



Course material

Course:

**ENG606 / PHYS 442**

Video:

**DOE\_lesson6\_part1\_Aliases**

Concepts (extracted from automatically generated subtitles):

**Part of the model. Alias matrix. Alias concept. Second degree. Much project. First step proposal. Alpha hat. Rest of the model. Linear coefficients. Interesting designs. Senior researcher. Model. Given design. Case study. Design of experiments.**



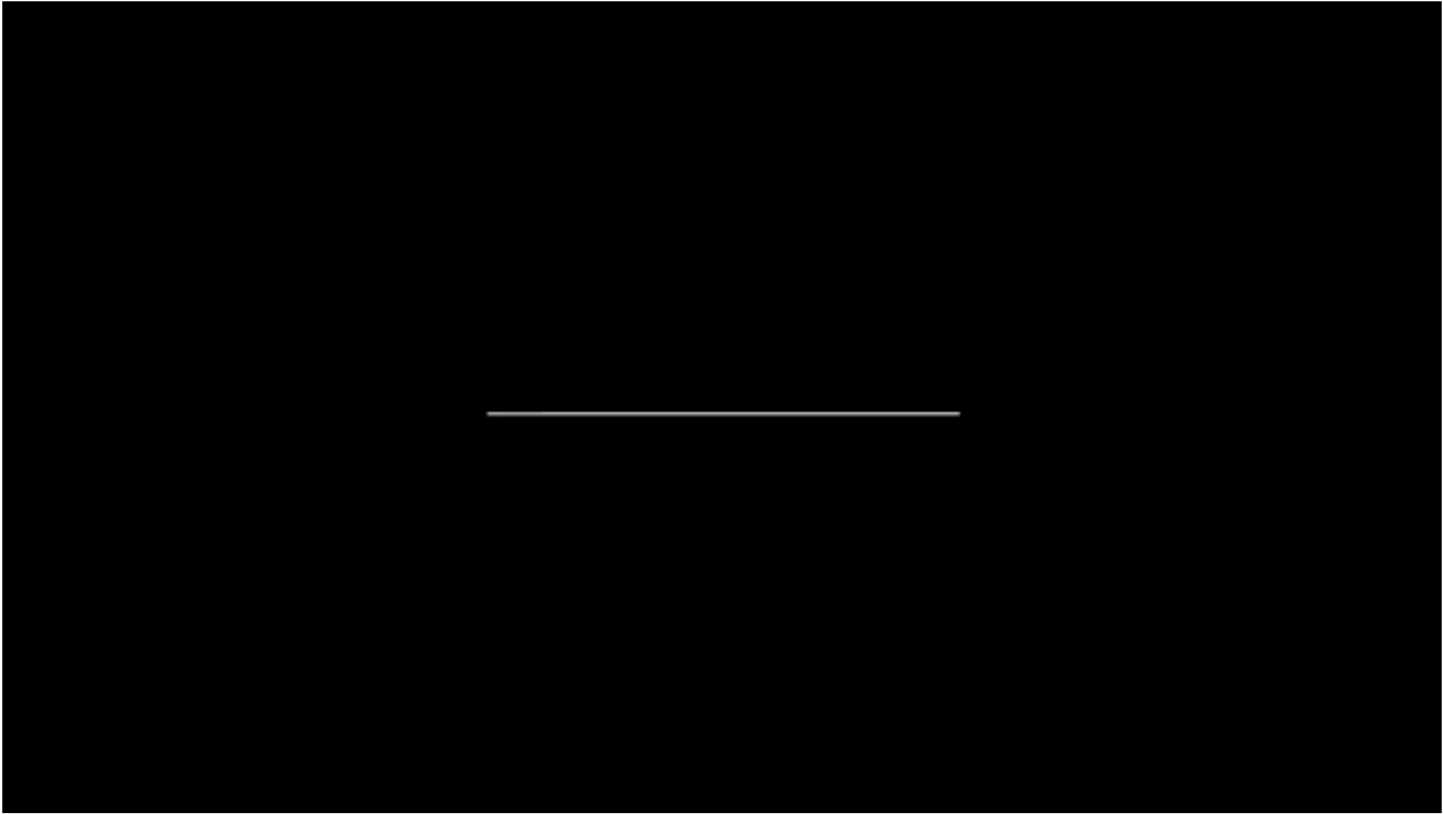
[to video sequence search](#)  
(within ENG606 / PHYS 442.)



[to video](#)

Center for Digital Education. More educational support material here:

<https://www.epfl.ch/education/educational-initiatives/cede/educational-technologies-gallery/boocs-en/>  
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Semestrial exam (MA students only)

Proposal and report (phD students only)

Project Rules

- Teamwork: the project is realized by a team of 2 to 3 persons. The teams must be formed for the submission of the proposal.
- Case study: the case study must be centered on an experimental or numerical situation. The discussion must analyse the DOE strategy design(s) strength and weaknesses, and the analysis of results.
- Guidelines: A document is given below with precise indications on the content of the proposal and of the report.
- The project proposal should be submitted by end of december (Extension of delay can be obtained)
- The project report should be submitted before mid June

Guidelines and example

Semestrial exam (MA students only)

Alier à...

EPFL

Contact EPFL CH-1015 Lausanne +41 21 693 11 11

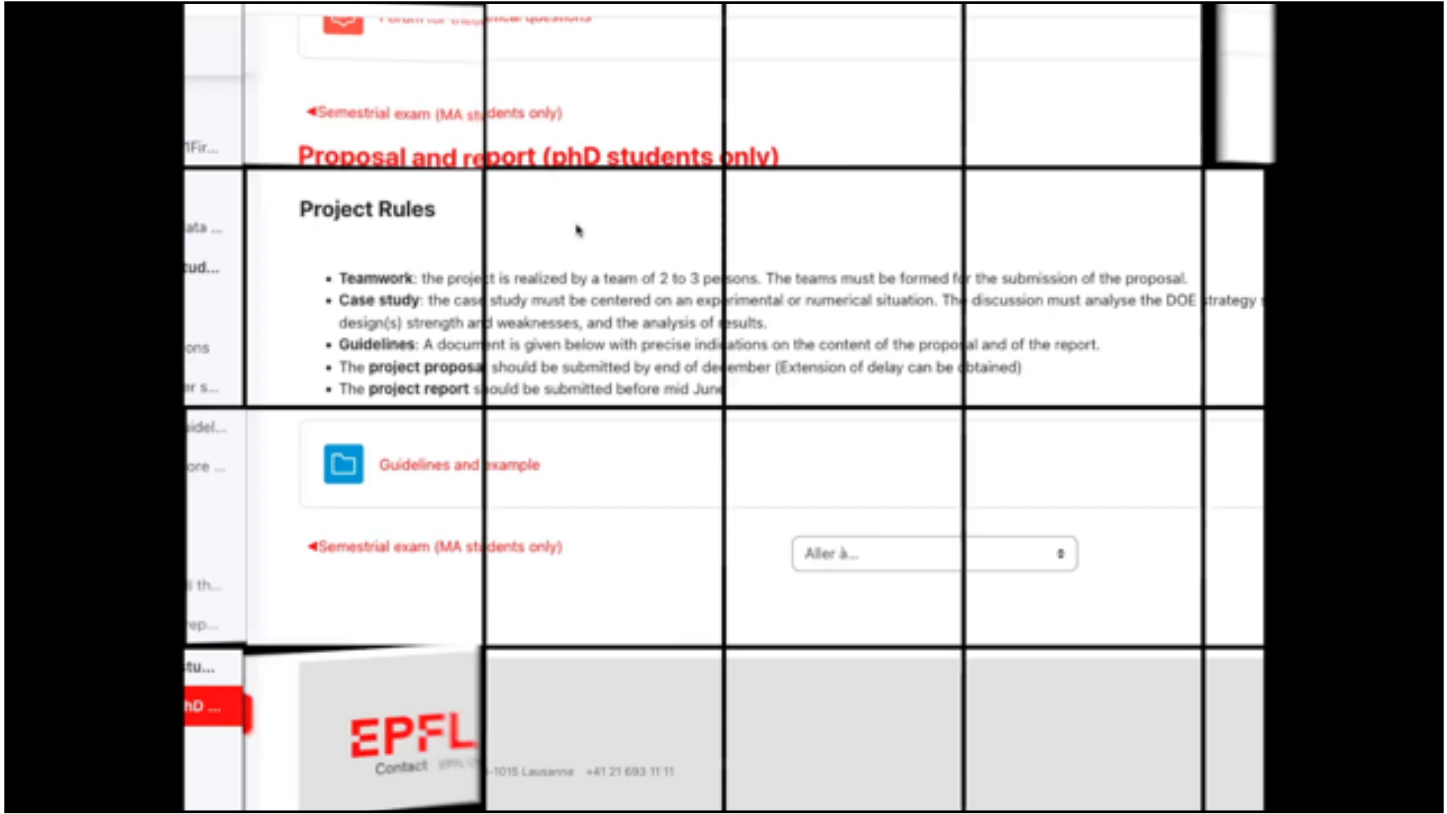
These subtitles have been generated automatically about the project. So this is for the PhD students. So for getting the credits, for

notes

summary

0m 1s

page 3/26 - DOE\_lesson6\_part1\_Aliases



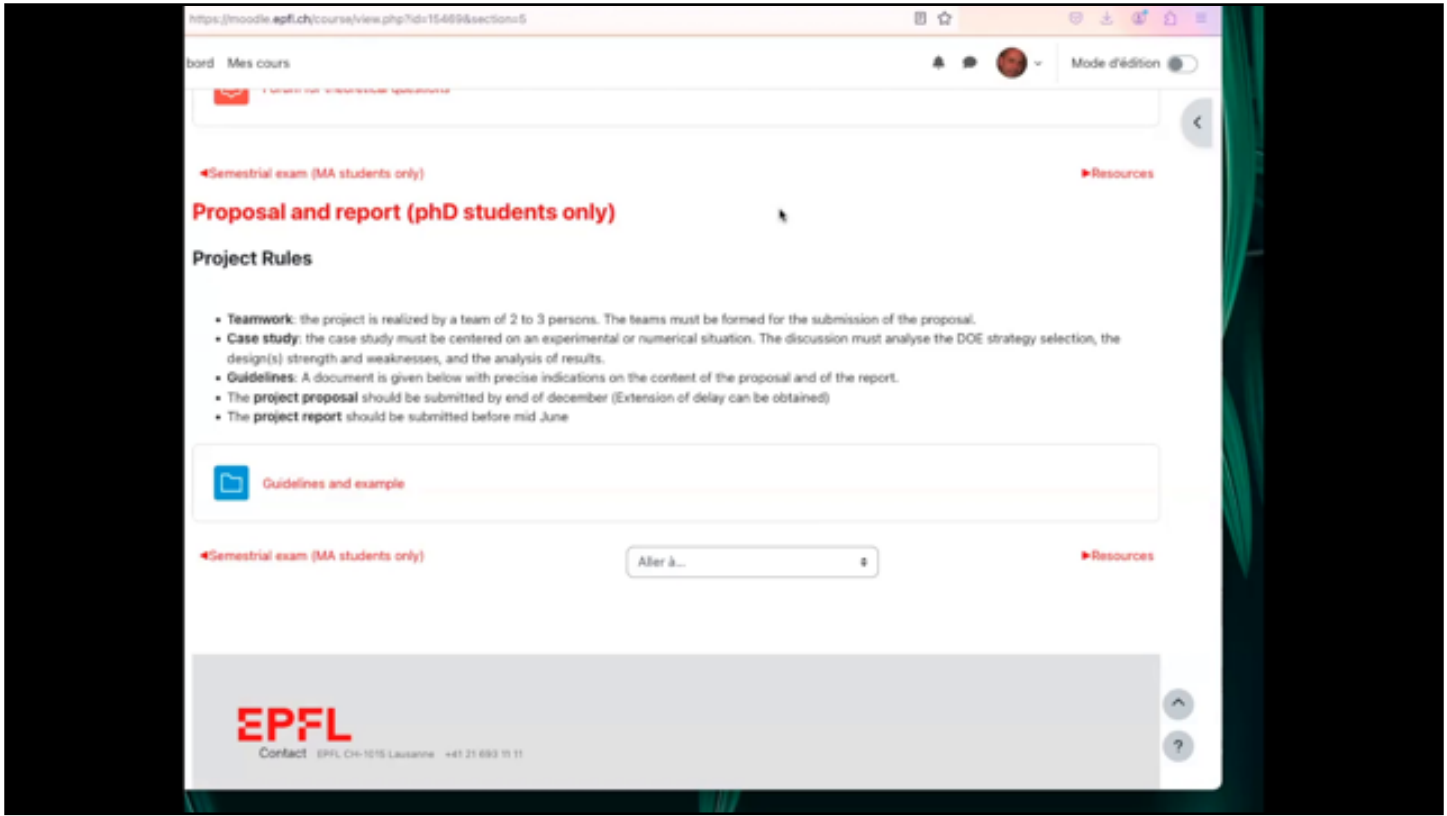
credit of this course, you have to present a report. You see here you have in the section Proposal and Report, you see the rules and a few documents. So it's a teamwork. Mainly I'm expecting that you get together two, three people. A group already asked me to be by four. It's okay. I accept exceptions. I also sometime accept exception. You could be a group of one person if you have a good reason for that. I just explained that I'm limiting the numbers of group for one person because I only have 44 hours per day and it became complicated for me to follow

notes

summary

0m 5s





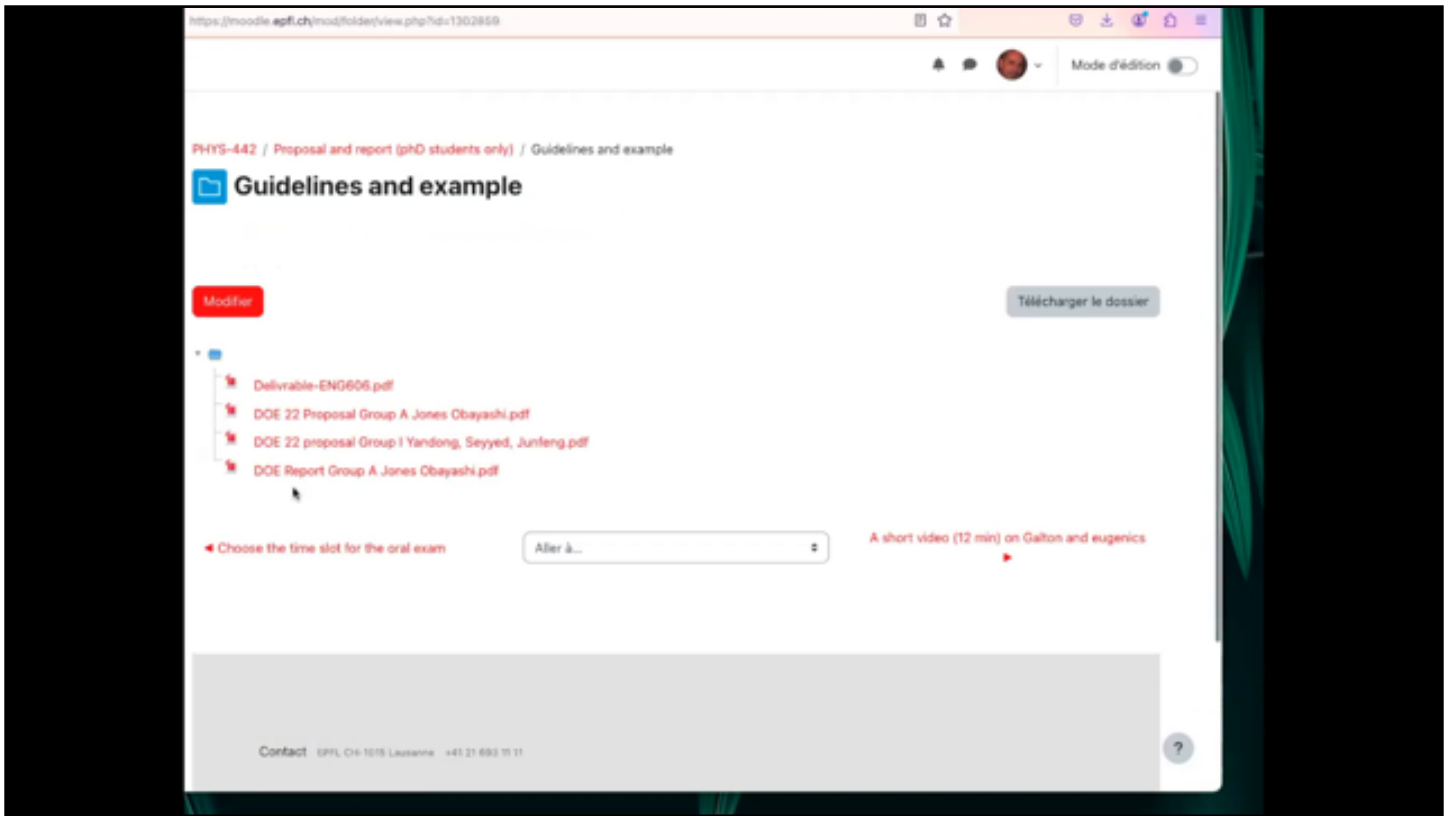
too much project. I asked also the group to be of more than one person because for me the project is also a pedagogical objective. The idea that you learn more competencies about DOE and when you

notes

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0m 58s





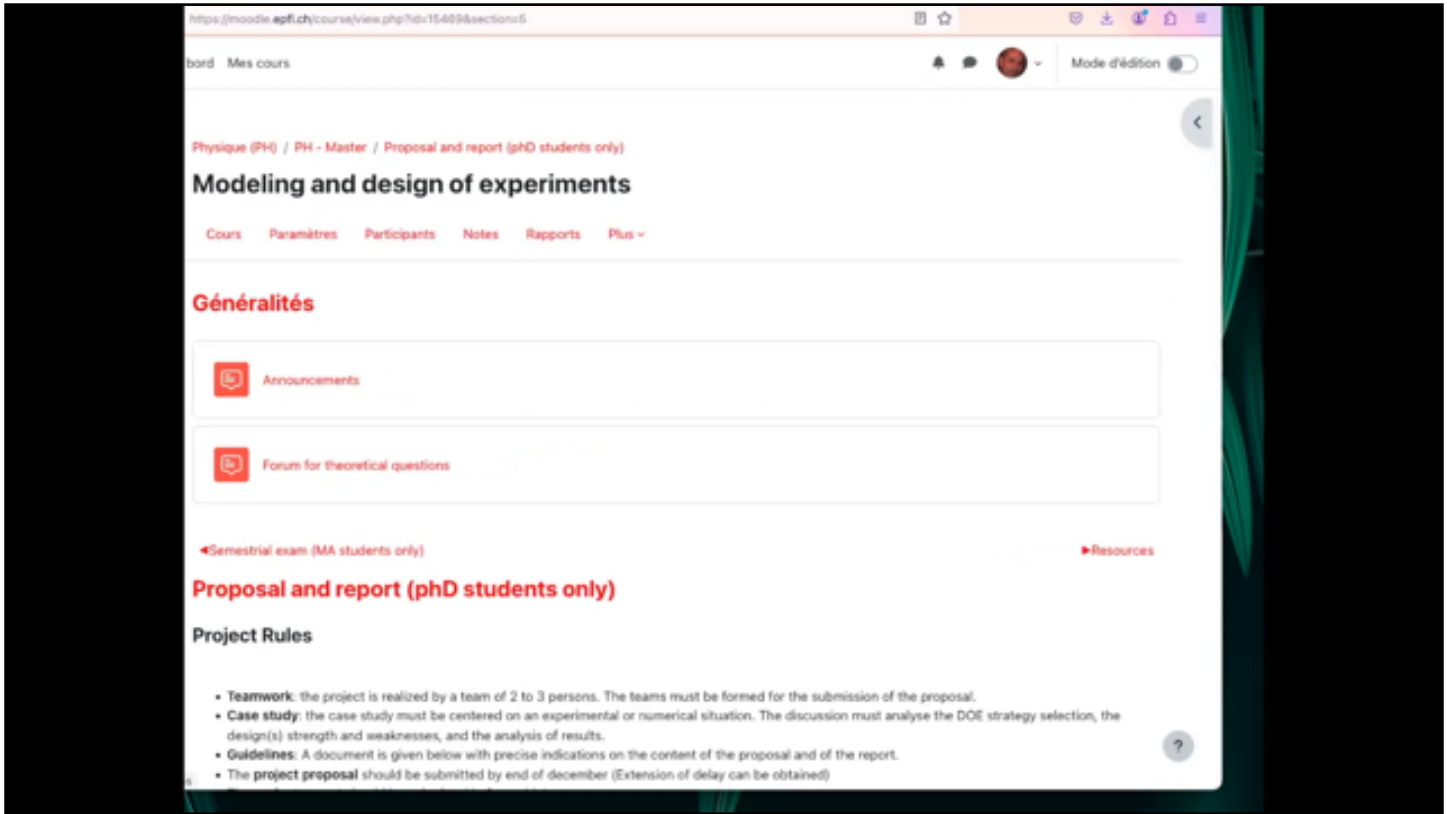
are two, three or four, you have questions between you and the learning is better. So it's also the objective of the course. So you need to have a case study. So for me, a case study is an experimental situation in which you have several factors. Please work with more than three factors because it's from three factors that things start to be interesting. I know it exists situation with one factor. It exists situation with two factors. But please choose for this course situation with more than three factors. No, three or more because it's where we start having interesting designs. So it could be a real experimental situation and you have factors and you would like to understand what are the main factors and to model your situation and to find an optimum of one or more responses. Or it could be also simulation. You can make a sensitivity analysis of computer code and you can use design of experiments also for doing that or you can simulate an experiment. So it's happened in some laboratory. So for me, that's open. It could be also a data you already have obtained and you are treating your data or the data that you get in the literature. In this case, it makes a little bit different because if it's your experiment or your simulation, so you design your experiments and you realize it and you analyze it. So if it's experiments that have already been already done, so you need to make a very interesting critics of the designs that have been applied and eventually propose a new design. You find in the document here below

notes

summary

1m 17s





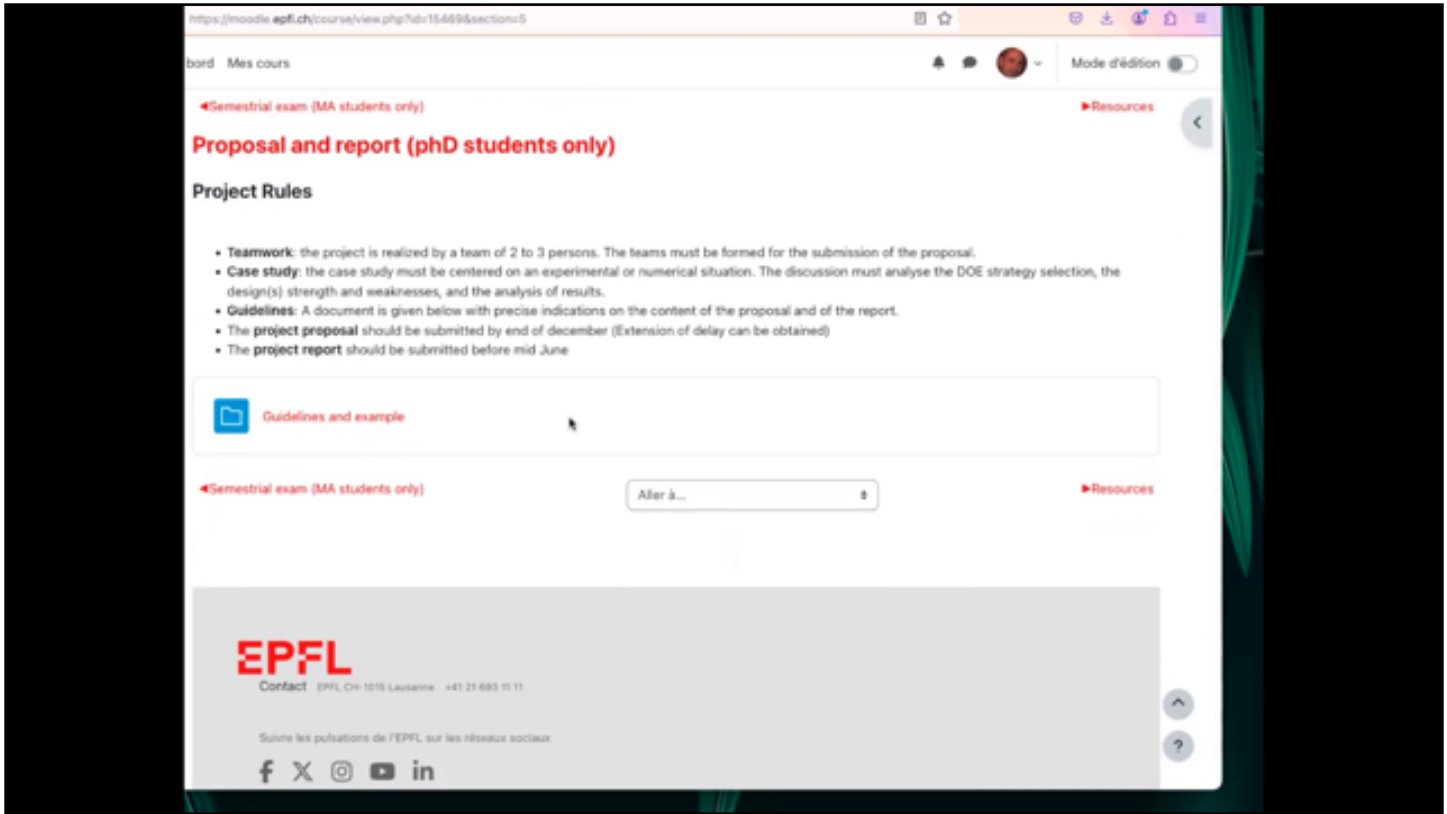
guidelines and example. I could put an S to example, but you see this document, deliverable engineering 606. So here you have the guideline for presenting the two documents

notes

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3m 29s





that you have to present in a first step proposal. So let's say for end of December, I'm not one there one week before later. It's not a problem, but you need to present me a proposal with the mind map of the case and explaining what you want to do and your groups. And also you have

notes

summary

3m 42s





### 3.3.1 What is an alias ?

- ▶ Alias : Zorro and Diego de la Vega
- ▶ The Concept of alias is useful for dealing with non-orthogonal situations
- ▶ It let compute the connection between two parts of a model
- ▶ Examples :
  - ▶ For a given design, what is the consequence of considering or not a regressor ?
  - ▶ For a given design, what is the consequence of considering the second degree coefficients on the first degree coefficients ?
  - ▶ In some situations we will see that the fact of considering the second degree coefficients will change the value of the constant, meaning that the second degree coefficients are aliased with the constant.

to choose a referee. I'm not at all knowledgeable in all domains or eventually I'm not knowledgeable at all in your domain. So I need somebody, a senior researcher, your thesis director, a senior researcher in your lab that could read your report at the end and tell me that it's okay. It's real stuff. It's something which is sounded and so it's the objective of that. So it just reads the report and write me a small message that is okay to support the report. It's things that have been done in the arts of the subject. It's also have for me another objective is a little bit of advertisement like that. I also share my course not only among students but also senior researcher discover about this course and is about this methodology. This too objective. So as I said this proposal should be really for let's say start of January. You do not have to wait too much. My answer if I forget you you write me back and ask for my feedback. Normally I say okay sometimes I make some comments. My main concern that I don't want you to throw you against the wall. So I just want to be sure that what you propose to do is not too much. Sometimes also I think it's not too much and I propose to make a little bit more. But most of the time my advertising, my comments are more that be careful. It's a lot of job that you are planning. So sometimes we reduce a little bit the objective and things like that. And that you can start before I say my okay. It's our problem we discussed and we find a solution. And so you have to produce a report of eight pages. Please respect that again. I do not have so so much time. So it's like a conference paper. The only

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#### summary

4m 13s



### 3.3.1 What is an alias ?

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  - ▶ For a given design, what is the consequence of considering the second degree coefficients on the first degree coefficients ?
  - ▶ In some situations we will see that the fact of considering the second degree coefficients will change the value of the constant, meaning that the second degree coefficients are aliased with the constant.

exception to that if you think your your your work is valuable for making a paper an official paper and you want to present it in the journal or in a conference it's okay. So you take the standard of where you want to present it and I find my I need in your in your in your paper in your in your report. But if it's just for me please eight pages. It's obliged you to summarize things. You cannot explain all details. So it's also an interesting exercise and writing a paper is also time and interesting. So if you need your credits very rapidly, you need time for making your project. But if you need it in February or March it's possible. Just hurry up and make your project. And when you send me your project be sure to indicate me that you are in a hurry and I need something let's say one week, 10 days for correcting it. And we find all the time. But if you do not have a precise delay on your side please finish it for the end of the semester at least you can finish it before but if I can close all the group before starting a new one in September I appreciate because after it's it's complicated. But I know that the job and the life of PhD student is complicated. So I am I'm known to be comprehensive. So don't hesitate to ask me for delays and we can talk. And during your preparation of the project I'm available. Usually I met each group one time. But it could be more and I really available write me emails ask for zoom or or meeting. I will be

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summary

## 3.2.11 Comparison between type I and type II

Source	SS*	DF	MS	F	P
$a_1$	37 033	1	37 033	678	0.001 %
$a_2$	9 074	1	9 074	166	0.021 %
$a_{12}$	16 122	1	16 122	295	0.007 %
Résidu 1	219	4	55		
Total	179 082	8			

Source	SS*	DF	MS	F	P
$a_1$	7 130	1	7 130	130	0.034 %
$a_2$	4 700	1	4 700	86	0.075 %
$a_{12}$	16 122	1	16 122	295	0.007 %
Résidu 1	219	4	55		
Total	179 082	8			

very happy to help you. If you remember in the start of this chapter we have seen how to make an ANOVA for model what how to deal when we want to separate the part of the model and the part

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summary

8m 25s



### 3.3.1 What is an alias ?

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  - ▶ In some situations we will see that the fact of considering the second degree coefficients will change the value of the constant, meaning that the second degree coefficients are aliased with the constant.

are orthogonal also when they are not orthogonal. And we finished last week with the difference between type one and type two during the week. I also read about type three. And so honestly for our case it doesn't make difference. So type two and type three are very close. The treatment of the interaction seems to be sometimes different. But even the MATLAB function I see no no difference. But just for summarizing type one is one factor after the other. So statistician like that when they really want to understand what a new factors is changing from what they have before. But the factors are not treated the same way. The first one gets more sum of square when the sum of square is shared between factors. The first one has the shared part and the last one not the shared part. Type two is putting all the factors on the same line. We treat all the factors as if they were the last ones. And in fact what I read this week type three usually you do the same for interactions. But honestly the algorithm from MATLAB I see no difference you have here. Type one and type three for my case. I do and I also calculate type three and it makes no difference. But okay. You will see perhaps the help of MATLAB is not very good on the subject that you can find. But typically remember the big difference treating the factor in some order and the order of an influence. So be careful to that or not order two or three and there are perhaps eventually a difference between type two and type three even if in MATLAB I didn't really observe it. So now we will see we will generalize those concepts and see a very interesting concept which is the alias concept. So alias I use this image is like Zoro

#### notes

#### summary

8m 46s



### 3.3.1 What is an alias ?

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  - ▶ For a given design, what is the consequence of considering or not a regressor ?
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and Diego de la Vega. If you remember this story you have Diego de la Vega that puts a mask and just things without we know is Diego de la Vega for bringing justice in California. And so it's an image but in fact in the alias is the difference between what we estimate and what is the reality could be a mixture. So this concept is really useful for dealing with non-orthogonal situation. Today I present the alias more as a problem but you will see in the next chapter that we can take profits of the alias for resolving the system step by step when we have an under determined system that means that we cannot determine each factor individually. We have to make linear combinations of factors for solving the undetermined systems. So we will play with that as I say today more trying to get protected from problems and after we will even use it as an advantage. So I give here an example I say so for a given design what is the consequence of considering or not considering a regressor.

#### notes

#### summary

### 3.3.2 The alias matrix

The alias matrix corresponds to the projection of the base vectors of the second subspace on the base vectors of the first sub-space

- ▶ Let's consider a linear model  $y = f(x_1, \dots, x_N, a_0, a_1, \dots, a_M)$  and a design with the model matrix  $X$
- ▶ Now let's part the model in two parts  $f = f_1 + f_2$  with the corresponding model matrix  $X_1$  et  $X_2$  so that  $X = [X_1, X_2]$
- ▶ The alias matrix  $A$  of  $X_2$  in relation to  $X_1$  is :

$$A = (X_1^T X_1)^{-1} (X_1^T X_2)$$

For example you can imagine that perhaps you have a second degree what will be the influence of taking into account the second degree or not on the other on the linear coefficients. If your regresses your factors are orthogonal it should have no influence by the second degree is never orthogonal to the rest of the model. So you all time have a little problem if your design is balanced or equilibrate usually this influence is on the constant but if your design is not equilibrated it could be on any other element and you would like to know what is the shadow this is not the statistical terms my turn but what is the shadow of the second degree on the first degree. What is the influence of calculating it or not on the coefficient of the first degree. Now shadows yeah it's it's a bias in fact shadow it's a bias and see if I would like to speak correct statistics it should be something as a bias but shadow for me is an image it's an influence is the fact that you don't see the picture completely it's changing the picture if you are calculating or not some of the some of the element of your of your model. For a given design I mean the second as a second point for a given design what is the consequence of considering the second degree coefficient on the first we already say it and in some the third one in some situation we will see that the fact of considering the second degree will change the value of the constant meaning that the secondary coefficients are aliased with the constant so sorry I didn't remember well my slide that's all I already explained orally what was written in my slide. The concrete element behind the alias is the alias matrix so alias is a concept and

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#### summary

12m 37s



### 3.3.2 The alias matrix

The alias matrix corresponds to the projection of the base vectors of the second subspace on the base vectors of the first sub-space

- ▶ Let's consider a linear model  $y = f(x_1, \dots, x_N, a_0, a_1, \dots, a_M)$  and a design with the model matrix  $X$
- ▶ Now let's part the model in two parts  $f = f_1 + f_2$  with the corresponding model matrix  $X_1$  et  $X_2$  so that  $X = [X_1, X_2]$
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we have an information on the alias in the alias matrix and I want to show you how we can calculate in a given situation an alias matrix so the alias matrix correspond to the projection of the base vectors of the sub-gamma space so we are dividing our result space in subspace is what we are doing when we are making a model you remember my 3d graphic about what is making a model that we have two sub-models depending the numbers of regressors because the numbers of regressors correspond to vectors geometrically and so we can have or not some some subspaces. So imagine that we have a model we call it y and it's depend on n factors and it's a way of writing models I also indicating that for each of the factors I have some coefficients representing the linear influence and even the interactions and eventually they second and third degree elements why I call this model f of x1 to the n of a0 to am because I could have more coefficients than factors because I could have interactions and second degree elements so if we have this model and we decide of an experiment we can build the model matrix we have learned in the previous chapter that we can write a model matrix that I usually I call x but now if we want to separate my model in two parts that make two subspaces I say okay f will be divided in f1 plus f2 so that

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#### summary



### 3.3.3 What is the alias used to ?

To make the link between the two subspaces:

$$\begin{aligned}\hat{Y}_1 &= X_1 \hat{\alpha} \\ \hat{Y} &= [X_1 \ X_2] \begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \end{bmatrix}\end{aligned}$$

Coefficients :  $\hat{\alpha} = \hat{\alpha}_1 + A \hat{\alpha}_2$

Orthogonal projection :  $X_{2,1} = X_2 - X_1 A$

Orthogonal decomposition :

$$\begin{aligned}Y &= X_1(\hat{\alpha}_1 + A \hat{\alpha}_2) + (X_2 - X_1 A)\alpha_2 + \epsilon \\ Y &= X_1 \alpha + X_{2,1} \alpha_2 + \epsilon\end{aligned}$$

means that I have my two model matrix that are my model matrix  $x$  is separated in two model matrix  $x_1$  for the first part of the model imagine it's the linear part of the model and  $x_2$  that could be the non-linear part of my model the instructions and the second degree of my model I'm allowed to do that I separate my  $x$  model matrix in two sub model matrix  $x_1$  and  $x_2$  so now we can compute the alias matrix I appreciate to give the the letter big  $a$  for alias but it's just for remembering so you see that it's the dispersion matrix of the first part of the model that it's for you the reference so the alias is all the time a reference and an additional subspace you or you don't treat both subspace the same way is the projection of one on the other is not equivalent you can change if you want but it doesn't give you the same the same matrix so is the dispersion matrix of the first part of your model and after you see it's a product of the two model matrix but one is transposed that is the same thing as making all the possible scalar product of my vectors so it's really the projection of  $x_2$  on  $x_1$  so first a few more algebraic calculation just representing what I just said and showing how we can use this alias matrix in some relation for explaining what's happened when we separate the model in two parts so imagine we have the first model  $y_1$  hat so is the estimate what we get from the experiment and we are able to calculate this estimate if we have the estimation of the coefficient be careful I'm playing with the indices a little bit and so I'm using  $\alpha$  hat and it's especially I'm not using  $\alpha$  hat

notes

summary

16m 49s





### 3.3.3 What is the alias used to ?

To make the link between the two subspaces:

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Coefficients :  $\hat{\alpha} = \hat{\alpha}_1 + A \hat{\alpha}_2$

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Orthogonal decomposition :

$$\begin{aligned}Y &= X_1(\hat{\alpha}_1 + A \hat{\alpha}_2) + (X_2 - X_1 A)\alpha_2 + \epsilon \\ Y &= X_1 \alpha + X_{2,1} \alpha_2 + \epsilon\end{aligned}$$

one because I will use alpha hat one when I have alpha alpha hat one and alpha two when I just have one I call it alpha because I'm estimating my coefficient alone without thinking that I have a second degree without thinking that eventually I have instructions so it's why you don't have an indices here okay and I'm comparing that to my model that I could separate it in two parts and one part correspond to the first one and my second part and now I'm able to write y hat is equal to the concatenation of my two part of my model x1 and x2 and then I have two parts in my coefficients I have the alpha one corresponding to my first part of my model and I have my alpha two corresponding to my second part and I would like to understand the relation between these alpha and these alpha one and alpha two if you remember the slide of last week when we have this geometrical separation when I show you the orthogonalization you see that the value of the constant was changing when I'm considering or not when I'm orthogonalizing so this is the same the same thing we are doing but this time we are doing with algebra I did it last week with just geometry now I'm doing the same thing with algebra so now you can see the relation between the coefficients I mentioned last day that when I make this mixture taking out the

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Analysis of variance

Analysis of variance of a model as a whole

Anova of the coefficients of a model

The concept of alias

Comparison between type I and type II

SS*	DF	MS	F	P
37 033	1	37 033	678	0.001 %
9 074	1	9 074	166	0.021 %
16 122	1	16 122	295	0.007 %
219	4	55		
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SS*	DF	MS	F	P
7 130	1	7 130	130	0.034 %
4 700	1	4 700	86	0.075 %
16 122	1	16 122	295	0.007 %
219	4	55		
179 082	8			

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Marie Fürbringer

Modelling and design of experiments

residue of the second step from the first step and things like that's okay we never do that if you

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summary

21m 1s



### 3.3.3 What is the alias used to ?

To make the link between the two subspaces:

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remember I'm referring sorry for your eyes I'm referring to these slides no so I'm doing quite the same operation only between one one layer and another but with algebra and not with the

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21m 11s



### 3.3.4 Example of an alias matrix

- Let's consider a design  $E$  and the model

$$y = a_0 + a_1x_1 + a_2x_2 + a_{12}x_1x_2$$

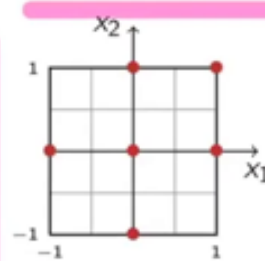
- the separation is between the linear and the interaction parts :

$$f_1(x) = a_0 + a_1x_1 + a_2x_2$$

$$f_2(x) = a_{12}x_1x_2$$

$$X_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{pmatrix} \text{ et } X_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$E = \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix}$$



numbers as I did it last week so now with the matrix alias you can explain express the relation between the different parts of your coefficients so alpha which is what you calculate when you don't consider the second part of the model can be calculated with this operation is the same thing as the alpha one what you estimate for the first part when you consider the second part and also the second part alpha two but multiplied by the matrix of alias because you are using the projection we are rebuilding some of the vectors and some of the projection using this rule of projection if you would like to calculate the projection of  $x_2$  if you remember in my graphic I was orthogonalizing the second vector in reference to the first one so  $x_2$  dot one so the part of  $x_2$  which is orthogonal to  $x_1$  the one which is bringing me information in this case is  $x_2$  minus  $x_1$  multiplied by the alias matrix and if you want to make an orthogonal decomposition you can also use this alias matrix so you see here that  $y$  is equal to the first part and another color a second part and if I do that my two parts of my model are orthogonal one to the other so let's see an example and I hope this example will make things more easy to understand so imagine that you have a matrix of  $s$  we have decided you have we have two two factors you see here the situation so this is the experimental situation you see we have a six muslim point it's quite a star design with one point at the center also one point in in in in a corner it's not a really good design but okay it's a design and the alias will help us to understand what is the consequence of this

#### notes

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21m 26s



### 3.3.4 Example of an alias matrix

- Let's consider a design  $E$  and the model

$$y = a_0 + a_1x_1 + a_2x_2 + a_{12}x_1x_2$$

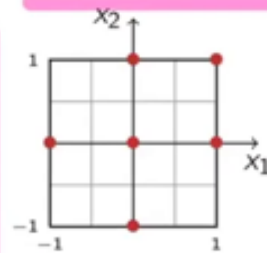
- the separation is between the linear and the interaction parts :

$$f_1(x) = a_0 + a_1x_1 + a_2x_2$$

$$f_2(x) = a_{12}x_1x_2$$

$$\text{► } X_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{pmatrix} \text{ et } X_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$E = \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix}$$



design we will rapidly see that this design is not orthogonal and we will understand what is the consequence of the non-orthogonality so let's imagine that we have in mind a linear model with instructions  $a_0$  plus  $a_1x_1$  plus  $a_2x_2$  plus  $a_{12}x_1x_2$  so if we want to separate in two parts because we are questioning that we need this the interaction or not or we just want to understand how these two parts of the model are related and then I'm able to define two subspace one with the constant and the linear coefficient and one with the interaction coefficient so the the dimension of the first one is two and the dimension of the second one is one you have here I have written the matrix of experiment and then

notes

summary

## 3.3.5 Example of an alias matrix (2)

$$\begin{aligned}
 A &= (X_1^T X_1)^{-1} (X_1^T X_2) \\
 &= \frac{1}{44} \begin{pmatrix} 8 & -2 & -2 \\ -2 & 17 & -5 \\ -2 & -5 & 17 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{pmatrix}^T \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
 &= \frac{1}{44} \begin{pmatrix} 8 & -2 & -2 \\ -2 & 17 & -5 \\ -2 & -5 & 17 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/11 \\ 5/22 \\ 5/22 \end{pmatrix} \begin{matrix} a_0 \\ a_1 \\ a_2 \end{matrix} \Rightarrow \begin{cases} l_0 = a_0 + \frac{1}{11} a_{12} \\ l_1 = a_1 + \frac{5}{22} a_{12} \\ l_2 = a_2 + \frac{5}{22} a_{12} \end{cases}
 \end{aligned}$$

A

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I'm able now to calculate my two model matrix for the two parts of my model so the first one you recognize a column of one is for the constant and after you have the repetition of my columns corresponding to my matrix E because for the linear coefficients I'm just reproducing the coordinates of my points and I have chosen space which is already normalized so I between minus one and plus one in this case it will not change a lot if you are in normalize and not normalized and I have a second model matrix or a second part of my total model matrix that I'm pulling x2 and at only one column because in the second part I just have one coefficient so it's the product of the two columns of my matrix E so 0 by 0 0 1 by 1 1 minus 1 by 0 0 etc strange column but it's like that at least you have not all 0 so it's not the best proof but you see in this case that you would be able to calculate the interaction at least I have the possibility with this one to calculate the interaction so let's calculate the alias matrix of this case so the alias matrix of this case is the dispersion matrix of the first part of my model and the product of my the transpose of my x of x1 multiply by x2 in this case it's just one column matrix so if you do this calculation you get first this result so I have simply put in evidence 44 for having an integer in my first part of my matrix my matrix of dispersion so this corresponds to a matrix of dispersion and the second part corresponds to the matrix x1 multiply by x1 x1 transpose multiply by x2 if you do this calculation you see step by step you

notes

summary

25m 13s



## 3.3.5 Example of an alias matrix (2)

$$\begin{aligned}
 A &= (X_1^T X_1)^{-1} (X_1^T X_2) \\
 &= \frac{1}{44} \begin{pmatrix} 8 & -2 & -2 \\ -2 & 17 & -5 \\ -2 & -5 & 17 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{pmatrix}^T \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
 &= \frac{1}{44} \begin{pmatrix} 8 & -2 & -2 \\ -2 & 17 & -5 \\ -2 & -5 & 17 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/11 \\ 5/22 \\ 5/22 \end{pmatrix} \begin{matrix} a_0 \\ a_1 \\ a_2 \end{matrix} \Rightarrow \begin{cases} l_0 = a_0 + \frac{1}{11} a_{12} \\ l_1 = a_1 + \frac{5}{22} a_{12} \\ l_2 = a_2 + \frac{5}{22} a_{12} \end{cases}
 \end{aligned}$$

A

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arrive as a result to this vector so this is my matrix of alias A how to understand it so it's tell me so each row correspond to a coefficient of x1 of my linear model so the first one is in relation with this A0 the second one to A1 and the third one to A2 and the columns represent are related to x2 so they represent what is x2 so they are representing A1 2 so this alias matrix tell me that in fact the vector that I'm using for estimating A1 2 is a linear combination of the vector I'm using for the previous part of the model is one eleventh of my vector 1111 plus 5 11 of my vectors that I'm using for calculating A1 and plus 5 I know not 11 but 20 seconds

notes

summary

### 3.3.3 What is the alias used to ?

To make the link between the two subspaces:

$$\begin{aligned}\hat{Y}_1 &= X_1 \hat{\alpha} \\ \hat{Y} &= [X_1 \ X_2] \begin{bmatrix} \hat{\alpha}_1 \\ \hat{\alpha}_2 \end{bmatrix}\end{aligned}$$

Coefficients :  $\hat{\alpha} = \hat{\alpha}_1 + A \hat{\alpha}_2$

Orthogonal projection :  $X_{2,1} = X_2 - X_1 A$

Orthogonal decomposition :

$$\begin{aligned}Y &= X_1(\hat{\alpha}_1 + A \hat{\alpha}_2) + (X_2 - X_1 A)\alpha_2 + \epsilon \\ Y &= X_1 \alpha + X_{2,1} \alpha_2 + \epsilon\end{aligned}$$

5 20 seconds of the vectors that I'm using for calculating A2 it's a balance of my collinearity between my new vectors that I'm interested for calculating A1 2 with what I already use in my different subspace I'm using for calculating the first part of my model so the consequence of this matrix of alias is that when I believe I'm calculating A0 in fact I'm calculating A0 plus 111 of A1 2 if I'm not calculating A1 2 if I'm calculating A1 2 I do not have this problem so if A1 2 doesn't exist no problem my constant is my constant and for that we use a concept that is the contrast so this is the L0 L1 L2 this equal is called a contrast so a contrast is what I get as a solution of my linear system especially when it is under determined what I wanted to explain is that this value is related is this is this value so what I get in my alias matrix is the components that I need to to write my linear combination of my coefficients when I'm not estimating the second part

notes

summary

29m 25s





## Example of an alias matrix

Consider a design  $E$  and the

$$+ a_1x_1 + a_2x_2 + a_{12}x_1x_2$$

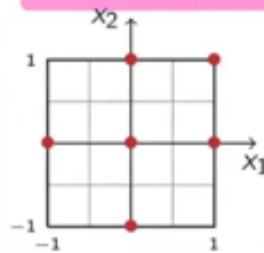
variation is between the linear  
interaction parts :

$$x) = a_0 + a_1x_1 + a_2x_2$$

$$x) = a_{12}x_1x_2$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 1 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} \text{ et } x_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$E = \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix}$$



is coherent with what I present here it's exactly the same thing but no I was showing you at the

notes

summary

31m 13s



## 3.3.6 ANOVA table for non orthogonal parts

Source	SS	SS*
$X_1$	$\alpha_1' X_1' X_1 \alpha_1$	$\alpha_1' X_1' X_1 \alpha_1 = (\alpha_1 + A\alpha_2)' X_1' X_1 (\alpha_1 + A\alpha_2)$
$X_2$	$\alpha_2' X_2' X_2 \alpha_2$	$\alpha_2' X_{2,1}' X_{2,1} \alpha_2 = \alpha_2' (X_2 - X_1 A)' (X_2 - X_1 A) \alpha_2$
Résidu	$\epsilon' \epsilon$	
Total	$SS_Y$	—

$N$  is the number of runs,  $A$  the alias matrix relative,  $\alpha$  the coefficients of the first part of the model when it is inferred alone and  $X_{2,1}$  is the model matrix of the second part of the model orthogonal to the first part.

level of vectors and matrices and no I'm showing you the results element per element coefficient so the same thing for the second one I'm like if I'm not calculating the interaction and that's interaction exists then it creates a bias to my constant a bias to my first coefficient and a bias to my second coefficient and the alias matrix let me calculate this bias so then I'm able to rewrite the slide it was quite at the start of the last lesson I can rewrite my corrected sum of square using the alias matrix again it's not a critical slide in my course but you just explain how you can calculate so you see here that for example if I want to separate between  $x_1$  and  $x_2$  so in this case  $x_1$  is the linear part of my model  $x_2$  is the part with an interaction it's big  $x$  now it's not small  $x$  it's big  $x$  and so you see that I want to separate my different part of my model and my sum of squares also and I'm able to use this coefficient so the not corrected sum of square would be  $\alpha_1'$  the products of  $x_1$  and you take the square of it but when you want to calculate the corrected sum of square I know it's a complicated concept I will use this concept of alias many times I will never come back on the calculation of it sometimes it's interesting you will see in one case we will calculate the alias it's useful to calculate it sometimes but people don't do this another table calculation by ourselves so in this case we will not do this calculation so not as usual I will stop here and give you a break

notes

summary

31m 21s

