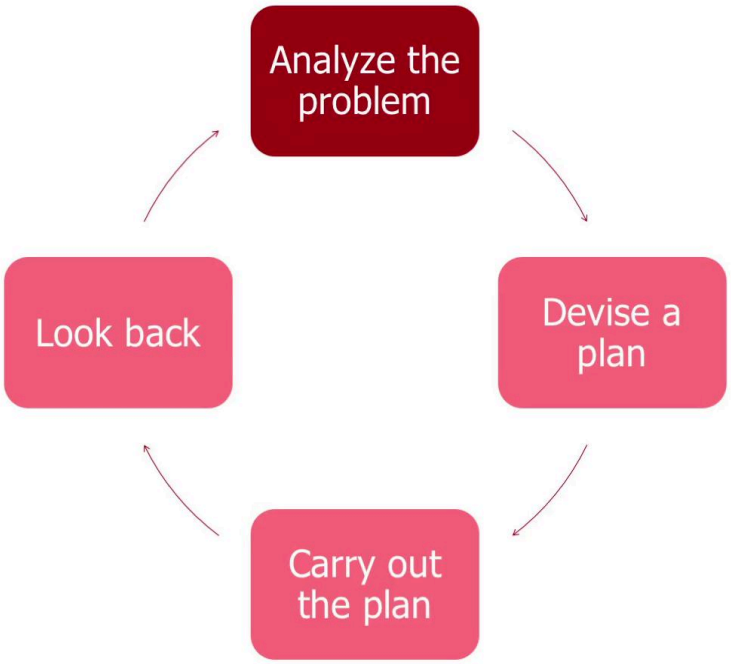
Notes

Summary

3m 11s

Polya's four-stage problem-solving process

- Are there words that you need to define?
- What do you need to find or show?
- Can you restate the problem in your own words?
- Is there a drawing that might help to clarify the problem?



There are many problems in mathematics or chemistry where actually drawing a picture might not help. On the other hand, in physics problems in statics and mechanics, a free body diagram will almost certainly help in the vast majority of cases. And so, these questions while useful remain maybe at still to general a level. Part of the job of the teacher and the teaching assistant is to take these questions and translate them into the phrases or the words which make most sense in the discipline in which they're being used.

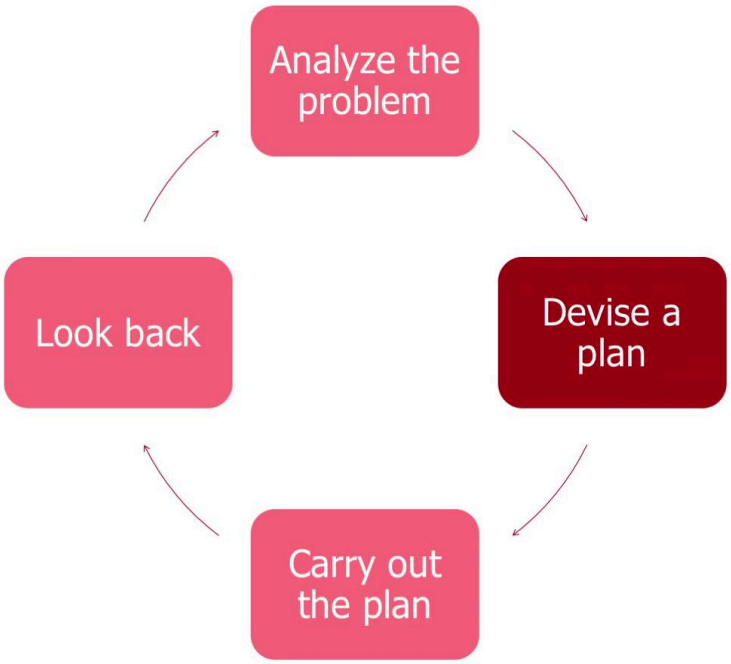
Notes

Summary

4m 30s

Polya's four-stage problem-solving process

- Can you use a problem that you have solved before?
- Can you use a problem with the same unknown?
- Work backward
- Solve a simpler problem



The second stage in Polya's four stage problem solving process for non-routine problems is to devise a plan. Again this is potentially a stage where students will have some difficulty. Students may well be hoping for an intuition. Instead they may need to actively pose questions to themselves in order to be able to allow that intuition to emerge. Polya proposes a number of different questions that students can use in order to help them devise a plan. Can you use a problem that you solved before? This is a question which maybe students implicitly use when they solve routine problems: they are looking for a similar looking problem. In this case, when faced with non-routine problems students may have to look deeper into the problem in order to see the similarity. Can you use a problem with the same unknown? Again, a very useful question because it's directing the students towards where they need to go. Likewise his third heuristic in this context is also useful because it's also directing the student towards where they want to go. Rather than start at the beginning of the question and try to work forward, is it possible for them to start at the end of the question and work backward from the goal to the beginning?

Notes

Summary

5m 05s

Polya's four-stage problem-solving process

- Can you use a problem that you have solved before?
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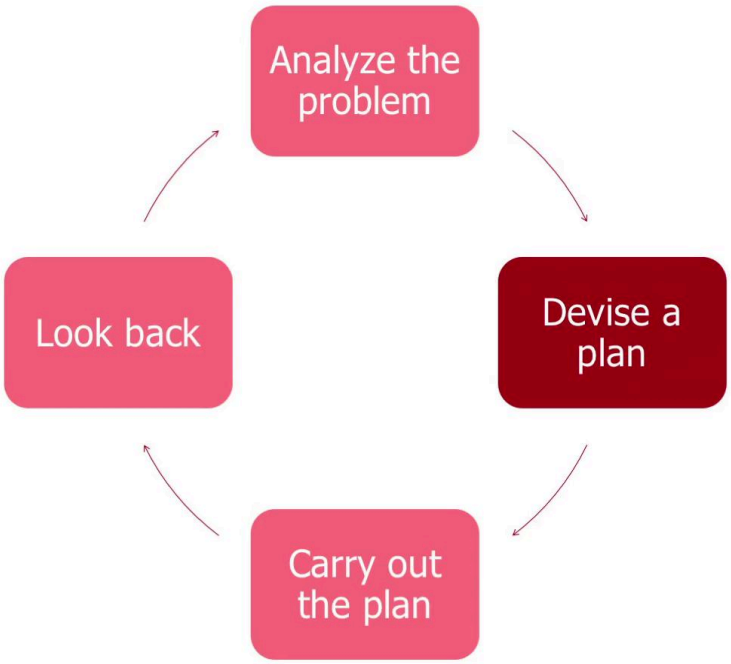


Sometimes he identified that the best way to devise a plan is to actually solve a simpler problem. If a problem has many different components to it, well perhaps a problem with fewer components can suggest a solution which can then be applied to the more complex problem.

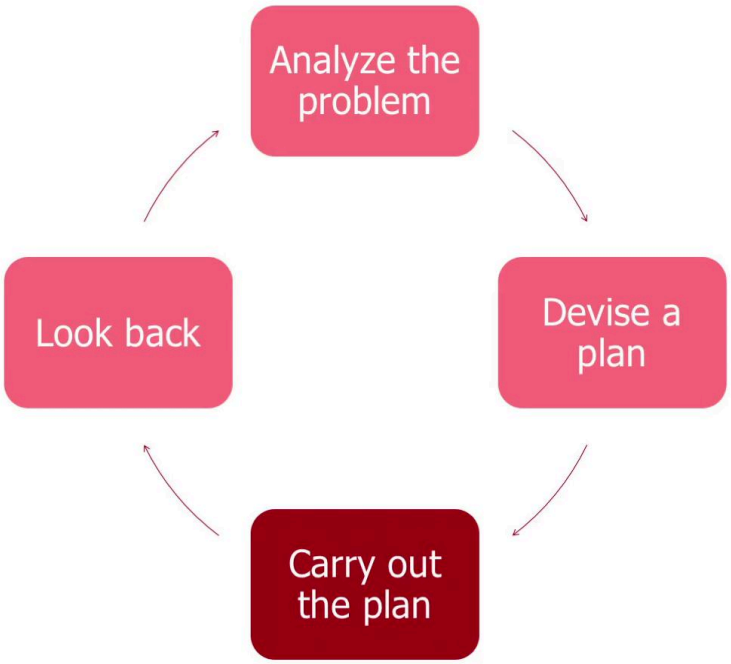
Notes

Summary

6m 30s

Polya's four-stage problem-solving process

- Check each step
- Check assumptions that apply in each step



The third stage for Polya is carrying out the plan if the plan is going to be successful then one needs to show due care and attention during this 'carry out' phase. Key strategies that he suggests at this stage are, number one, check each step of the process. And number two, check the assumptions that are being made in each step in the process. This second one perhaps is particularly useful in the case of mathematical proof type questions The fourth stage in the problem-solving process is looking back.

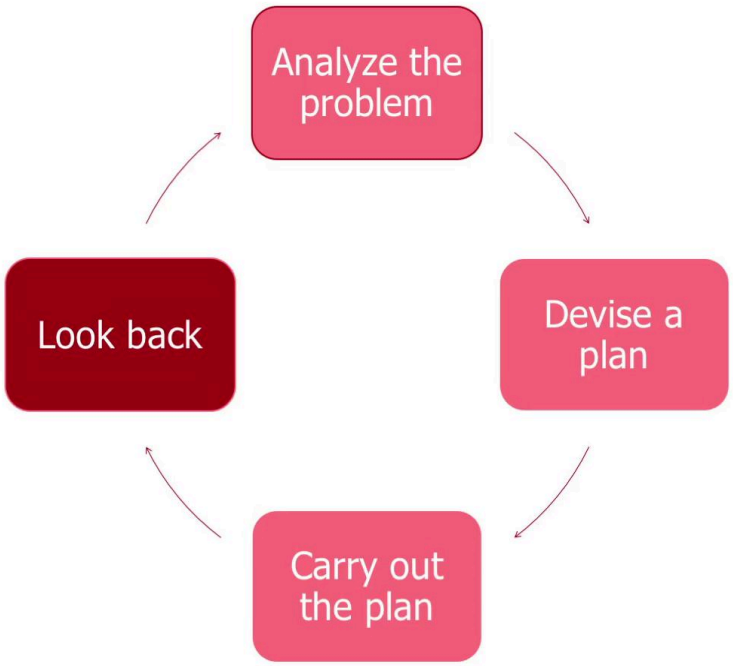
Notes

Summary

6m 49s

Polya's four-stage problem-solving process

- Can you check the result (against reality)?
- Can you check the argument?



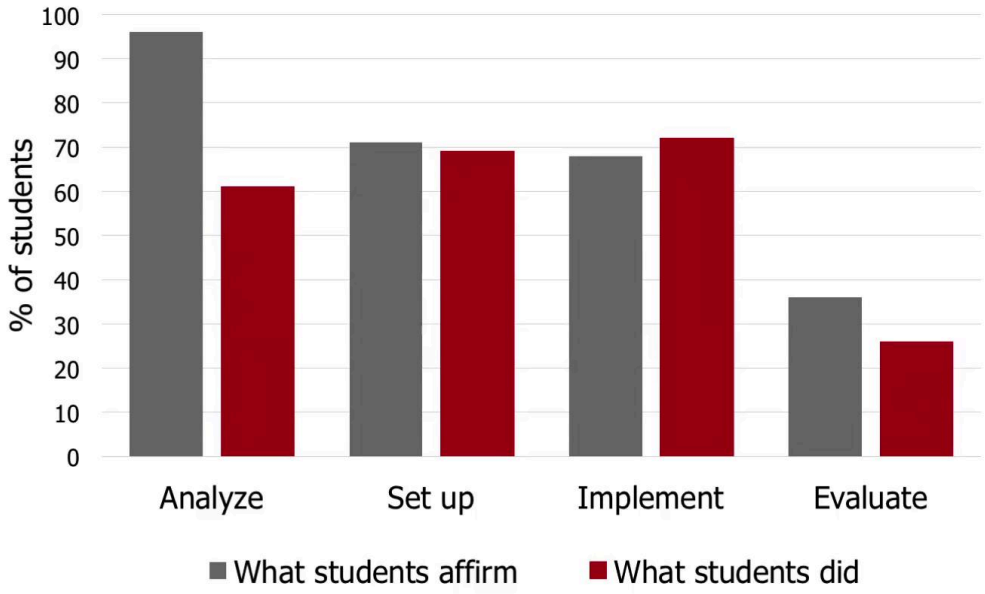
Once you've arrived at a solution you then need to check that solution. In fact, it turns out that one of the differences in problem solving behavior between experts and novices is that experts typically spend more time checking their solutions afterwards than do novices, Again, just as in the analysis phase, the strategies which are involved in looking back, in checking the result, may differ enormously between one discipline or another. The way in which you will check the results of a question about Eigenvectors in linear algebra will be very different to the way in which you will check the physical reality of a question in mechanics. Nonetheless the broad principle is the same. There is a need to check the results once the problem-solving process is finished.

Notes

Summary

7m 25s

Do students systematically solve problems?



Student's understanding of the extent to which they completed each stage of Polya's problem resolution approach (grey), as compared to an observation of their problem solving (red). N=28

L. Duré, P. Leroux, H. Meng, and C. Baril (2016), How do students solve problems

So do students use a Polya like systematic approach to solving non-routine problems. In order to look at that we carried out this study here in EPFL. 28 students were asked to solve a problem they were observed while solving the problem and the observers identified to what extent students used strategies associated with each of the four stages that Polya identifies. At the end of the process students were then given a questionnaire in which they identified to what extent they felt they had used each of the strategies. A couple of things are evident from this. First is that when we look at the data on the analysis phase it turns out that students thought that they had analyzed the problem to a much greater extent than they actually had. Students self-report data is in gray whereas student's actual behavior data is in red. And so here we have a mismatch. Students may think that they've analyzed the problem sufficiently but in many cases they may not have. Secondly, we can identify that very few students actually said that they looked back or evaluated the problem effectively afterwards and even fewer students actually did. So, this suggests that there are a number of different places where students will need help in terms of applying a problem-solving methodology for non-routine problems.

Notes

Summary

8m 14s

Putting it into practice

- Polya's heuristics are quite general
  - E.g., In what circumstances does it help to draw a picture? What kind of picture?...
- But we know students should pay more attention to (at least):
  - Analysis
  - Looking back
- We know students will solve problems better if they
  - Look at the (more general) process
  - Ask themselves how they are doing
  - Believe problem analysis takes time

OK. So, what does all this mean, if you're in an exercise session and you're trying to help a student learn - not just how to solve the problem in front of them - but learn how to solve problems in general. Well from this video we have a couple of key ideas. First of all, Polya's heuristic model is potentially useful but at the same time quite general. We know it's useful to often ask a student, "Is there a diagram you could draw here?" but in what circumstances is it appropriate to draw a diagram? And what kind of diagram do they need to draw to solve this kind of problem? So, the heuristic model will be helpful in general terms but it will need to be made specific to the context in which you're working. Secondly, we know that students probably need to pay more attention to particular parts of the four-stage process. It seems that students are particularly weak in terms of the analysis phase and in terms of a looking back phase they may not be aware that they're weak at the analysis phase. And so, this is something that you as an assistant can help the students to start to see in their own process. Thirdly, we know some useful things from the first video on problem solving.

Notes

Summary

9m 46s

Putting it into practice

- Polya's heuristics are quite general
  - E.g., In what circumstances does it help to draw a picture? What kind of picture?...
- But we know students should pay more attention to (at least):
  - Analysis
  - Looking back
- We know students will solve problems better if they
  - Look at the (more general) process
  - Ask themselves how they are doing
  - Believe problem analysis takes time

We know that students will often transfer their learning better if, rather than focusing in on the specific task in front of them, they also think about the general principles which apply in a situation like this. We know that they will often learn better if they ask themselves what they're doing and how they're doing when they're solving a problem. And finally, we know that students often believe that the solution should come to them very quickly and that this belief is something that they may need help to change.

Notes

Summary

11m 04s

What you can do?

- Draw attention to the 4 phase model
- Clarify if the student is using a heuristic strategy appropriate to the type of problem
  - Routine
  - Non-routine but defined
  - Ill defined
- Ask Polya's questions in the context of the discipline
- Highlight these as transferable heuristics
- Work on student's beliefs

So, what can you do in an exercise session to help the student learn better problem-solving skills? First of all you can draw their attention to the four stage process model you can identify to them that it's useful for them to think in terms of analyzing a problem, setting up a plan, following through on the plan, and then looking back. Secondly, you can help the student to clarify if the heuristic strategy they're using is appropriate for the type of problem they have. Is the problem in front of them a routine problem in which case looking to see if they've already seen a problem like this before will probably be the right strategy if the problem is non routine however, well then they may need to use a wider set of questions in order to analyze the problem and in order to devise a plan. Thirdly, you can actually help them by asking the questions you can use Polya's questions in the context of the discipline, to help the student analyze the question, and to help them set up the plan. Fourthly you can highlight that these questions are transferable. They are not simply questions that you ask the student. They are questions that the student can ask themselves. And finally you can help the student by helping them work on their beliefs about problems.

Notes

Summary

11m 37s

Conclusion

- Problem solving can be explicitly taught and learned
- Get the right balance between a general problem-solving model and discipline specific strategies
- Get students to think about "high road" transfer
- Ask (Polya's) questions of students …
- …and get them to ask these questions of themselves

In particular you may need to help the student recognize that it takes time to analyze a problem effectively. So, to conclude, over these two videos we've seen that problem solving is something which can be explicitly taught to students, and the students will generally learn it better if it is explicitly taught. Doing so will involve getting the right balance between a general problem-solving model which works in any situation, like that presented by Polya, and the way in which that needs to be enacted in the specific context of the discipline and the questions which the students are facing. Students will be able to solve problems better if we help them to think about high road transfer if we have to help them to think about actively transferring their learning from one situation to a different situation. In doing so we can help them by asking Polya's questions of students and ultimately by getting them to ask these questions of themselves.

Notes

Summary

13m 04s