Course material

Course:

ENG606 / PHYS 442

Video:

DOE_lesson7_part1_PBdesign

Concepts (extracted from automatically generated subtitles):

Adamar matrices. Good designs. Quality of the call. Name of adamar. Mathematical structure of those matrices. Least square feet estimate. Most efficient design. Main effects. Very optimal designs. Transpose matrix. Brumann design. Type of matrices. Lot of different concepts. Domain of experiments. Little bit.

to video sequence search
(within ENG606 / PHYS 442.)

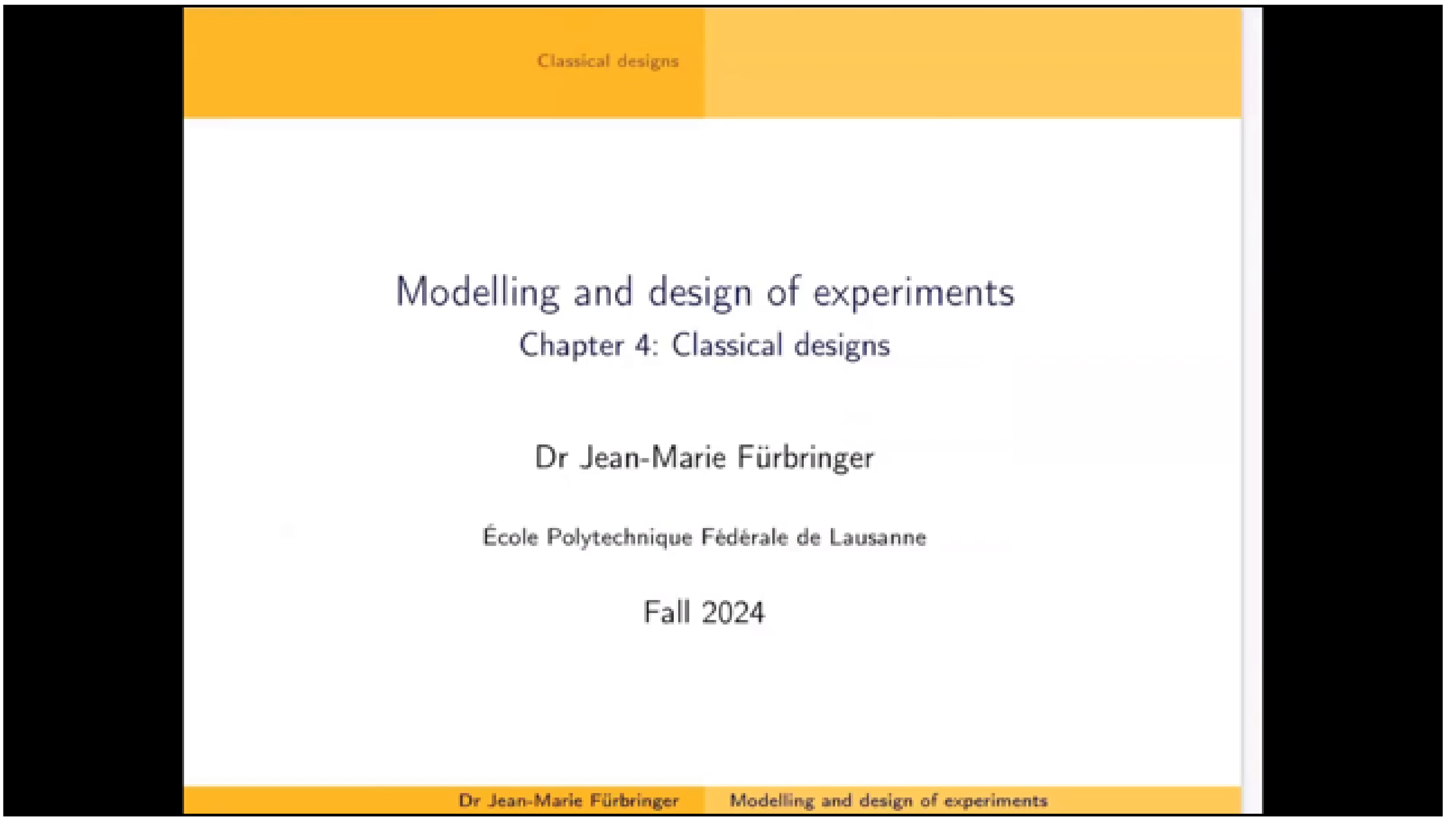
to video

...

notes

summary

0m 0s

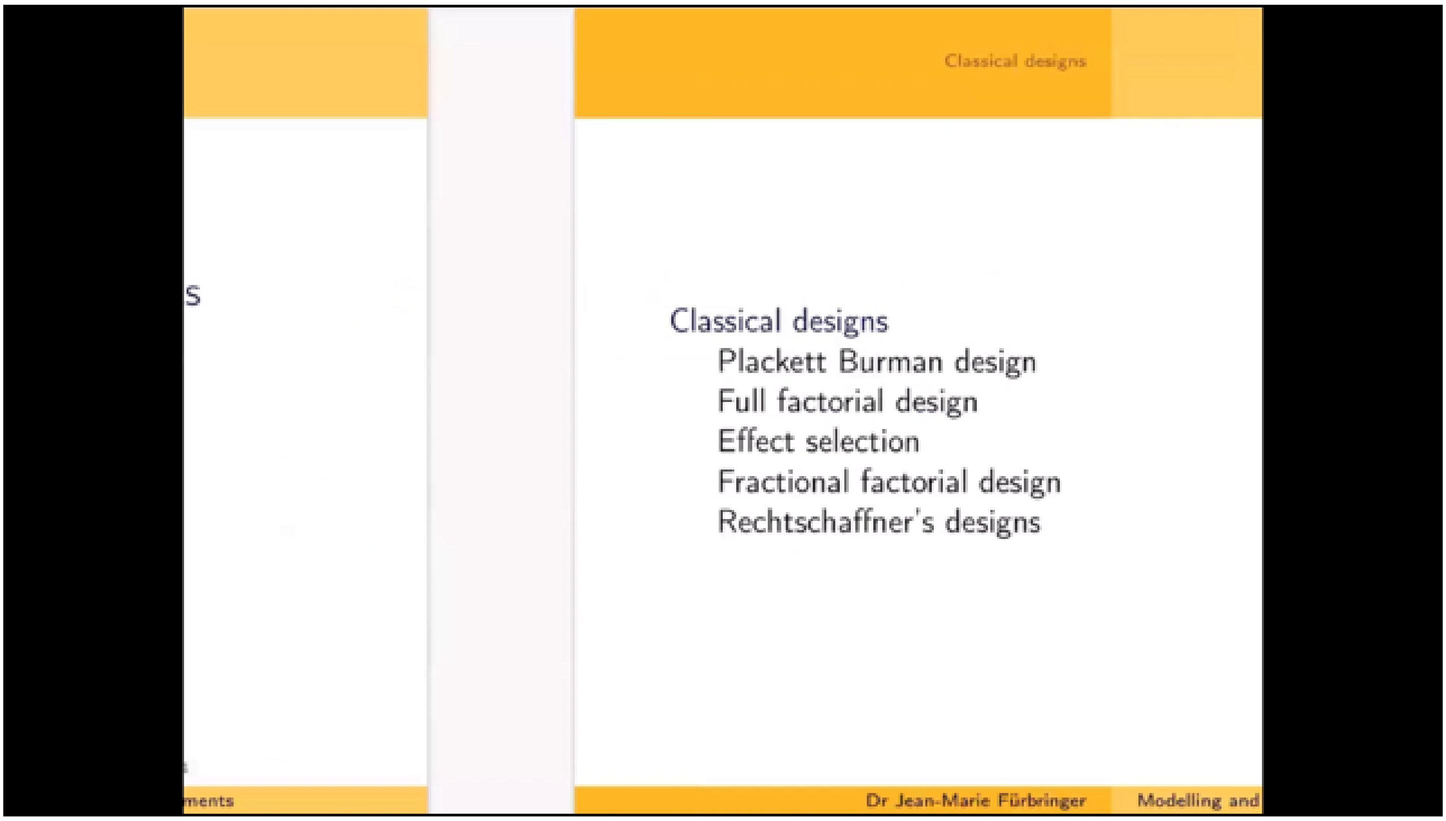


These subtitles have been generated automatically

notes

summary

0m 1s

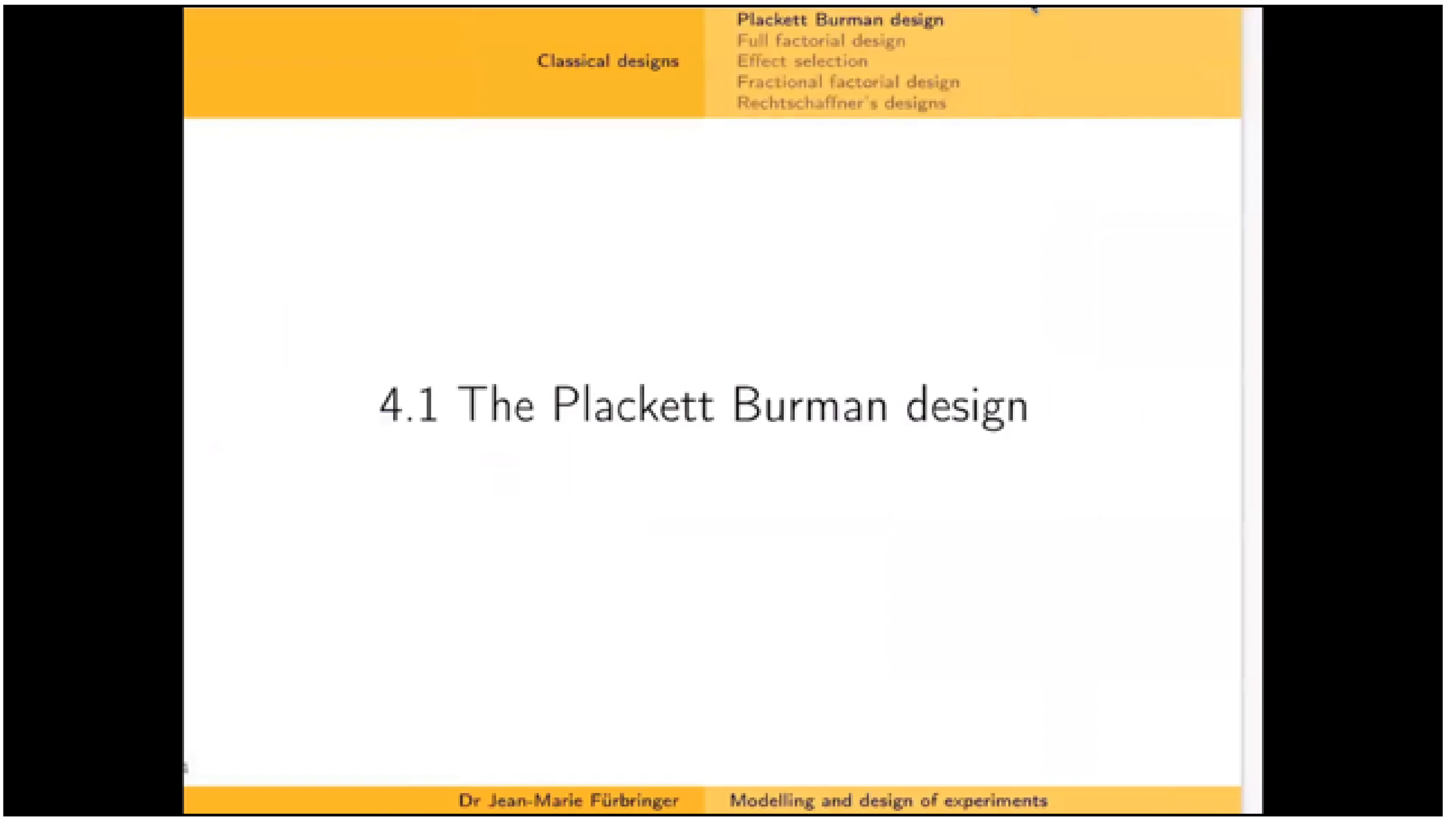


of experiment and of modeling with ANOVA and all those concepts. Remember, there are a lot of different concepts. But now today we start with discovering some good designs, some very optimal designs. So we will see today what we called Plackette and Brumann design. And after we will discover full factorial design, fractional design. So we will first have designs that are adequate for model without interaction and with interaction, linear model with interaction and without interaction. And we will see after what we call a response surface designs that are more adequate when you want to estimate second degree coefficients that is necessary. If you want to understand how your answer have curvature. So now the design we will see this two or three weeks will be exclusively dedicated to discovering what are the main effects and the possible interaction between factors. And later we will see how to discover the curvature.

notes

summary

0m 5s

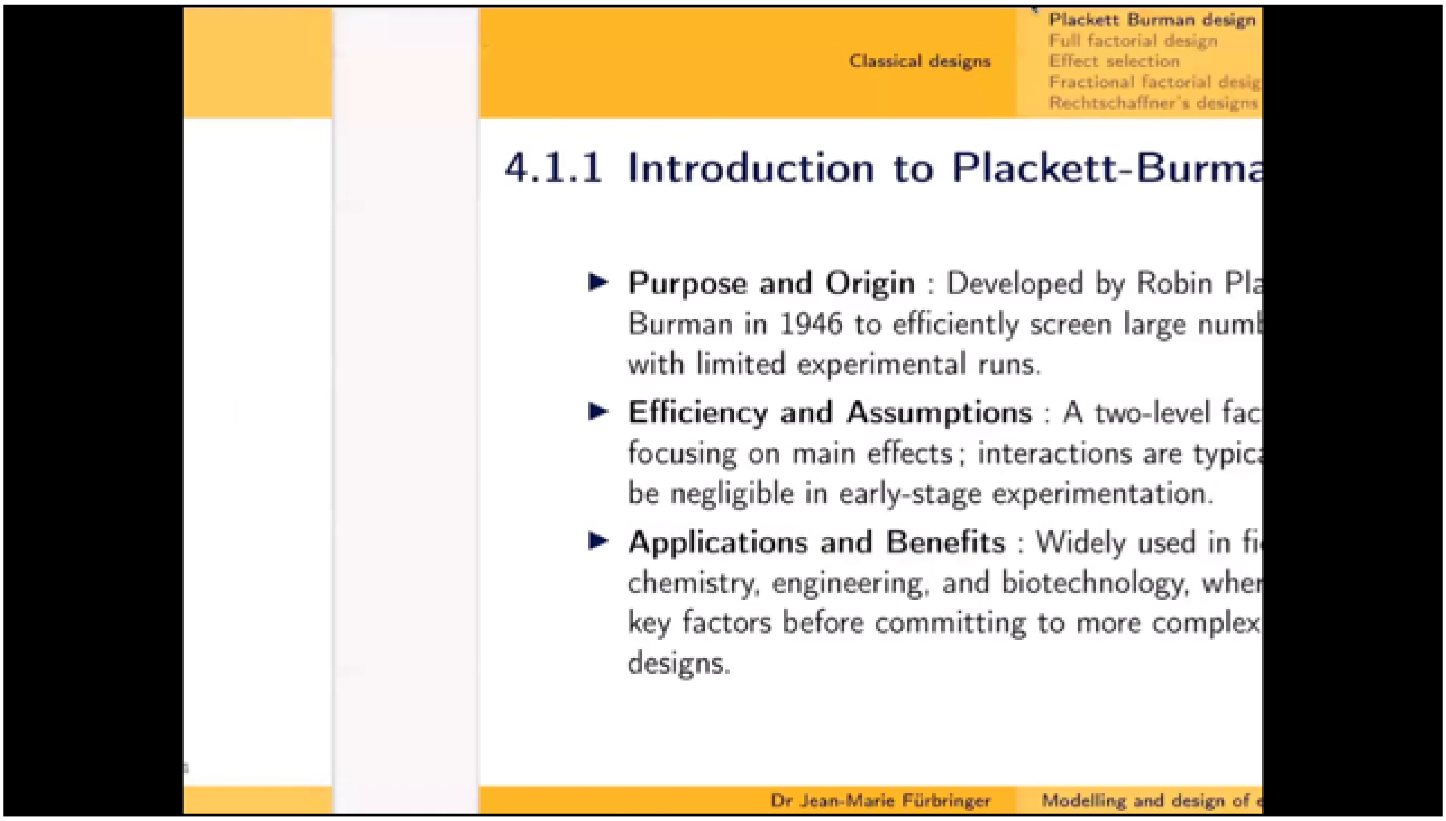


So Plackette, Brumann design, full factorial design. After I will show you a few elements of statistics that help you to analyze those effects to separate between the significant and the non-significant effects. And I will also show you fractional factorial design and reshaffner designs are designed especially to put in evidence economically the two by two interactions.

notes

summary

1m 51s

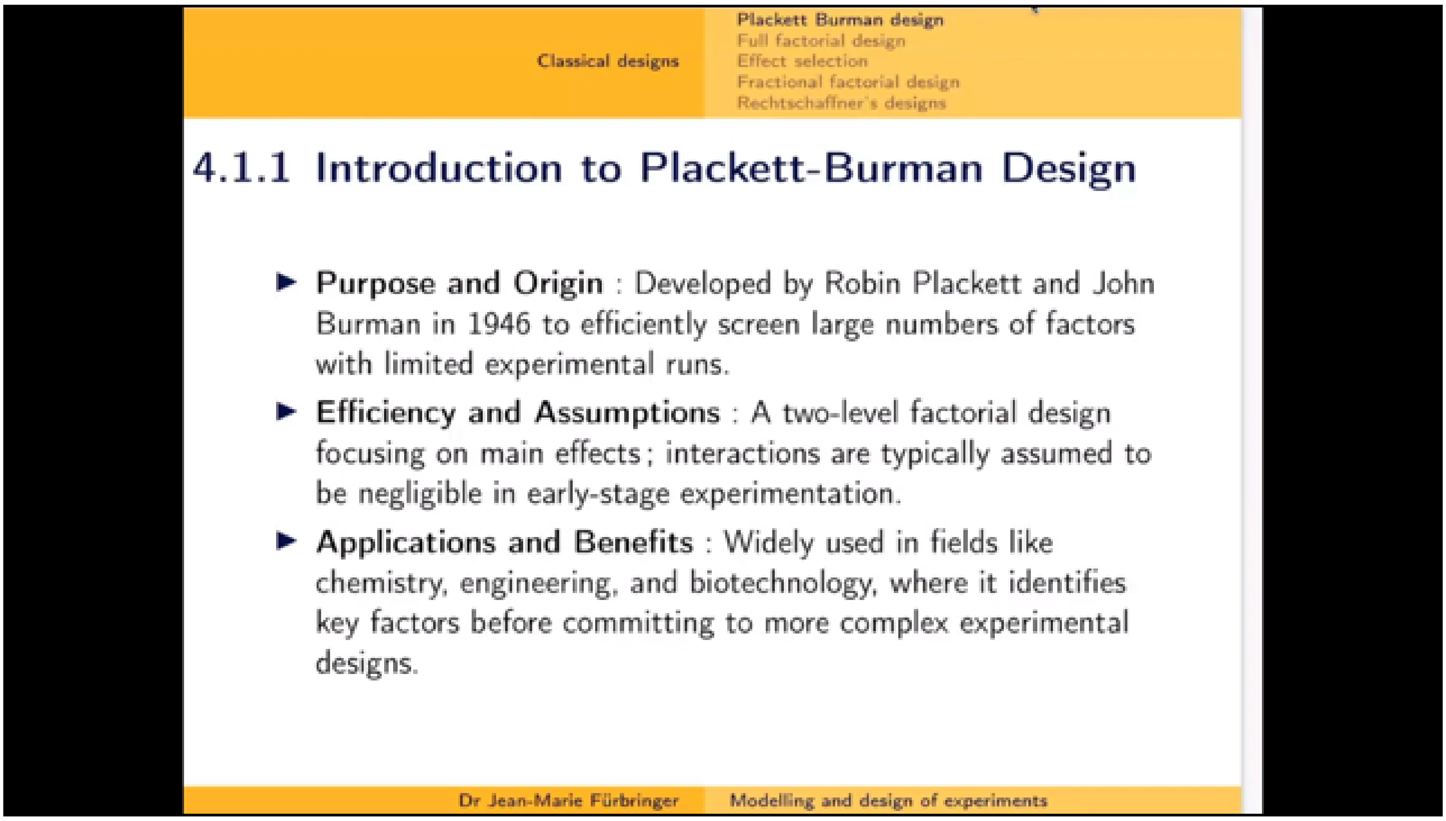


OK, so let's start with the Plackette, Brumann design. Plackette and Brumann were two British statisticians who have developed most of the approach during the Second World War. And in 1942, 43 Churchill has the university to help a lot in the war efforts of England. And they were they were present and they put in some tools very efficient. And after they published that after the war. So the papers we are still using are what is quite not so common in physics, but in math that's useful, is useful and common to use very old papers. And so there are papers from 1948 and those years.

notes

summary

2m 30s

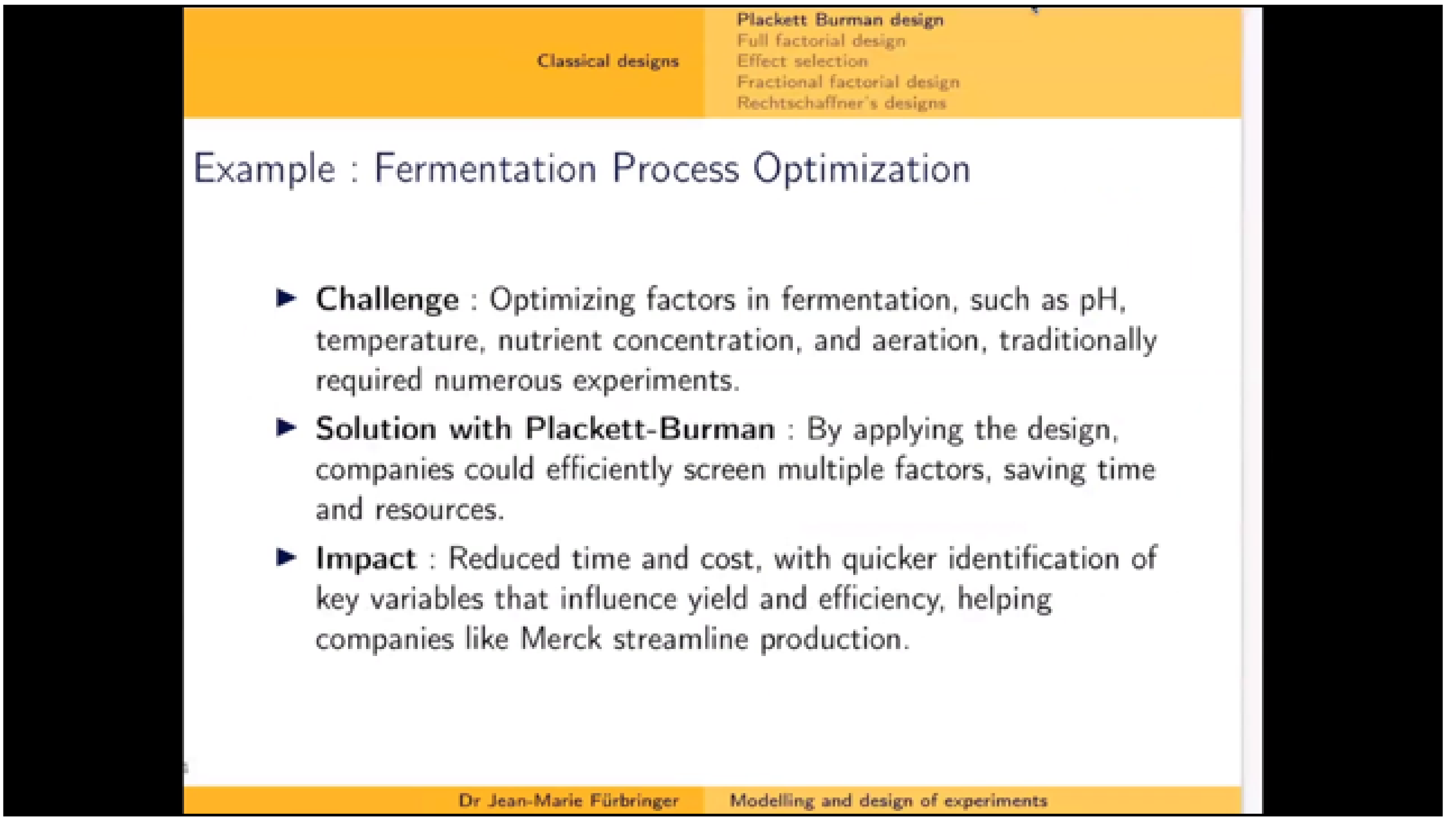


Plackette, Brumann, they are the most efficient design for evaluating main effects. So it was exactly a Plackette, Brumann design that we have used. If you remember the case I present you for the scale, we want to wait three objects. It was in fact, without I was saying it, it was a Plackette, Brumann design. They are efficient and they work when we think that we do not have interaction. But was the case when you wait objects, except if you make chemistry and you have something going as as a gas. Usually when you have two objects on the scale, the results is the sum of the weight of each of the objects. So additive model. It's work also to discover. Main effects when you have small interactions. So the way sometimes to be sure that we have only small interactions is to restrict the domain of experiments,

notes

summary

3m 30s

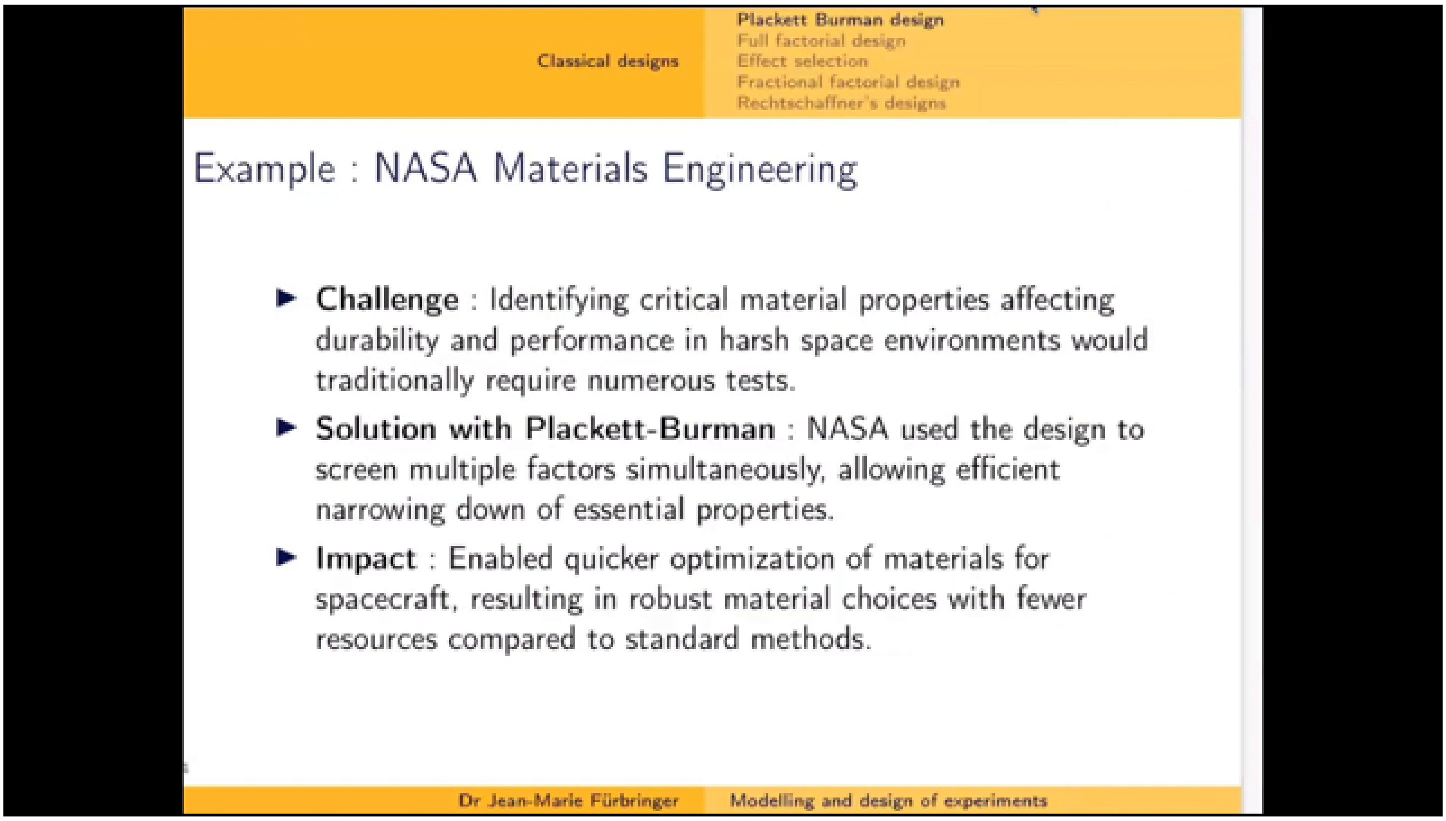


just to be sure that interaction are not so so so important. So it's a really interesting tool, which is used commonly in biotechnology, engineering, chemistry, anything. And I really invite you to to use them when you when you go.

notes

summary

4m 37s

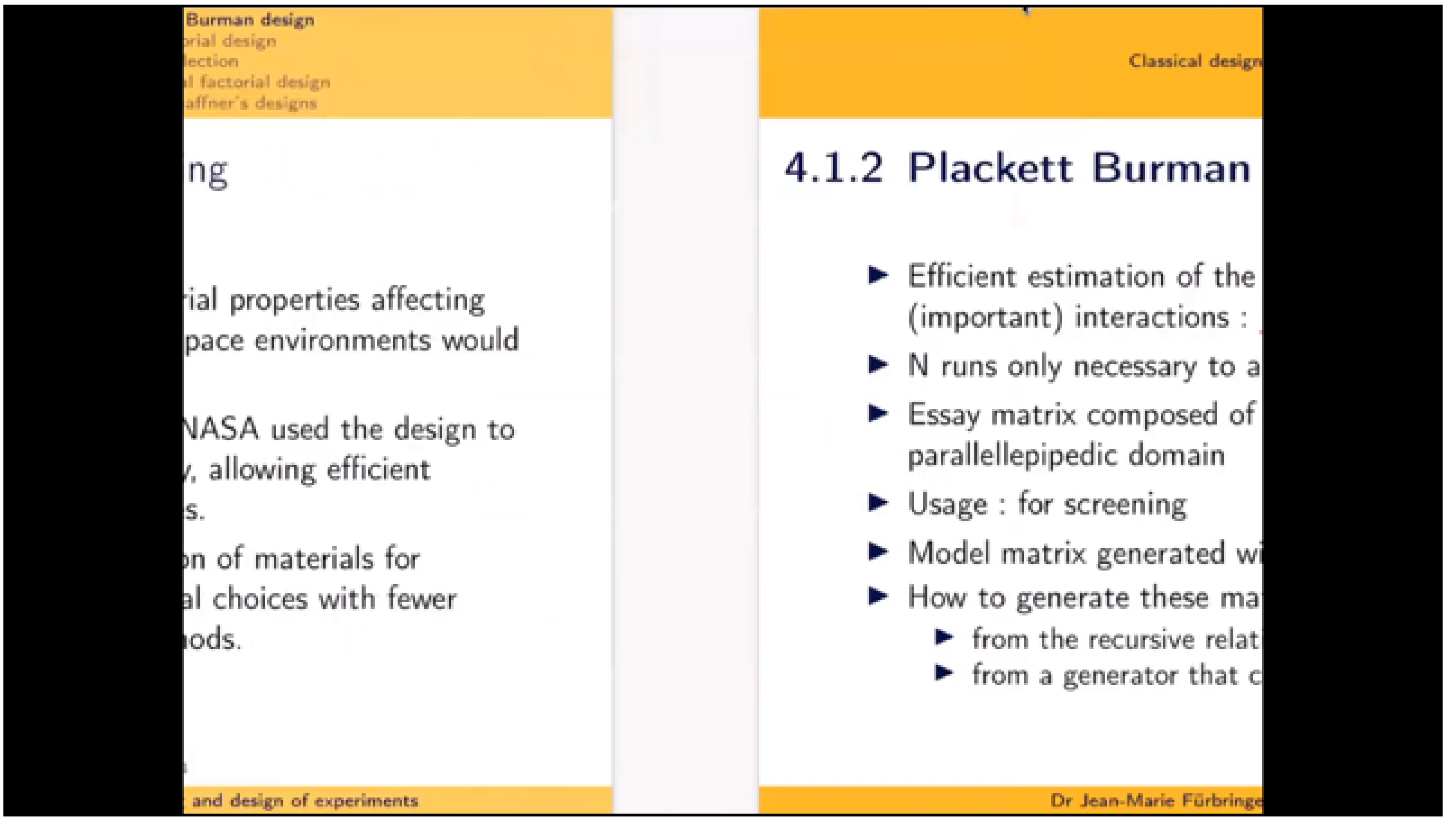


So I wanted to show you just two examples for illustrating. In fact, I have asked Chaggipiti to give me those examples. So there are fresh new slides from yesterday that Chaggipiti made for me. So, for example, for the fermentation process optimization, you want to make a bricotine in in Valais or Williamine. So you need to test different aspects of that. You see how it is important. No, it's a very important subject. And so the challenge is optimizing the factors in the fermentation. You have the pH, the temperatures, the nutrient concentrations, the iration. So if you really want to optimize the taste of your product and the other quality of your product, you need to make a lot of experiments. So with Placket and Berman, so it's looked like an advertisement, with Placket and Berman, you can diminish the numbers of experiments and put very rapidly the main effect in evidence. So it's sort of Pareto analysis. Let's discover first the perhaps 20 percent of the facts that have 80 percent of the consequences. Of factors that have 80 percent of the effects. So it's really interesting to do that. It's let you diminish the time for developing. So it's also let you be quicker in developing new products, which is quite critical in engineering and also in food engineering.

notes

summary

4m 57s

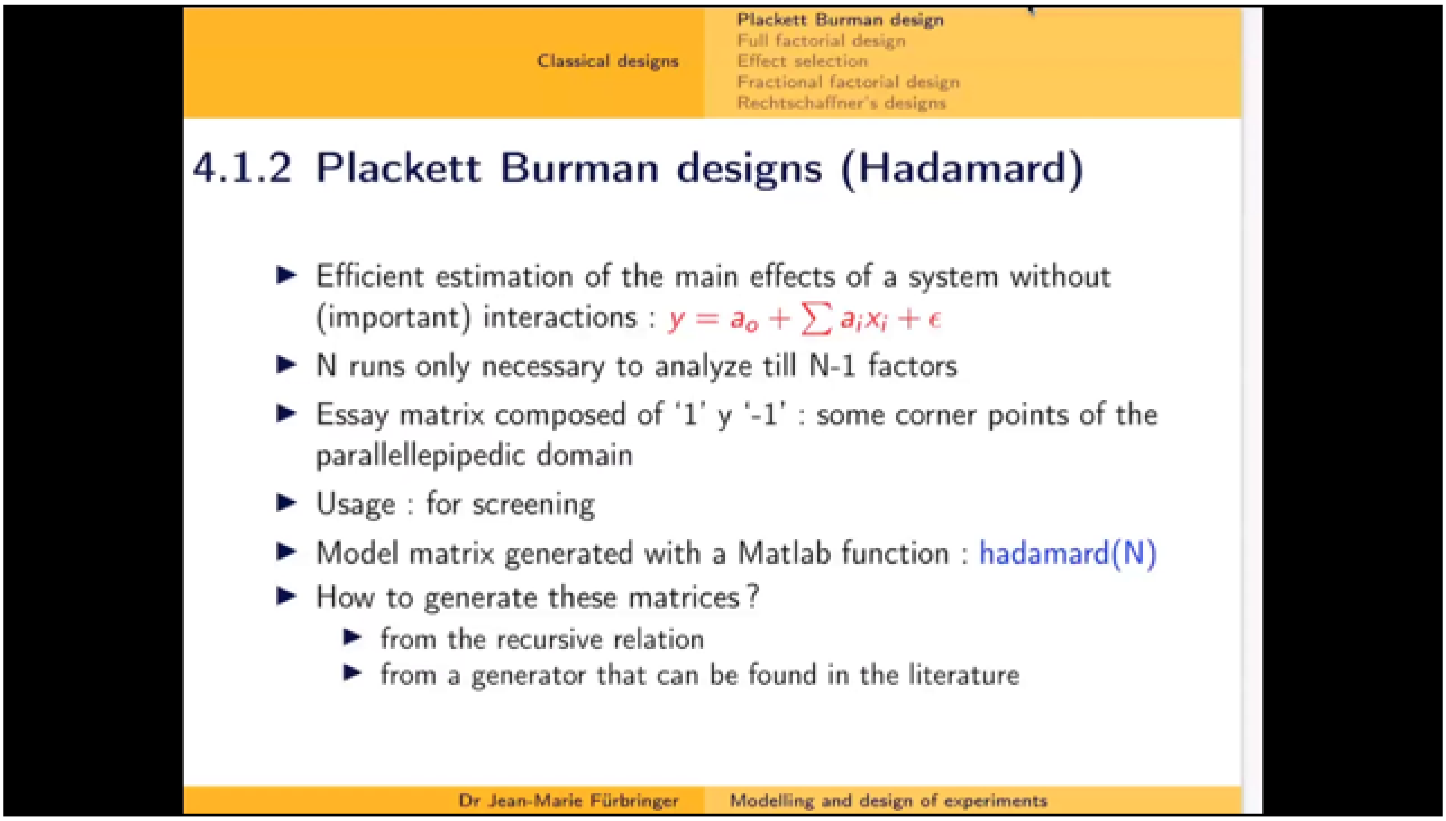


I find also on Internet, other examples, NASA is also using Placket and Berman design for testing new materials. So it's really the first type of experiments that it's very useful to do when you have a lot of factors and you want to diminish the number of factors. We call that screening because it's very complicated to manage 10 factors or 20 factors. So it's very useful if you can diminish the numbers of factors. We usually without too much effort can manage three factors, four factors. We quite understand. Well, more it's became quite complicated. And if you add to that instruction, if you add to that curvature, it became quite a complicated model. So it's very useful to diminish rapidly. And this is the ideas of the Pareto principle.

notes

summary

6m 45s

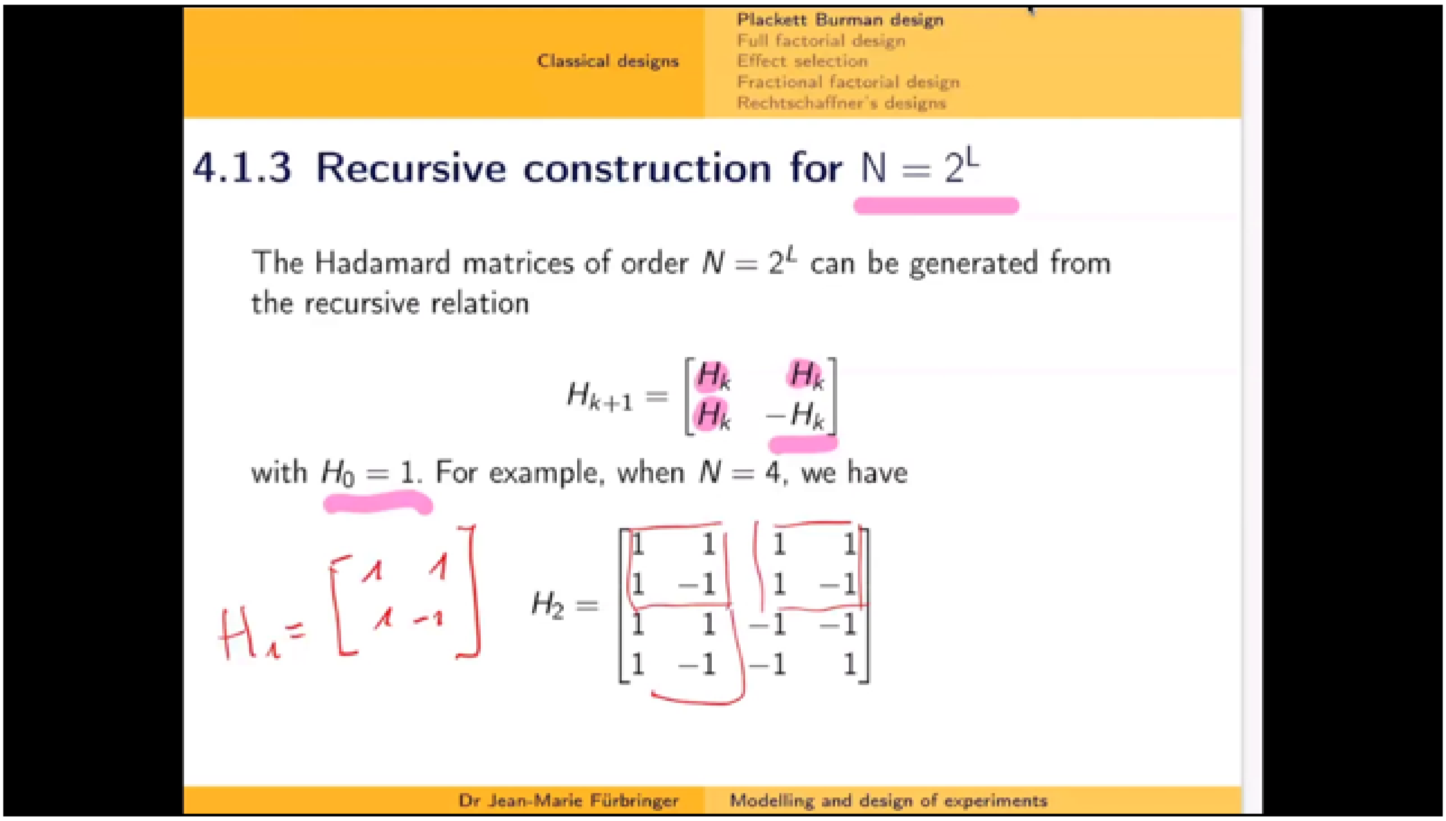


So Placket and Berman design, you will see in my in my mouse and in publication also the name of Adamar because we are using the Adamar matrices. So Adamar was a French statistician. He didn't have didn't make design of experiment. It was the mathematical structure of those matrices that interest him. And so the French that like to put in evidence of French science used to say Adamar design. I'm also using sometimes Adamar design, usually in the Anglo-Saxon literature in the world literature. So it's called Placket-Berman PB, PB design. So it's the very fine, efficient estimation of the main effect. So that means that the model is a constant plus the sum of effect plus an error is what we have to do. Sometimes we can if we have a lot of effects that are not important and we have only a few interactions, it's sometimes possible also to evaluate a few interactions. It's what the Japanese are. I have to say Toyota used to do with what we call the Taguchi method that is using this type of matrices with a

notes

summary

7m 44s

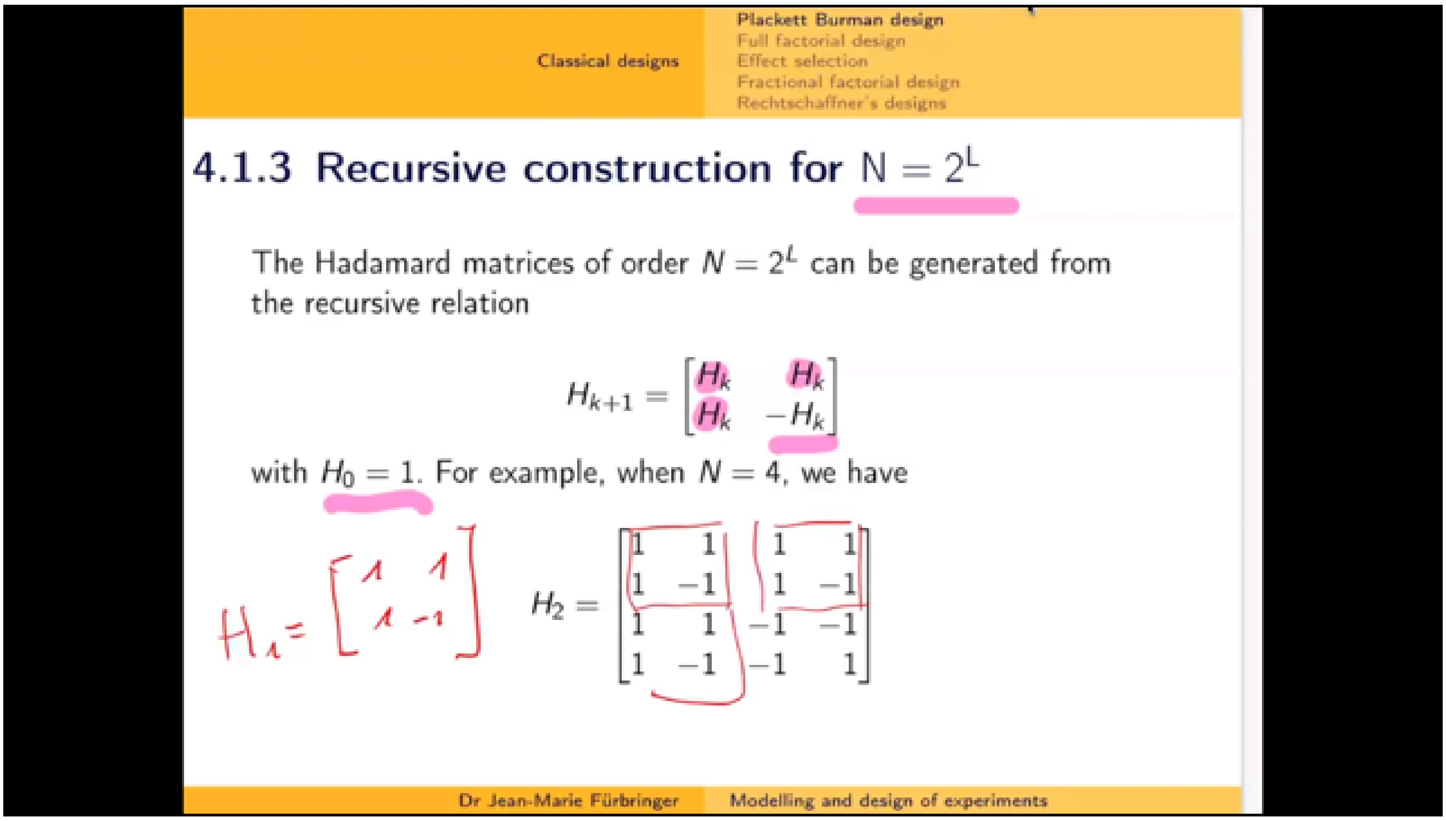


quite different philosophy in analyzing, but it's the same mathematical structure which is behind is the Adamar matrices. The Adamar matrices you will see later as a very particular property that the product was orthogonal matrices. So the product with the transpose matrix give diagonal matrices. So it's very interesting for that. So with that, with this type of design, you only need n runs for analyzing n minus one factor. You do not have all the possible n's. So it's work with Adamar design, Adamar matrices of four runs or four experiments and you can analyze still three factor of eight runs. So you can analyze till seven factor. You have 12, 16, so multiple of four, not all multiple of four for I try 100. So it doesn't work with 100, but it works with 108 or something like that. So you need to be close to a multiple of four and a few other properties should be also a number divided by 12 and a few things. It's different also of the algorithm. It exists a few other placate Berman designs. If you're interested out to build them so you have measures of books of mass in the library. So if you really want to understand, but what in this course, we're not interested. How we discover new Adamar matrices, we are just interested in how we use them. So there are matrices made with extreme of our domain. So when it's normalized is only plus one and minus one. So we are making measurement at the corner of the domain, not all the corner, but a few of the corner, but only on the corner of the domain. So very good for screening. You can generate easily those matrices with the function in MATLAB in Python. You will find also because they are very useful for analysis of signal and things like that. So you find

notes

summary

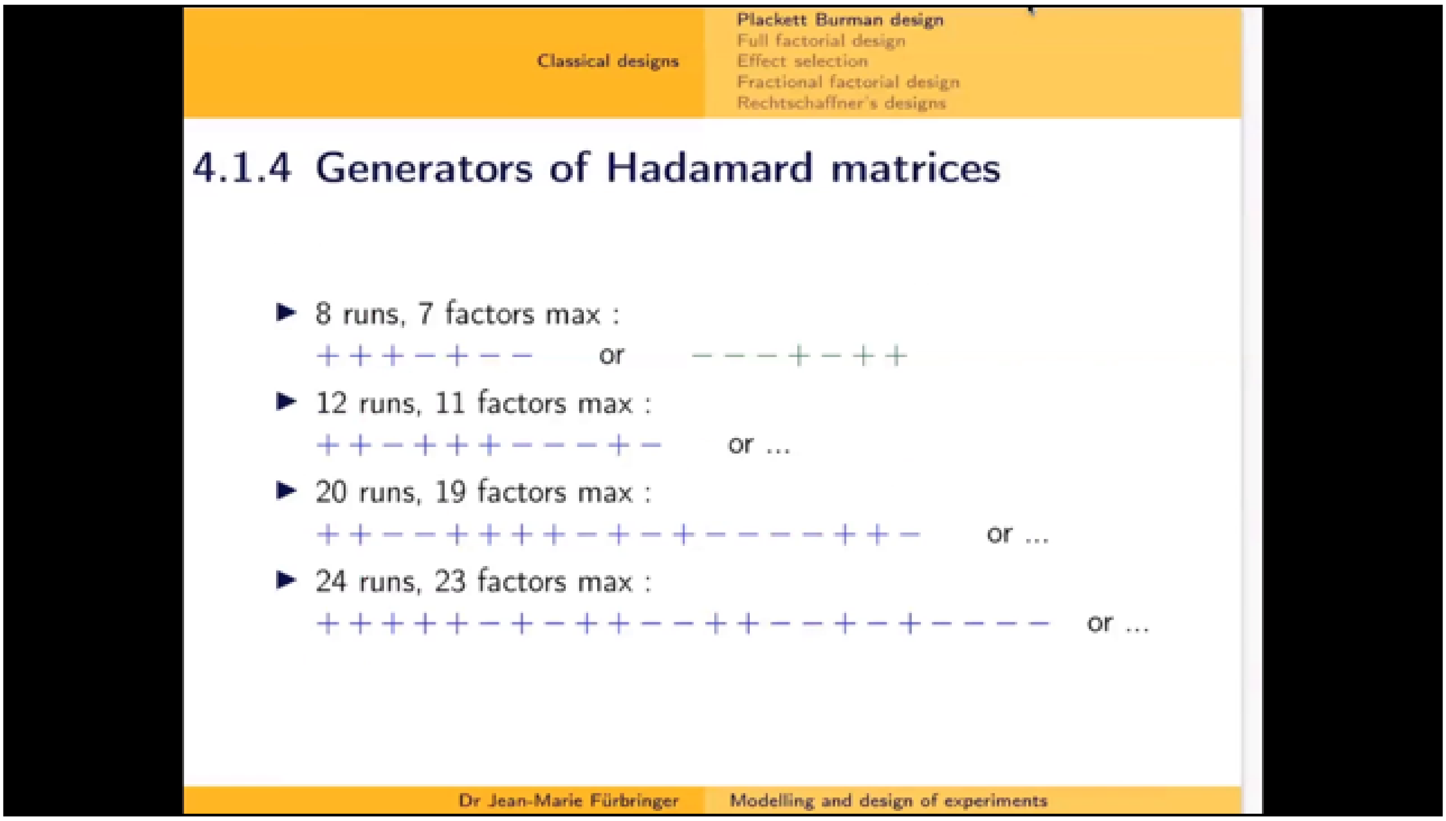
9m 25s



them, it's routines in all the statistical library. You find them. Remember, N is the numbers of run is not the number of factors. You can also generate yourself those matrices and I'm offering you two ways of generating them. We are a recursive algorithm for building the matrices. This work very well is the one probably used by MATLAB. It's work very well for the multiple of four. And we can also find in the literature what we call generators that give you one line of the matrix and from one line of the matrix to can generate the rest of the matrix. So I just illustrate that with a few slides so you can build recursively the Adamar matrices for N, which is power of two. N is the numbers of run. L is the power of two that we are considering. And you see that you can build the new matrix for so K must be more case and indices for classifying them. So it's different from N and L. So you can generate a new matrix from the previous one. If you make this construction with the previous one, so you have three times a previous one and one time. The negative, the opposite of the previous one and like that you can build, but you need to start somewhere and the first situation is one. So with this H zero, you can build H one and H one will be the matrix one, one, one, minus one.

notes

summary

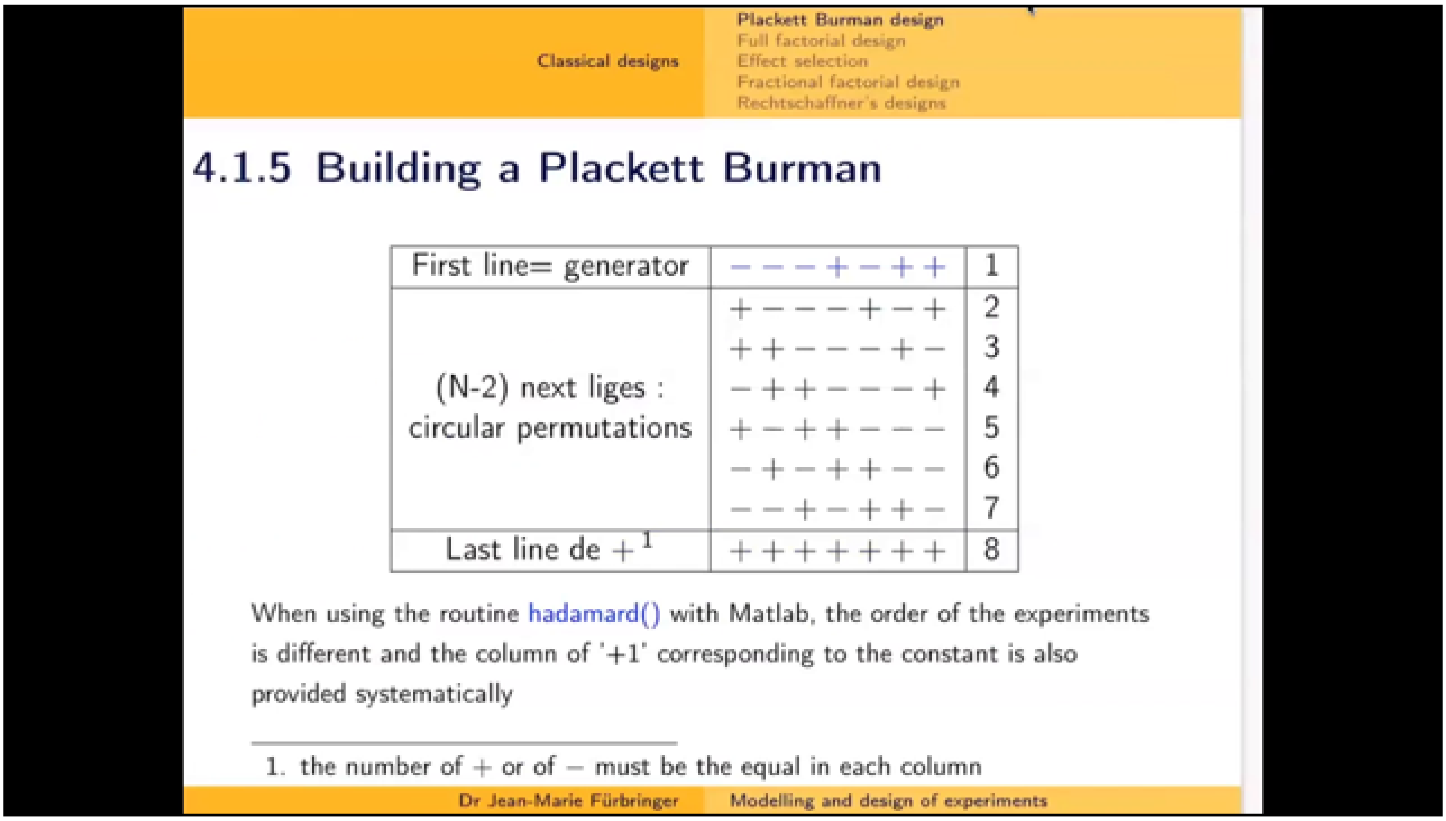


And from H one, you can build H two because you have H one, one time here, you have H one, one time here, H one, one time here and you have the opposite. I want to change my color. You have the opposite of each one here and like that you can build your mattress.

notes

summary

14m 13s

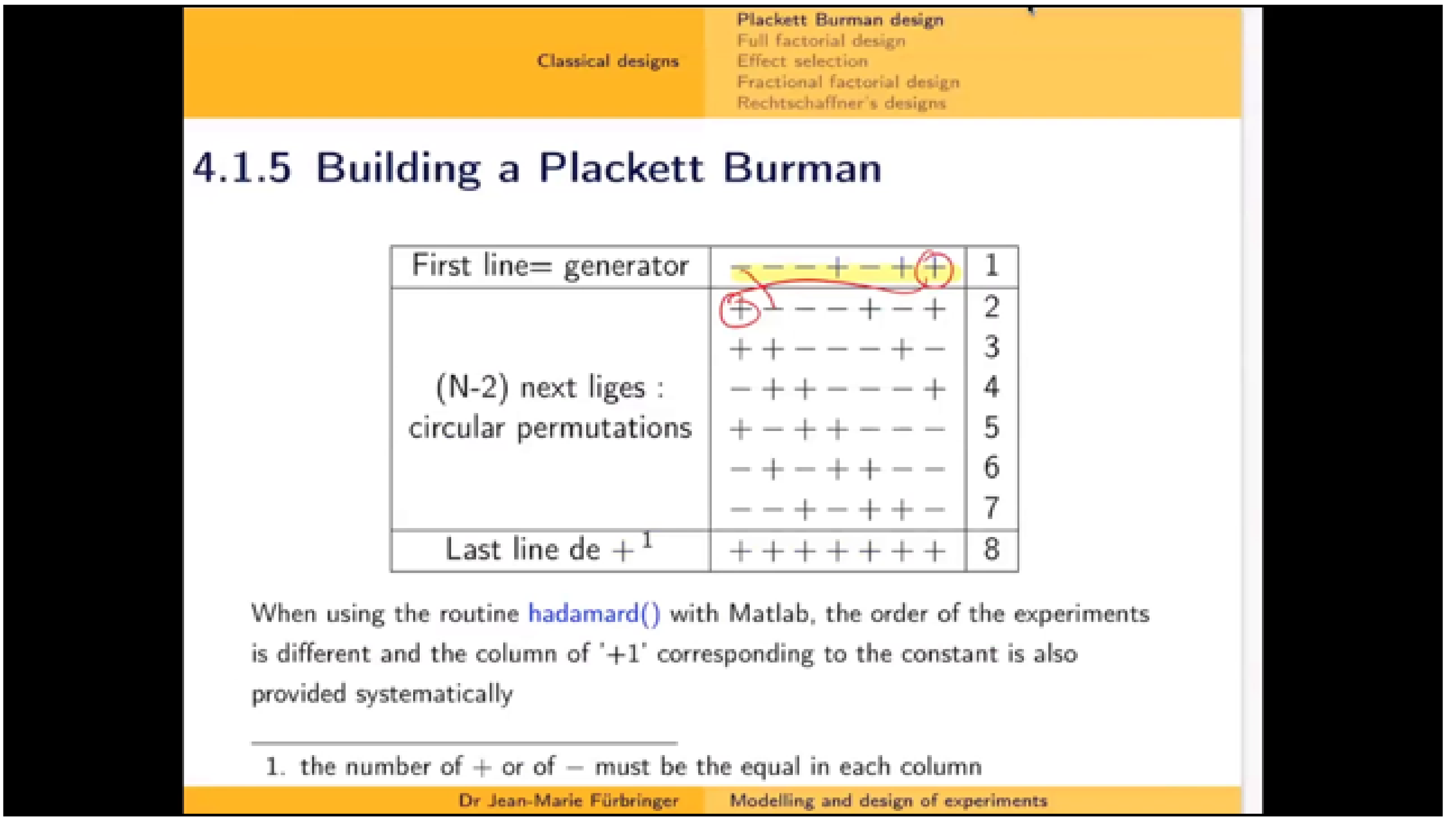


You can also find in the literature some generators. So here I give you, for example, the generator for the eight run matrix for the 12 runs matrix, which is not 12 is not the power of two. So it's interesting. There are a few more cases with this method 20, 24. It's exist also for 100, etc. With this method. So how it works with this generator. So first, understand that you can have a generator, but you can also have its opposite. It's work also you can change the plus one with the minus one and reversely and it's it's okay. It works also to just you just need one generator.

notes

summary

14m 26s

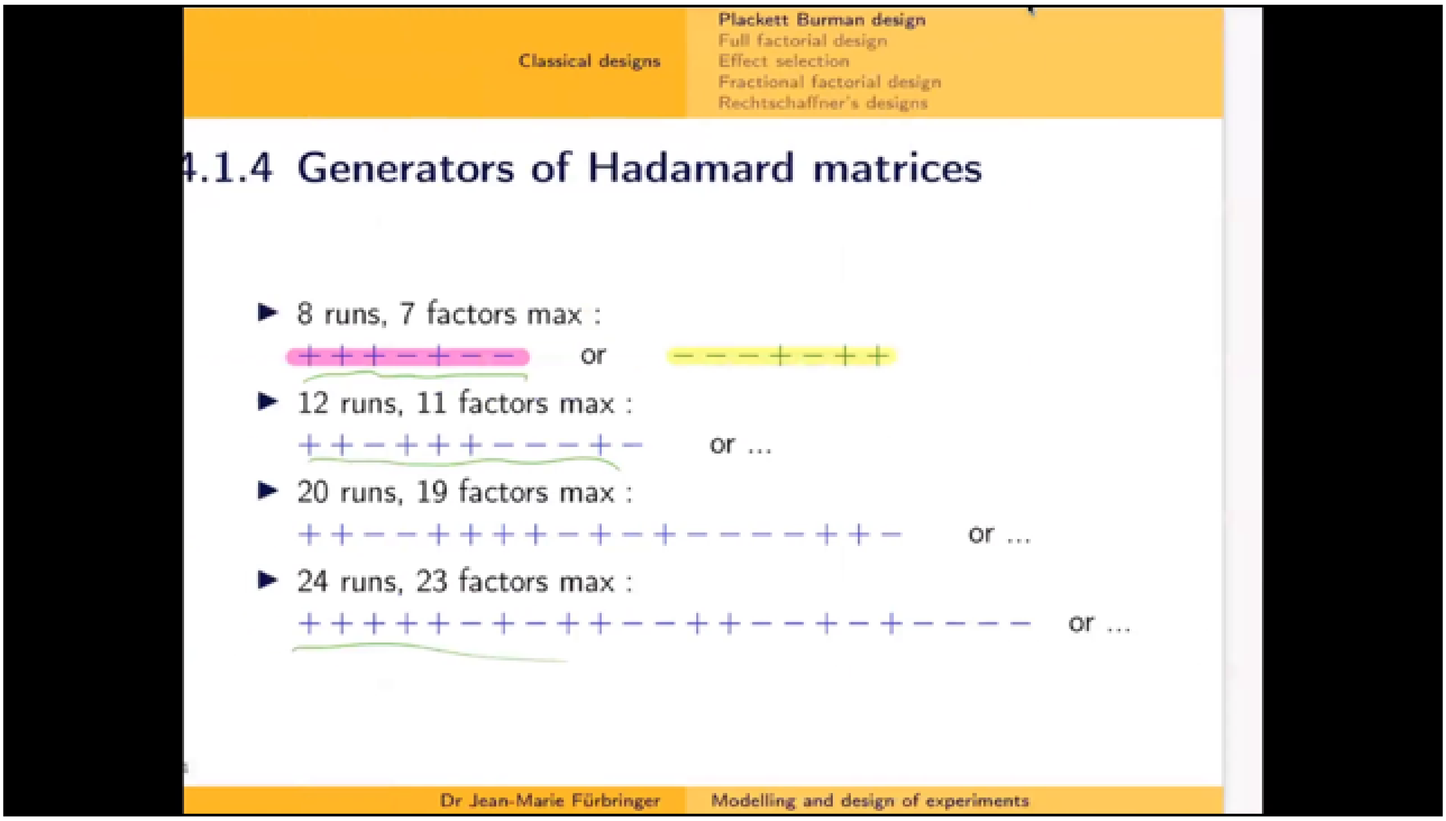


You take the generator and you consider it's your first row of your matrix. We are talking about the matrix of essay. After you can make a circular permutation. This element became this one and the first one became the second.

notes

summary

15m 19s

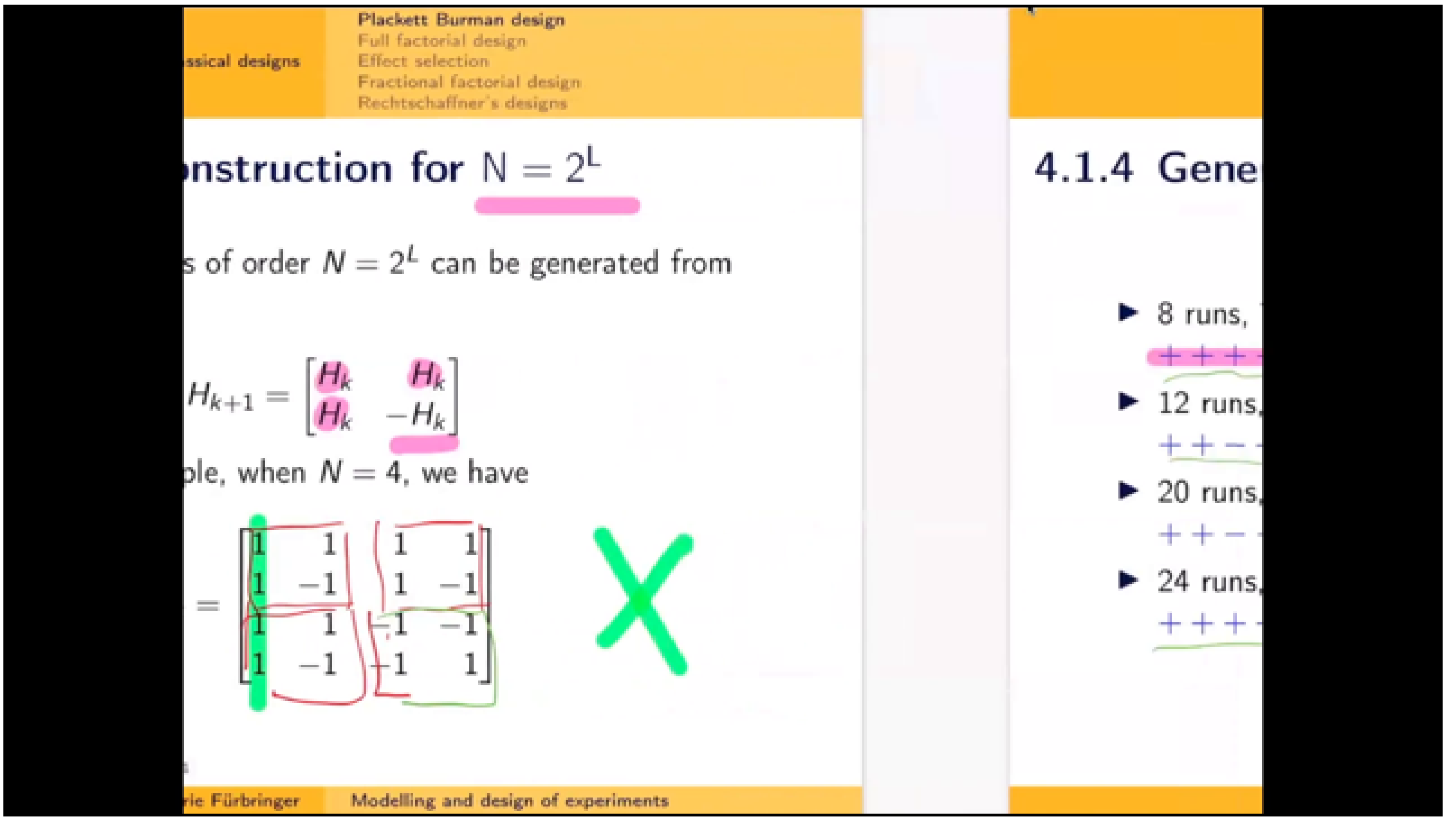


The second became the third, etc. You just you could do also in the other sense. And with that with this permutation, you can make in this case seven row and you have to add a last row that must be all plus one or all minus one. You can check it depends of your generator, but in each column, you must have the same numbers of plus and the same numbers of minus. So you see here you have one, two, three plus and you have one, two, three, four minutes. So my last row must be a row of plus for balancing. So it's a balanced design, which is very practical.

notes

summary

15m 45s



If you see this algorithm is giving you the matrix that I call the matrix of essay. This routine is give you the matrix of the model. You have the column of one, one, one in the first one.

notes

summary

16m 52s

Classical designs

Plackett Burman design
Full factorial design
Effect selection
Fractional factorial design
Rechtschaffner's designs

4.1.6 LSF estimates for orthogonal systems

The system $Y = X\alpha + \epsilon$ has for solution

$$\begin{cases} \hat{\alpha} = (X'X)^{-1}\, X'Y \\ \hat{\epsilon} = Y - X\hat{\alpha} \end{cases}$$

However $(X'X)^{-1} = \frac{1}{N_{exp}}\, I_{N_{exp}}$, thus $\hat{\alpha} = \frac{1}{N_{exp}}\, X'Y$

Example : H_4

$$(H_4' H_4) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

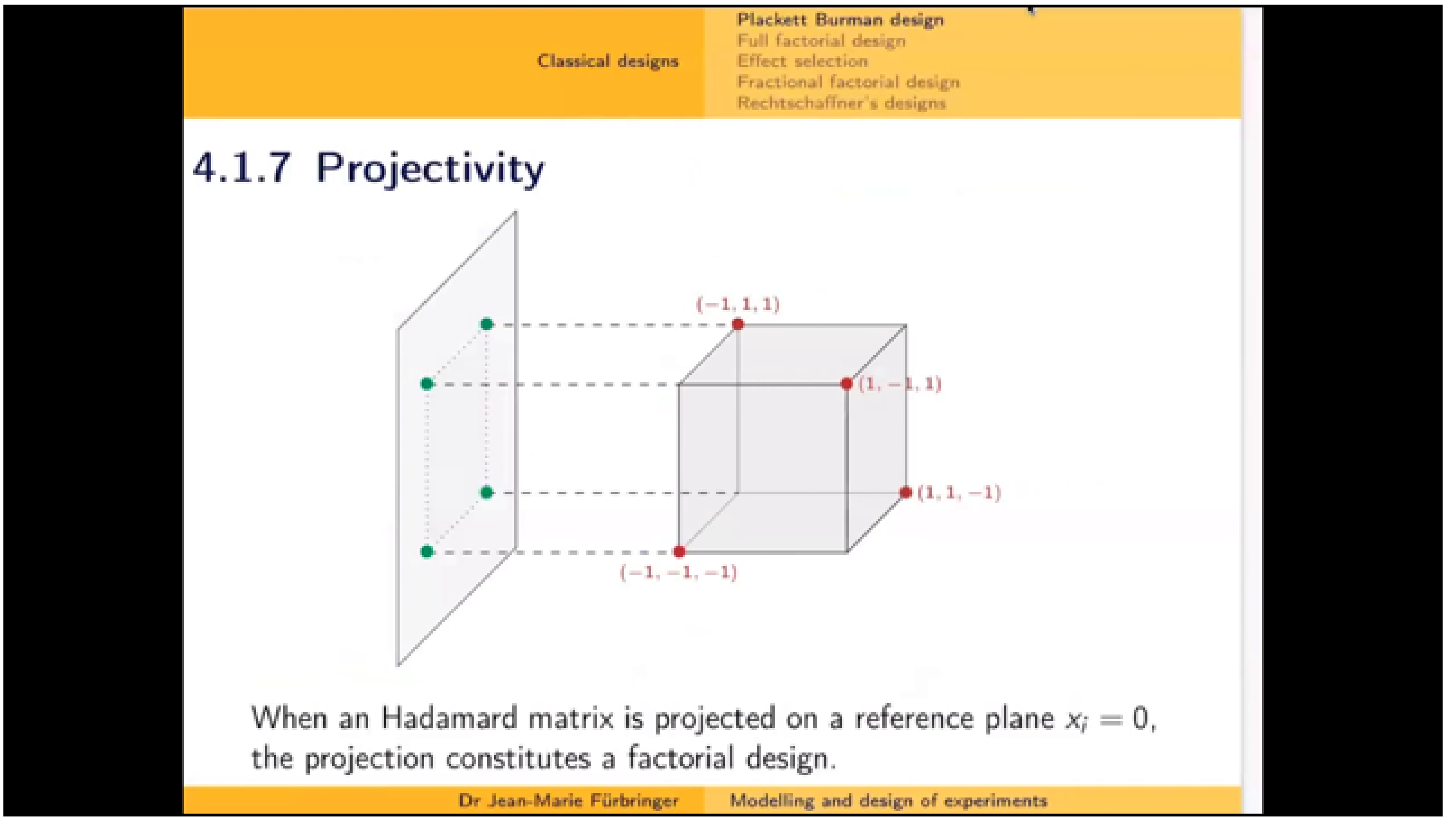
Dr Jean-Marie Fürbringer — Modelling and design of experiments

So the result is the same except that one is giving you the matrix of the model X when the other give you the matrix of essay. But it's easy to go from one to the other, adding a column of ones or subtracting, eliminating a column of ones, depending the one you have. The algorithm of MATLABs give you the matrix of the model. So it will give you the first column with ones. So when you use it, you have to forget the first column because it doesn't correspond to the variation of a factor. So understand also that when you have this matrices, so each column plus minus means plus one minus one. That means that when you have plus, you use the maximum of your factor and when you have minus, you use the minimum of your factor. There are an interest that perhaps today is less important if the least square feet estimate doesn't need to make a matrix inversion. This was when it has been settled in 1945 and those years very practical because inversing a matrix in those days was something quite complicated today. It's not any more a problem. But look at that. So if you have your system, which is my answer, y is equal to a model, a matrix of the model X multiplied by alpha, the coefficients plus an error. You have your standard estimate of the effect, alpha hat, which is this formula that I ask you to have in mind now and never forget it till the end of your life.

notes

summary

17m 11s

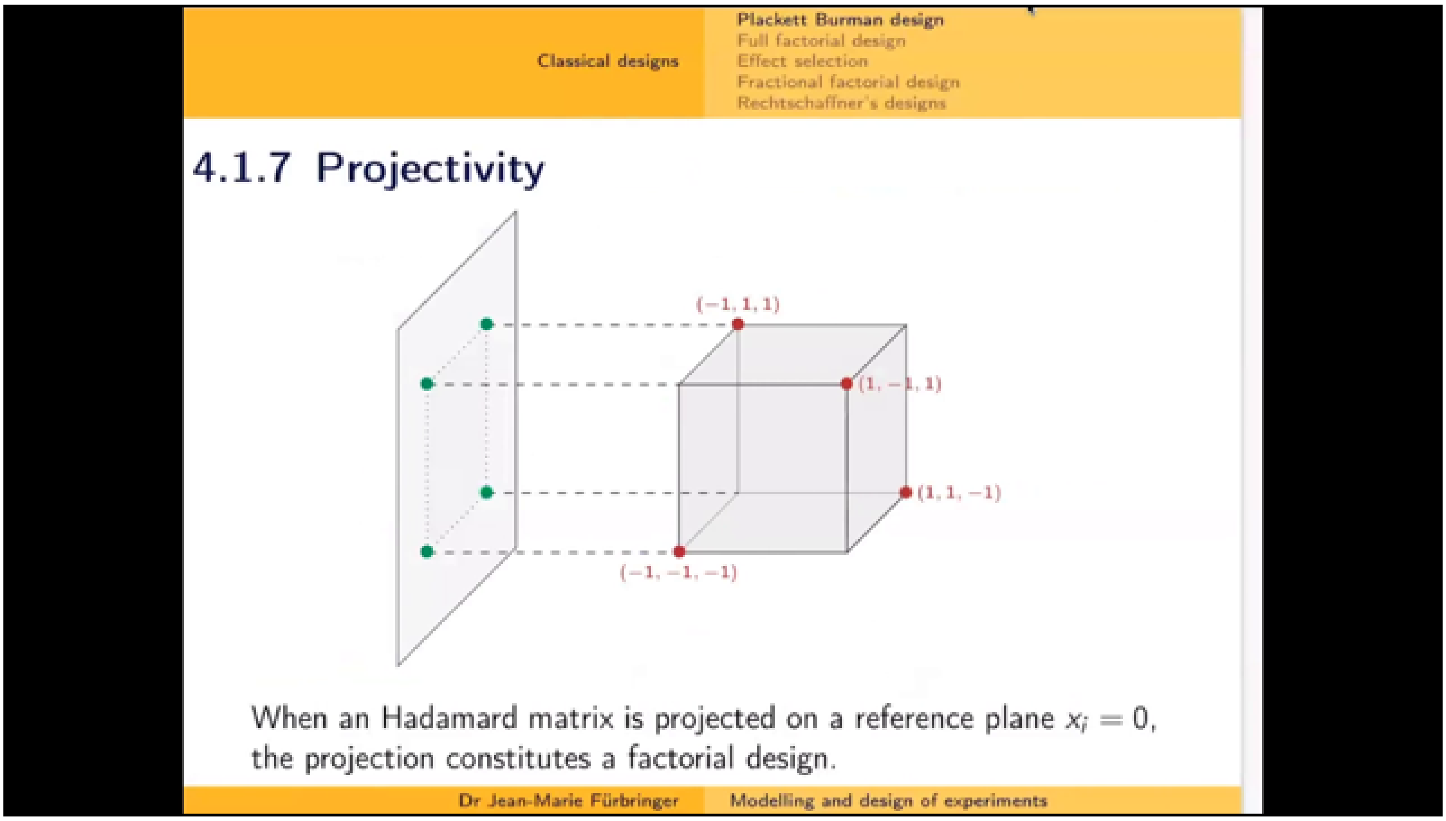


And perhaps even in the heavens, you can use it. X transpose X minus one, X transpose Y. This is the least square feet estimate. And the estimate of the residue, which is the difference between your measurement Y minus your estimate of your measurement X multiplied by alpha hat. But now because this Adamar matrices have a special structure, the inverse of the product with a transpose give the identity matrix divided by the number of run n and X. So that makes that the algorithm became a lot more simple, what I call it simplified least square feet algorithm. You just need to multiply the matrix of the model, the transpose of the matrix of the model with your vector of measurement and divided by the numbers of experiments. And that's it. So it's make very simple. As explained before, it was a very important advantage for the calculation before the generalization of the computer. But with that became a trap. Imagine that you start making some experiment with Adamar design. You have these algorithms for estimating the effects. And after you say, OK, I want to make a few more experiments. So you add a few more experiments. You repeat not all, you repeat only one or two of this experiment. It's not anymore an Adamar. You are not allowed to use this simplified algorithm. I made this error a few times. You will make it a few times. So you arrive with effects that you don't understand. So when you have that, go back to your algorithm, check if you are using correctly the least square feet algorithm. If you are using the simplified algorithm, when you are not allowed, because it's not exactly another matrix. A factor was not exactly the values that you're seeing. So you change it a little bit and this invalidates the possibility to use a simplified algorithm. You have to

notes

summary

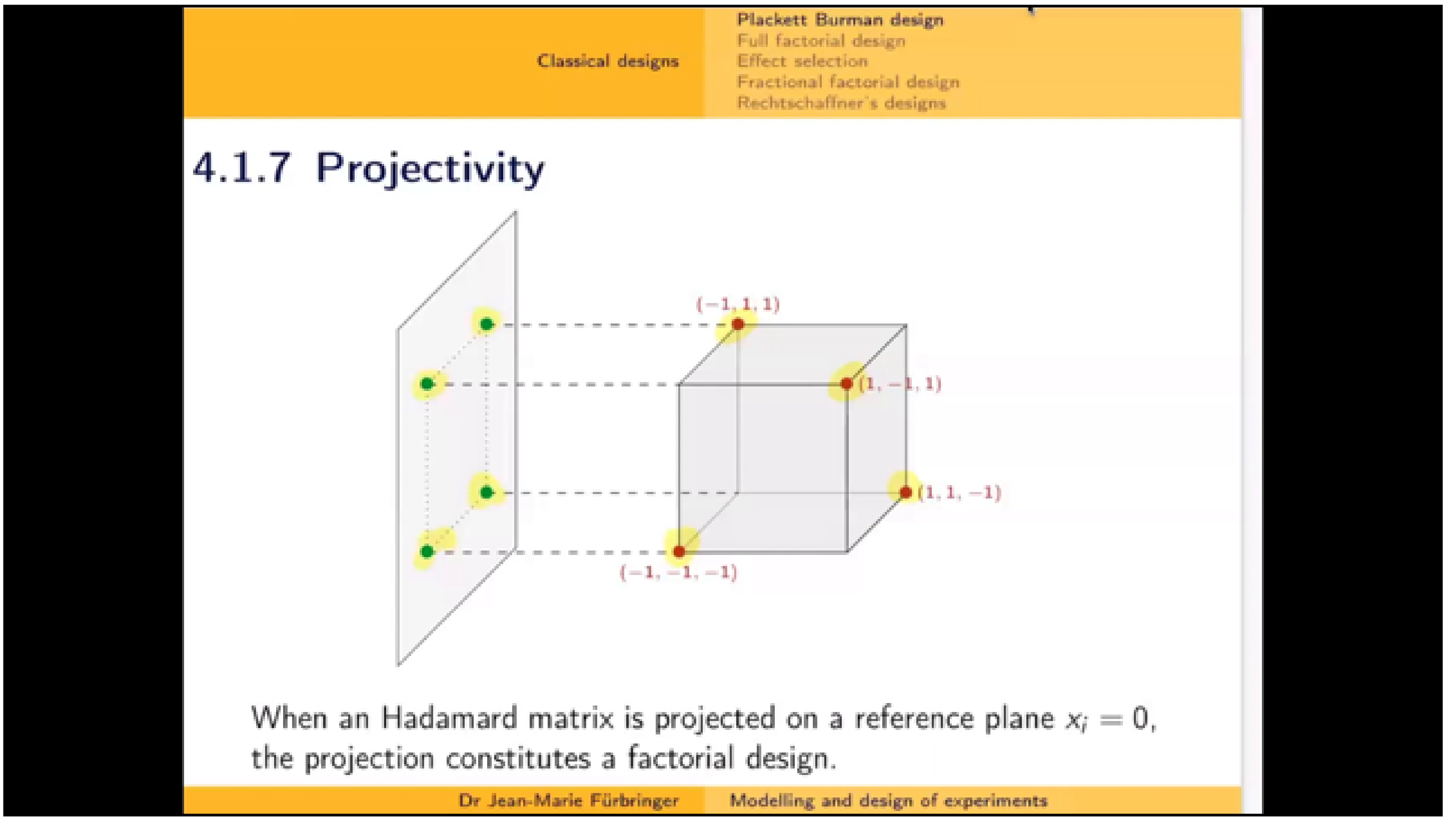
19m 1s



go back to the global algorithm X transpose X. So just be careful when you use this simplified algorithm and you want to go further. Here, just an example, an illustration. So if you take the force, Adamar matrix with four runs, you just want to see you that if you multiply the transpose of it with this matrix, you obtain a diagonal matrix with four in diagonal. And if you inverse it, so it will be one one quarter in the diagonal.

notes

summary

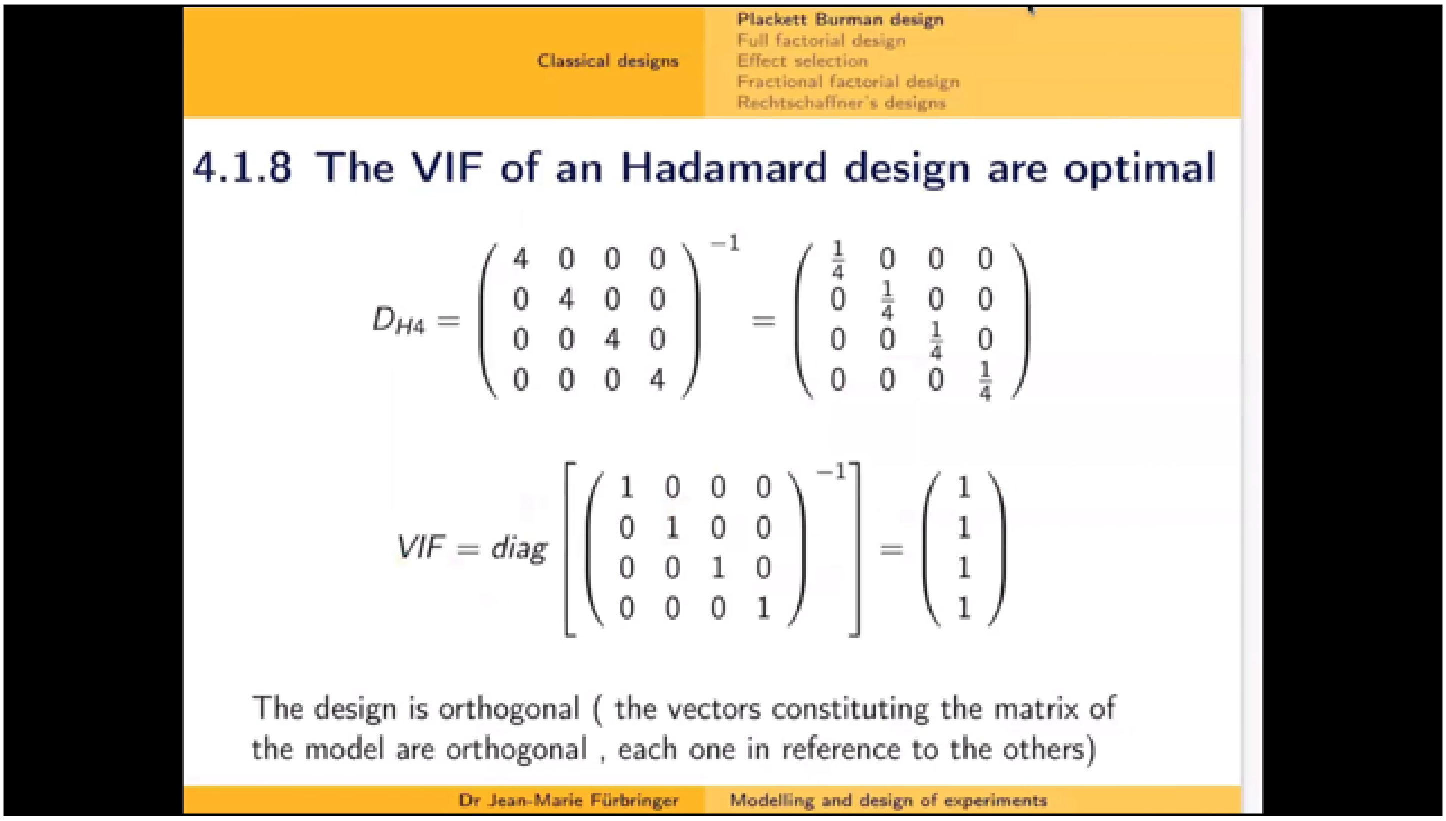


This matrices have an interest. They are a way of explaining why they are so powerful in the design of experiment because they have something we call projectivity. You see here, I have presented what could be draw in a piece of paper. So three D diagram in two days. So if you have this, these points, you can project them in the different two factor planes. So you understand so projecting in a two factor plane, that means that the third factor will not be important. You are scratching one dimension and you see when you are scratching one dimension, you finish with points distributed as a full factorial design. So this is the same thing with more dimensions is the interest. We are using that because we make the hypothesis that in fact, not all factor we have in mind are important. So it's give us the possibility to projecting. So it's why I'm telling you that the Japanese approach is using that we make bigger Adamar matrices than necessary with more dimension without some dimensions are not used. And this allow finally to eliminate some factors that are typically not important and then

notes

summary

22m 10s

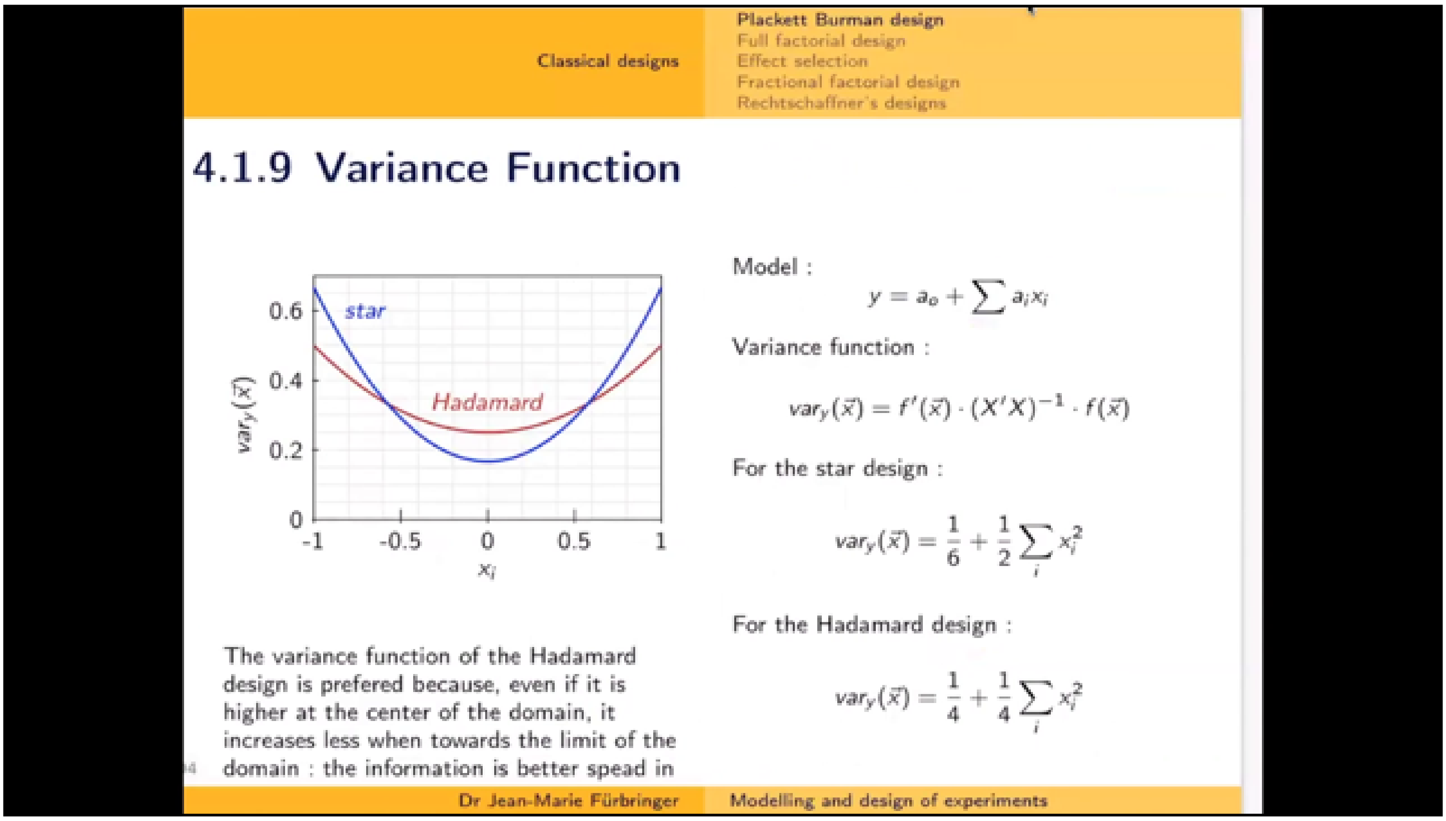


interpret the result also in term of interactions. We will not do that, but I tell you that this is possible. It's called the Taguchi approach the Taguchi methods. Very, very interesting.

notes

summary

23m 37s

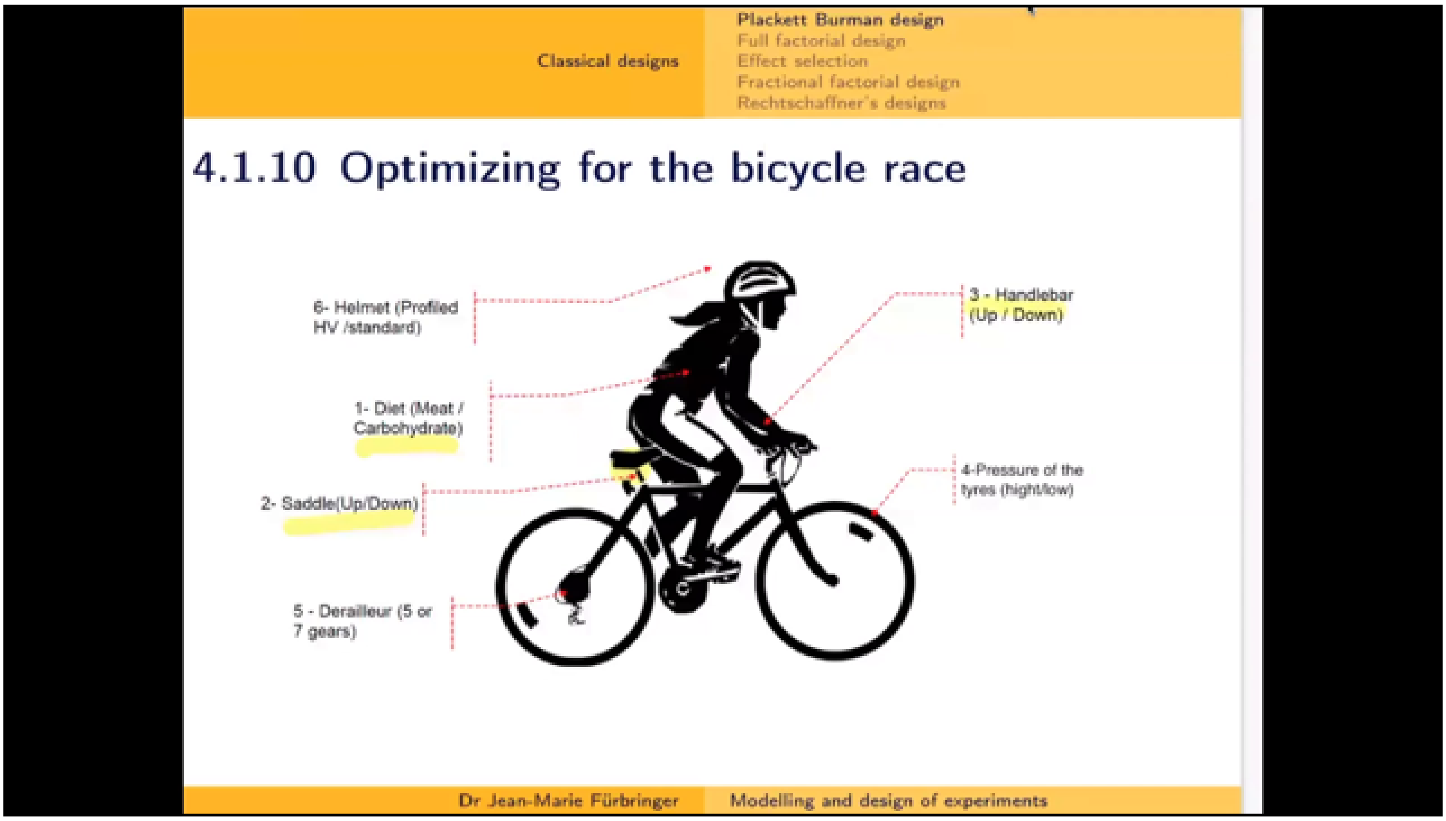


Just for proving that it's interesting, I just have calculated the variance inflation factor. So you don't need to do it from now because it's evident the variance inflation factor are one everywhere. It's the best possible design for that after you can improve the design repeating experiment. But if your objective is to put in evidence the main effects of a phenomena. Only one product. Placket and Berman design. You do not have better than that. If you accept to make both and runs. There are a few techniques for going step by step and avoiding to make all those end designs. I'm not including that in the course. So but if you accept that and design is your budget, your boss is OK to spend this money and time is the best possible. The variance inflation factor are one. You remember the variance inflation factor are indicating the quality of the call in the area that you will have in your factors at the end. So the rate of the size of the confidence interval related to the variance of your measurement. So the minimum is one. It could be higher. And when it's more than six, you start having problems and you start to avoid experiments. So this is a very good design.

notes

summary

23m 49s

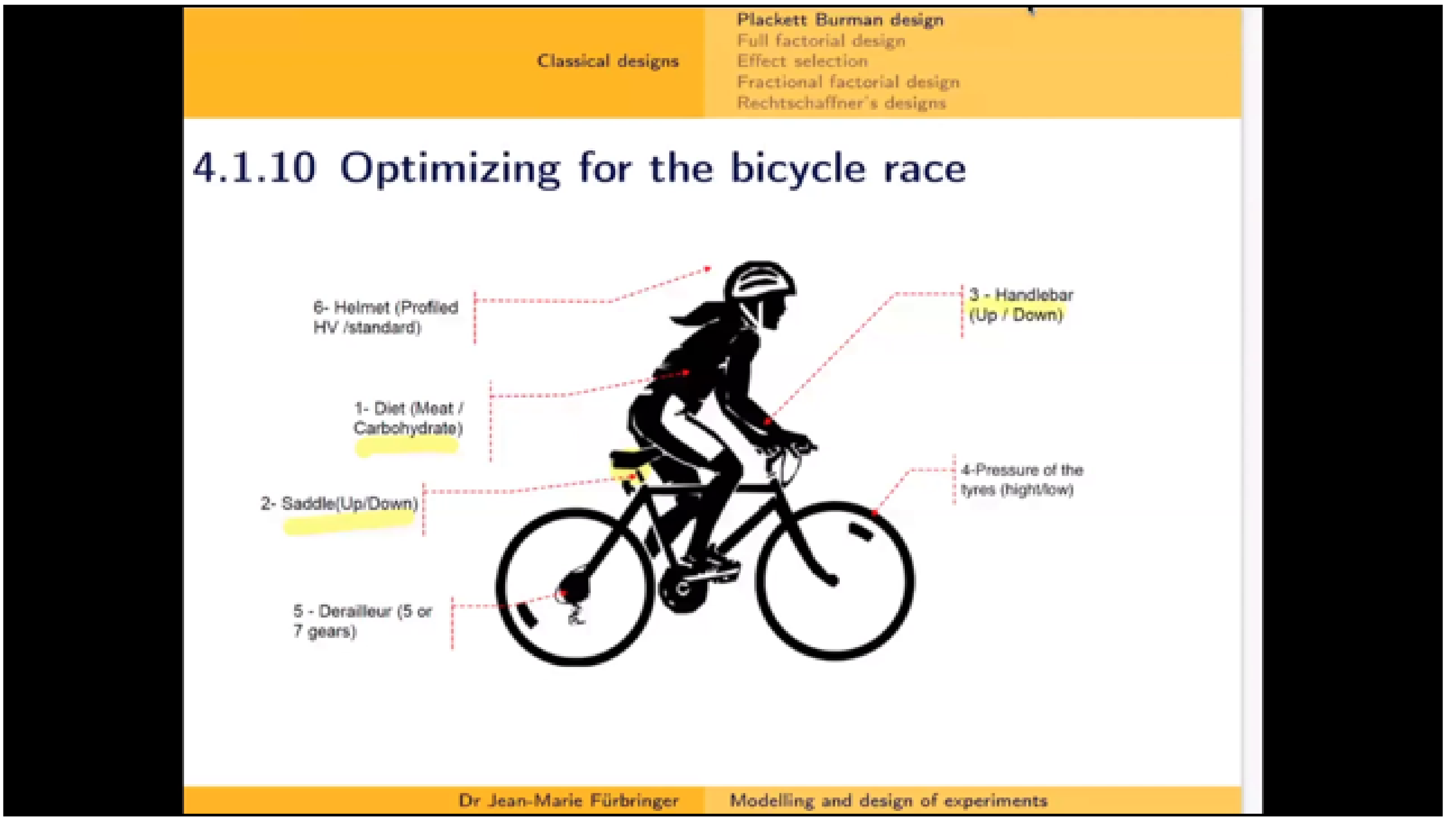


If you look at the variance function, I made it in two dimension is perhaps not the most critical situation. But it's a bit more easy to show it. So in fact, what I call the star design in blue is if you use the point from the center, you just vary one after the other, the different factor, which could represent more or less the same numbers of experiment. If you do that, it's clear that the center of your domain, you see that the blue. Is better because you have a lot of information which is concentrated on the center of your domain, but on the domain in the whole, you have a better result with the red curves. That means with the Adamor for all the domain, the placate and Berman strategy is better than one factor at a time. And after you just have here is a calculation of what I just explained you better. So in the star design, you have the variance, which is one six, representing the value here at the center plus one half of the square of your of your range. And in the other, you have one quarter at the center, which is bigger than one six. But after you just adding one one quarter, not one half for the additional variable. So let's check how to use this design in a comprehensive example. Imagine that the doctoral school or the section of physics is organizing a race of bicycle. If you win the race, you have one additional semester free. So you really want to win the race, but as the student, PSE, you are not so rich, you cannot buy the last technology in bicycles. So you have to use your bicycle and you want to improve, optimize your bicycle. What can you optimize in your bicycle? Well, you can even really start by optimizing you. So the

notes

summary

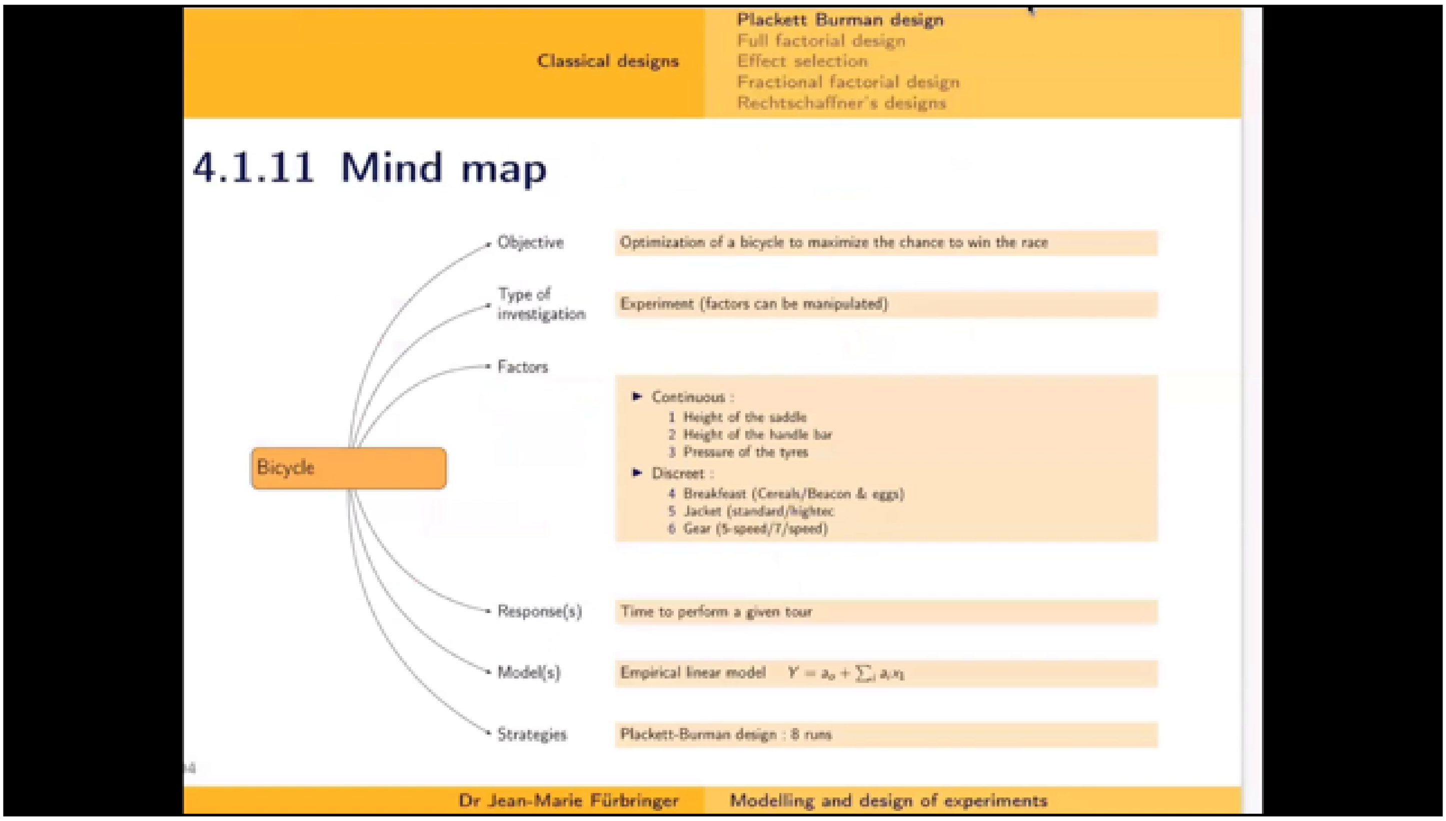
25m 27s



diet, you read a few things about diet and you see that you can have a carbohydrate diet, or perhaps you can have a diet with meats or different diet. Imagine we just have two, you want to decide between these two diet, which one is better. You can look at your saddle, the eighth of your saddle.

notes

summary

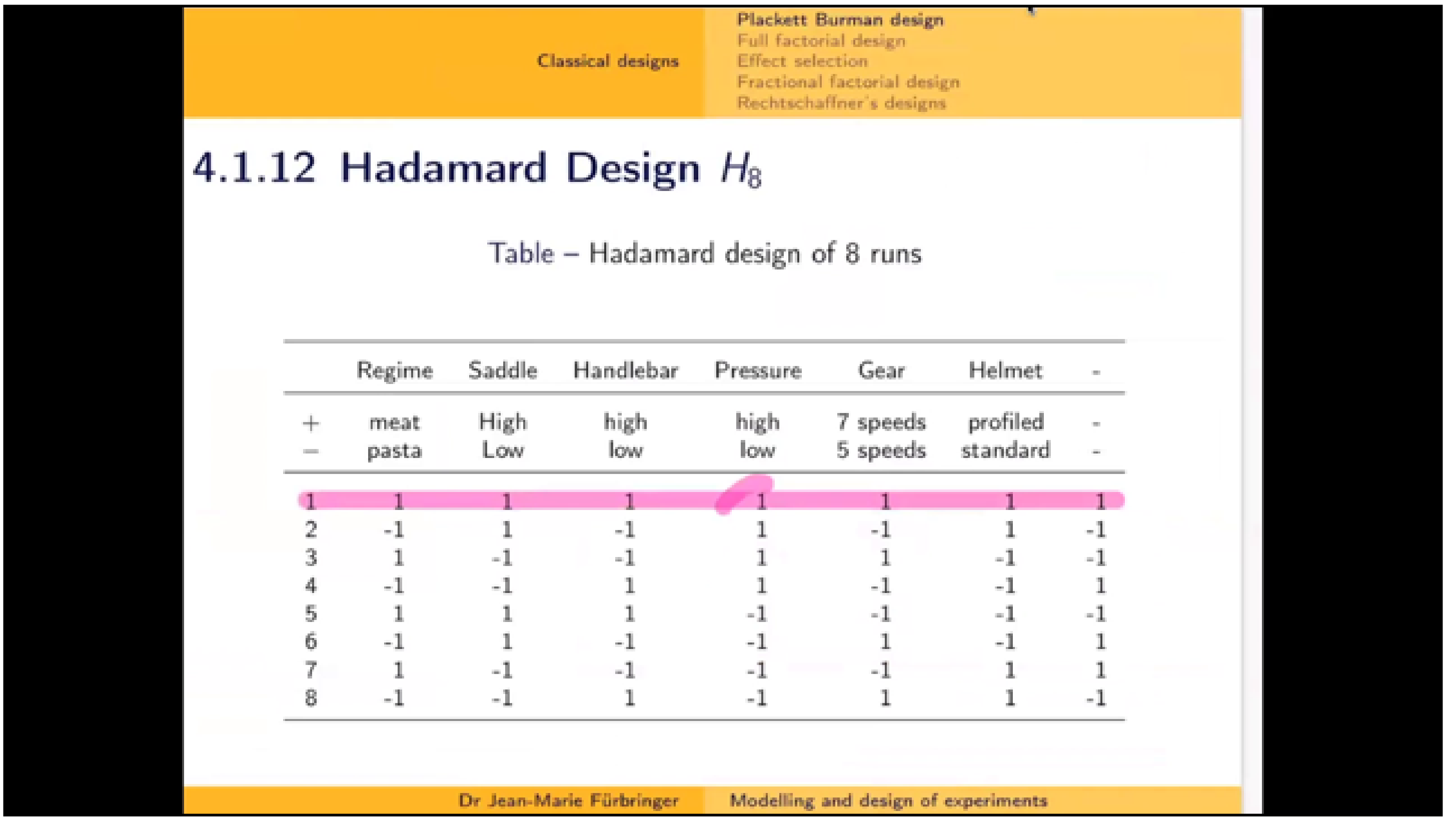


You can also change the position of the handlebar. You perhaps understand that there are an interaction between these two, probably related to your body and your physiology. There are eventually an interaction between these two. I'm not sure there are an interaction between the diet and between the saddle eight and not between the diet and the handlebar. You can also change the pressure of your tire. It could be more hard or a little bit softer. You can change the derailleur, which can have, you have two derailleur. You can have one there with five gears and you have one with seven gear. In some situations, it could help. One could be better than the other. And you can also have a standard helmet, your helmet you use every day for coming in a PFL. Or you have a friend who has aerodynamic helmets and eventually it could help.

notes

summary

28m 25s



4.1.12 Hadamard Design H_8

Table – Hadamard design of 8 runs

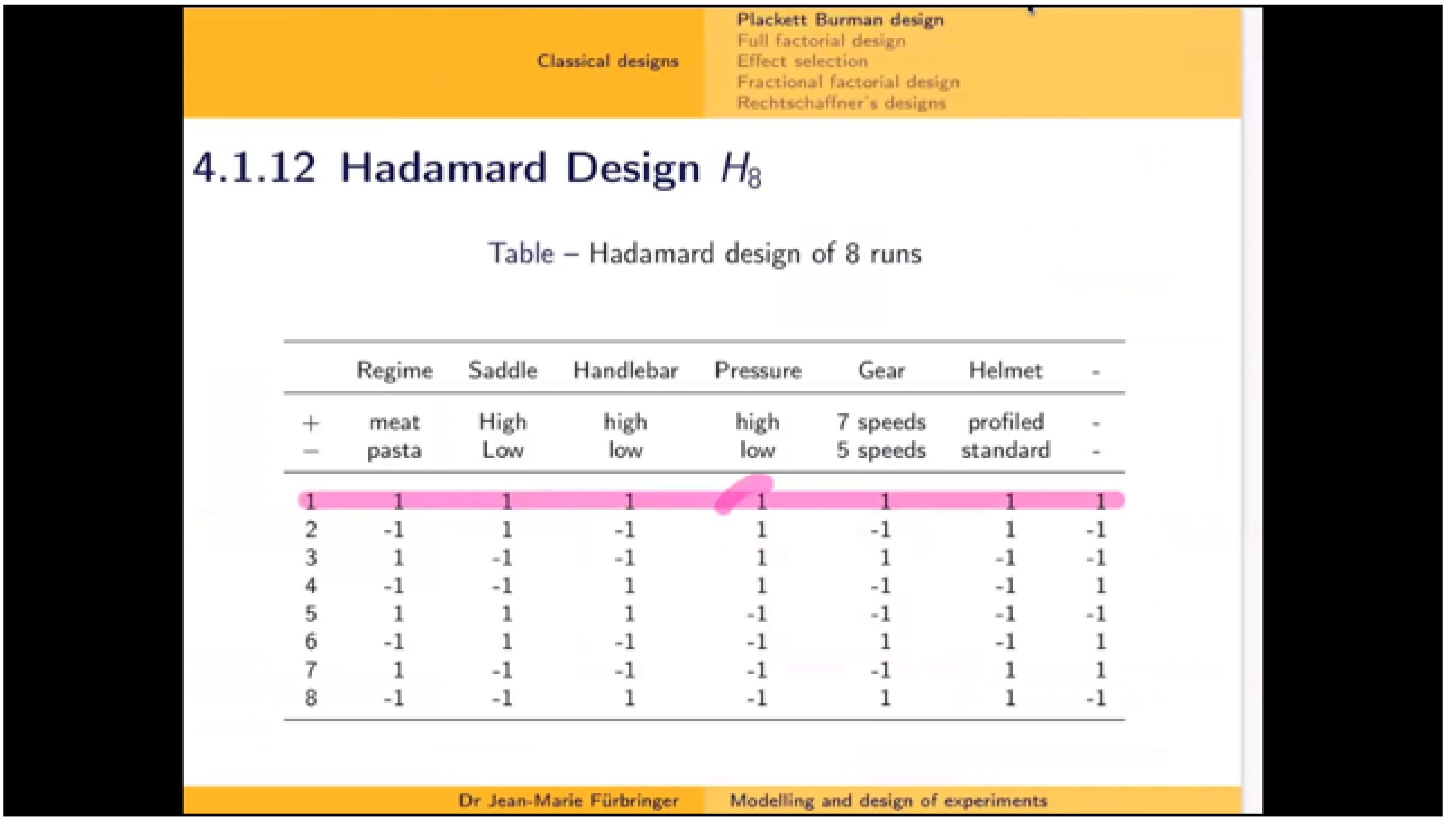
	Regime	Saddle	Handlebar	Pressure	Gear	Helmet	-
+	meat	High	high	high	7 speeds	profiled	-
–	pasta	Low	low	low	5 speeds	standard	-
1	1	1	1	1	1	1	1
2	-1	1	-1	1	-1	1	-1
3	1	-1	-1	1	1	-1	-1
4	-1	-1	1	1	-1	-1	1
5	1	1	1	-1	-1	-1	-1
6	-1	1	-1	-1	1	-1	1
7	1	-1	-1	-1	-1	1	1
8	-1	-1	1	-1	1	1	-1

Mind map for analyzing the situation. What you want is the optimization of a bicycle to maximize the chance to win the race. This is your objective. It's, in this case, an experiment. You could also go in the literature check if somebody is telling things, but it's not your bicycle. It's not you. So it's better to really make the experiment. It's also make training for you. But too much training is not good. You will arrive tired to the race. So you better have a middle intensive training. What are the factors? So you have some factors that are continuous. The eighth of the saddle in millimeter, you can regulate this eight. The eighth of the handlebar, the pressure of the tire. There are three continuous factors that could have continuous value. In this case, we will use extreme. We will try to be careful not having a second degree influence or not too high and not too low. So try to be in the interesting range. The same with the handlebar, the height of the handlebar and the pressure of the tire. But we have also some discrete factors. Your breakfast is one or the other is with cereals or with bacon and eggs. Sorry, some factors have changed. So now this is jacket when mine was helmet. And the gear. And you can also have a third discrete factor is the gear that you are using. Seven speed or five speed. As a response, the time to perform so you understand more or less what will be the tour for the race. And you can train your different solution in the same itinerary. You don't want to start with a mechanical sophisticated optimization of your bicycle. Just model was main effect and the constant could be already OK to tell you to where to put the factor and thinking that eventually when you discover something after

notes

summary

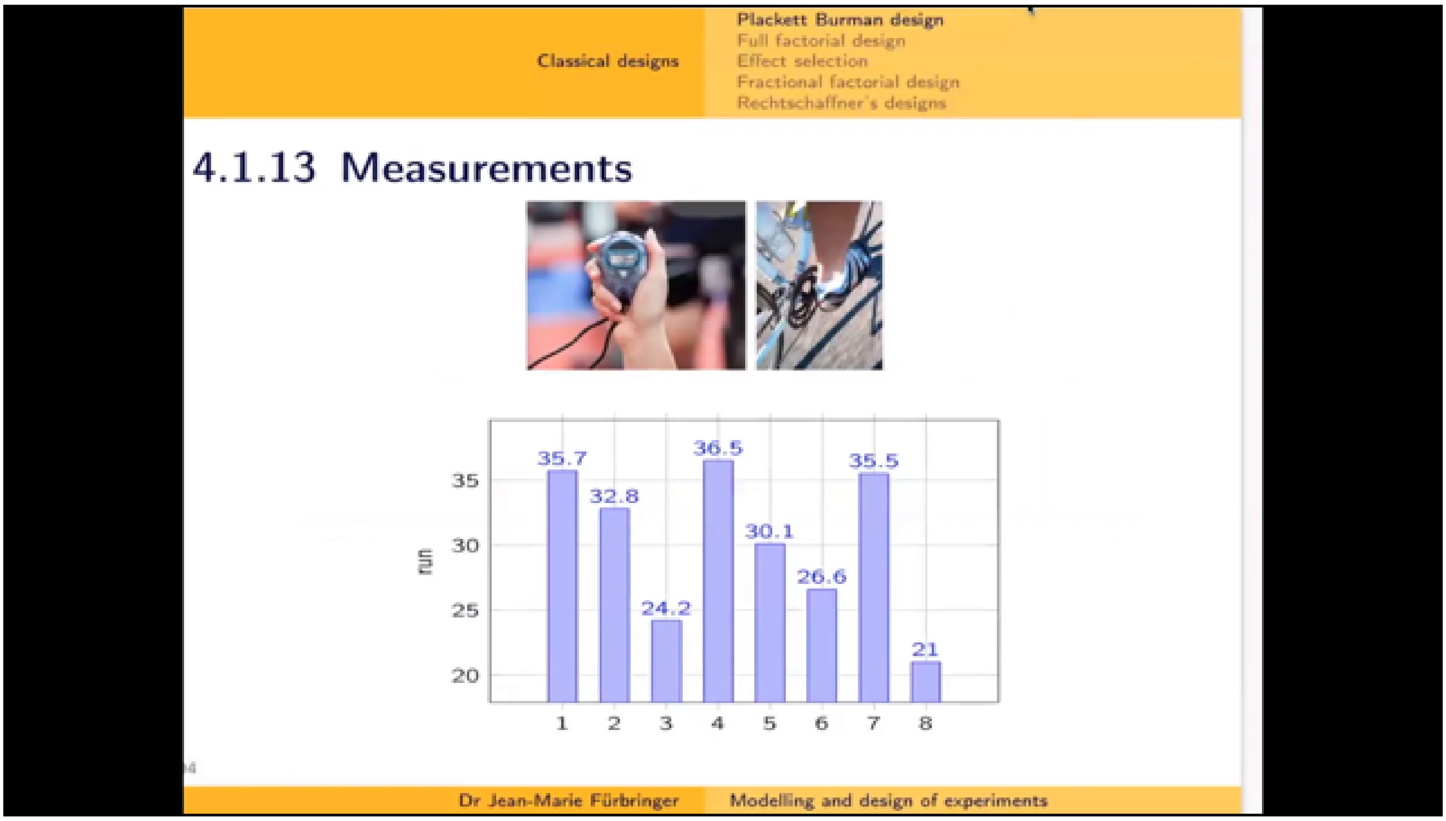
29m 33s



you can adapt more, more finally, the continuous factors. So solution, Placket-Burman, eight runs. I have six factor. So I do not have a Placket-Burman for seven runs. So I have no choice. I have one with four runs and one with eight runs. So you have to take the one with eight runs. So with the help of a friend who have the key and the chronometer, you start making your different itinerary. You be careful to have the same days, the same not rainy day, some day in some sunny days or try to control the factors that we are not taking into into into account. And then you have the first situation. So in reality, I call this runs one, two, three, four, eight, but you have to execute them in the random order.

notes

summary



You do that because if you don't do that, you see that you have for the pressure, you have all the positive first and all the negative first. So if you are, for example, running the first days when you are not tired and it's sunny days and after it's you are tired and it's rainy, you will have an influence. You will have a bias on those effects. So it's why you want to execute. So you have to execute those experiments, not in the order you draw it. You have to make it random. So first experiment, second experiment, etc., etc. You follow that, you adapt your handlebar, you adapt your regime, your diet, you add on your saddle, etc. You make these eight experiments.

notes

summary

33m 13s

4.1.14 Determination of the effects

$$\hat{\alpha} = (X'X)^{-1}X'Y = \frac{1}{N}X'Y$$

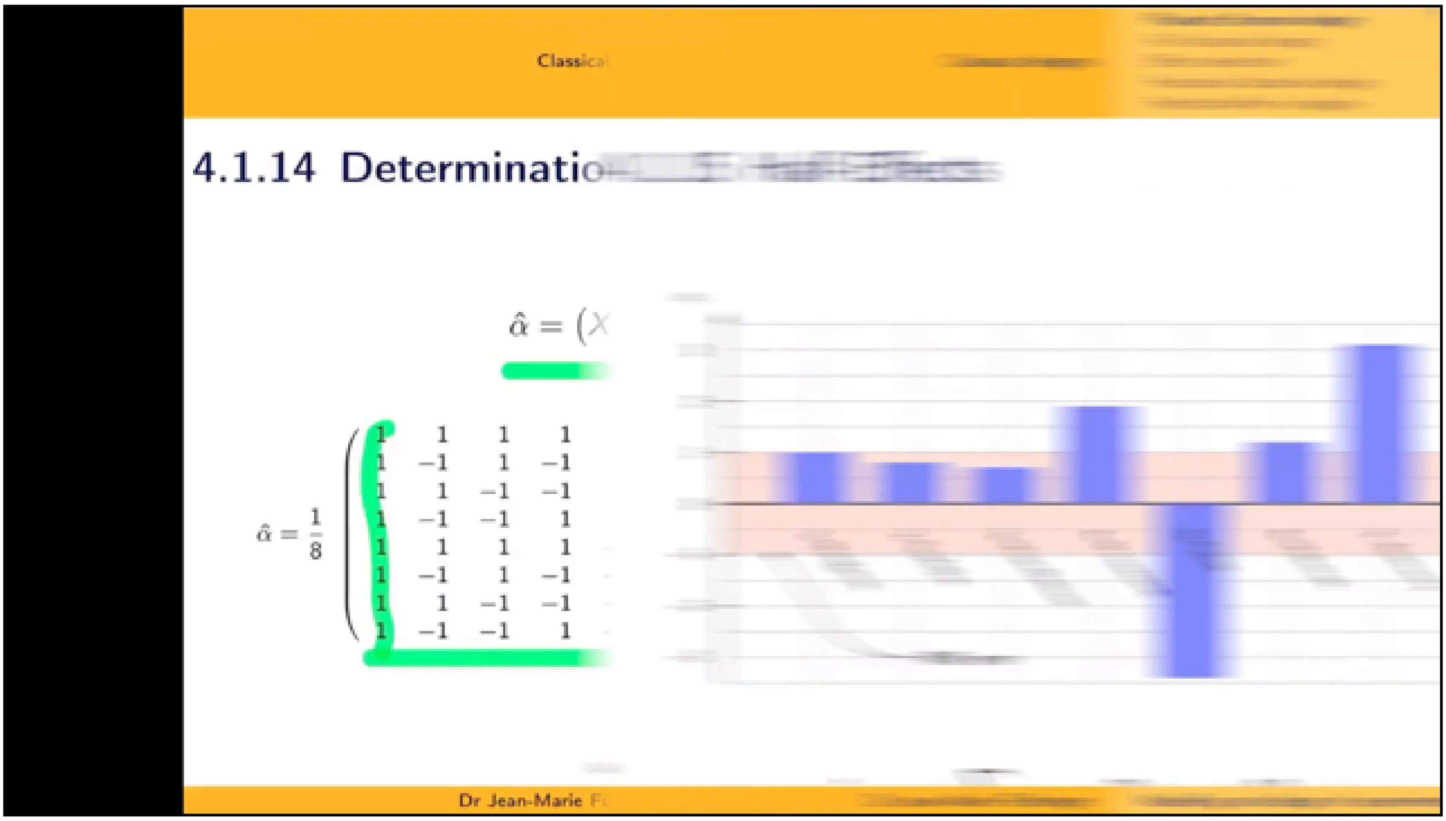
$$\hat{\alpha} = \frac{1}{8}\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{pmatrix}^T \begin{pmatrix} 35.7 \\ 32.8 \\ 24.2 \\ 36.5 \\ 30.1 \\ 26.0 \\ 35.5 \\ 22.1 \end{pmatrix} = \begin{pmatrix} 30.4 \\ 1.0 \\ 0.8 \\ 0.7 \\ 1.9 \\ -3.4 \\ 1.2 \\ 3.1 \end{pmatrix}$$

And after you get this time. Imagine that in minutes what you do in your different races. First experiment, 35.7 minutes, 32.8 minutes, etc., etc. You have your eight results. We treat the continuous factor as the same way as the discrete factor. And it's okay because we are looking for linear relation. If you make measurement at the center, you just discover it's linear or not. But you are not changing the linear relation. The best way of determining the linear relation is making measurement at the extreme.

notes

summary

34m 5s

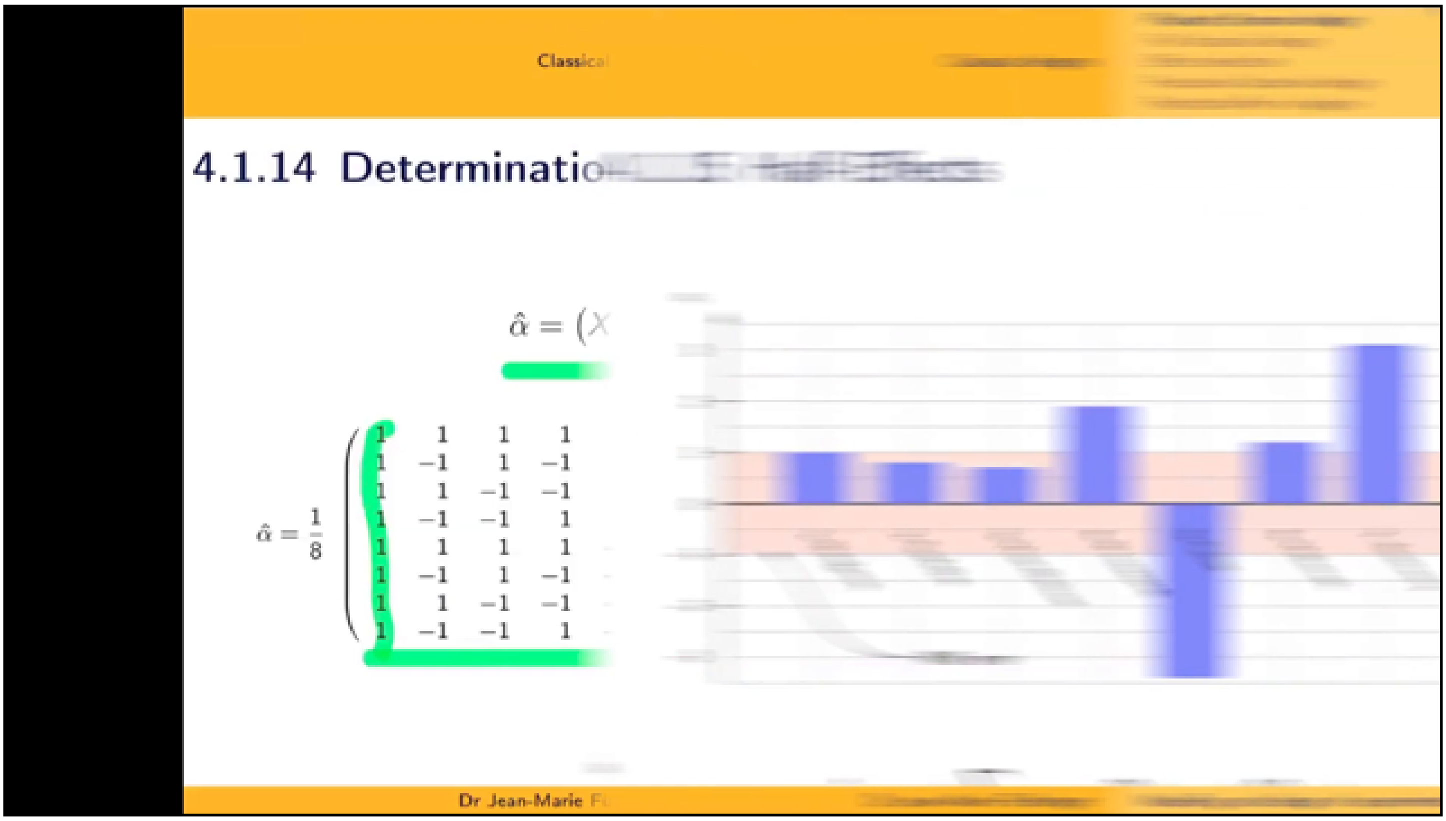


You remember your least square feet algorithm, X transpose X minus one. You remember that it was a simplified algorithm. So it's X transpose Y divided by N is sufficient. So you have here your matrix of the model. You can observe that I have added the columns of one. It's transpose multiplied by your vector of measurement. It's very easy to make the calculation. So now you see here what could be a first result. You see that you have the gear that seems to have a big effect. The factor five. You have the pressure of the tire, which is quite a significant effect. Forget the value of the alpha E unit. Just look at the big picture. And we have also the helmet that seems to have an effect. The diodes, the saddle, the handlebars, I'm not having. But we have one tricky situation. We have what I call the dummy factors. We have only six variables. Nevertheless, in my matrix of Adamar, I have seven columns that let me vary factors. I'm not talking about the first one with one, one, one. This is just for the constant, but I have seven columns of variation. I call that when I'm not using them, I call that dummy factors. So that means a factor that doesn't exist. The problem with this dummy factor is the most important. Why? What's happened? Something additional. Here I have draw in pink the confidence interval. When you are using a balanced design, the confidence interval of all the coefficients are the same. When you have normalized, balanced and normalized design. So it's easy to see what is significant and what is not significant. And I have used, in this case, the root mean square error. I made my fit and I get a root mean square error. And this helped me to discover what could be the confidence interval. OK, so

notes

summary

34m 54s



now explain me why my dummy factors is so important and what can I do for that? What's happened? What is the statistical cause? Because it's a statistical cause because the dummy factor doesn't exist.

notes

summary