



Course material

Course:

ENG606 / PHYS 442

Video:

DOE_lesson4_part2_YoungModulus

Concepts (extracted from automatically generated subtitles):

Random coefficient. Carbon concentration. Young modulus. Matrix of experiments. First coefficients. Standard temperature. Data points. Effect of temperature. Book flow. Sulfur concentration. Linear model. Variance function. Most complicated model. Mechanical engineer. New coefficient.



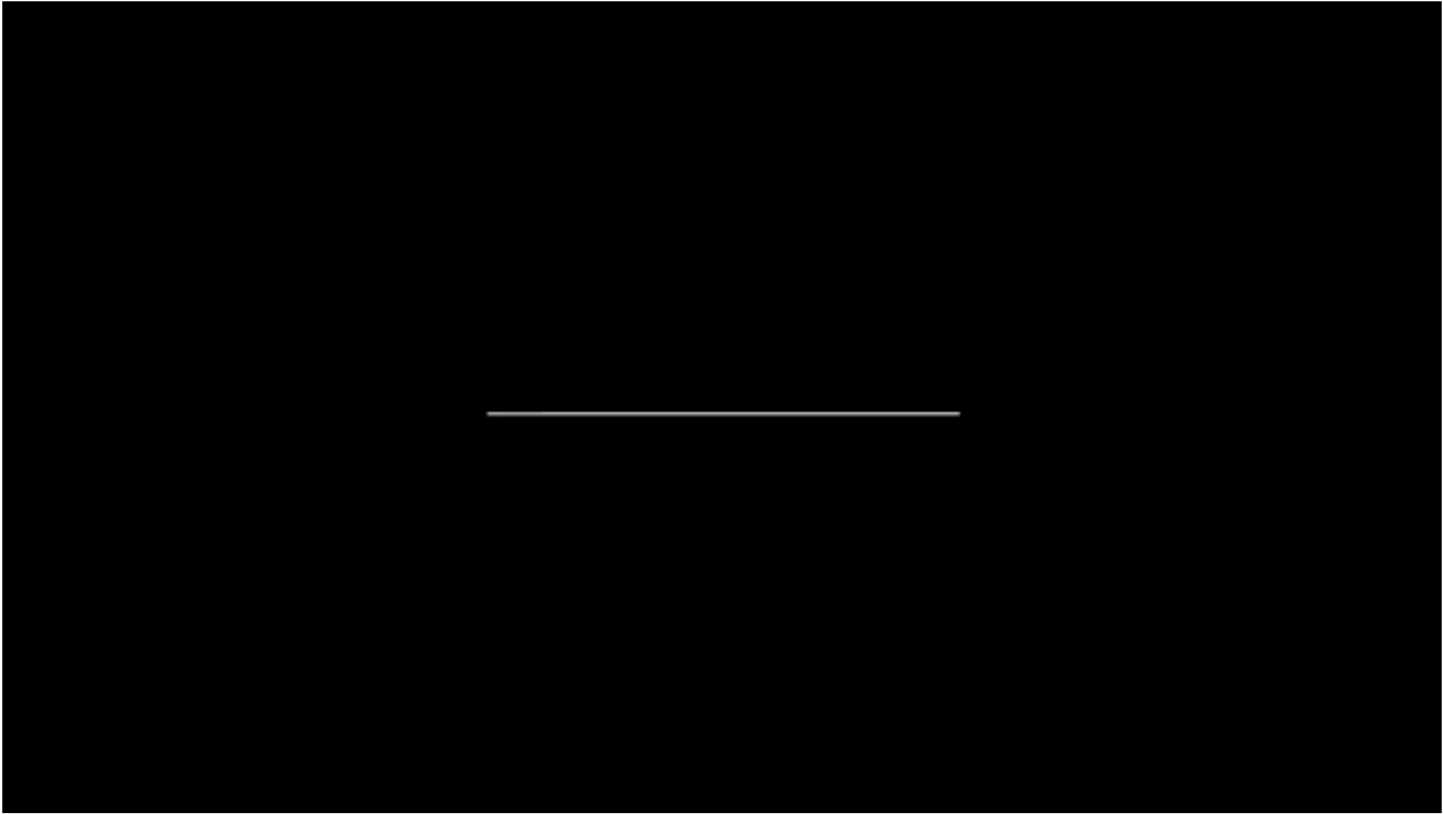
[to video sequence search](#)
(within ENG606 / PHYS 442.)



[to video](#)

Center for Digital Education. More educational support material here:

<https://www.epfl.ch/education/educational-initiatives/cede/educational-technologies-gallery/boocs-en/>
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notes

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summary

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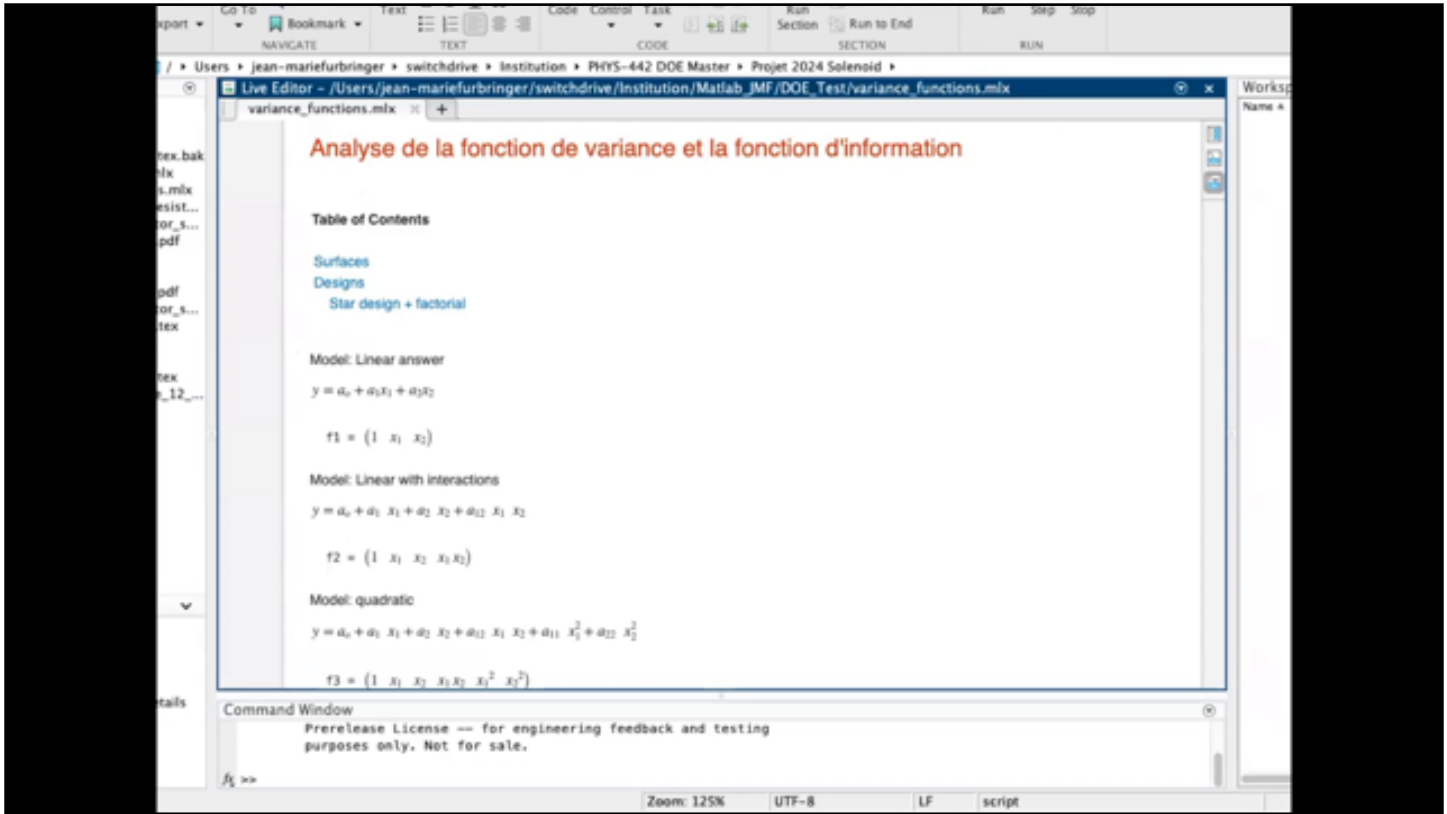
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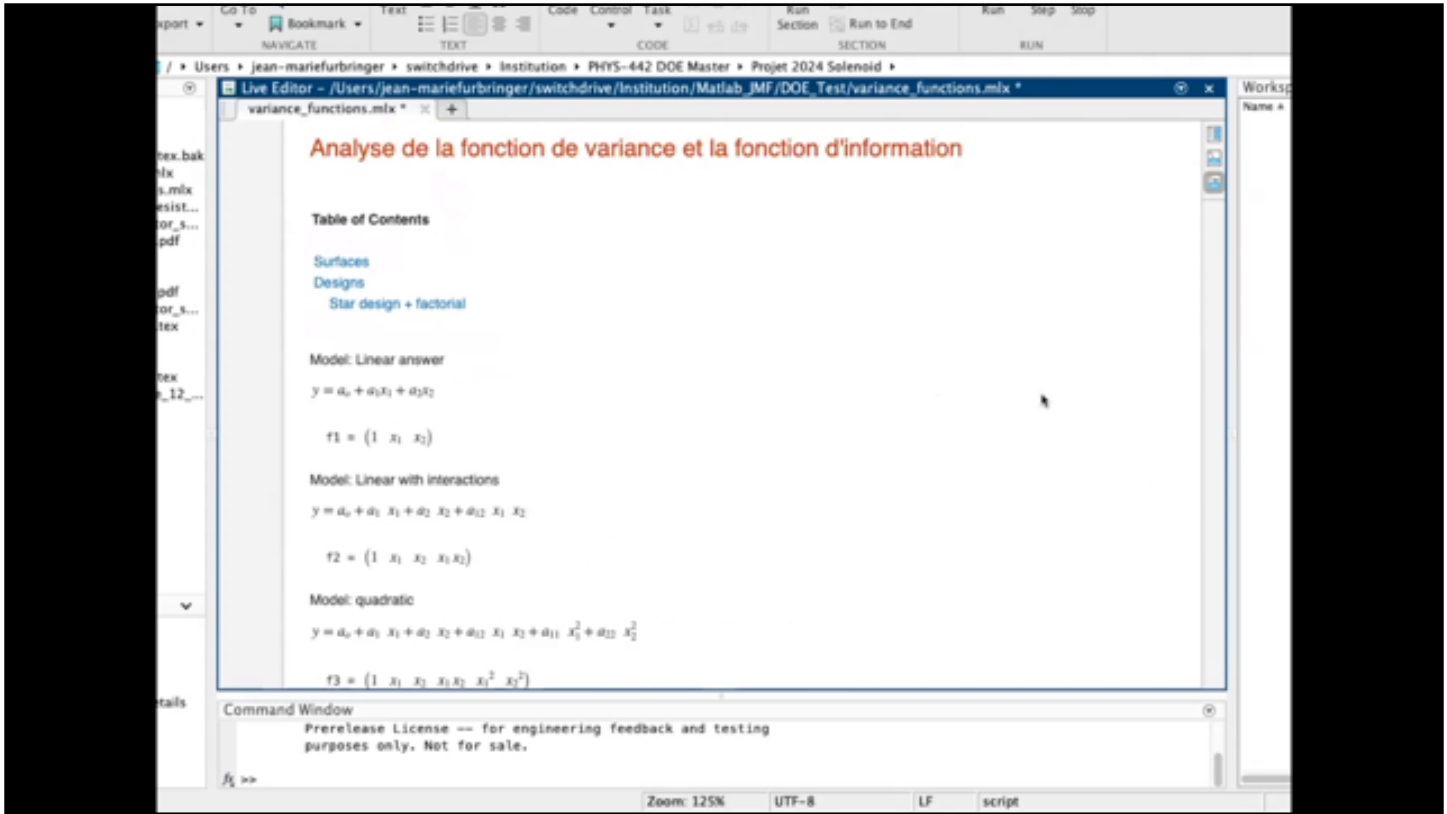
These subtitles have been generated automatically In the model, you can find a file, an Excel file called variance function.

notes

summary

0m 1s





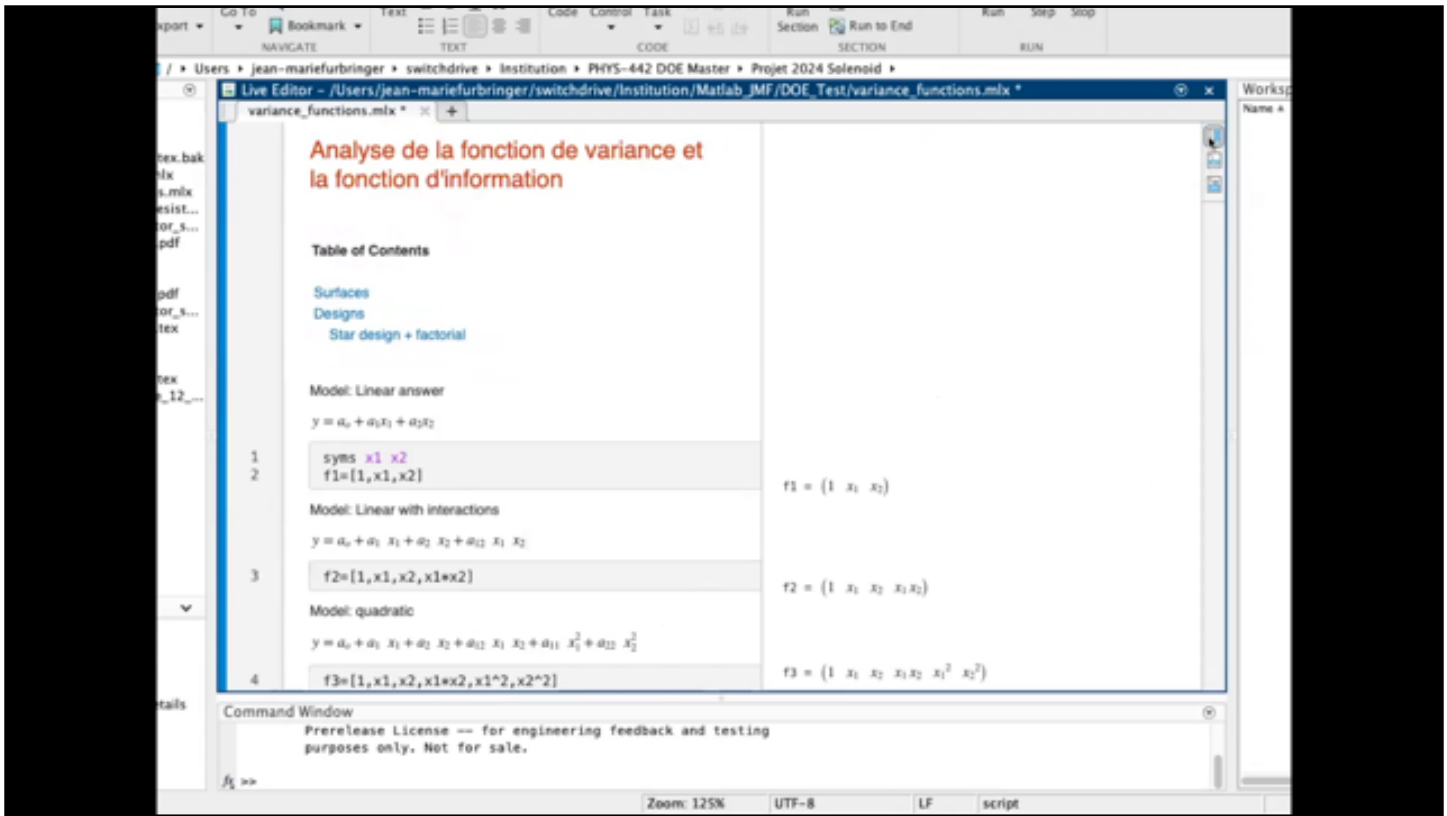
And so you can play with it if you want.

notes

summary

0m 5s





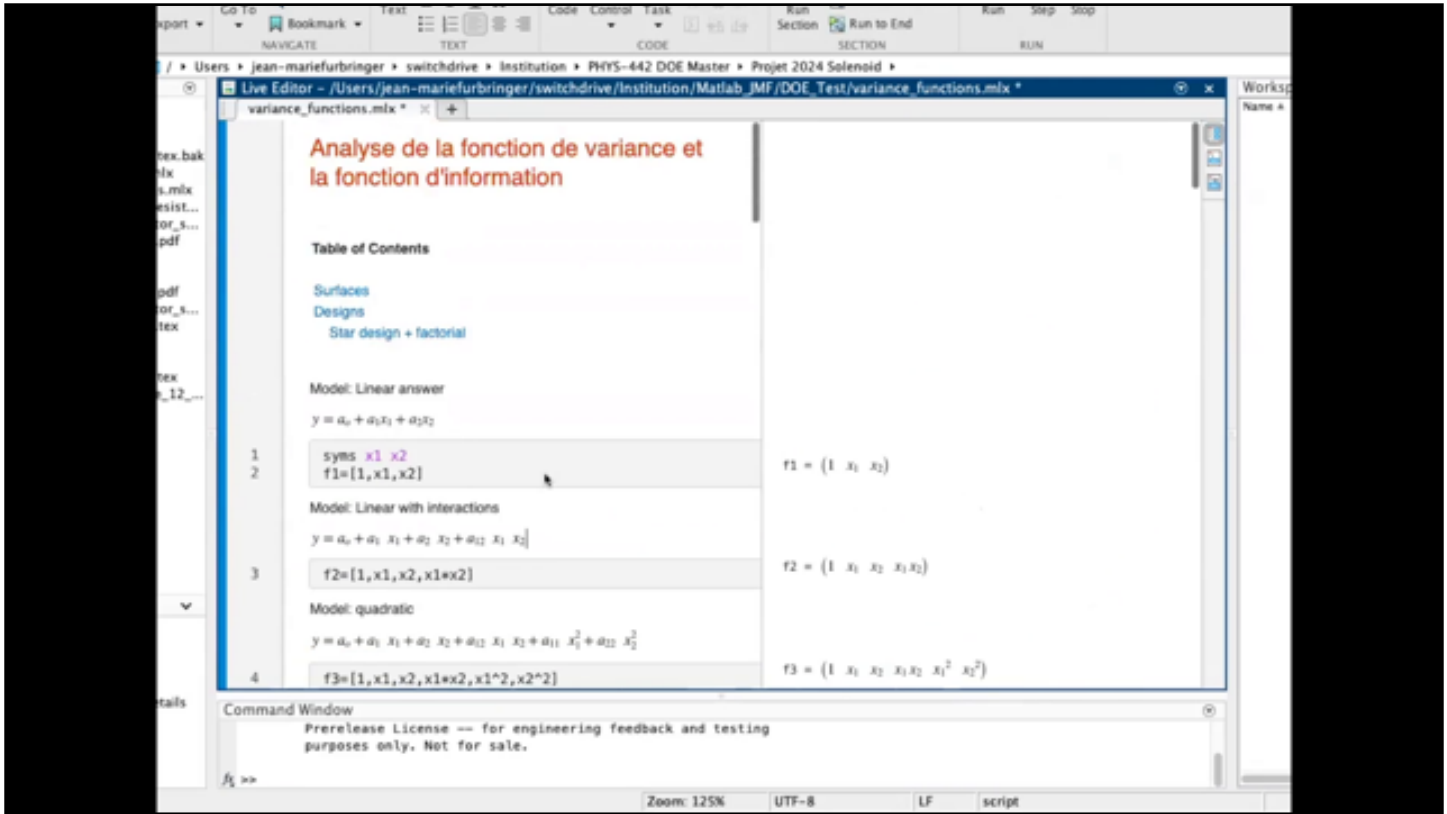
I just want to show you one or two things with that. So just playing with these functions and so you can see how to treat them, what is the function I'm using. So just something about Matlab.

notes

summary

0m 17s





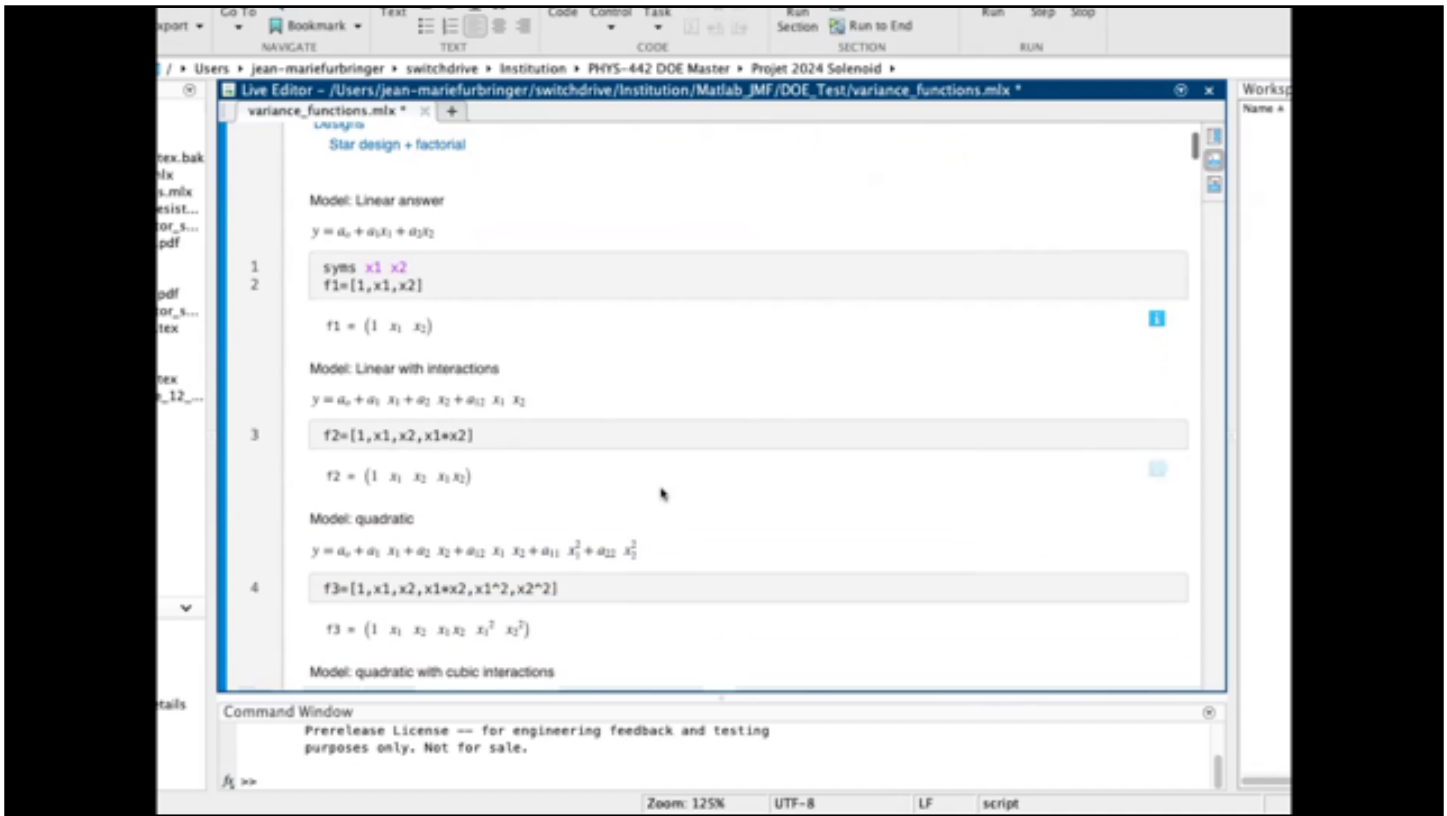
I don't remember if I mentioned it already, but in Matlab, if you use the live script,

notes

summary

0m 30s





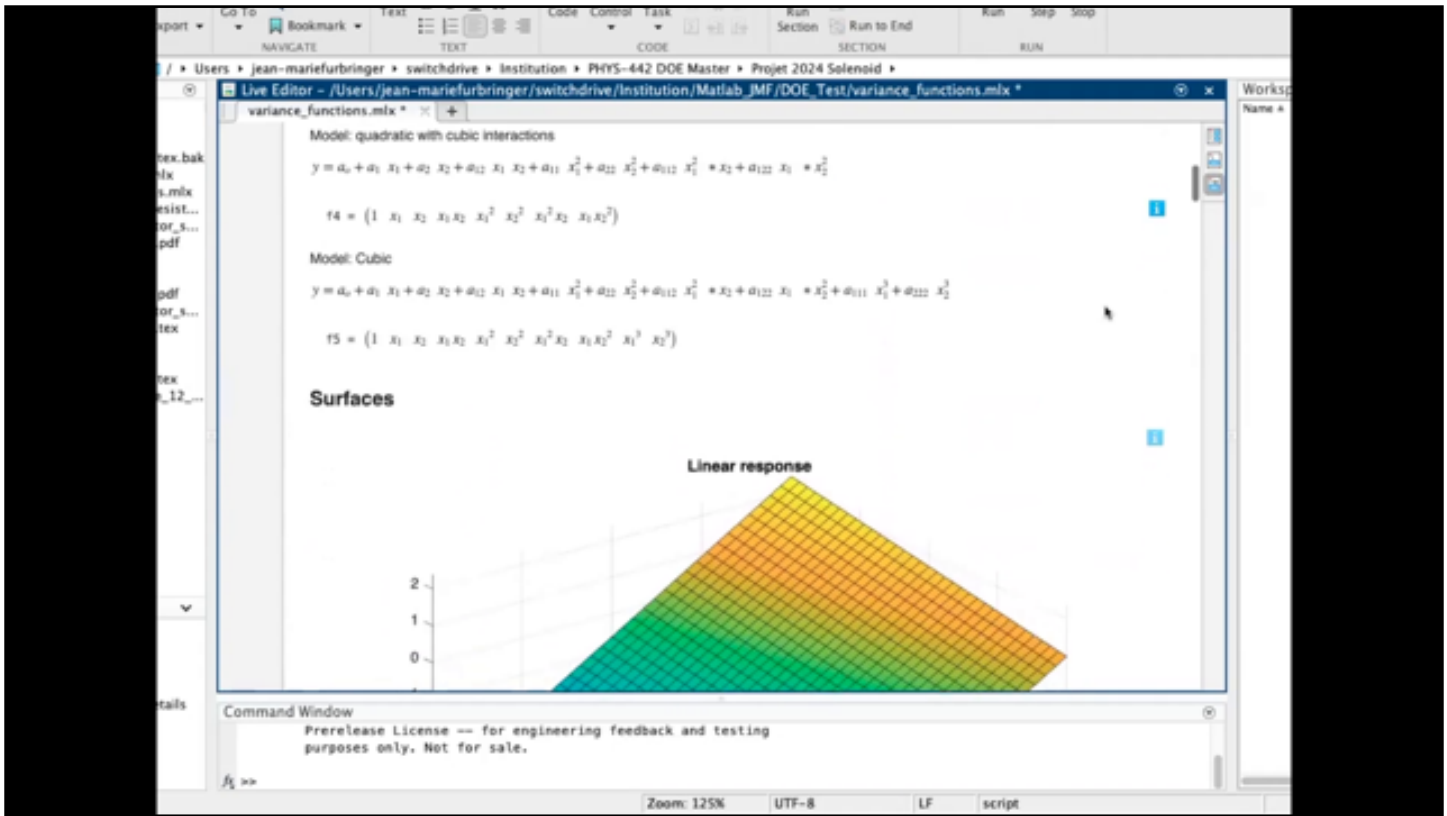
you can have your code on one side and you can have the result of your code on another side. On the right corner, you have a three button for selecting how you want to see things. So you can see like that, right and left. You can see like a block. So you have the codes.

notes

summary

0m 39s





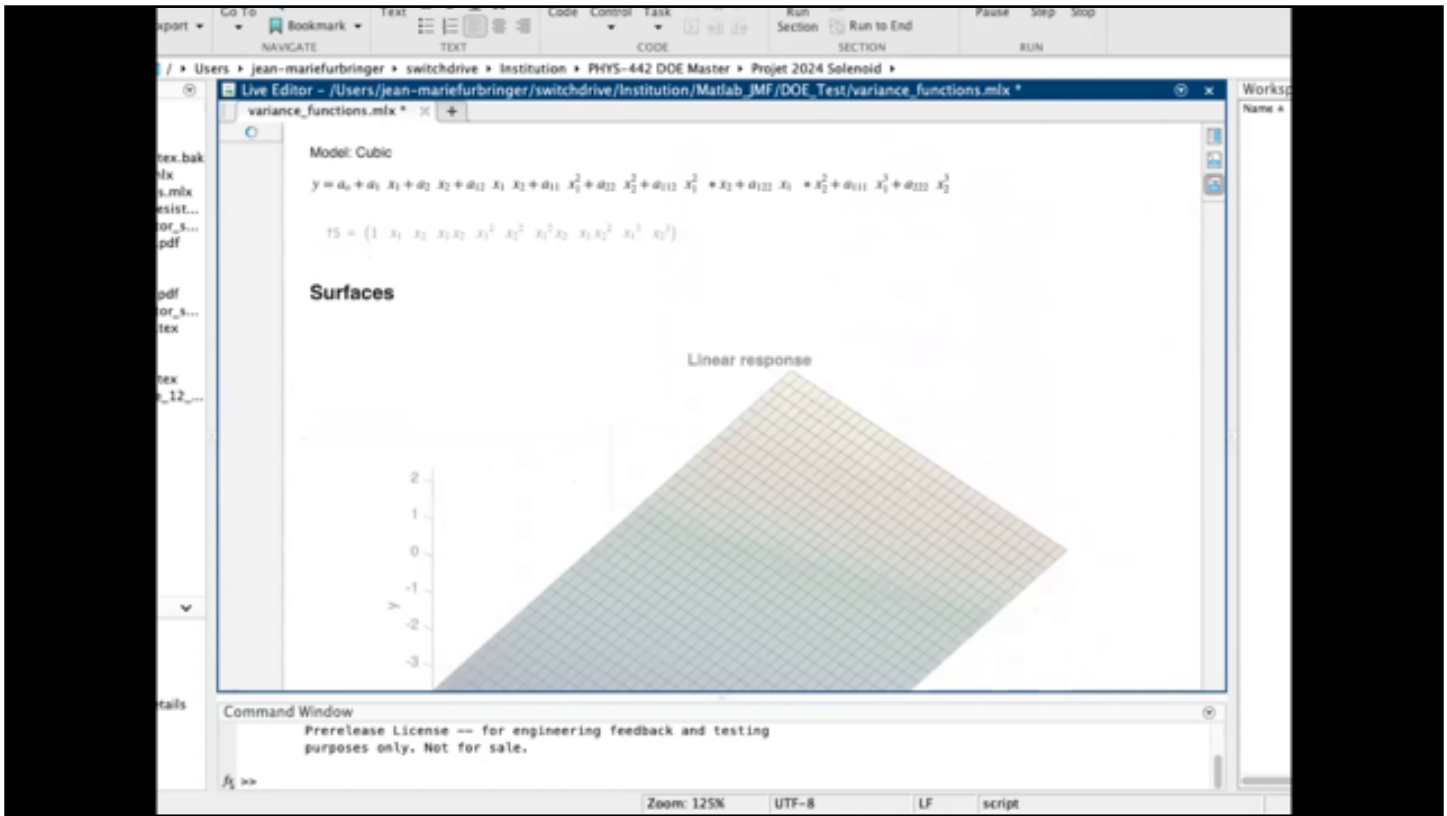
When it's gray, you have the code and when it's right, it could be your comment or the result of your calculation. And you can do the third one. You can even hide the code and you have, you can present things.

notes

summary

1m 0s





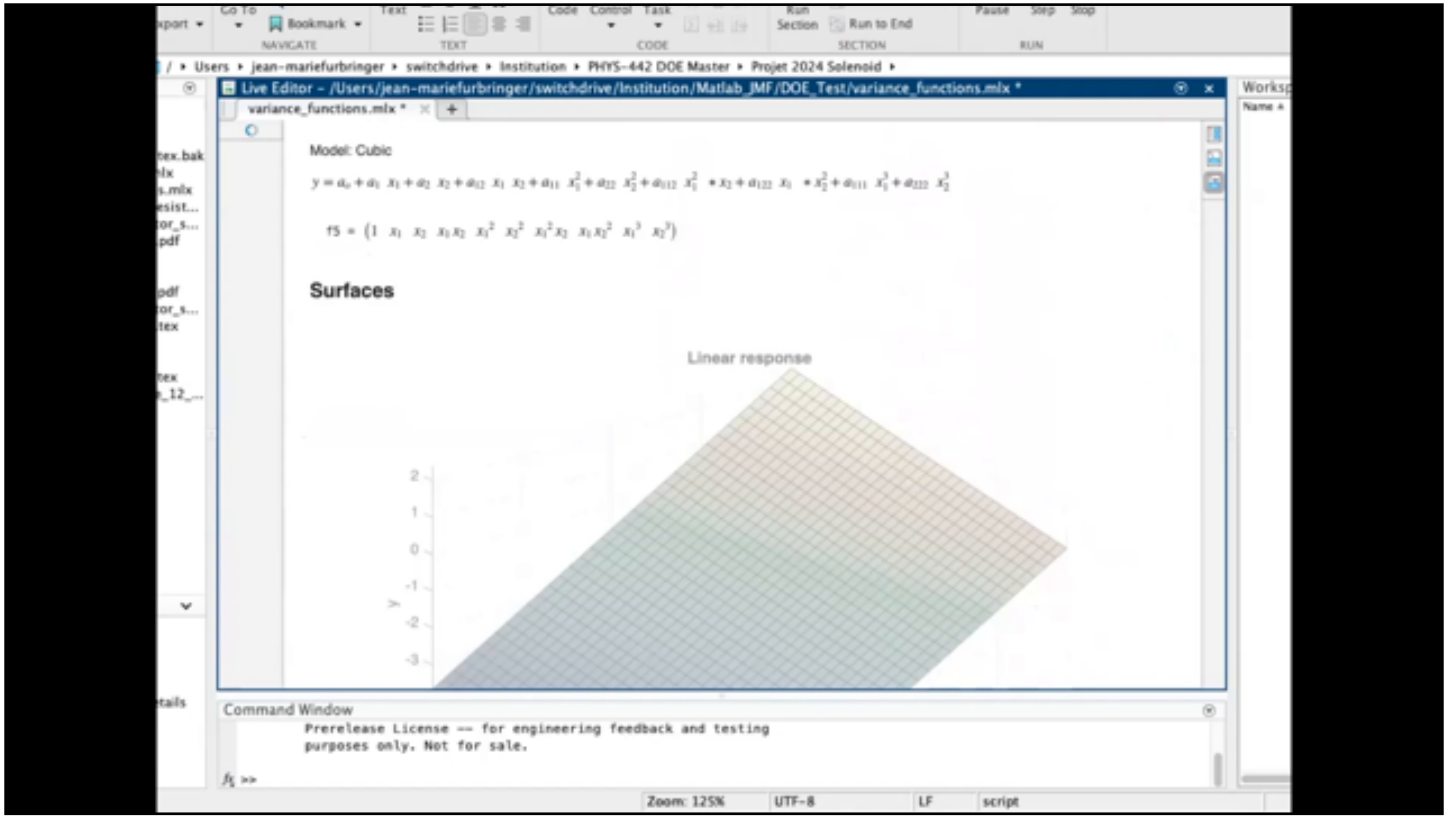
So very simply, I just wanted to represent the different surface I present you.

notes

summary

1m 13s





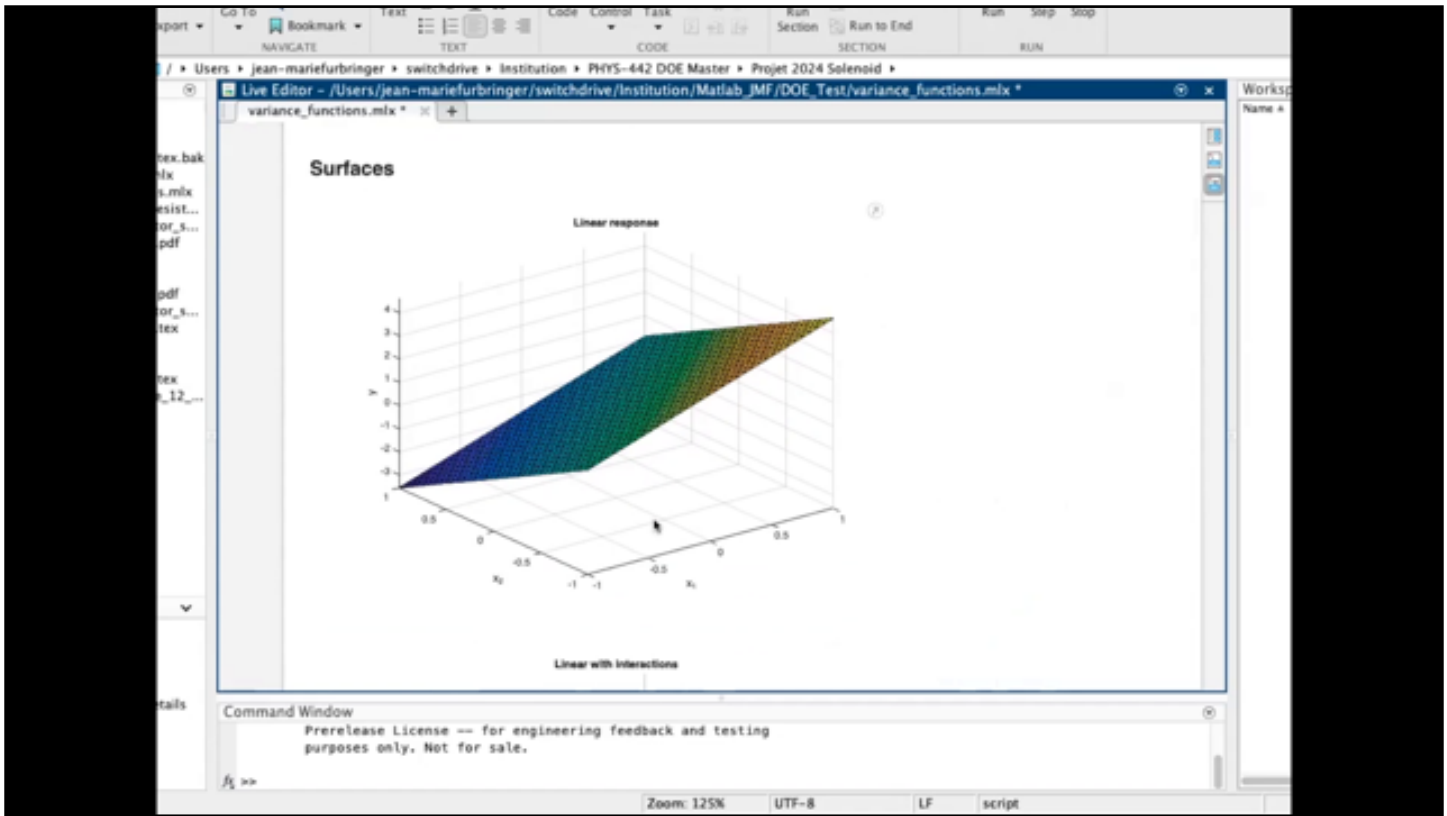
So what I did, I use random coefficient just for showing you what's happened with my surfaces and each time I have a most complicated model using the same first coefficients and having

notes

summary

1m 21s





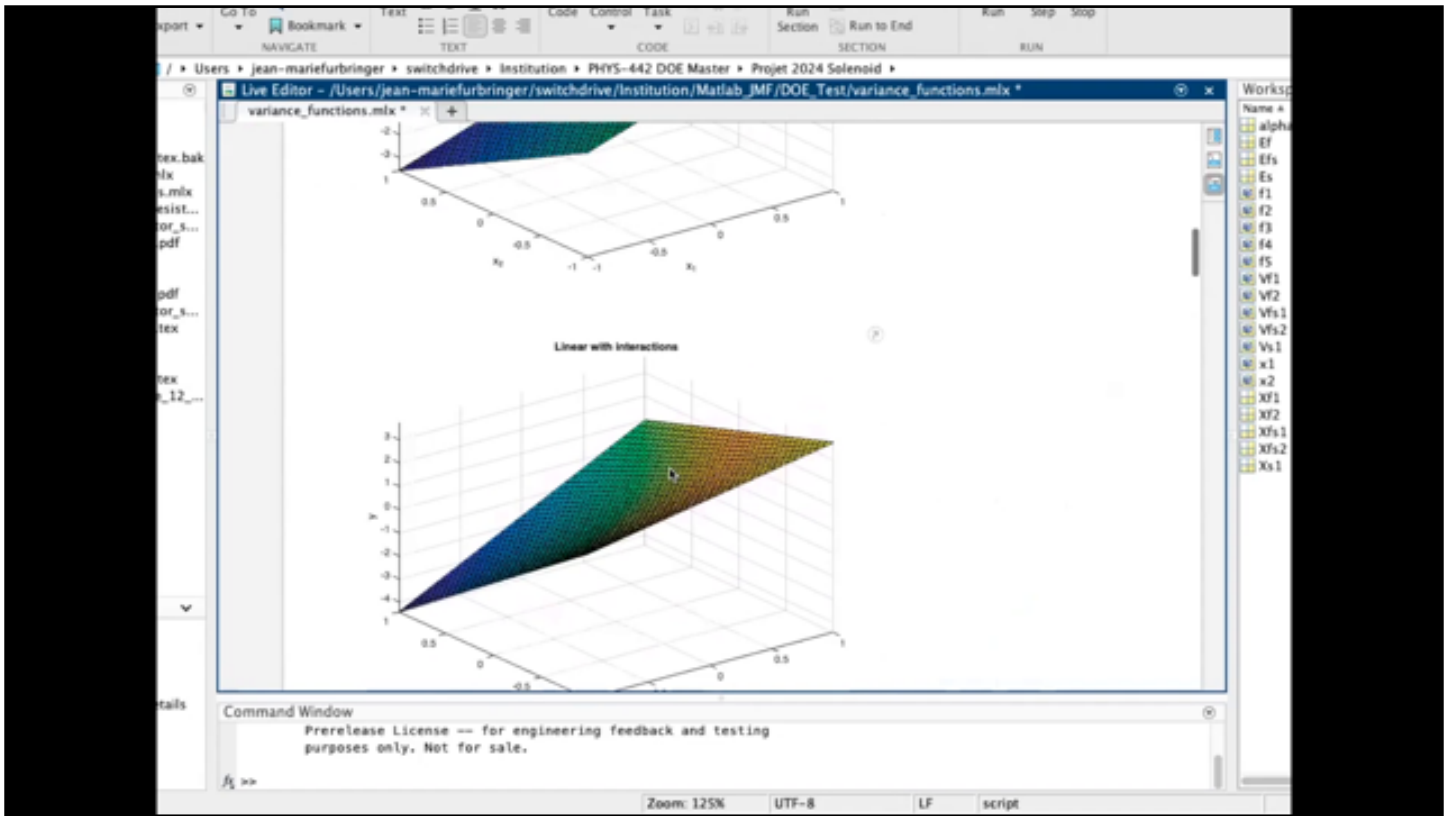
a new coefficient for the complexification of your model. So you see here, the linear model is an hyperplane.

notes

summary

1m 33s





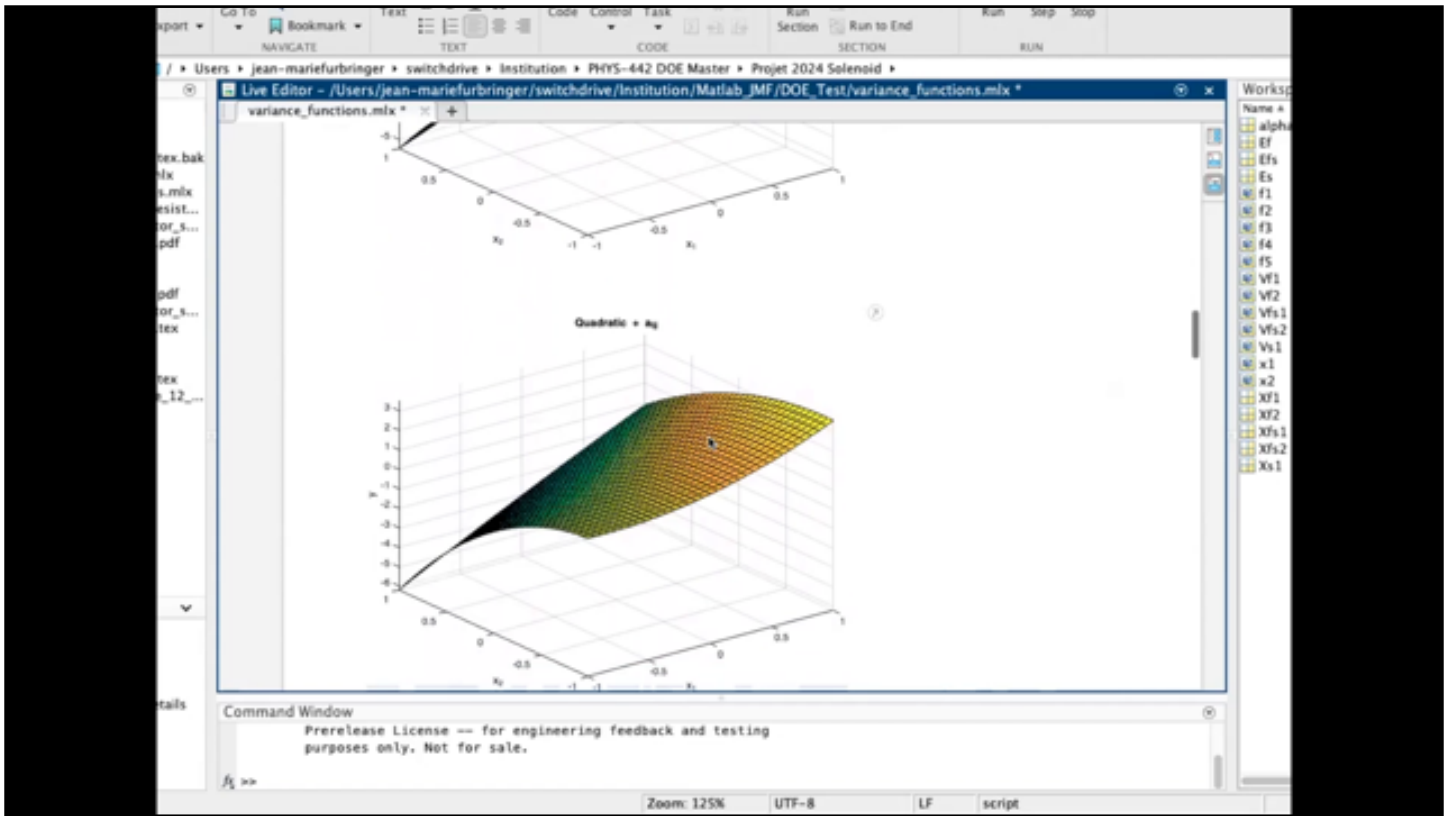
After with the same first coefficient, so the first linear coefficient and just adding

notes

summary

1m 42s





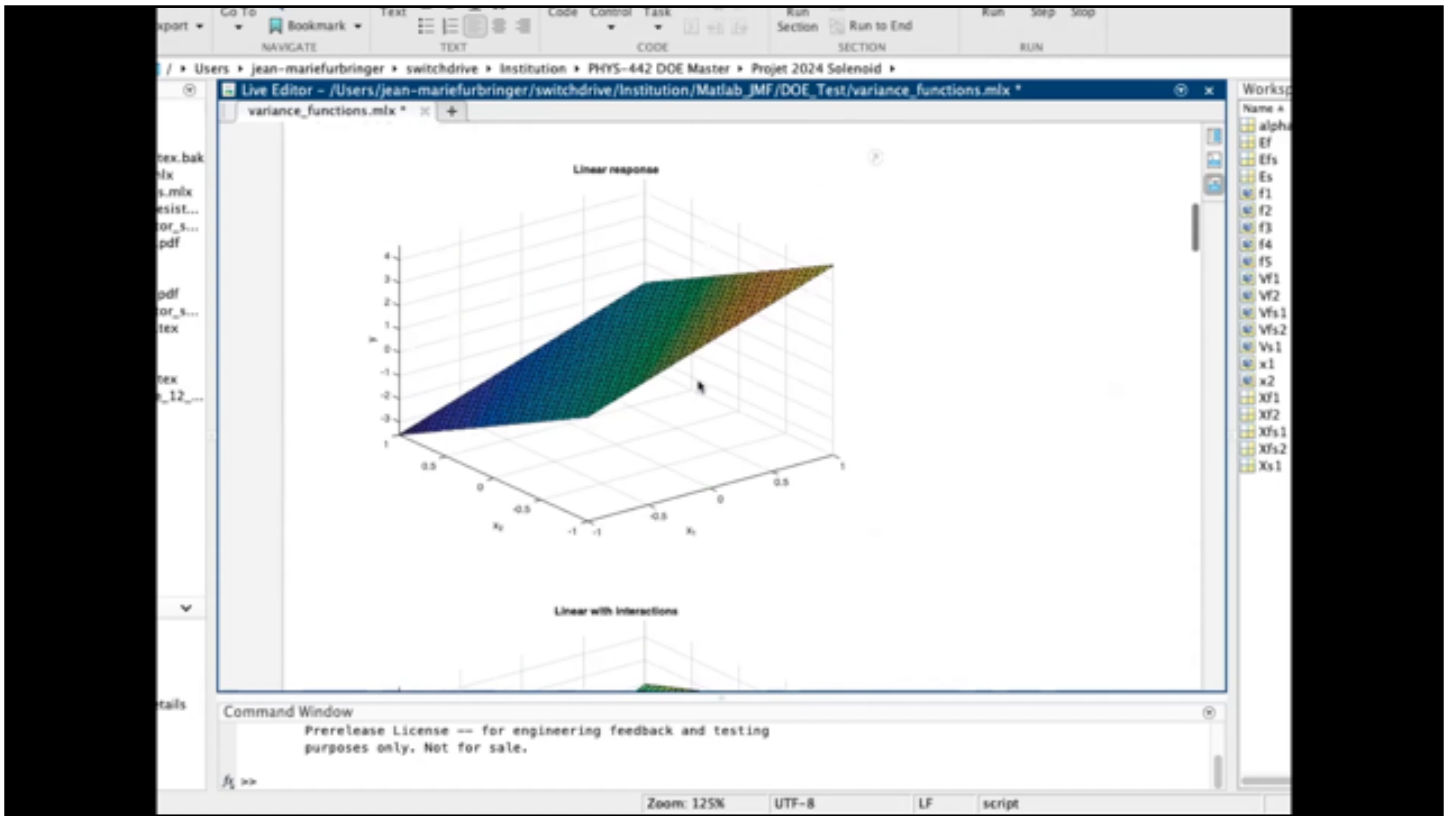
an interaction coefficient, you see, well, the shape is quite the same. Just my plane now is a little bit shifted. And after I can add a quadratic and you see is still quite the same. But now not only shifted, but only I have some spline or something like that. I have a curvature and all creating and that even go to a third degree interaction coefficient.

notes

summary

1m 51s





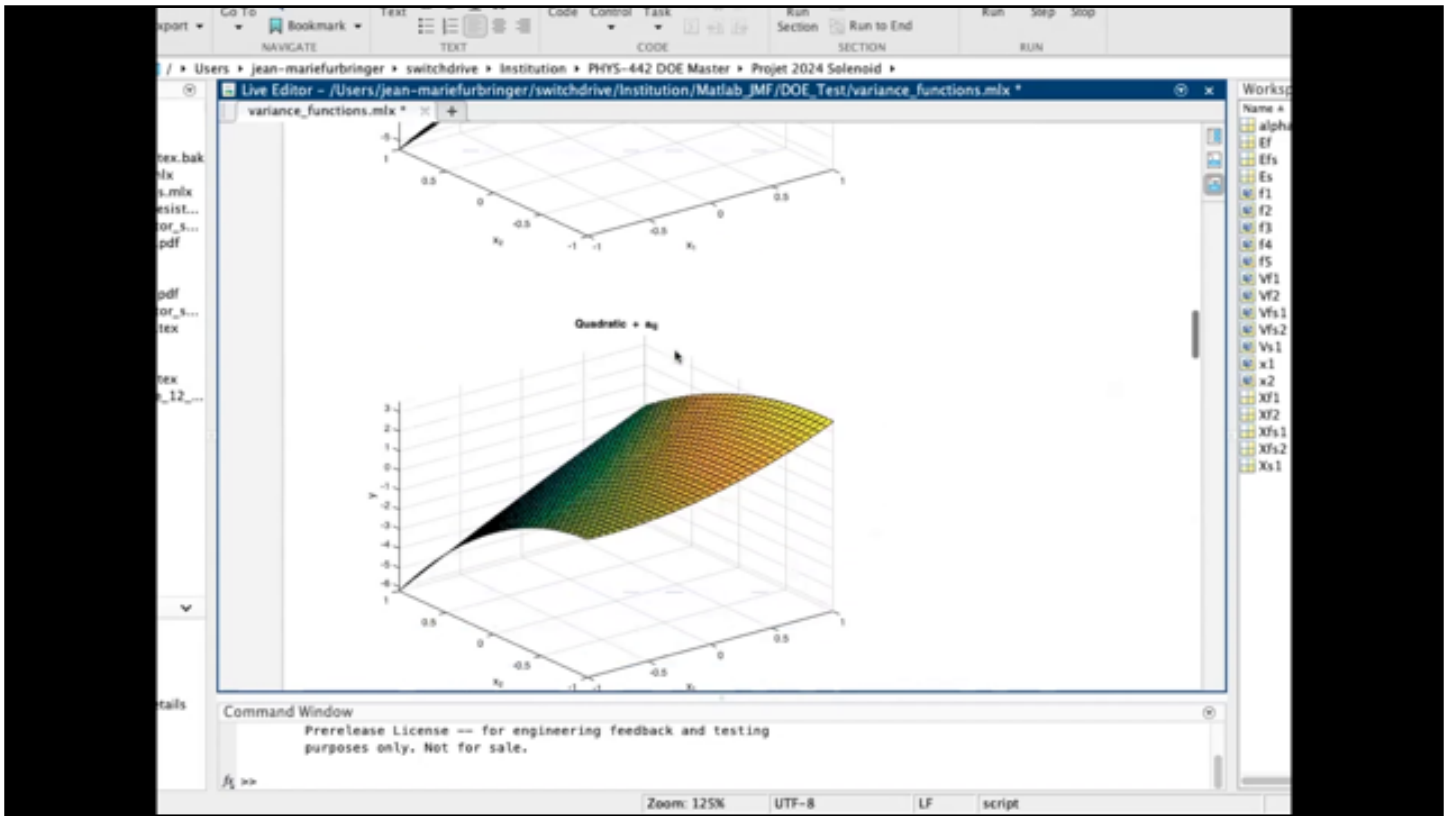
So you see, it's just a deformation of the surface step by step. So it makes sense what we have done with this Taylor polynomials, complicifying each time our model. And this is a cubic model.

notes

summary

2m 17s





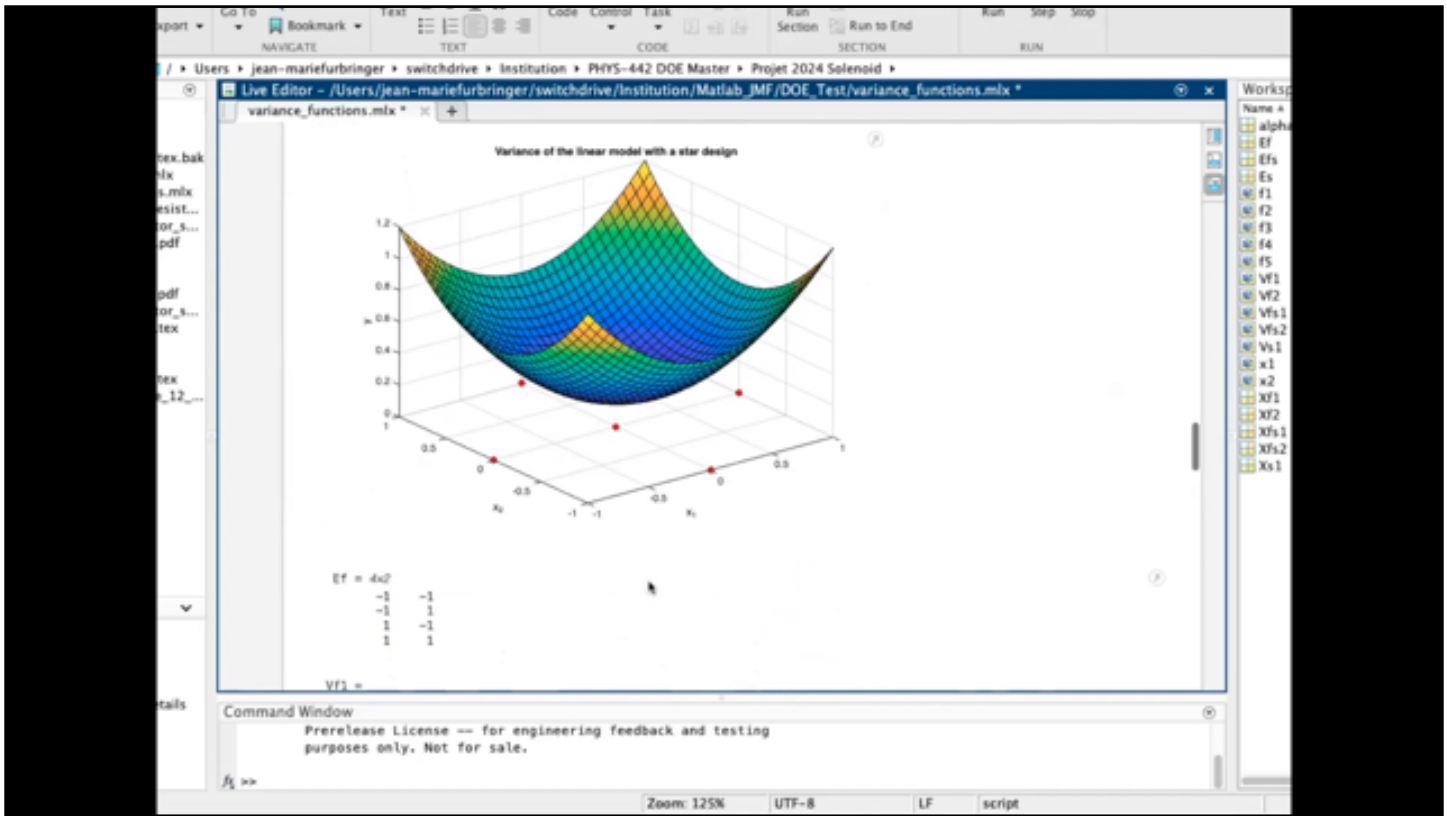
So it's looked like the first one, except that now you have some curvature a little

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2m 36s





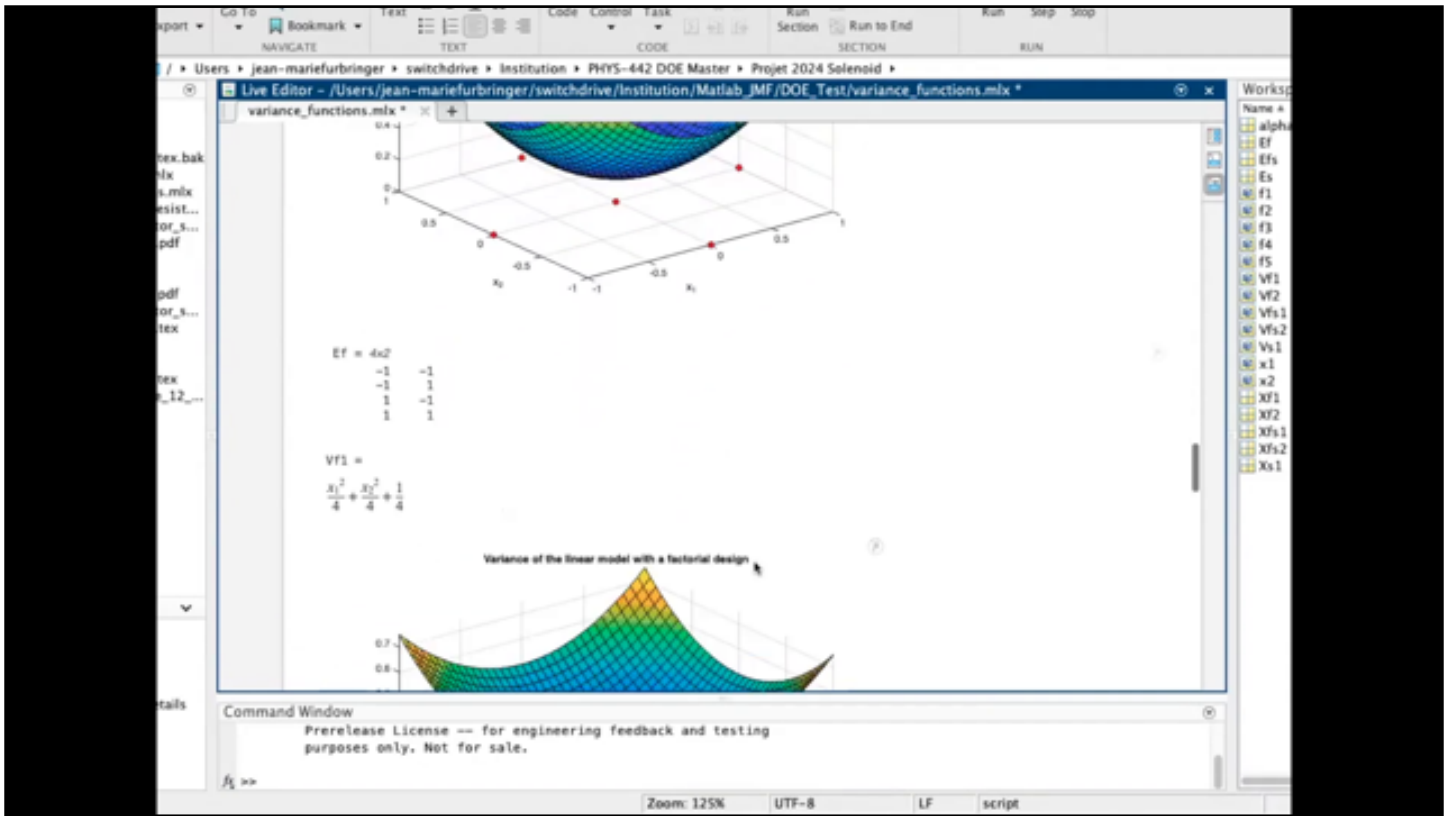
bit more in the corners. And I also make the same thing for the variance function.

notes

summary

2m 38s





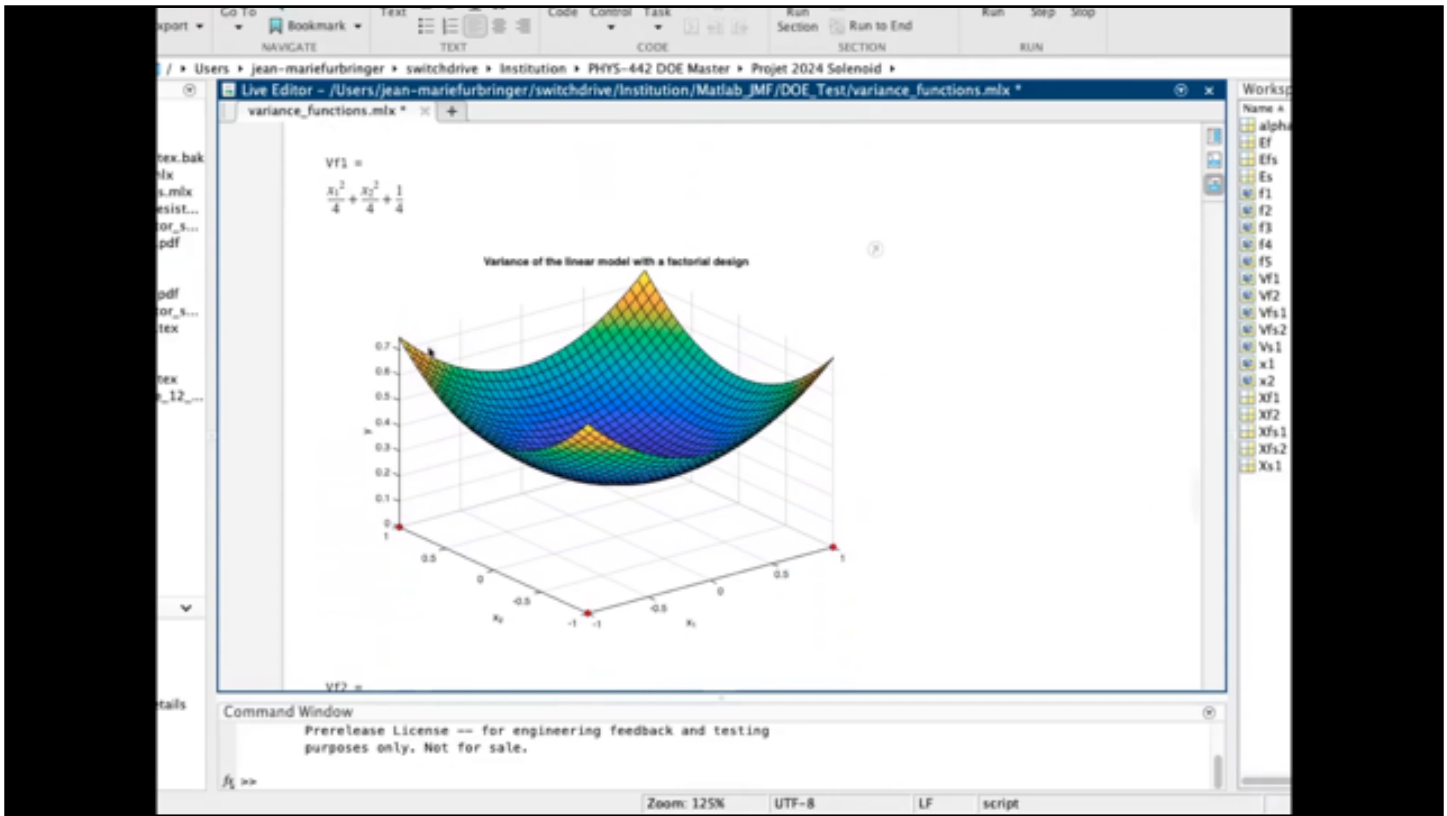
And you see here in the variance function, you see my data points. So my data points are the red points on the floor. So I try first the same type of model changing my design to see what's happened. So the shape will be all time a quadratic function. So the shape of the function will not change because the variance is a quadratic function. And you understand that when you are going away from your data points, so the variance is increasing. So no, no, but what could be interesting is the value. So if you use here five measurement points in, I call it star point, but star point is my words. You don't find that it's a literature. It's just my way of calling them one factor at a time. So you see that here you have the extreme at one point to the variance is around one point two.

notes

summary

2m 51s





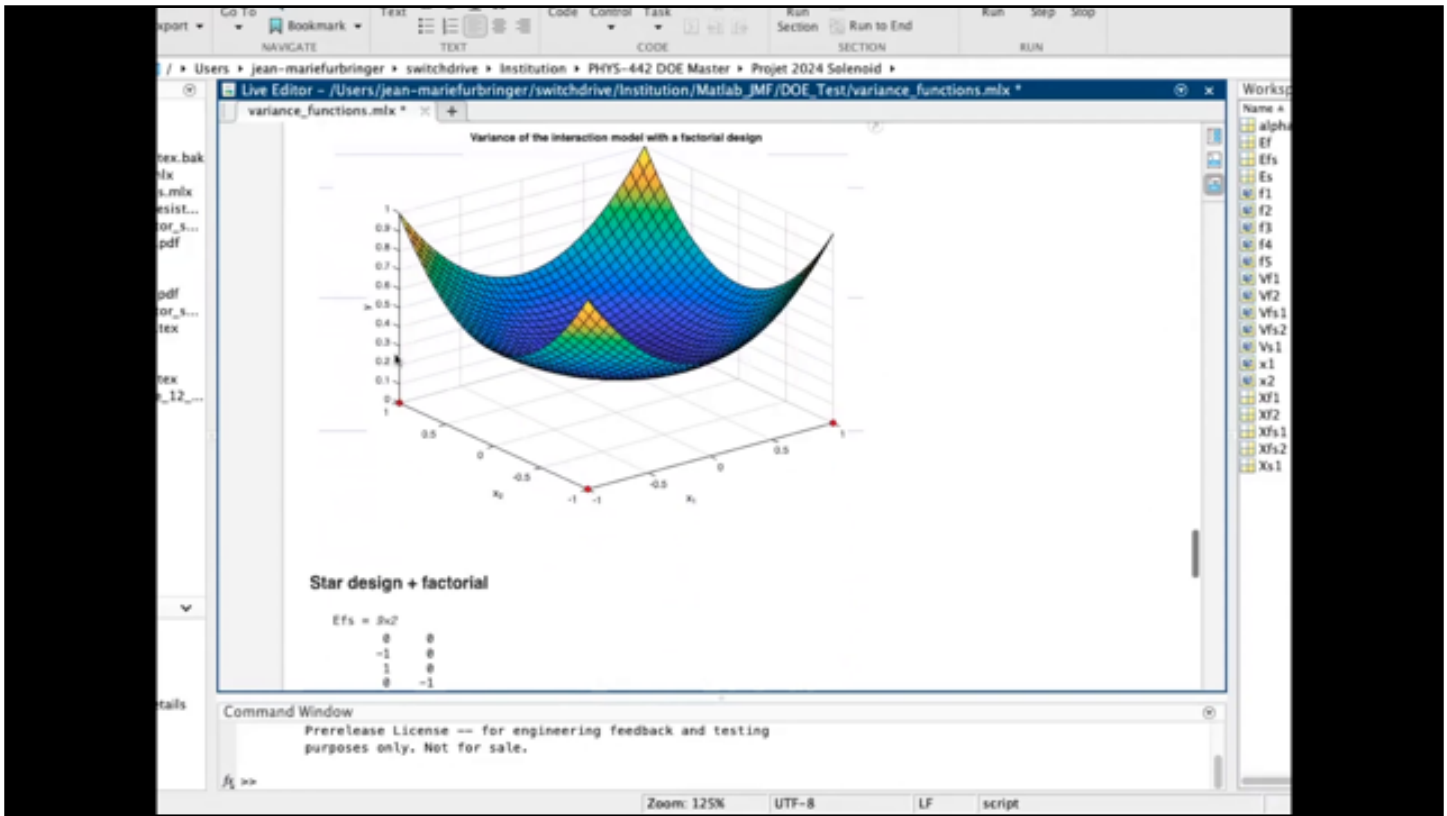
When I'm making measurement on the corner, it's what we call the factorial design.

notes

summary

3m 46s





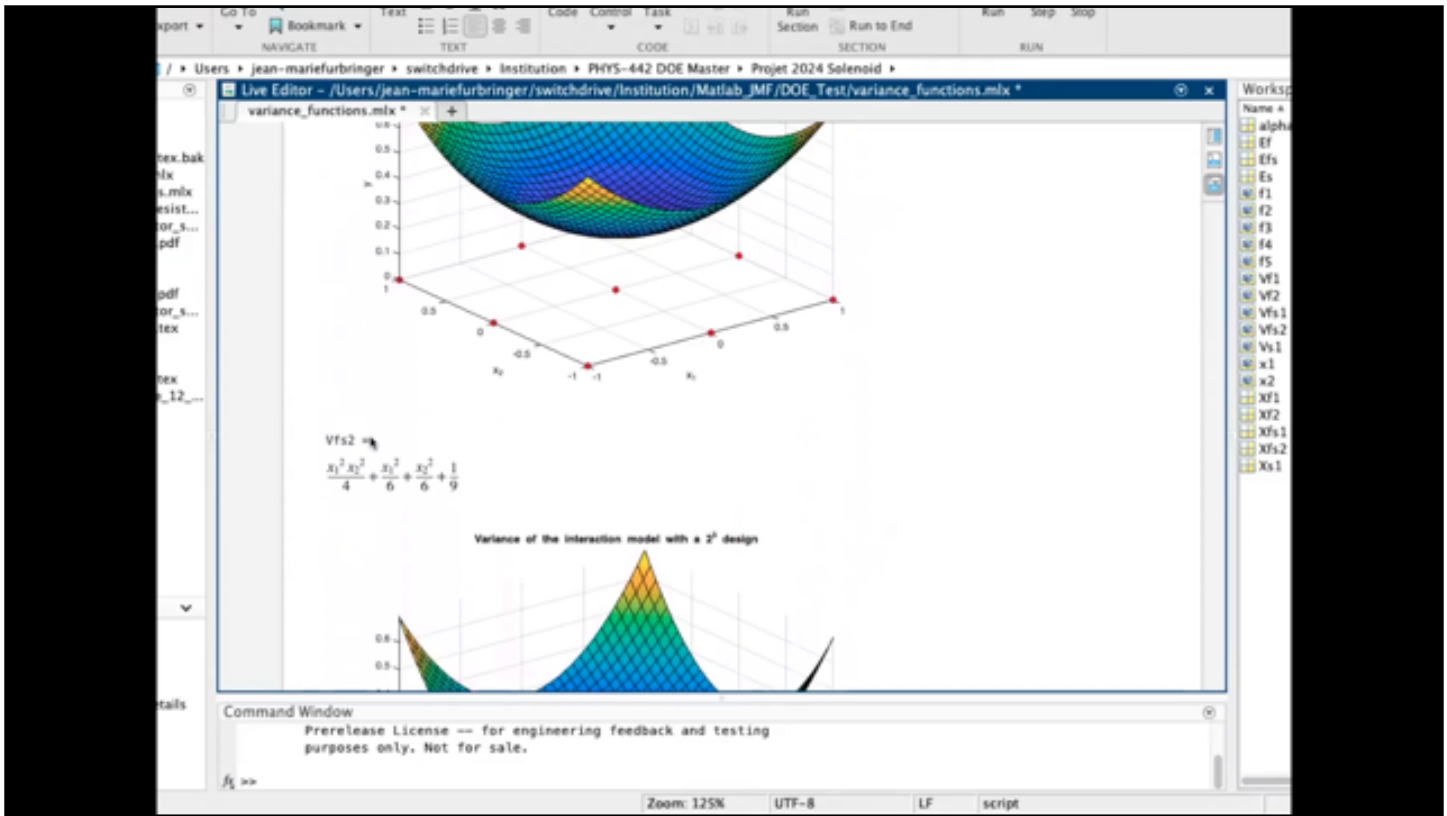
You see now that my variance is 70 something percent. So I'm improved quite half of the quality of my variance just changing and even making less measurement points here at four points before I have five points of measurement.

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summary

3m 54s





And it's clear now what was that after it was interaction. Okay. So it's again, not so well and after I'm improving, et cetera. So if you can, you can, you see also the variance function.

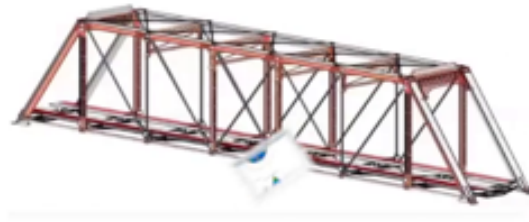
notes

summary

4m 13s



2.5.1 Young modulus of steel



A company is in charge of the design of a bridge and then has to choose the steel that will be used for the beams. An important characteristic of a material when analysing its elasticity is the Young modulus E . In this case, the engineer want to test the Young modulus in function of the temperature and the concentration of carbon and sulfur characterizing the steel available in the market.

It's a quadratic function you see everywhere. You can play with it if you want. So I just put it there for you as a toy.

notes

summary

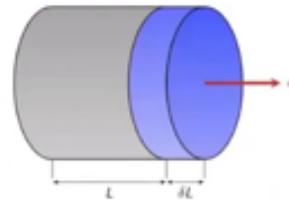
4m 25s



2.5.2 Example of multilinear regression

Hooke law : $\epsilon = \frac{\delta L}{L} = \frac{\sigma}{E}$

1. ϵ : relative strain [-]
2. σ : stress [kpa]
3. L : length of the sample [m]



Sensitivity of the Young modulus : $E = f(T, C, S)$

T : temperature [$^{\circ}\text{C}$]

S : concentration of sulfur [%]

C : concentration of carbon [%]

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I would like to show you an example. Imagine you are in a company, you are a mechanical engineer, you are in a project that would like to build a bridge and you have to choose the steel and to qualify the quality of the steel for your bridge or railroad.

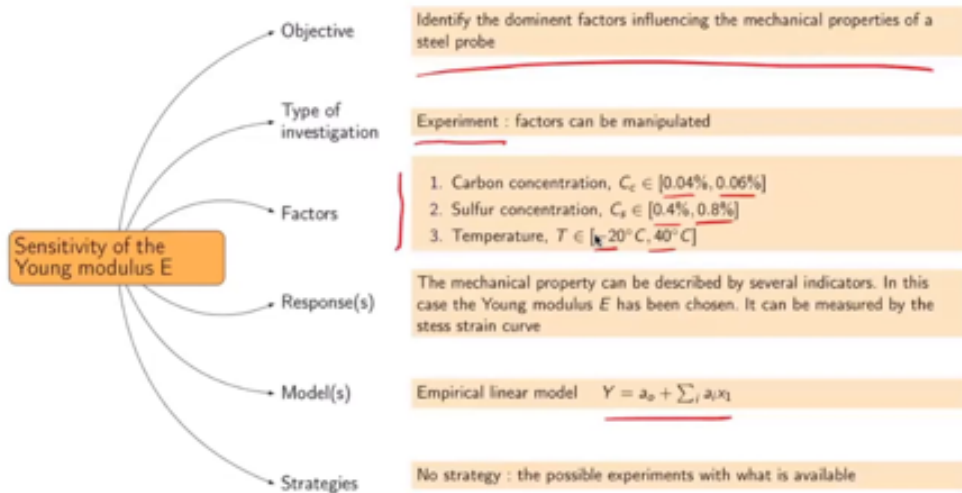
notes

summary

4m 34s



2.5.3 Mindmap



So you have to remember your book flow for elasticity and you remember from the book flow that you have the relative elongation, which depends on the ratio between the strain and between the young modulus E . So we are interested to understand how this young modulus depends on different factors. So I would like to know how it's dependent on temperature, how it depends on the carbon concentration, how it depends on the sulfur concentration. So my young modulus depends, I would like to know about the temperature, the carbon and the sulfur. So I make my map. So what do I want? I want to identify the dominant factors influencing the mechanical property of a steel probe because I want to choose the right steel. What is the type of investigation is experiment. I will go in a laboratory. I will manipulate this concentration of carbon of sulfur on the temperature. I'm not just taking measurement observation from other bridges and things like that. I'm making experiment, not observation experiment. My factor, I have three factors. I have a carbon concentration and imagine that the data are fake, but they are quite correct. I check with a mechanical engineer. So imagine that what you have, what you can buy and what is used usually for this type of constriction is steel. I was a concentration of carbon between 0.04% and 0.06% of carbon. On the sulfur, you see that it's 10 times higher. It's 0.4 and 0.8 as a range of the concentration. And in the temperature, you are interested between minus 20 degrees Celsius and 40 degrees Celsius. It's probably what you imagine being the range of temperatures that interest you. The responses is the mechanical responses. So there are a lot of possibilities to measure that we won't decide for one for the sake of this example. But it's possible sometimes that you have some experiment

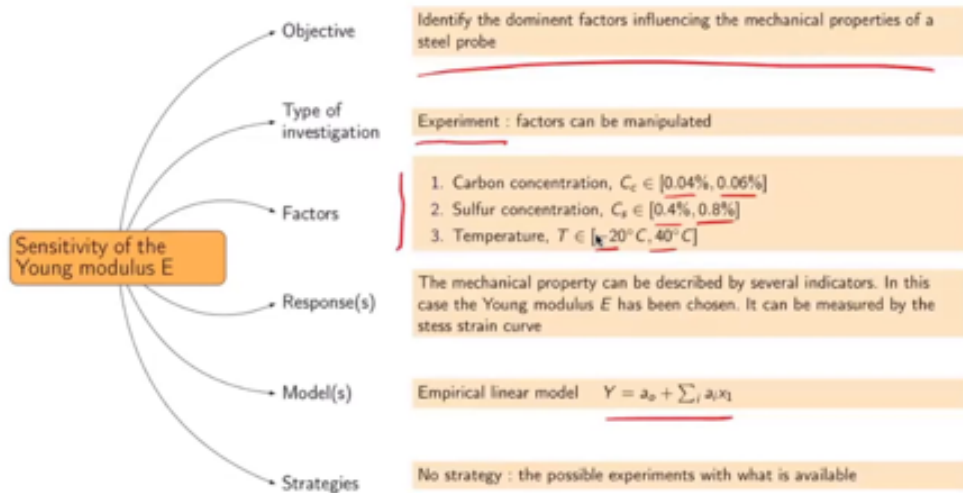
notes

summary

4m 54s



2.5.3 Mindmap



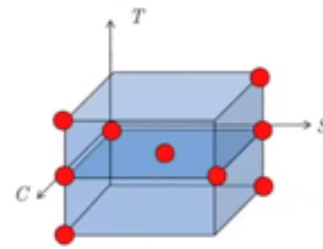
and several outputs. You don't only have one Y , you can have several Y . You can only one. And it's clear that it exists in metallurgic models for understanding with how is the temperature. You can open a book of material and perhaps it'll give you some curves, but you really would like to know what is it and you don't want to make a PhD thesis on it. So empirical models are quite interesting. So having a model telling, OK, I have a constant plus effect of temperature, effect of the carbon concentration and effect of the sulfur is something which is shorter than making a thesis, a PhD thesis on it, and it will predict what I want and let me choose what are the correct steel to choose. The reality is more complex.

notes

summary

2.5.4 Data Young modulus

	Carbon [%]	Sulfur [%]	Temperature [°C]	E [kpa]
1	0.04	0.4	-20	210.31
2	0.04	0.4	0	210.37
3	0.04	0.4	20	210.28
4	0.04	0.8	0	209.18
5	0.05	0.5	0	210.31
6	0.06	0.4	0	210.81
7	0.06	0.8	-20	209.70
8	0.06	0.8	0	209.58
9	0.06	0.8	20	209.67



Eventually there are more than two elements carbon and sulfur in carbon to consider, molybden, anything, but let's make a simple situation. As we are at the start of the course, you don't know any strategy and we will just do what usually people do, trying experiment here and there to get information.

notes

summary

8m 37s



2.5.5 Linear system

Model : $E = \alpha_o + \alpha_c C + \alpha_s S + \alpha_T T + \epsilon$

Matrix equation : $Y = X\alpha + \epsilon$

$$\begin{pmatrix} 210.31 \\ 210.37 \\ 210.28 \\ 209.18 \\ 210.31 \\ 210.81 \\ 209.70 \\ 209.58 \\ 209.67 \end{pmatrix} = \begin{pmatrix} 1 & 0.04 & 0.4 & -20 \\ 1 & 0.04 & 0.4 & 0 \\ 1 & 0.04 & 0.4 & 20 \\ 1 & 0.04 & 0.8 & 0 \\ 1 & 0.05 & 0.6 & 0 \\ 1 & 0.06 & 0.4 & 0 \\ 1 & 0.06 & 0.8 & 0 \\ 1 & 0.06 & 0.8 & 0 \\ 1 & 0.06 & 0.8 & 20 \end{pmatrix} \times \begin{pmatrix} \alpha_o \\ \alpha_c \\ \alpha_s \\ \alpha_T \end{pmatrix} + \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \\ \epsilon_7 \\ \epsilon_8 \\ \epsilon_9 \end{pmatrix}$$

This is the result of the experiment. I don't know if you know how we do this type of experiment. We take sample and we put a strain on the sample and we check the dimension of the sample is changing. And we are able from that to estimate the young modulus, the elasticity coefficient. So this is typically what's look like the pros. This is not steel, it's copper, but usually we have things of that shape and you put a force here and you put a force here in the machine here. And you try to see all this element is behaving when you put strain on it. And here you have the experimental space that we have in the laboratory. We have a few steel probes and we test them at different temperature. It's not so easy because you have to put the device in a controlled temperature environment. So we have one first probe we test at three temperature, vertically you have the temperature. We have a second probe that we test which have different concentration of carbon and sulfur that we test at three temperature. And just before publishing, I don't know deciding and making my report, I have by a few sample of steel and I have time to test them at the standard temperature, but I didn't have time to test them everywhere. So it's why you have these points that are standard temperature, but with different concentration. OK, so reasonable scenario, it's not a good scenario, but it's something I find in a lot of the situation. So now I'm able to have my matrix of experiments. So it could be an E is not the same either to young modulus, but it's usually the matrix I call E. So you see the values. I have made a few experiment with the carbon concentration as 0.4 some at 0.06 and one in

notes

summary

9m 1s



2.5.5 Linear system

Model : $E = \alpha_o + \alpha_c C + \alpha_s S + \alpha_T T + \epsilon$

Matrix equation : $Y = X\alpha + \epsilon$

$$\begin{pmatrix} 210.31 \\ 210.37 \\ 210.28 \\ 209.18 \\ 210.31 \\ 210.81 \\ 209.70 \\ 209.58 \\ 209.67 \end{pmatrix} = \begin{pmatrix} 1 & 0.04 & 0.4 & -20 \\ 1 & 0.04 & 0.4 & 0 \\ 1 & 0.04 & 0.4 & 20 \\ 1 & 0.04 & 0.8 & 0 \\ 1 & 0.05 & 0.6 & 0 \\ 1 & 0.06 & 0.4 & 0 \\ 1 & 0.06 & 0.8 & 0 \\ 1 & 0.06 & 0.8 & 0 \\ 1 & 0.06 & 0.8 & 20 \end{pmatrix} \times \begin{pmatrix} \alpha_o \\ \alpha_c \\ \alpha_s \\ \alpha_T \end{pmatrix} + \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \\ \epsilon_7 \\ \epsilon_8 \\ \epsilon_9 \end{pmatrix}$$

the middle at 0.05. The sulfur also and the temperature I made a few experiment at minus 20 some at plus 20. I didn't go till 14 and I get a few experiment to zero. It could be also standard temperature. Let's check what we get. Let's fit. It's also what we have tried to do with the data with our master project and the master course. So we have a model. So our model is a linear model. So I will say that my experimental young modulus E in this case is would be a constant alpha zero plus a coefficient multiplying the concentration of carbon plus a coefficient alpha S multiplying the concentration of sulfur and plus alpha T is a coefficient multiplying the temperature. Usually, you know, for thermodynamics, we prefer absolute temperature, but in this case, we could accept any temperature.

notes

summary

2.5.5 Linear system

Model : $E = \alpha_o + \alpha_c C + \alpha_s S + \alpha_T T + \epsilon$

Matrix equation : $Y = X\alpha + \epsilon$

$$\begin{pmatrix} 210.31 \\ 210.37 \\ 210.28 \\ 209.18 \\ 210.31 \\ 210.81 \\ 209.70 \\ 209.58 \\ 209.67 \end{pmatrix} = \begin{pmatrix} 1 & 0.04 & 0.4 & -20 \\ 1 & 0.04 & 0.4 & 0 \\ 1 & 0.04 & 0.4 & 20 \\ 1 & 0.04 & 0.8 & 0 \\ 1 & 0.05 & 0.6 & 0 \\ 1 & 0.06 & 0.4 & 0 \\ 1 & 0.06 & 0.8 & 0 \\ 1 & 0.06 & 0.8 & 0 \\ 1 & 0.06 & 0.8 & 20 \end{pmatrix} \times \begin{pmatrix} \alpha_o \\ \alpha_c \\ \alpha_s \\ \alpha_T \end{pmatrix} + \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \\ \epsilon_7 \\ \epsilon_8 \\ \epsilon_9 \end{pmatrix}$$

So if I rewrite my model in a matrix equation, I have an answer why in this case is E but I'm multiplying a matrix of the model X multiplying a coefficient of vector of coefficient. I call alpha and I have some residual some errors. I call epsilon. So that means that no, I have this results that correspond to why I obtain after making calculation of my strength, my strength experiment. I get this value. Usually it's around 200 kilo Pascal. As young modulus, I have my model of the matrix. So my model of a matrix has one column of ones for the constant. And after I have my experiments is a matrix. So this part, let's do like this. So this.

notes

summary

12m 49s



2.5.6 Computing the effects

Estimator : $\hat{\alpha} = (X^T X)^{-1} X^T Y$

Matrix of dispersion :

$$(X^T X)^{-1} = \begin{pmatrix} 3.28 & -58.3 & -0.42 & 0 \\ -58.3 & 1666 & -41.6 & 0 \\ -0.42 & -41.67 & 4.17 & 0 \\ 0 & 0 & 0 & 0.0006 \end{pmatrix}$$

Coefficients :

$$\hat{\alpha}^T = (210.6[kpa], 20.9[kpa/\%C], -2.9[kpa/\%S], -5 \cdot 10^{-5}[kpa/K])$$

Differential : $dE = 20.9dC - 2.9dS - 0.00005dT$

Effects (variation of the output between the extremes of the experimental range) :

$$\text{effect}_C = 2\alpha_C \Delta C = 0.84[kpa], \text{effect}_S = -2.3[kpa], \text{effect}_T = -0.004[kpa]$$

Correspond to my matrix of experiment. And in this case, my matrix of the model was just a matrix of experiment plus a coefficient because I do not have considered interaction between my coefficients. My factors and multiplying a vector of coefficients alpha zero alpha C alpha S and alpha T plus. A vector of residual. So it's how I would like to interpret my experiment. So I use the estimator formula alpha equal alpha X transpose X minus one X transpose. Why you see here the matrix of dispersion. Okay, not really easy to say I sell you that usually you can see something you see it's not diagonal. But because of the units, you have a diagonal value, which is three. You have a value three for the constant. You have the value 1066 for the for the further carbon coefficient. You have one value for the solution coefficient and you have one value zero point zero zero six is a problem of the matrix. In fact, I can say nothing in this situation. I will propose you something. But what I the only thing I can observe it's not a diagonal matrix. So I'm able to make this calculation. X transpose X, etc. And if I do that, I obtain my different coefficient alpha. I would obtain for my constant 210.6 kilo Pascal is my a zero. After I obtain a coefficient for the carbon, it will be 20.9 kilo Pascal per percent of carbon. And I will obtain a coefficient for the sulfur. It will be minus 2.9 kilo Pascal per percent of sulfur. And I will obtain a number for the temperature and it's minus five 10 power minus five kilo Pascal per degree centigrade or Kelvin. What what you want. The only thing you can observe that you we are probably around 200 kilo Pascal. So it's standard for young modulus and that you

notes

summary

13m 50s



2.5.6 Computing the effects

Estimator : $\hat{\alpha} = (X^T X)^{-1} X^T Y$

Matrix of dispersion :

$$(X^T X)^{-1} = \begin{pmatrix} 3.28 & -58.3 & -0.42 & 0 \\ -58.3 & 1666 & -41.6 & 0 \\ -0.42 & -41.67 & 4.17 & 0 \\ 0 & 0 & 0 & 0.0006 \end{pmatrix}$$

Coefficients :

$$\hat{\alpha}^T = (210.6[kpa], 20.9[kpa/\%C], -2.9[kpa/\%S], -5 \cdot 10^{-5}[kpa/K])$$

Differential : $dE = 20.9dC - 2.9dS - 0.00005dT$

Effects (variation of the output between the extremes of the experimental range) :

$$effect_C = 2\alpha_C \Delta C = 0.84[kpa], \quad effect_S = -2.3[kpa], \quad effect_T = -0.004[kpa]$$

have a negative value for the sulfur. Are you really able to tell me just like that what is the most important factor? You have to make a few calculation. You have to calculate the effects. So here you don't have really effects. You have coefficient of a fitted model. But the problem that you cannot compare the value between them because the units are not the same.

notes

summary

2.5.7 Let's take stock

- ▶ We have started with a matrix of experiments : A
- ▶ We have chosen a model with a linear response :

$$E = \alpha_o + \alpha_c C + \alpha_s S + \alpha_T T + \epsilon$$
- ▶ A model matrix is built :
- ▶ The least square fit method is used to compute the model coefficients
- ▶ The quality of the fit is evaluated with the statistic $R^2 = \frac{\hat{Y}^T \hat{Y}}{Y^T Y}$
- ▶ The theorem of the confidence interval is used to verify the quality of the coefficients

And they are dependent of the metric. They are dependent of the range of variation. You can calculate a differential, but it's just mathematics of what we have calculated. We have a difference of young modulus, which is proportional. It was our model of the difference of carbon, the difference of sulfur and the difference of temperature. But if you want to understand the effect of each factor, you have to multiply these numbers by the range of them. Or at least half of the range of them for understanding what really vary, what you can vary for changing your steel. But what import you is are you vary E ? The young modulus, the values of the coefficient doesn't interest you for itself. So if you do that, that means that you have to multiply the coefficient by two times the delta. The delta is half of the range. So you have to multiply by the range. Delta C is half of the range of my carbon. And then you see that just changing the carbon from the highest to the smallest value of the carbon, you can vary of 0.84 kilopascal. The same thing, the same calculation for the effect of the sulfur. You have to multiply the coefficient by the range of the sulfur. And you see that you are able to vary your steel elasticity coefficient by 2.3 kilopascal. In this case, it's negative. That means that increasing the sulfur will decrease the young modulus. That means that I will make your steel blander. And what is interesting also is you do the same thing with the coefficient of the temperature. And you see that what was very, very small because the numbers of temperature, the change was a lot. It was 40 degrees of variation. So it's still a small coefficient, but not as small as 10 minus 5. Because when you see 5, you say,

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summary

17m 13s



2.5.7 Let's take stock

- ▶ We have started with a matrix of experiments : A
- ▶ We have chosen a model with a linear response :

$$E = \alpha_o + \alpha_c C + \alpha_s S + \alpha_T T + \epsilon$$
- ▶ A model matrix is built :
- ▶ The least square fit method is used to compute the model coefficients
- ▶ The quality of the fit is evaluated with the statistic $R^2 = \frac{\hat{Y}^T \hat{Y}}{Y^T Y}$
- ▶ The theorem of the confidence interval is used to verify the quality of the coefficients

OK, forget it. It's not important. It could be, in fact, more important than you believe because of this question of ranges that are very different. If you want to get the effect, you have to multiply your coefficient by the delta C. But the problem of the delta C is from 0 to the extreme. Usually when you write your differential, so if you want to have the effect, it's two times your DC. It's possible that my numbers are not correct. It's absolutely possible.

notes

summary

2.5.7 Let's take stock

- ▶ We have started with a matrix of experiments : A
- ▶ We have chosen a model with a linear response :

$$E = \alpha_o + \alpha_c C + \alpha_s S + \alpha_T T + \epsilon$$
- ▶ A model matrix is built :
- ▶ The least square fit method is used to compute the model coefficients
- ▶ The quality of the fit is evaluated with the statistic $R^2 = \frac{\hat{Y}^T \hat{Y}}{Y^T Y}$
- ▶ The theorem of the confidence interval is used to verify the quality of the coefficients

Let's say a few things. So we have started with the matrix of experiments. I don't know why I call it A. We choose a model. It was a linear model.

notes

summary

20m 21s



2.5.7 Let's take stock

- ▶ We have started with a matrix of experiments : A
- ▶ We have chosen a model with a linear response :

$$E = \alpha_o + \alpha_c C + \alpha_s S + \alpha_T T + \epsilon$$
- ▶ A model matrix is built :
- ▶ The least square fit method is used to compute the model coefficients
- ▶ The quality of the fit is evaluated with the statistic $R^2 = \frac{\hat{Y}^T \hat{Y}}{Y^T Y}$
- ▶ The theorem of the confidence interval is used to verify the quality of the coefficients

We build a linear matrix.

notes

summary

20m 35s



2.5.7 Let's take stock

- ▶ We have started with a matrix of experiments : A
- ▶ We have chosen a model with a linear response :

$$E = \alpha_o + \alpha_c C + \alpha_s S + \alpha_T T + \epsilon$$
- ▶ A model matrix is built :
- ▶ The least square fit method is used to compute the model coefficients
- ▶ The quality of the fit is evaluated with the statistic $R^2 = \frac{\hat{Y}^T \hat{Y}}{Y^T Y}$
- ▶ The theorem of the confidence interval is used to verify the quality of the coefficients

We use the least square feet method for calculating the alpha coefficient. And we can evaluate, eventually, the quality of our feet with an air square. And we have a theorem of confidence interval we can calculate.

notes

summary

20m 41s



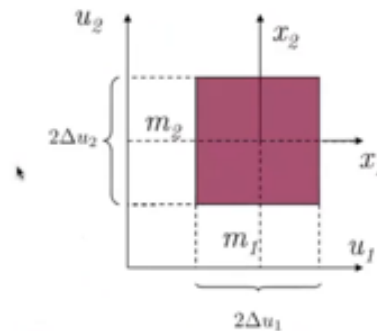
2.5.8 Standardisation of the experimental domain

- ▶ Original variables : u_i
- ▶ Standardized variables x_i
- ▶ Midway of the ranges :

$$m_i = \frac{\max(u_i) + \min(u_i)}{2}$$
- ▶ half ranges :

$$\Delta u_i = \frac{\max(u_i) - \min(u_i)}{2}$$
- ▶ Linear relations : $x_i = \frac{u_i - m_i}{\Delta u_i}$

$$u_i = m_i + x_i \Delta u_i$$



sur 47

So what we have done is correct, but I would like to propose a better way of doing this.

notes

summary

20m 57s



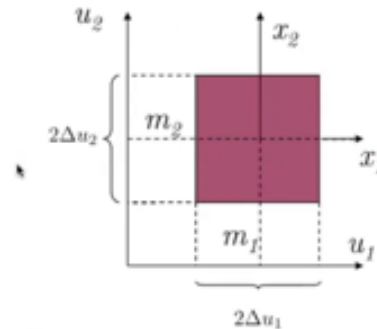
2.5.8 Standardisation of the experimental domain

- ▶ Original variables : u_i
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$$m_i = \frac{\max(u_i) + \min(u_i)}{2}$$
- ▶ half ranges :

$$\Delta u_i = \frac{\max(u_i) - \min(u_i)}{2}$$
- ▶ Linear relations : $x_i = \frac{u_i - m_i}{\Delta u_i}$

$$u_i = m_i + x_i \Delta u_i$$



It's the standardization that lets you see better what are the dominant factors and see you also better what you can do for improving your experiments.

notes

summary

21m 4s



2.5.9 How to standardised the matrix of experiments

$$E = \begin{pmatrix} 0.04 & 0.4 & -20 \\ 0.04 & 0.4 & 0 \\ 0.04 & 0.4 & 20 \\ 0.04 & 0.8 & 0 \\ 0.05 & 0.6 & 0 \\ 0.06 & 0.4 & 0 \\ 0.06 & 0.8 & -20 \\ 0.06 & 0.8 & 0 \\ 0.06 & 0.8 & 20 \end{pmatrix} \rightarrow E_{st} = \begin{pmatrix} -1 & -1 & -1 \\ -1 & -1 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

MATLAB

```
E_st = rescale(E,-1,1,'InputMin',min(E),'InputMax',max(E))
```

So when we do standardization, so there are very standard standardization of statistics is going to the mean and multiplying or dividing by the variance for having a variance of one. It's not this standardization that we do. We will go to the median. We will go to the middle of our range and not the mean of our data. And we will go from minus one to plus one is another way of standardization, but it's very practical for comparing designs. And also after you have a standard space in which we play, it's more easy. So we start from the original variable in this slide. I call them U. So is the percentage of carbon is the temperature is a percentage of of sulfur. And I would like to use standardized value. I call them X. So what is important is the midway of the range, the median. So is, as we said, maximum plus minimum divided by two, and I used to call it M. And I also using the half range, I use a capital letter delta. And it's the maximum minus the minimum divided by two. And then I can do is a linear transformation. I can do the transform as a reverse transformation. So if I want to go from the. So if I want to calculate the standardized value, it will be the natural value. Minus the midrange and divided by the half range. And if I go the reverse way, I will say that my natural value is the one the value I need in the laboratory. Because if you go to the laboratory and you say, oh, make the measurement in plus one minus one, you know, the variable. People will not do what to do. So you need to go from one to the other. But it's a linear transformation. So it doesn't change the curvature of your model. It

notes

summary

21m 25s



2.5.9 How to standardised the matrix of experiments

$$E = \begin{pmatrix} 0.04 & 0.4 & -20 \\ 0.04 & 0.4 & 0 \\ 0.04 & 0.4 & 20 \\ 0.04 & 0.8 & 0 \\ 0.05 & 0.6 & 0 \\ 0.06 & 0.4 & 0 \\ 0.06 & 0.8 & -20 \\ 0.06 & 0.8 & 0 \\ 0.06 & 0.8 & 20 \end{pmatrix} \rightarrow E_{st} = \begin{pmatrix} -1 & -1 & -1 \\ -1 & -1 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

MATLAB

```
Est = rescale(E,-1,1,'InputMin',min(E),'InputMax',max(E))
```

doesn't change nothing. It's just you are moving your coordinates of your domain so that you know where you are, where you are more easily. And we will travel between minus one and plus one. So the natural value will be the midrange plus the standard value. XI multiplied by the half range. And you see this small graphic, we just fit my window of interest around where I have data.

notes

summary

2.5.10 Computing the standardised effects

Estimator : $\hat{\alpha} = (X^T X)^{-1} X^T Y$

Dispersion matrix : $(X^T X)^{-1} = \begin{pmatrix} 0.11 & 0 & 0 & 0 \\ 0 & 0.17 & -0.083 & 0 \\ 0 & -0.083 & 0.17 & 0 \\ 0 & 0 & 0 & 0.25 \end{pmatrix}$

Half effects : $\hat{\alpha}^T = (210[kpa], 0.245[kpa], -0.63[kpa], -0.053[kpa])$

Model : $E = 210 + 0.24x_C - 0.63x_S - 0.053x_T$ avec $x_i \in [-1, 1]$

Relative effects relatifs :

$ER_C = 0.11[\%]$, $ER_S = -0.3[\%]$, $ER_T = -0.07[\%]$

ER = half effect divided by the constant a_0

So if I do that, my original matrix of experiment became a matrix with values that normally are between minus one and plus one. And anything in between, in this case, I have zero, but I can have any value in my domain. I have a standardized domain between minus one and plus one. You can do that very easily with MATLAB, with a rescale function. You have to tell what you would like to rescale. In this case, E is the matrix I would like to rescale, my matrix of experiment, not my value of my young modulus. Sorry for having this E. You say that you want to go between minus one and plus one. And after you say what you would like to change, so you're saying that your input minimum will be the minimum of your matrix. And you will say that the maximum will be the maximum of your matrix. So now it's become more interesting analyzing some steps. Now I do the same operation, but with my matrix of experiment standardized. I do not standardize my experiments, my result, my why. I do not standardize it. It's just my coordinate of my domain that I'm studying.

notes

summary

24m 23s



2.5.9 How to standardised the matrix of experiments

$$E = \begin{pmatrix} 0.04 & 0.4 & -20 \\ 0.04 & 0.4 & 0 \\ 0.04 & 0.4 & 20 \\ 0.04 & 0.8 & 0 \\ 0.05 & 0.6 & 0 \\ 0.06 & 0.4 & 0 \\ 0.06 & 0.8 & -20 \\ 0.06 & 0.8 & 0 \\ 0.06 & 0.8 & 20 \end{pmatrix} \rightarrow E_{st} = \begin{pmatrix} -1 & -1 & -1 \\ -1 & -1 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

MATLAB

`E_st = rescale(E,-1,1,'InputMin',min(E),'InputMax',max(E))`

So now when I'm calculating my matrix of dispersion, now I'm able to compare the element because the unit are everywhere the same because they are related to the range. All those values are related to the range and the covariance between my different variable. So what I observe, I have 11% as a diagonal element for the constant 17% and it's correct to talk about percent in this case because it is percent of my range. 17% for the concentration of carbon and the concentration of so that the linear the linear effect of the concentration of carbon and the linear concentration of. Sulphur and I have 25% of the temperature. So here I'm able to understand the quality of my measurement my experiments. I understand 11% is how many that I have. I don't remember.

notes

summary

25m 49s



2.5.10 Computing the standardised effects

Estimator : $\hat{\alpha} = (X^T X)^{-1} X^T Y$

Dispersion matrix : $(X^T X)^{-1} = \begin{pmatrix} 0.11 & 0 & 0 & 0 \\ 0 & 0.17 & -0.083 & 0 \\ 0 & -0.083 & 0.17 & 0 \\ 0 & 0 & 0 & 0.25 \end{pmatrix}$

Half effects : $\hat{\alpha}^T = (210[kpa], 0.245[kpa], -0.63[kpa], -0.053[kpa])$

Model : $E = 210 + 0.24x_C - 0.63x_S - 0.053x_T$ avec $x_i \in [-1, 1]$

Relative effects relatifs :

$ER_C = 0.11[\%]$, $ER_S = -0.3[\%]$, $ER_T = -0.07[\%]$

ER= half effect divided by the constant a_0

sur 47

Nine so 11% is one divided by nine.

notes

summary

26m 59s



2.5.10 Computing the standardised effects



Estimator : $\hat{\alpha} = (X^T X)^{-1} X^T Y$

Dispersion matrix : $(X^T X)^{-1} = \begin{pmatrix} 0.11 & 0 & 0 & 0 \\ 0 & 0.17 & -0.083 & 0 \\ 0 & -0.083 & 0.17 & 0 \\ 0 & 0 & 0 & 0.25 \end{pmatrix}$

Half effects : $\hat{\alpha}^T = (210[kpa], 0.245[kpa], -0.63[kpa], -0.053[kpa])$

Model : $E = 210 + 0.24x_C - 0.63x_S - 0.053x_T$ avec $x_i \in [-1, 1]$

Relative effects relatifs :

$ER_C = 0.11[\%], ER_S = -0.3[\%], ER_T = -0.07[\%]$

ER= half effect divided by the constant a_0

So for my constant, I'm making my best. It's normal because my constant is there and I'm measuring several times and I'm able to calculate it as. And it is I see that for the carbon and the sulphur I have 17%. It's not the best possible is the best possible would be 11% but it's not so bad. I'm just losing 5% of quality, but it's even a lot better than my measurement. My measurement is one. So that means that I will have a variance of my constant which will be 11% of my variance of my measurements. I will have the variance of my coefficient of sulphur and my coefficient for the carbon that will be 17% of my quality of my measurement of my variance. S square. Sorry, it's not the right s square. It's the variance of my measurement. So it's quite okay. It's not a bad measurement and I have 25% to quarter for the quality of my wisdom. So I can do better, but nevertheless, my experiment is not bad at all. I'm really having coefficients. There are more precise than my measurement itself. Okay. What I would appreciate is having 11% everywhere and zero outside. You also see that the non-diagonality is less severe now and I'm observing it. Nevertheless, I see that I have one non-diagonal element of 8%. So I have some covariance between the influence of the sulphur and the influence of the carbon. So that means that in fact, okay, I've divided it's not as my example with the solar cells that I'm measuring. It's when it's dry and hot and cold and wet. But okay, I have a small influence. 8% is not so bad. Okay. No, I'm able to calculate my coefficient and no, I'm able to compare them. Because the units of my coefficient is kilopascal. It's not kilopascal per concentration or per temperature. So

notes

summary

27m 3s



2.5.10 Computing the standardised effects

Estimator : $\hat{\alpha} = (X^T X)^{-1} X^T Y$

Dispersion matrix : $(X^T X)^{-1} = \begin{pmatrix} 0.11 & 0 & 0 & 0 \\ 0 & 0.17 & -0.083 & 0 \\ 0 & -0.083 & 0.17 & 0 \\ 0 & 0 & 0 & 0.25 \end{pmatrix}$

Half effects : $\hat{\alpha}^T = (210[kpa], 0.245[kpa], -0.63[kpa], -0.053[kpa])$

Model : $E = 210 + 0.24x_C - 0.63x_S - 0.053x_T$ avec $x_i \in [-1, 1]$

Relative effects relatifs :

$ER_C = 0.11[\%]$, $ER_S = -0.3[\%]$, $ER_T = -0.07[\%]$

ER= half effect divided by the constant a_0

I'm able to understand also the ratio between them.

notes

summary

2.5.11 Coefficients in the original units

To have the coefficients in the units of the laboratory it is necessary to go back to the non standardised system. It is different for each model. Here is the case of a linear model.

$$\begin{aligned}
 y &= a_0 + \sum a_i x_i \\
 x_i &= \frac{u_i - \bar{u}_i}{\Delta u_i} \\
 y &= a_0 + \sum a_i \frac{u_i - \bar{u}_i}{\Delta u_i} \\
 &= \left(a_0 - \sum a_i \frac{\bar{u}_i}{\Delta u_i} \right) + \sum \frac{a_i}{\Delta u_i} u_i \\
 &= b_0 + \sum b_i u_i
 \end{aligned}$$

So you see that they are all smaller than the units. So I'm not changing a lot my Jung modulus. You see directly that the most important for what I'm varying is the sulphur, which is 0.63, which is three times something as three times bigger. Negative, but three times bigger than the influence of the carbon. And you see that the influence of the temperature is smaller than the other, but not 10 power 5 smaller than the other. No, I'm able to compare. But it's clear for my analysis. It's not for my lab. In the lab, I have to come back to the other model. If I really want to say, okay, for this concentration, what do I get as a Jung modulus? So in fact, we need both that mean inverse that means that when I'm increasing the sulphur, I'm making my Jung modulus smaller. So I make a blender, a blender still. Something that you can also calculate that I find interesting. Having handling well your, your, your, your stuff. Yes, calculating the relative effect. So you divide the effects, the half effect in fact, by the a_0 . And that gives you a percentage of variation. So rapidly you understand not only what are the most important factor you also see what how they're effective. And you see that in this case, if you talk about a few percentage or even 10s or hundreds of percentage that you can move, you don't change a lot. You see the Jung modulus of your case, because you see that per percentage of change, the effect is 11% for the carbon is 0.3% for the sulphur, also 30% of the change in percent. And it's seven, seven percent of change. This is I have because I have a big, a big range of it. So you see that you cannot be very, very effective in this game.

notes

summary

30m 1s



2.5.11 Coefficients in the original units

To have the coefficients in the units of the laboratory it is necessary to go back to the non standardised system. It is different for each model. Here is the case of a linear model.

$$\begin{aligned}
 y &= a_o + \sum a_i x_i \\
 x_i &= \frac{u_i - \bar{u}_i}{\Delta u_i} \\
 y &= a_o + \sum a_i \frac{u_i - \bar{u}_i}{\Delta u_i} \\
 &= \left(a_o - \sum a_i \frac{\bar{u}_i}{\Delta u_i} \right) + \sum \frac{a_i}{\Delta u_i} u_i \\
 &= b_o + \sum b_i u_i
 \end{aligned}$$

We know that the Jung modulus of still is quite fixed.

notes

summary



Again, I can finish the course with these slides. So now it's very important that you understand each of those elements and you are able to place them and to use them and to understand how you calculate them from one another. Matrix of experiment, matrix of the model, matrix of dispersion, variance, inflation, coefficients are the most important. So in the exercise today I propose you use exercise three. It will be this calculation. I believe the data is a little bit different. I don't remember that nevertheless you will make this math lab or Python or Excel. You will redo this calculation.

notes

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summary

33m 31s

