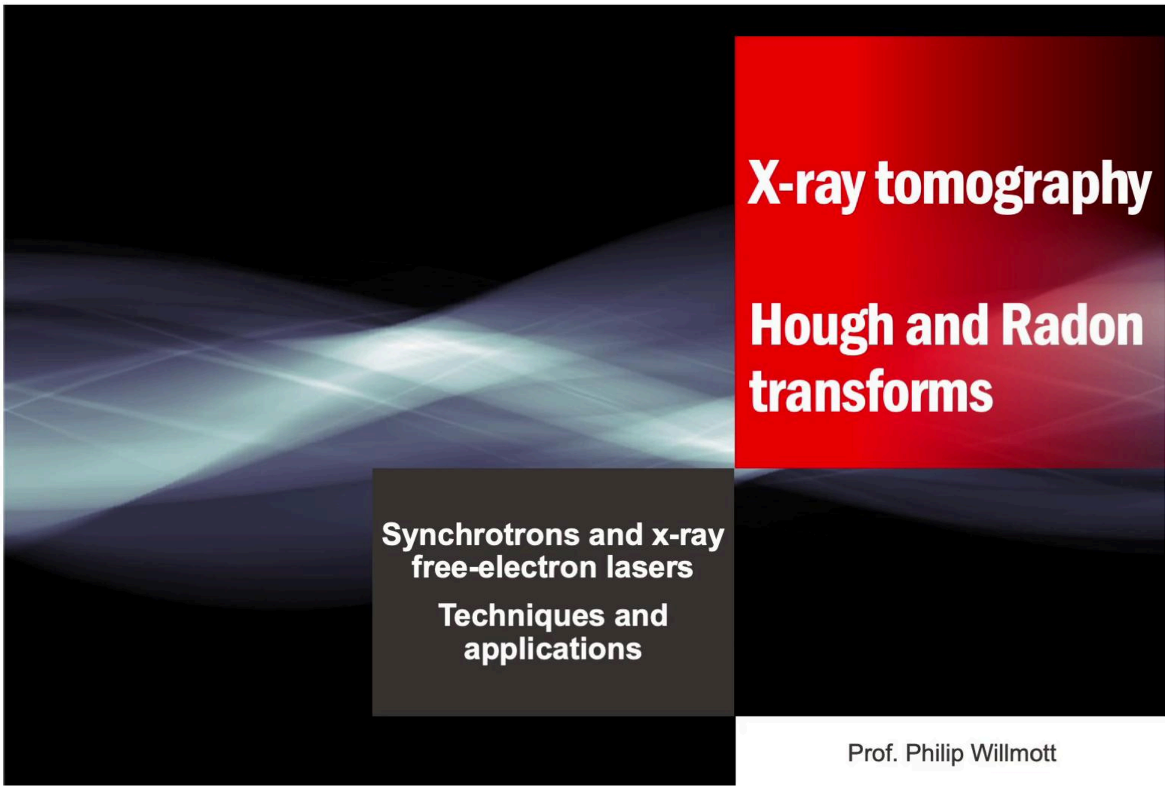
EPFL



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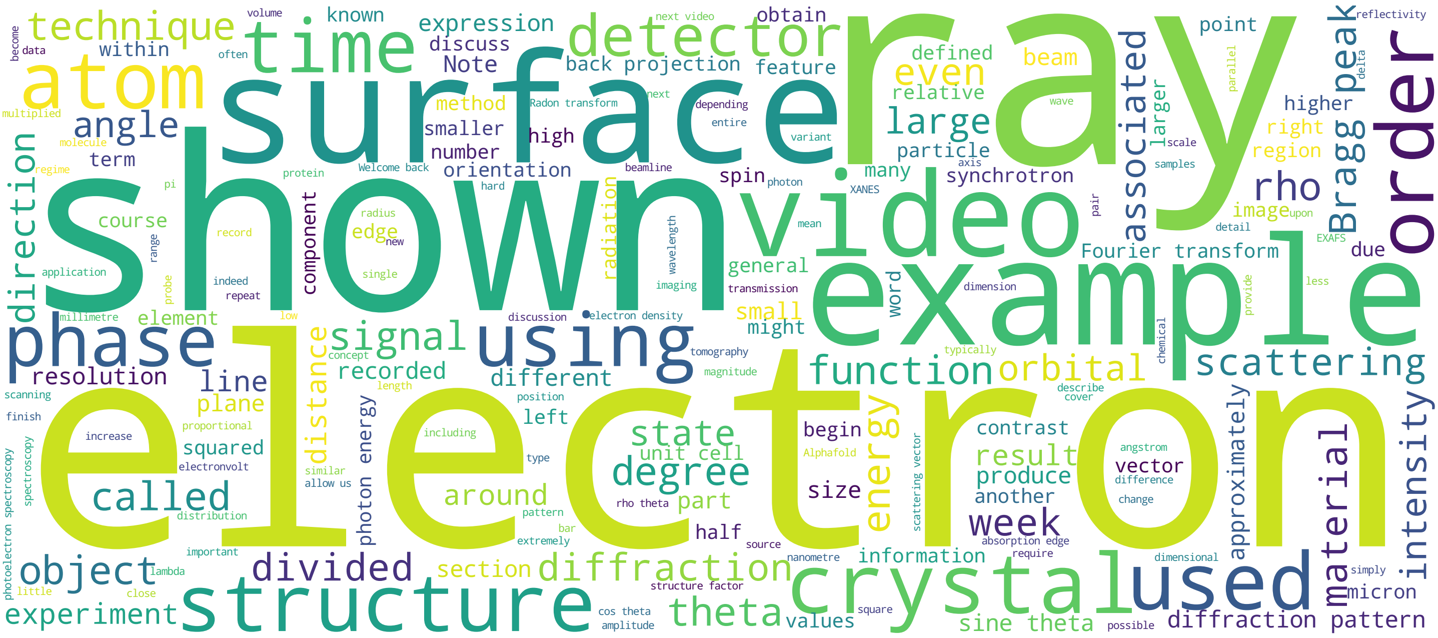

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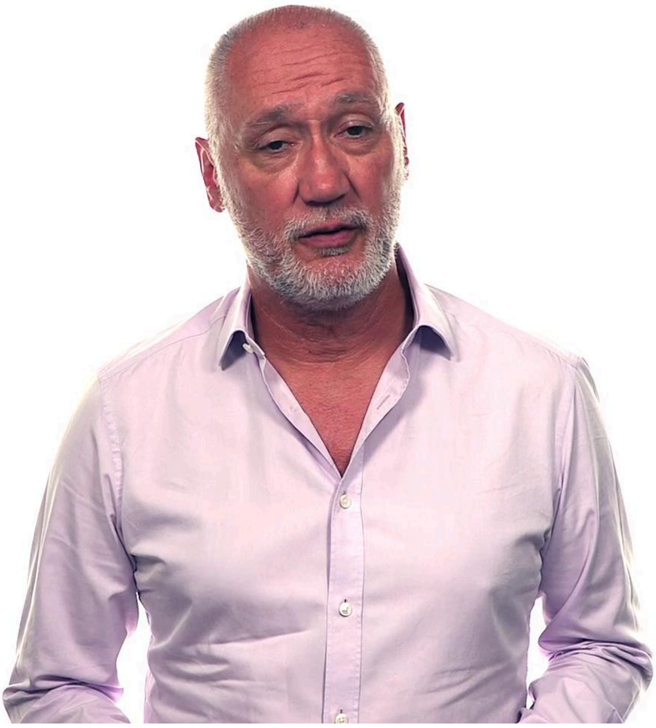
Video

EPFL

Contents and objectives of this video



- The Hough transform
  - Why bother?
- The Radon transform (1917)
- Sinograms
- Back projections described in Radon transforms

Hi again. In this video, we continue to discuss back projections in mathematical terms. We begin with the conceptually simple Hough transform, followed by the Radon transform, a little more complex in nature. This will allow us to describe sinograms mathematically and back projections in terms of the Radon transform. This will allow us to obtain faithful image reconstructions, from sets of projections.

Notes

Summary

0m 04s

Parameterizing lines – the Hough transform

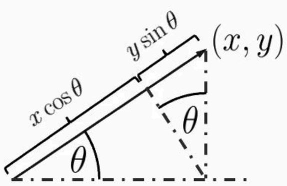
- We all know

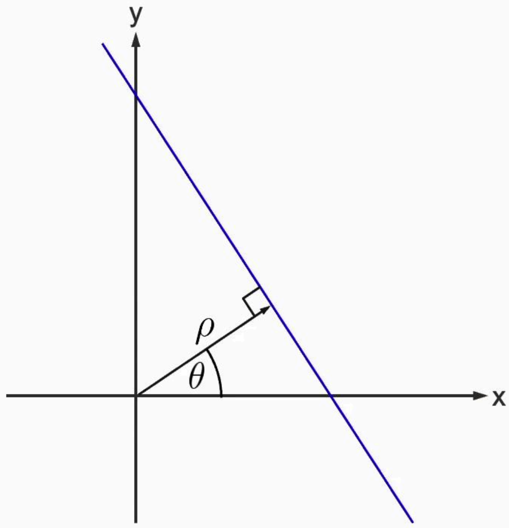
  $$y = mx + c$$

- But what if line is vertical?
  - m = ∞, y undefined

- Define line through the Hough transform

  $$\underline{\underline{\rho}} = x\cos\theta + y\sin\underline{\underline{\theta}}$$





Now, most commonly, when we describe a straight line, we tend to do this using the very well-known and simple expression that y is equal to mx plus c. m is the gradient and c is the constant. But unfortunately, if the line is perfectly vertical, m is infinite and y is completely undefined. Now to get around this problem, we can instead define any line through the so-called Hough transform, given by rho equals x cos theta plus y sine theta. Note that even for vertical lines for which theta equals 0 degrees, the second term on the right equals zero, even if y is undefined, as sine theta is equal to zero. So essentially, rho is a vector between the origin and the line touching that line at its end at 90 degrees to that line. We see this makes sense by breaking rho down into two components, one given by x cos theta, the other by y sine theta. Hence, for a fixed rho and a fixed theta, our line is completely defined.

Notes

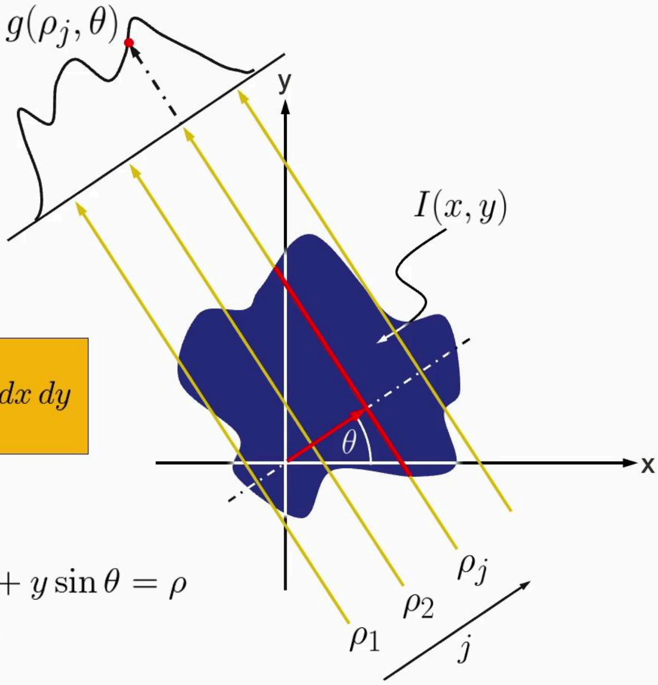
Summary

0m 35s

The Radon transform

$$g(\rho, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(x, y)\, \delta(x \cos\theta + y \sin\theta - \rho)\, dx\, dy$$

$$\delta(x\cos\theta + y\sin\theta - \rho) \begin{cases} = 1 \text{ if } x\cos\theta + y\sin\theta = \rho \\ = 0 \text{ otherwise} \end{cases}$$



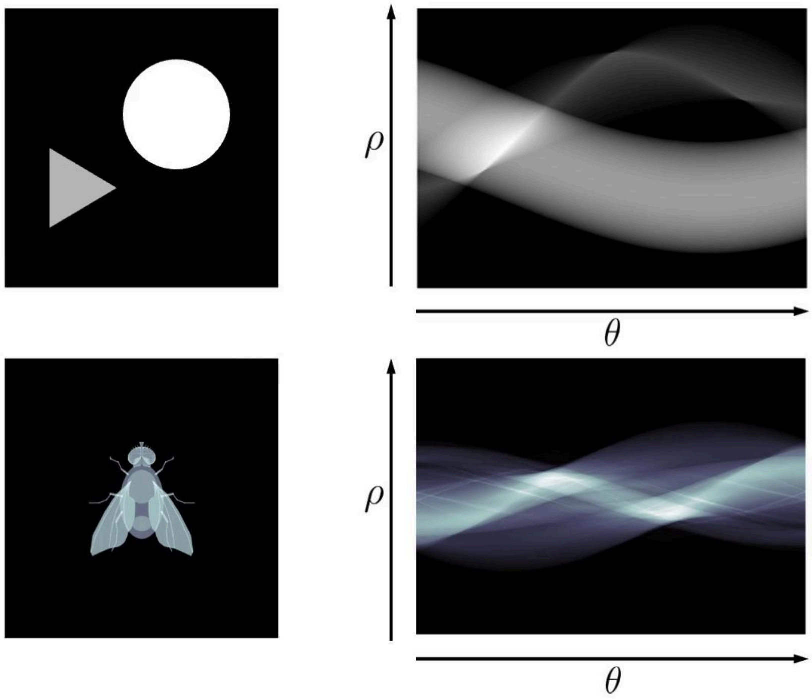
Now let's consider a slice through an object, with a varying absorption intensity I as a function of x and y, this is being irradiated by a beam of X-rays at a certain angle theta, which we break down into a set of parallel rays, each one defined by rho 1, rho 2, rho 3, etc, etc, etc. The index for rho in general, we label j. So for the line defined by rho j, the transmission through the object will result in the data point G, a function of rho and theta. So rho j, and theta in this case, for a given rho and theta. Our challenge now is how to describe mathematically G rho, theta. We can see that it is the integral along the thick red line defined by rho and that part of the entire line that lies within the absorbing object. What we do is integrate along the entire x-y plane, including an expression that ensures that only those x,y pairs, which are associated with rho, are selected. This expression looks like this. It is the delta function expression that selects only the relevant x,y pairs. It's defined as being equal to 1, if x cos theta plus y sine theta is equal to rho, and zero otherwise. G rho theta is the Radon transform.

Notes

Summary

1m 51s

Visualisation of the Radon transform – sinograms



So we can produce a 2D visual output for G, as a function of both rho and theta. This is the sinogram we met in the second video of this section. Two examples are given here for a triangle and circle of differing absorption strength, and a cartoon fly.

Notes

Summary

3m 27s

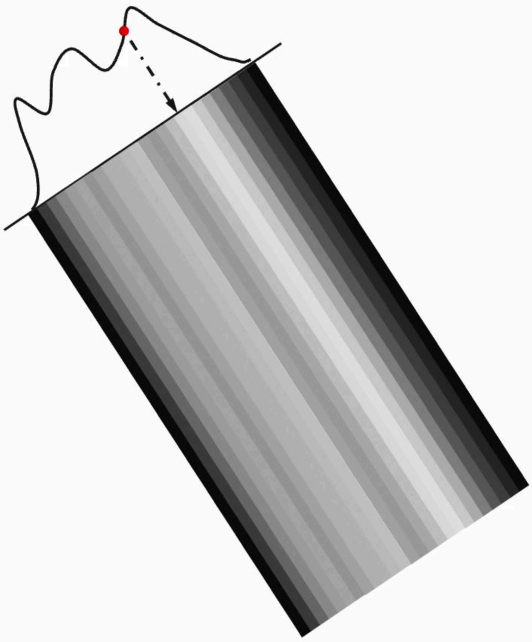
Back projections in terms of g(ρ,θ)

- For a given pair of values ρ and θ project back the value $g(\rho,\theta)$ along the corresponding row $\rho = x\cos\theta + y\sin\theta$
- ρ and θ completely determine the values of x and y

$$F_\theta(x,y) = g(x\cos\theta + y\sin\theta,\ \theta)$$

- Now do this for all lines ρ for a given θ to obtain a back projection in one angular direction
- Then sum up these over a set of θ between 0 and $\pi = \{\theta_k\}$ (e.g., 0, 1, 2, … 179°)

$$\sum_{\{\theta_k\}} F_\theta(x,y)$$ "Laminogram"



Now we've got our expression for a sinogram, how do we next use this, to generate back projections? This is fairly straightforward. For a given pair of values for rho and theta, we should project back the value G rho theta along the line rho. So the back-projected image at theta, which we will call big F theta as a function of x and y, is equal to the value g x cosine theta plus y sine theta, theta. We repeat this for all the lines rho for a given theta.

Notes

Summary

3m 46s

Back projections in terms of g(ρ,θ)

- For a given pair of values ρ and θ project back the value $g(\rho, \theta)$ along the corresponding row $\rho = x \cos\theta + y \sin\theta$
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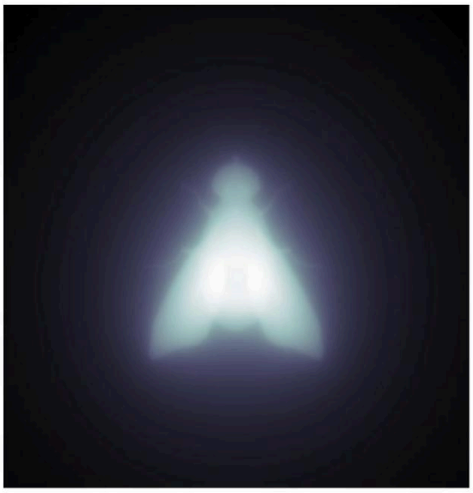


Lastly, we change theta to a new value, and repeat the above and sum these back projections for all values of theta in a set, say between 0 and 179 degrees in one-degree angular steps. This results in a so-called laminogram.

Notes

Summary

4m 23s

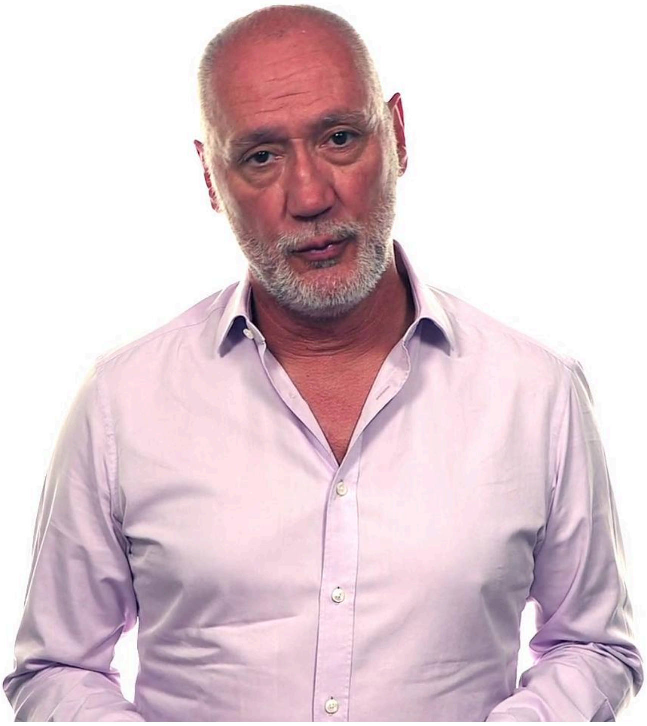
So to summarise this video, you might be wondering why on earth, did we bother to derive this mathematical expression for a laminogram, also known as an unfiltered back projection. The clue is in the word unfiltered. We want to get rid of the halo effect, and this is best done via a filtering process.

Notes

Summary

4m 41s

In the next video...



So in the next video, we will build on all we have learnt in this video, and this will allow us to recognise the correct approach to remove the halo.

Notes

Summary

5m 03s