Course material

Course:

ENG606 / PHYS 442

Video:

DOE_lesson9_part1_FractionalFactorialDesign

Concepts (extracted from automatically generated subtitles):

Fractional factorial design. Last column. First possibility. Full factorial design. Main effects. Typical design of experiment course. Numbers of factors. First column. Concept of the contrast. Sort of a game. Minimum cases. Lot of time. Design of experiment. First factors. Canonical form of the generator.

to video sequence search
(within ENG606 / PHYS 442.)

to video

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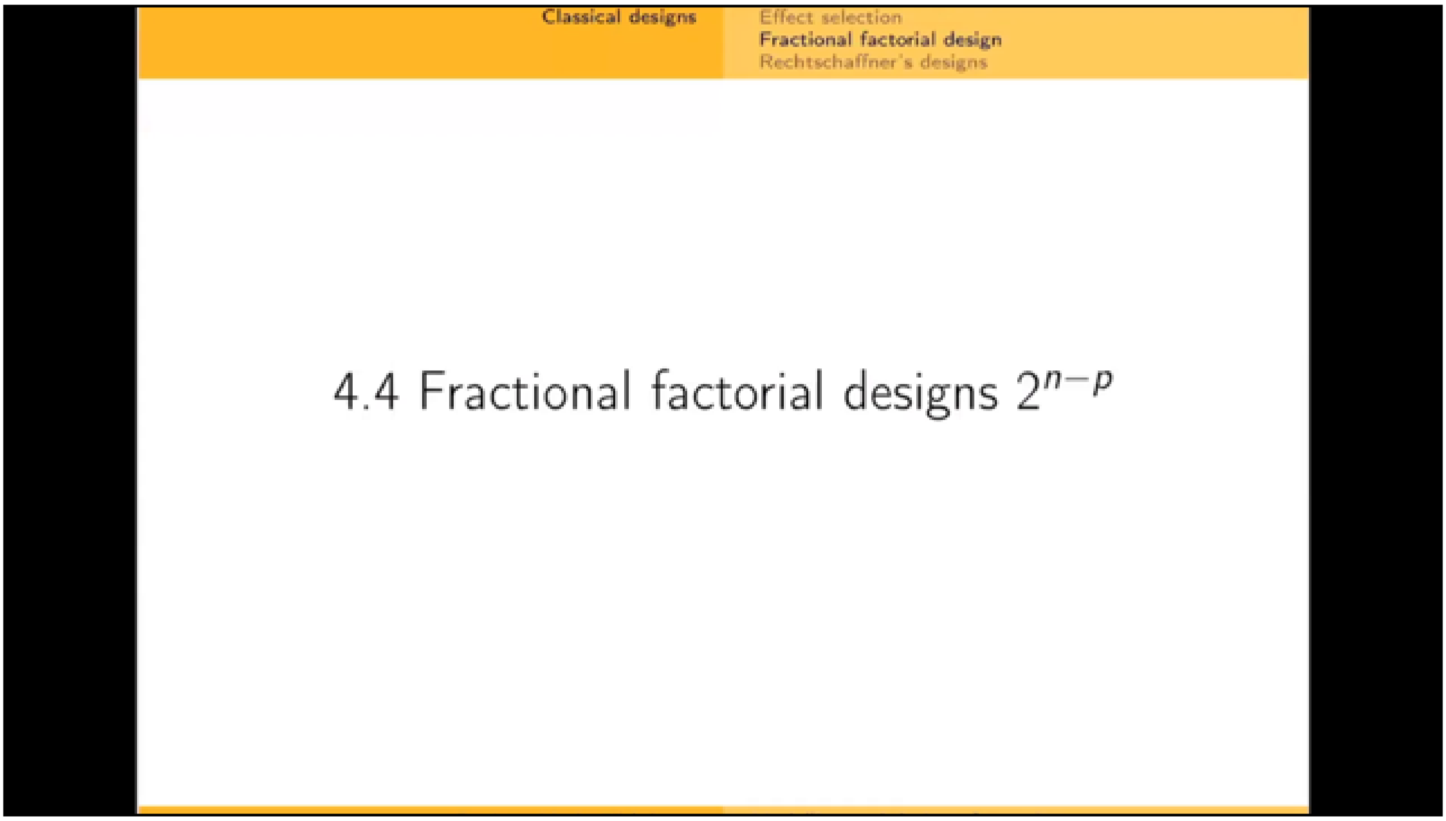
notes

summary

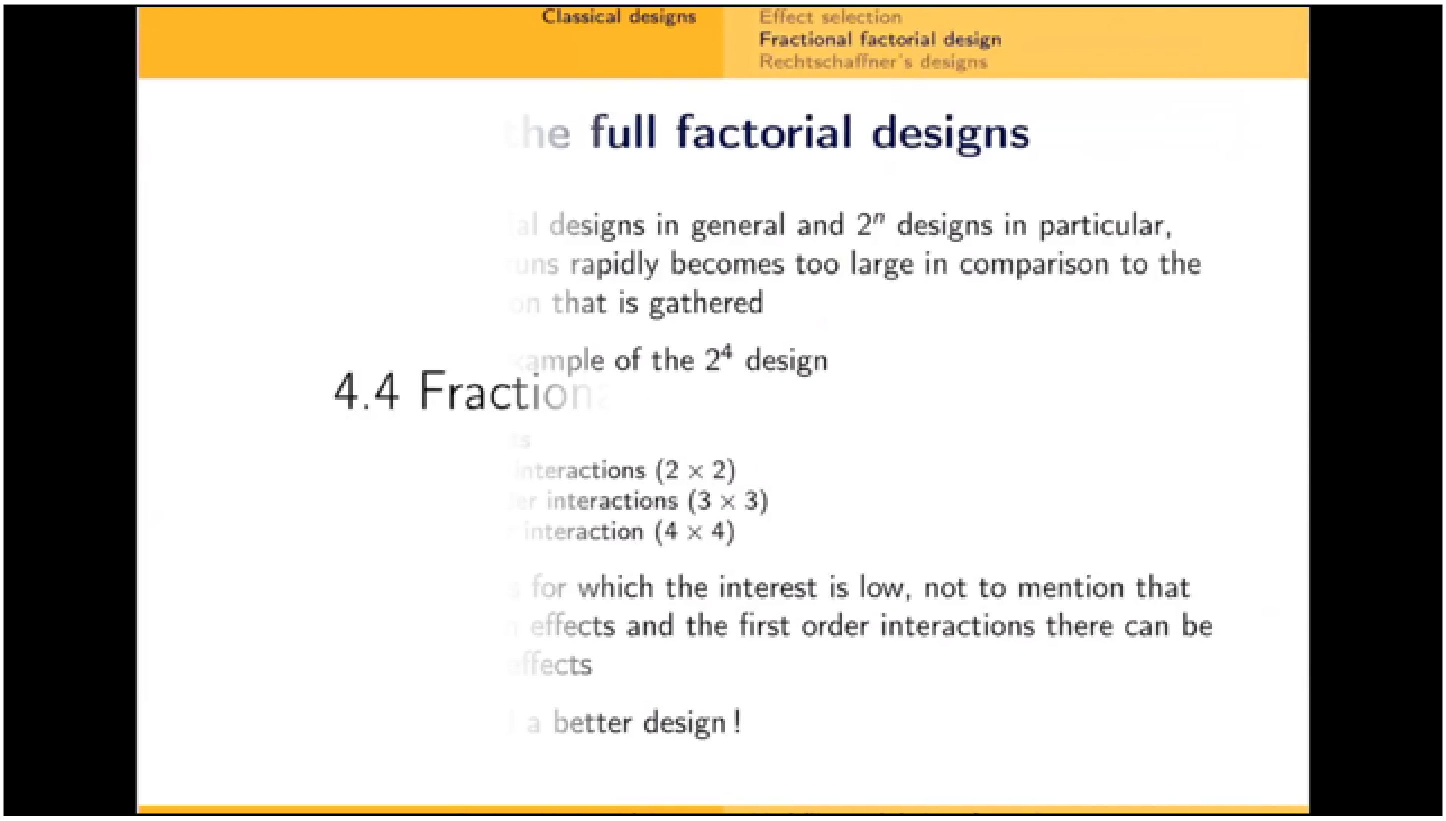


These subtitles have been generated automatically Good morning to everyone.

notes

summary

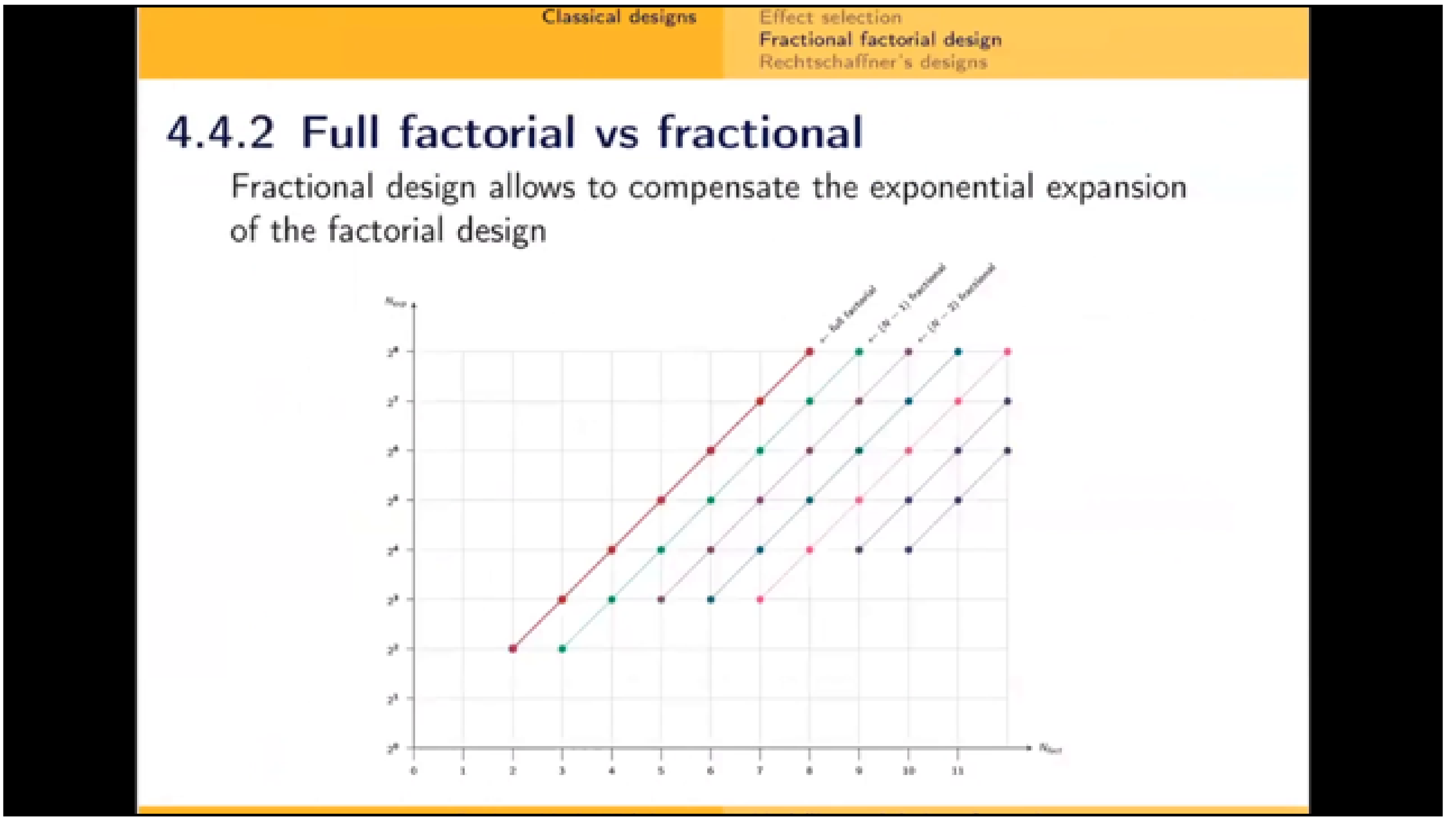
0m 1s



So today we will look at the fractional factorial design.

notes

summary

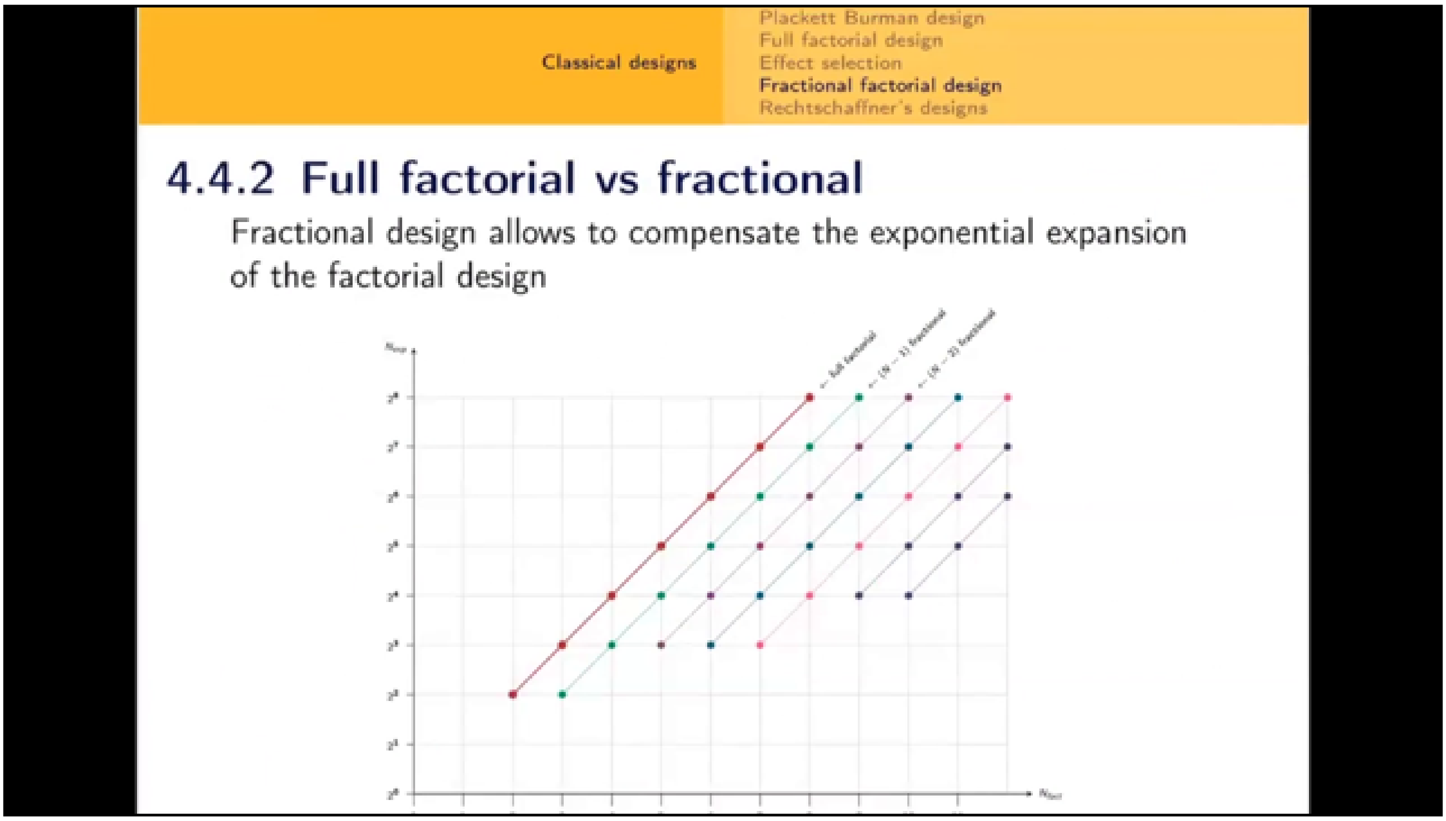


So this is a very, very typical design of experiment course.

notes

summary

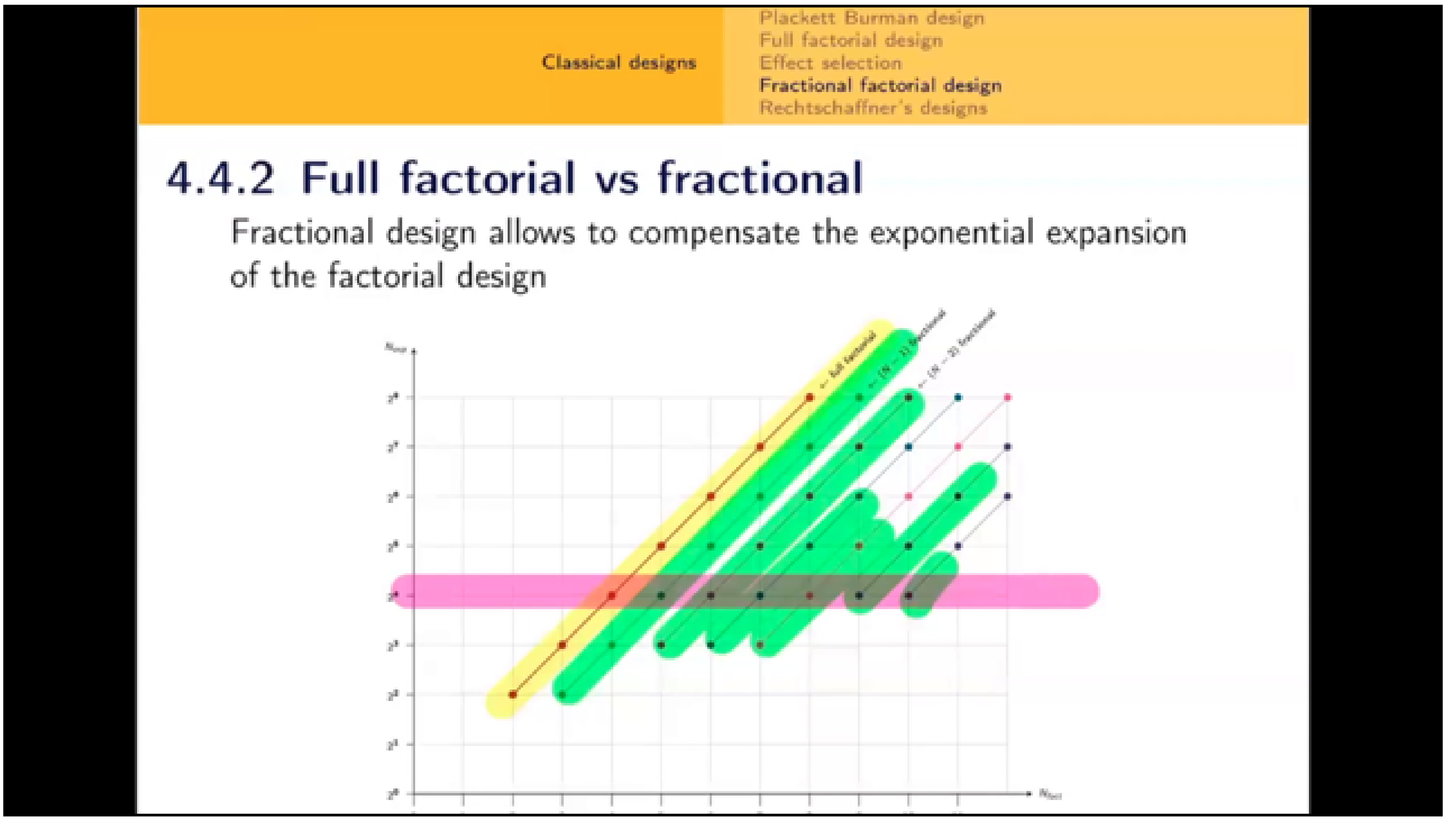
0m 5s



It's a course, really what a lot of people live in mind when they think to design of experiment. We have seen the Adamar design, the Placket Blument design, very useful for estimating main effects. And we also have seen last week the situation with full factorial design, quite an expensive design in which you can estimate all the instructions, all the main effects, the constant the main effects. But we are still with this approach of the Taylor polynom, this Taylor development that let us estimate a complicated relation between factor but by a polynomial. And so we have seen the two extremes. Placket Blument are very efficient, but only for main effects, but sometimes we need more things that just main effects. The full factorial design are efficient, but quite expensive, giving the possibility to estimate all the factors. So fractional factorial design will offer a possibility to do things in between. They are called 2 power n minus p. So 2, because we are still working with two level factorial design, it exists also fractional design or 3 power something, but we don't see them in the course. In the book of Montgomery, you can find some example, but we are in this course only working with two level designs. And so n is the numbers of factors and p is the reduction that we are using. p could equal to n, tanger could be 1, 2, 3, depending if we are divided our full factorial design by 2, by 4, by 8.

notes

summary

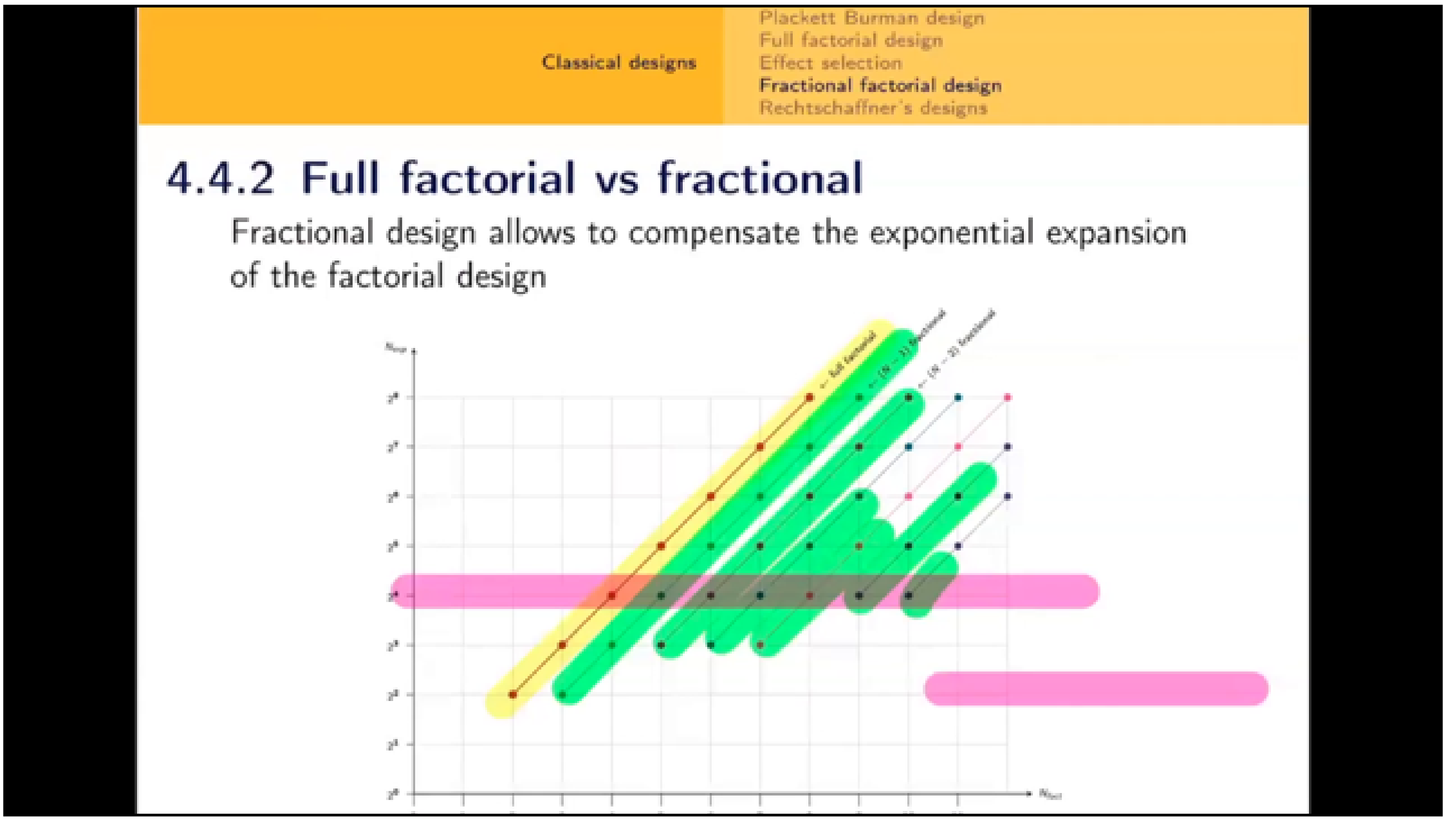


Okay, so with a factorial design in general and especially with 2 power n, the number

notes

summary

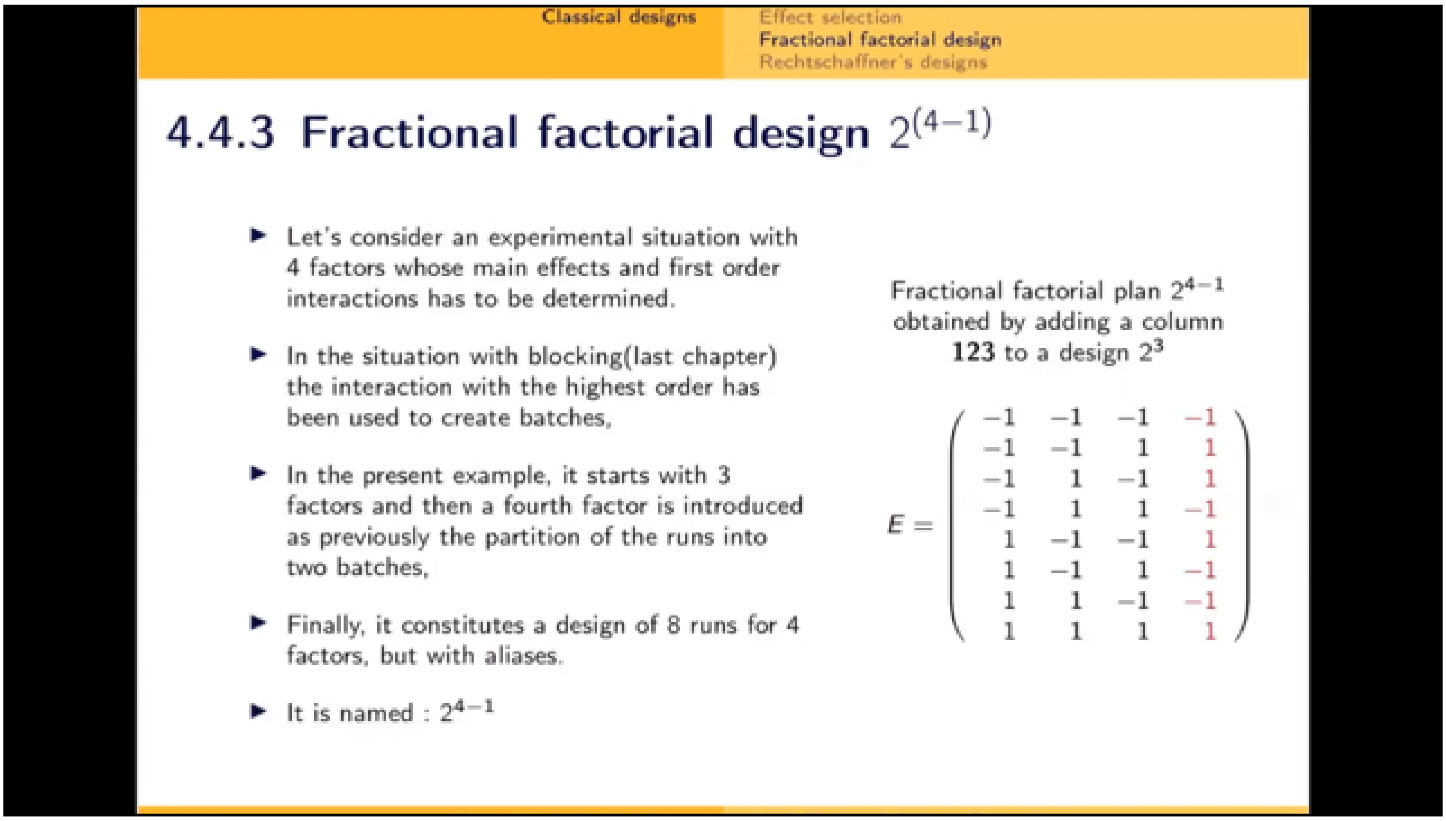
2m 22s



of run rapidly increased. It was an exponential increase. So this is not very appreciated by your boss because it's represented a lot of time for making experiment, a lot of money. Look at a small example with a 2 power 4 design. So it's 16 runs. So we can evaluate one constant, four main effects, six 2 by 2 interactions, four 3 by 3 interactions and one third order interactions, four by four. But what happens that usually we don't consider the 3 by 3 and 4 by 4, that means that we are interested mainly by 11 coefficients. So we made four experimentants more than the minimum. Okay, in some situation it could be okay because you say at least I will have good, I have sufficient degrees of freedom. So it's let me diminishing the confidence interval. Nevertheless, sometimes you are interested to rapidly get what you want without too much degrees of freedom. This example is just with four factors. It became more dramatic when you have more factor than the numbers of runs that you are doing in comparison with what interests you. The difference will be bigger. So let's find a better design.

notes

summary



Okay, so look at that.

notes

summary

4m 6s

4.4.3 Fractional factorial design $2^{(4-1)}$

- Let's consider an experimental situation with 4 factors whose main effects and first order interactions has to be determined.
- In the situation with blocking(last chapter) the interaction with the highest order has been used to create batches,
- In the present example, it starts with 3 factors and then a fourth factor is introduced as previously the partition of the runs into two batches,
- Finally, it constitutes a design of 8 runs for 4 factors, but with aliases.
- It is named : 2^{4-1}

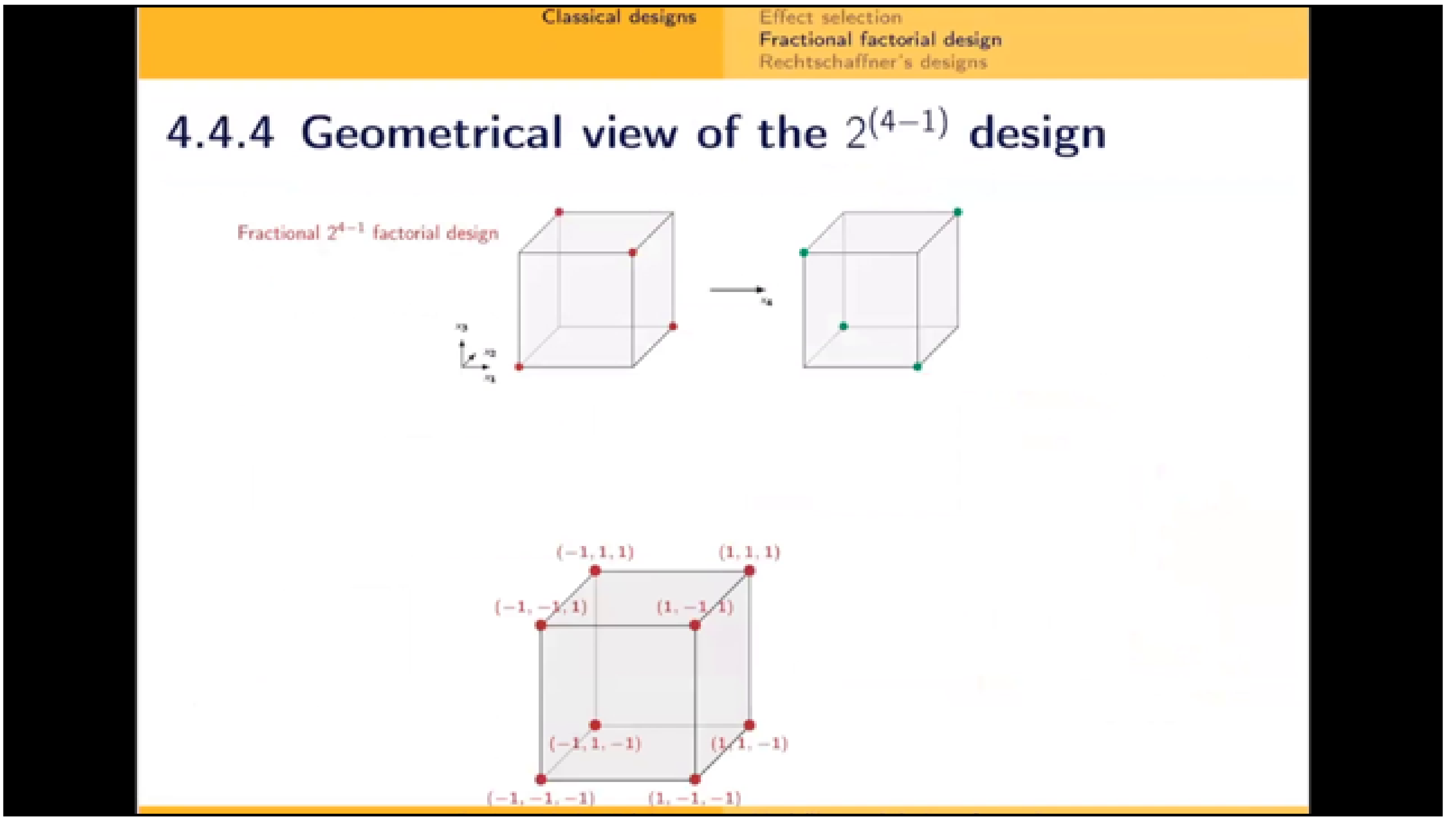
Fractional factorial plan 2^{4-1} obtained by adding a column **123** to a design 2^3

$$E = \begin{pmatrix} -1 & -1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

You can see in this graph the different design we can have.

notes

summary

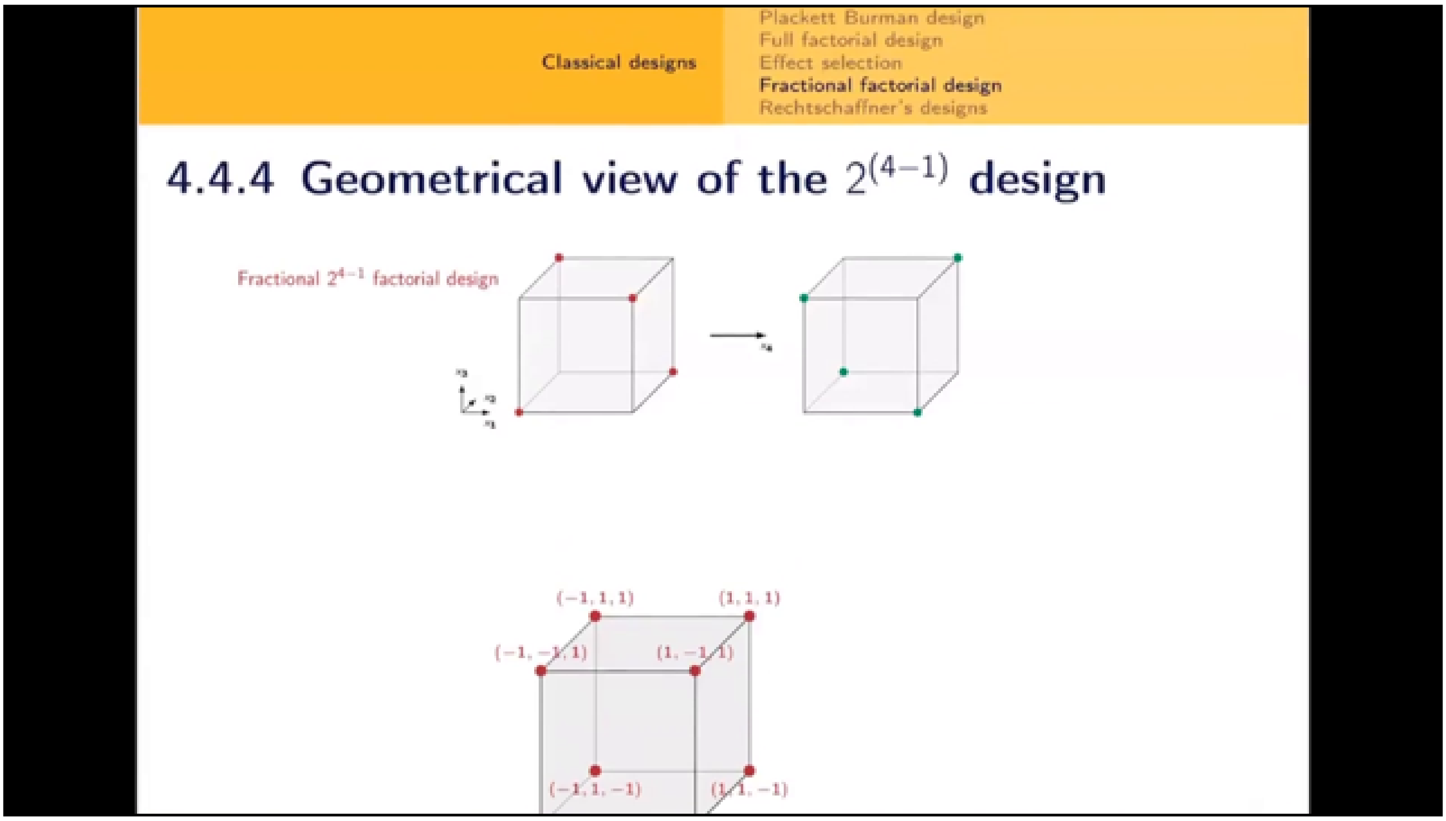


So in this line you see it's semi-log graphics or it's a linear evolution, represent in fact

notes

summary

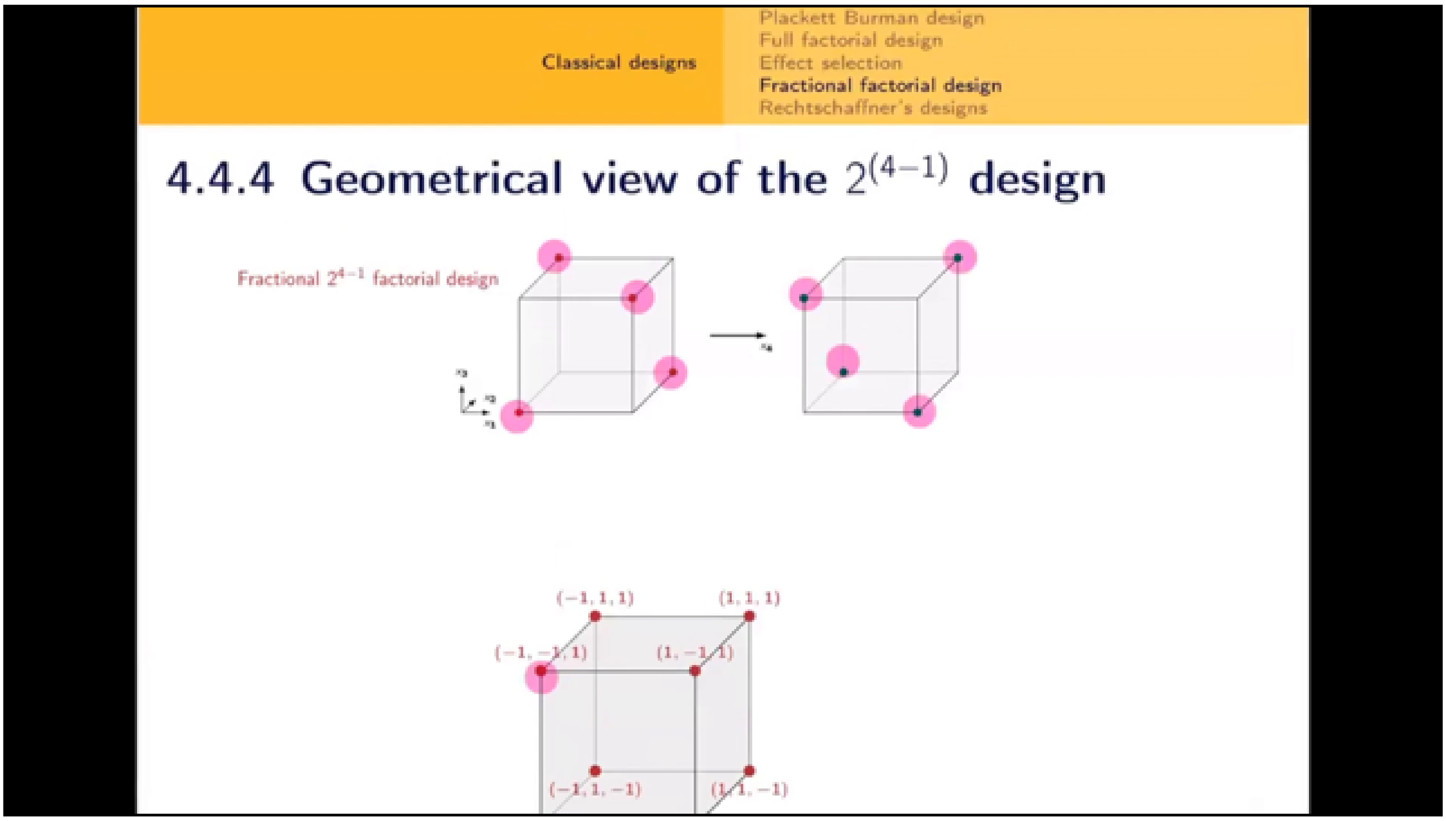
4m 15s



an exponential evolution. So in red what I have put now is a yellow stabilobus. It's the full factorial design. So by dividing the number of runs, I can divide it by 2. This is the first possibility by 4 by 8 by 16 by 32 and by 64. So you see the power with the number of experiments. If you are interested of having, I don't know, you have the money for only performing all the time, only 16 experiments, you can still evaluate more factors.

notes

summary

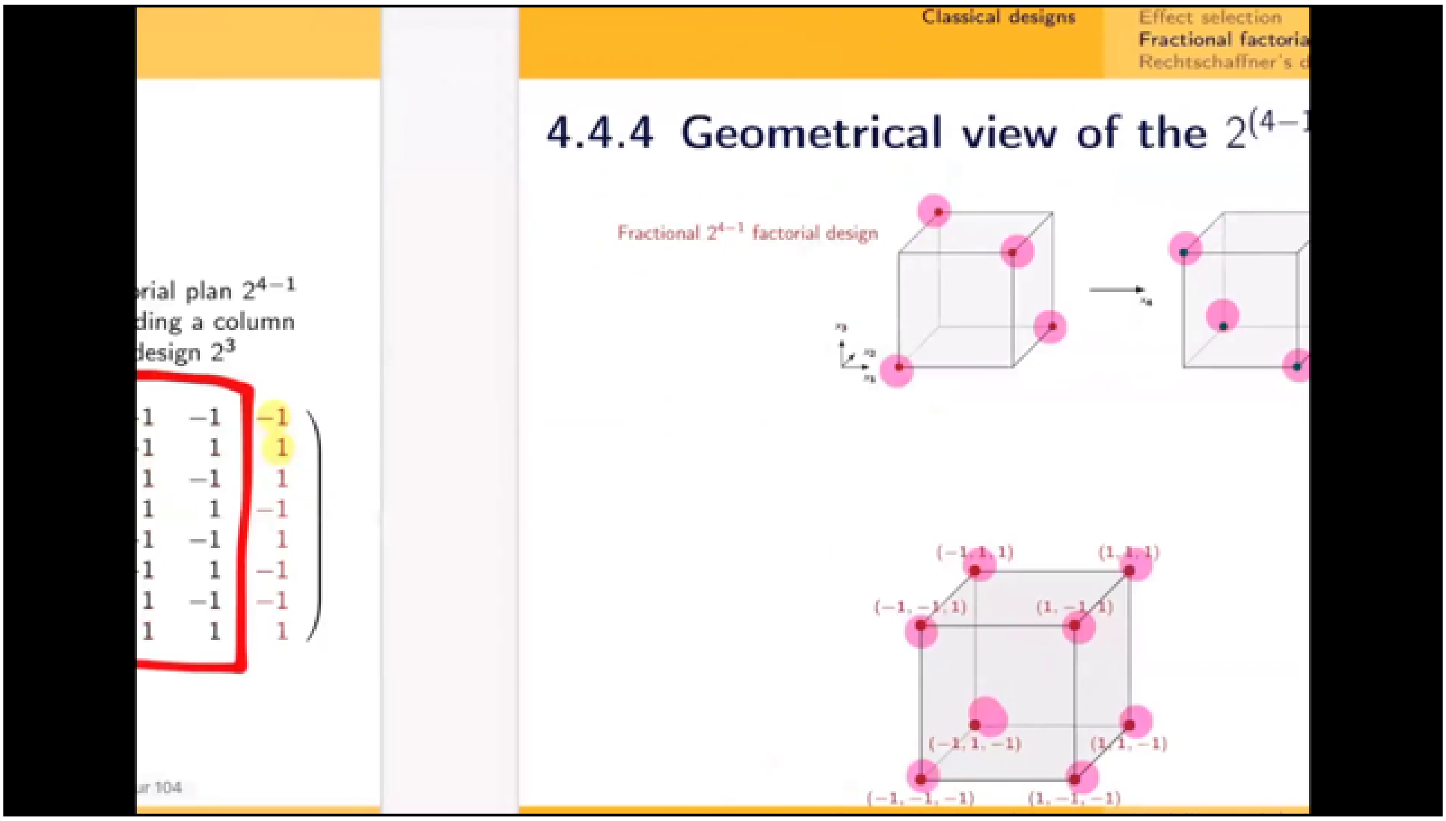


So it's why it offers you the fractional factorial design.

notes

summary

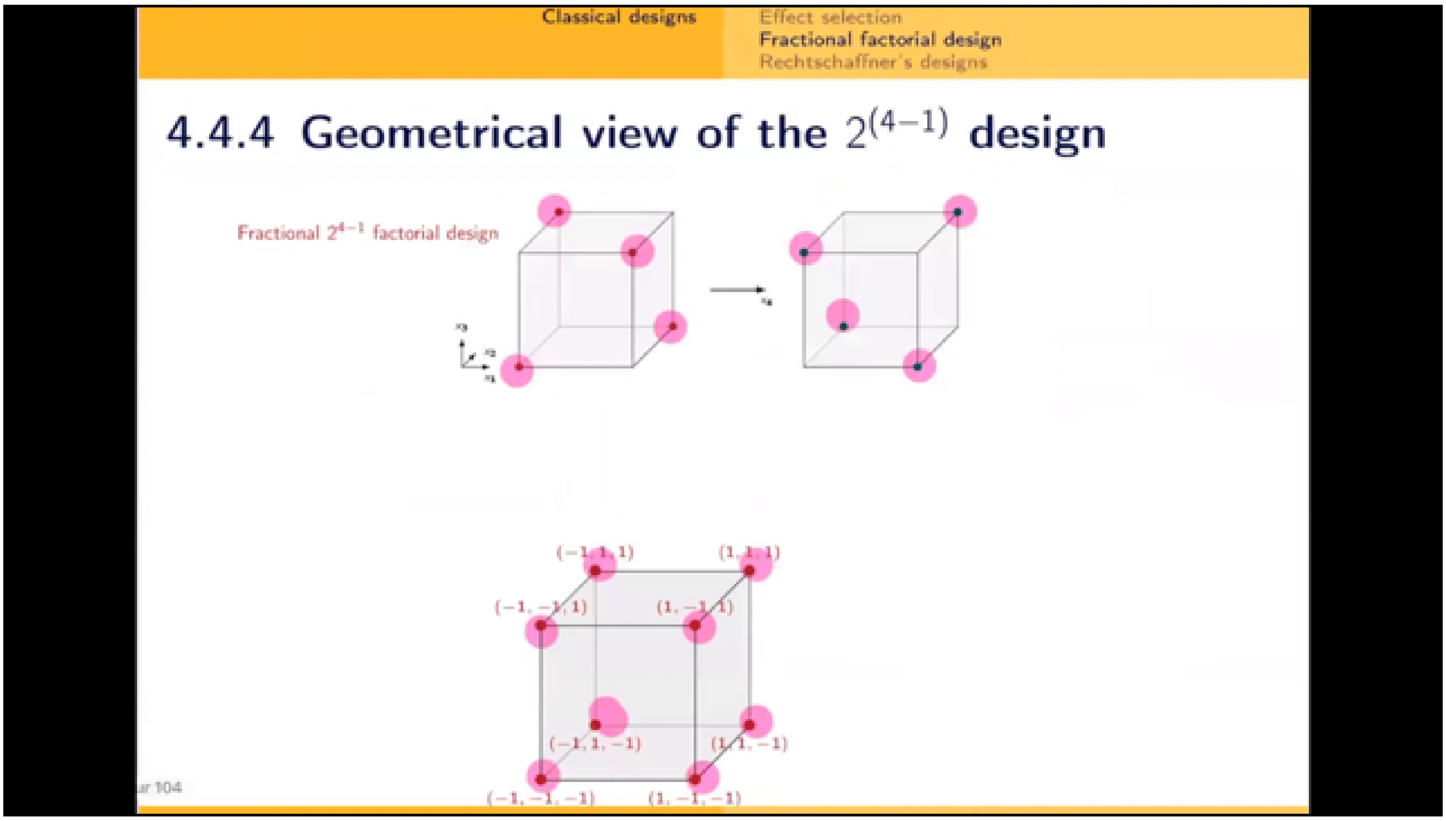
5m 13s



You are not obliged to follow this exponential expansion of the numbers of runs. You can make less experiment, let's imagine 16 and with these 16 runs you can evaluate more factors. But you have to understand, I already mentioned it that in mathematics there are no free meals. That means that there are costs. You will lose something. So it's a trade-off. So it's this trade-off that is the objective of this course today is to explain you well how to perform this trade-off, understand what you get and what you are losing. So what we will get are the main effect and the interactions 2 by 2. And what we are losing is the fact that the interactions 3 by 3, 4 by 4, 5 by 5 will be aliased with the one that interests us. So we have to manage this aliasing and be able, when necessary, to de-alize making a few more experiments. Usually it's a double of experiment or something like that. So when you are dividing by 2, doubling is making 2 power n experiment. But when you are dividing by 16, the double will be at the place of having a 16th. You make an ace, it's still interesting. Like that, step by step, we will be able to choose the next step and just de-alizing what interests us depending on what we have obtained as information and also what we knew about the possible interactions that exist. Why we do not have the minimum cases? Because these matrices doesn't exist. Because it makes a too important mismatch between your factor and you see nothing.

notes

summary

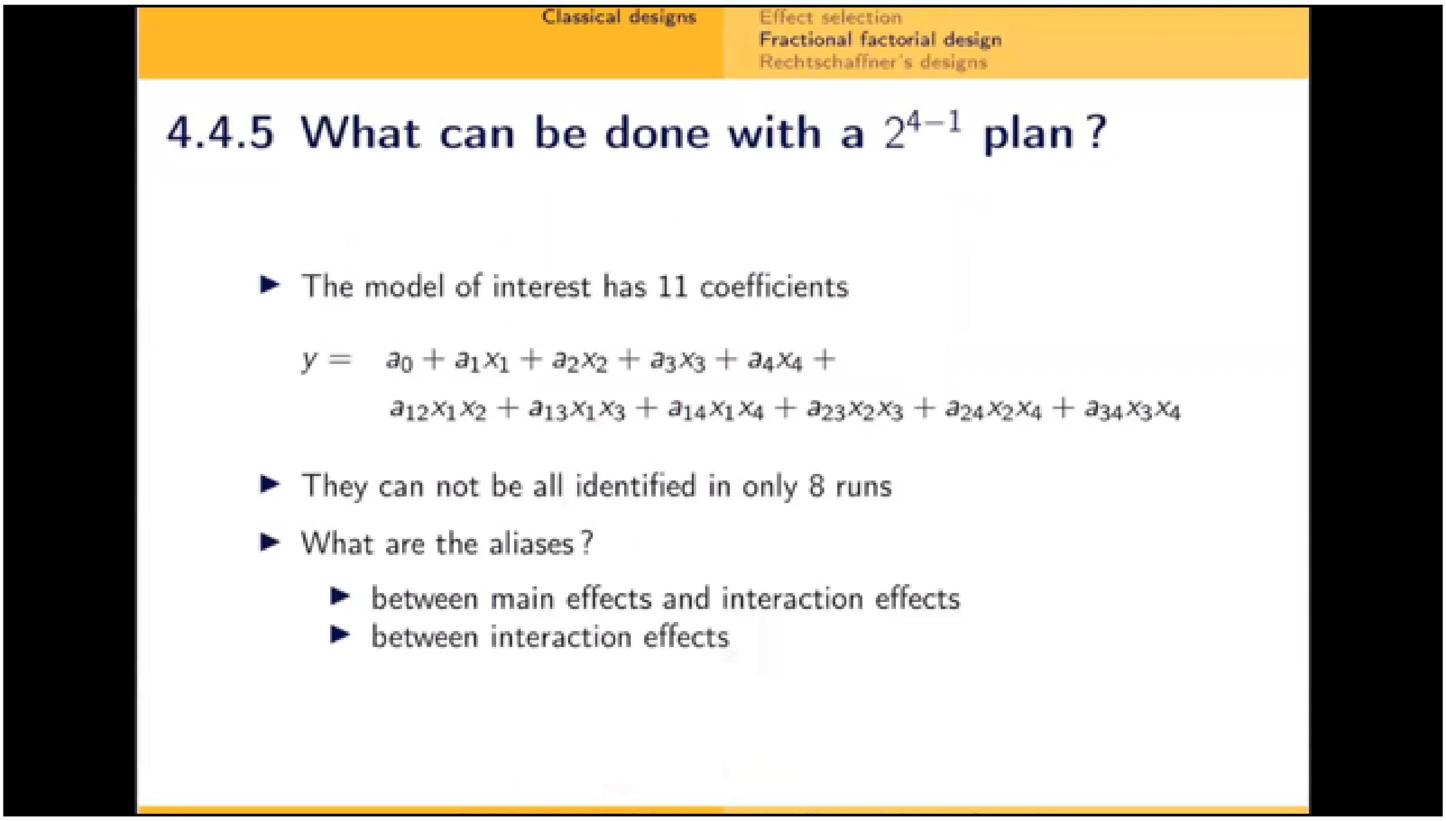


Things will become clear step by step.

notes

summary

7m 9s



So don't panic. Even a few things could appear quite complicated at the start. When everything is in place, you understand better.

notes

summary

4.4.5 What can be done with a 2^{4-1} plan ?

- The model of interest has 11 coefficients

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{14}x_1x_4 + a_{23}x_2x_3 + a_{24}x_2x_4 + a_{34}x_3x_4$$

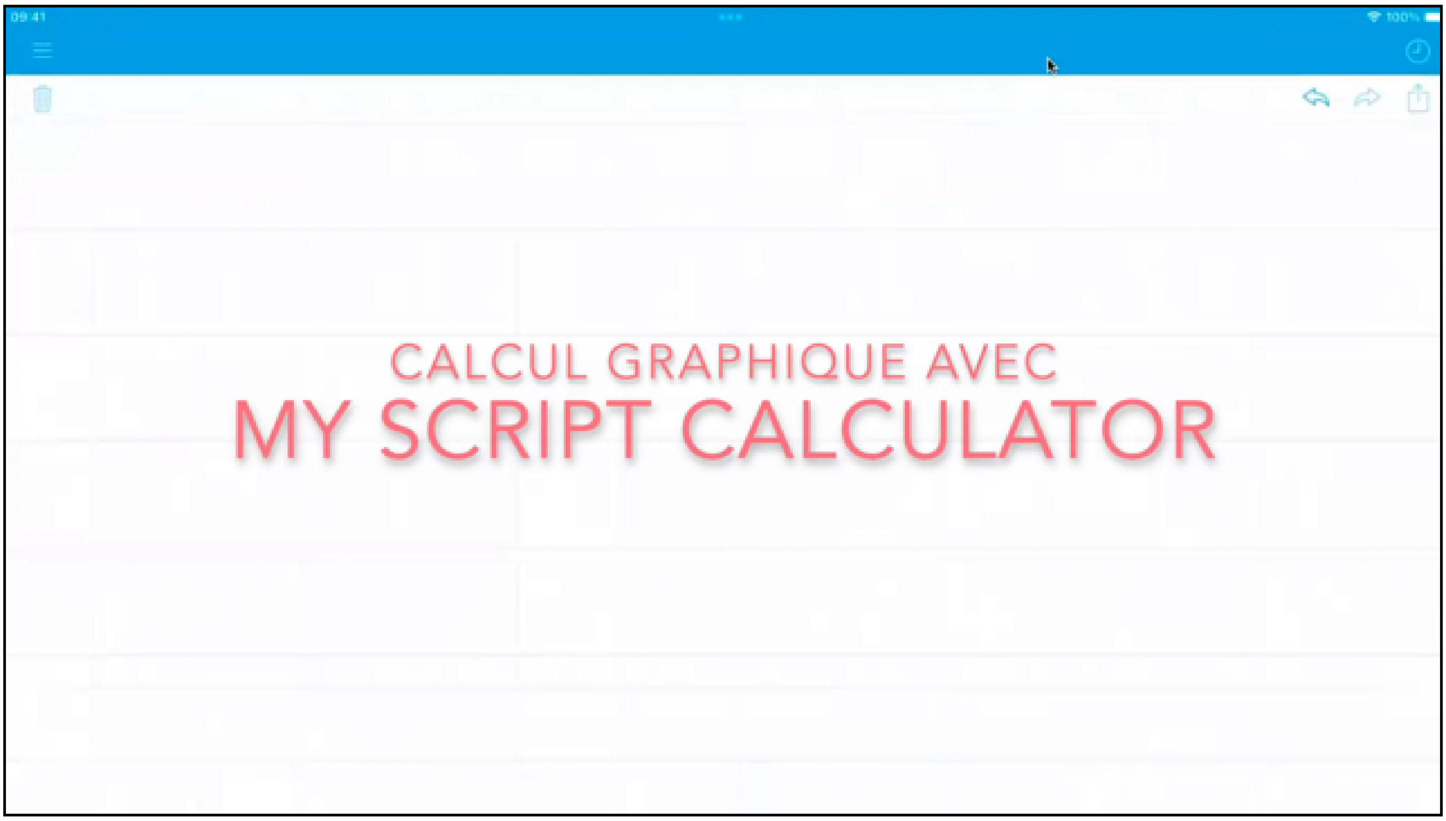
- They can not be all identified in only 8 runs
- What are the aliases ?
  - between main effects and interaction effects
  - between interaction effects

So let's first discover one example.

notes

summary

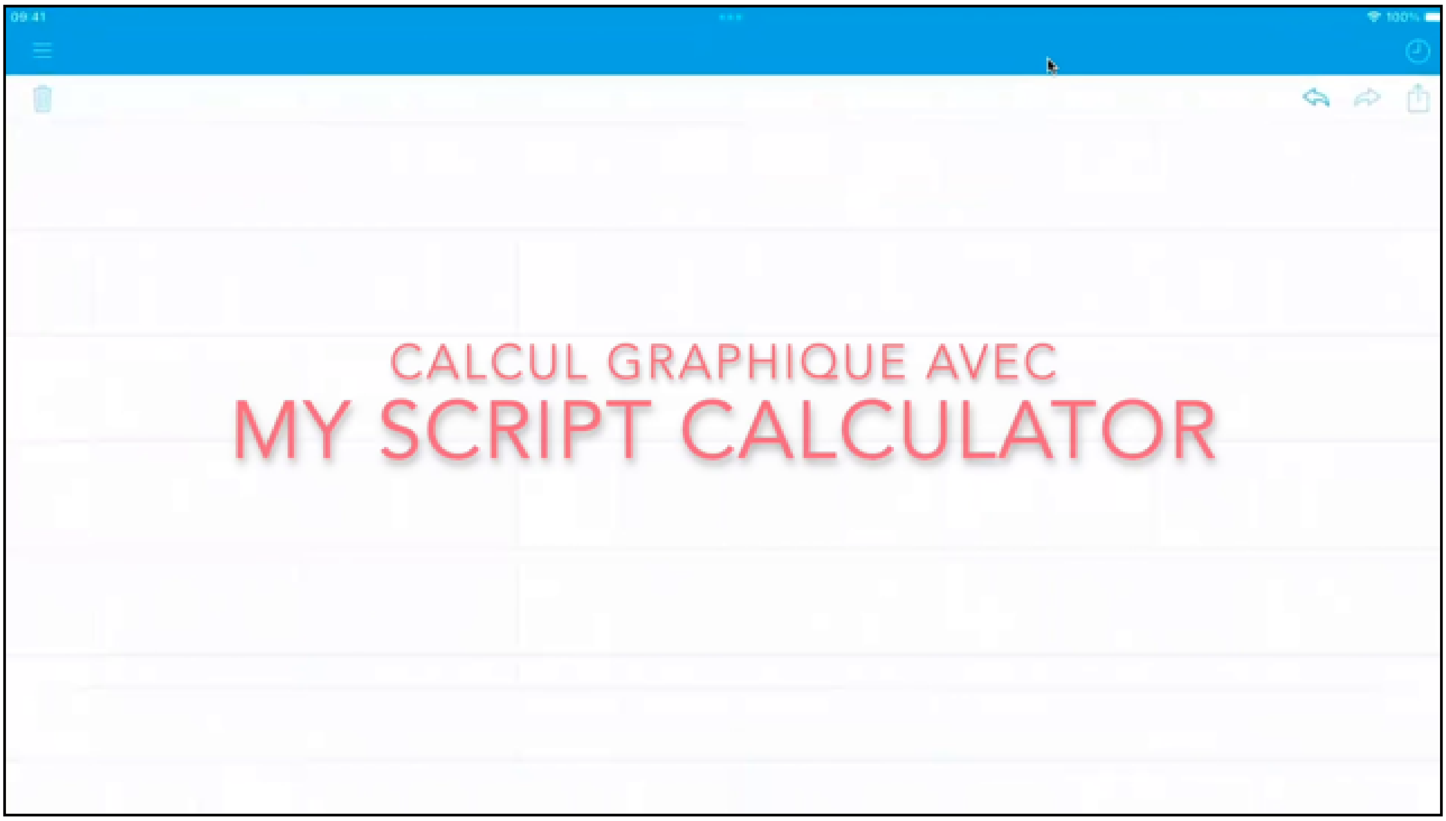
7m 23s



A design I will call 2 power 4 minus 1. So this is also the nomenclatura, the name of the design. So that means that I'm playing with two levels for each factor. I have four factors and I'm dividing the 2 power 4 by 2. I'm diminishing the numbers of experiments dividing by 2. That means I have one generator. I need a relation for telling me how I go from 16 experiments to 8 experiments. 2 power 4 is 16 and 2 power 3 is 8. So if you remember, we have done something like that last week with the blocking. I was just explaining how you can separate your experiments in two groups because you have two batches to constitute. You have two laboratories, two things that doesn't interest you in fact, but you are obliged to separate in two. But now it's the same situation except that we are interested by the effect of batch. So if you remember last week, we have taken a full factorial matrix. We can consider a full factorial matrix and we had considered the batch was the 3 by 3 interactions. In fact, we do the same. So we will introduce a force factor. But how we will vary it? Will we vary this force factor according to the interactions 3 by 3 of the three first factors? And this last column, which is in red in my slide, you can check is the product of the three first column. Minus one, minus one, minus one makes minus one. Minus one, minus one, plus one make one, etc. So it's the way I have introduced an additional variable. I'm just making eight experiments. What usually is necessary to determine all the main effect and interaction effect of the three factors, but I would like to analyze four factors. So I'm adding in my matrix of essay, one additional column that

notes

summary



come from the matrix

notes

summary

4.4.5 What can be done with a 2^{4-1} plan ?

- The model of interest has 11 coefficients

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{14}x_1x_4 + a_{23}x_2x_3 + a_{24}x_2x_4 + a_{34}x_3x_4$$

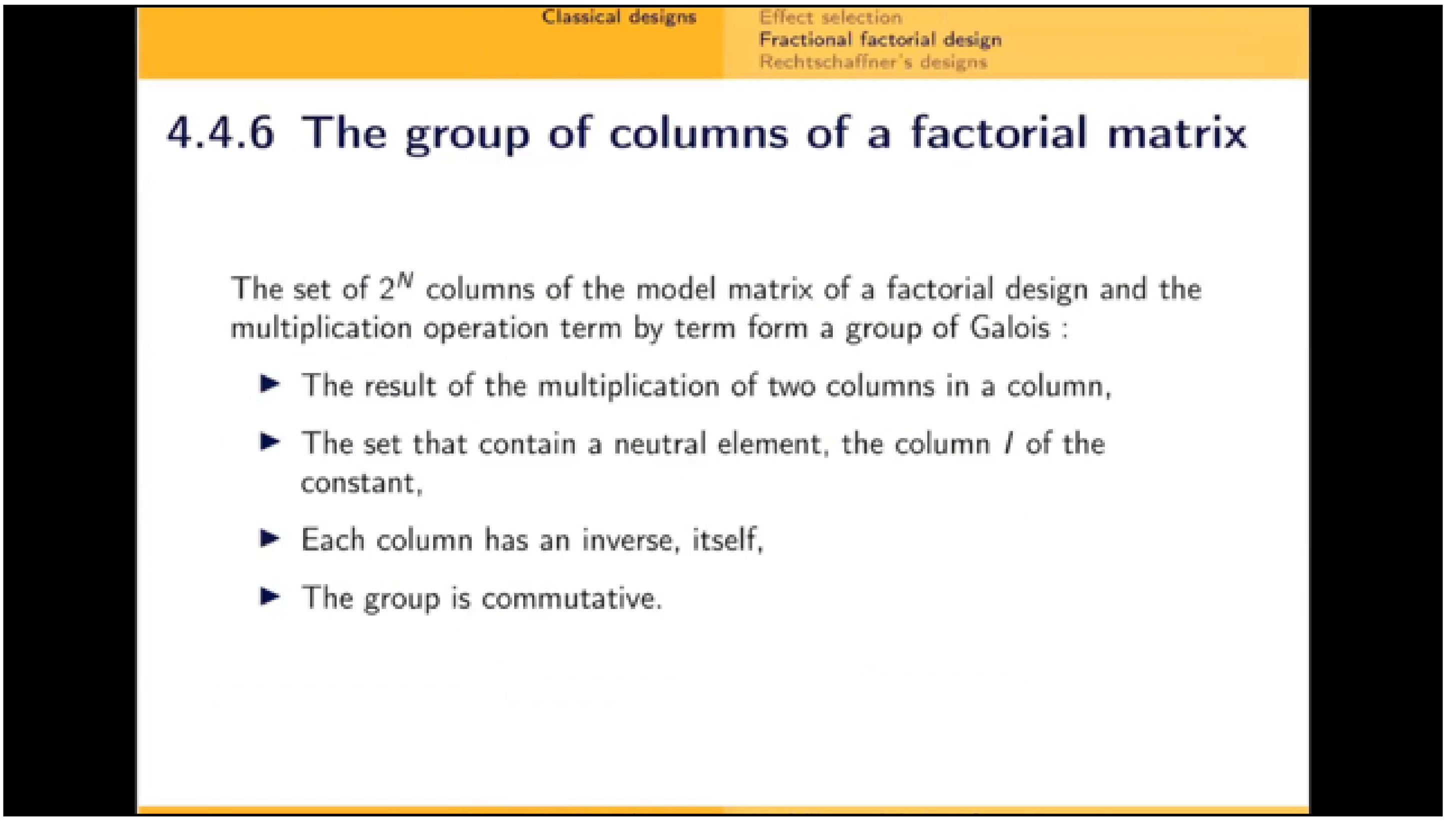
- They can not be all identified in only 8 runs
- What are the aliases ?
  - between main effects and interaction effects
  - between interaction effects

of the model of the three factor model.

notes

summary

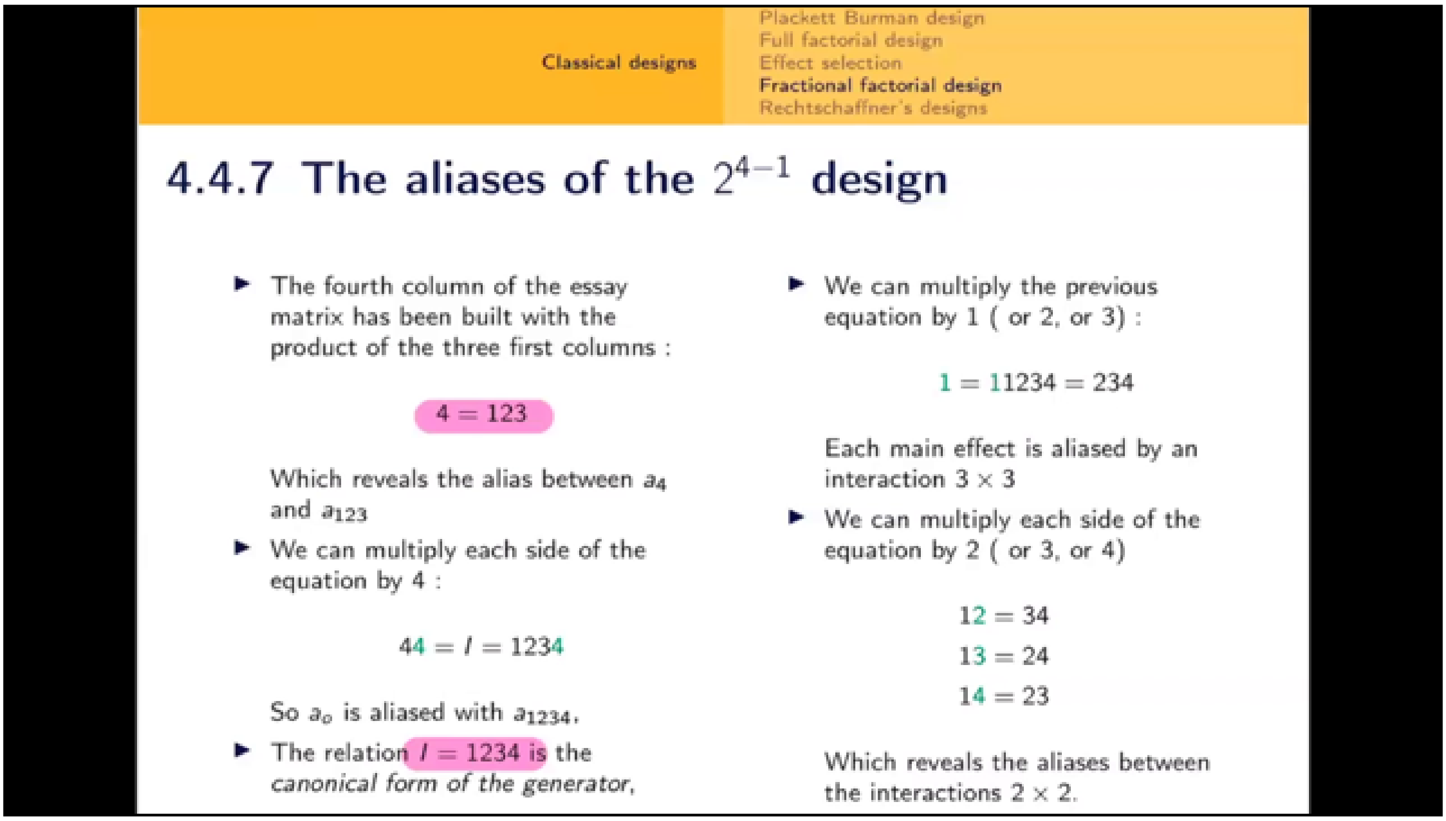
10m 25s



And I say this is my fourth factor is the way I will vary my fourth factor.

notes

summary

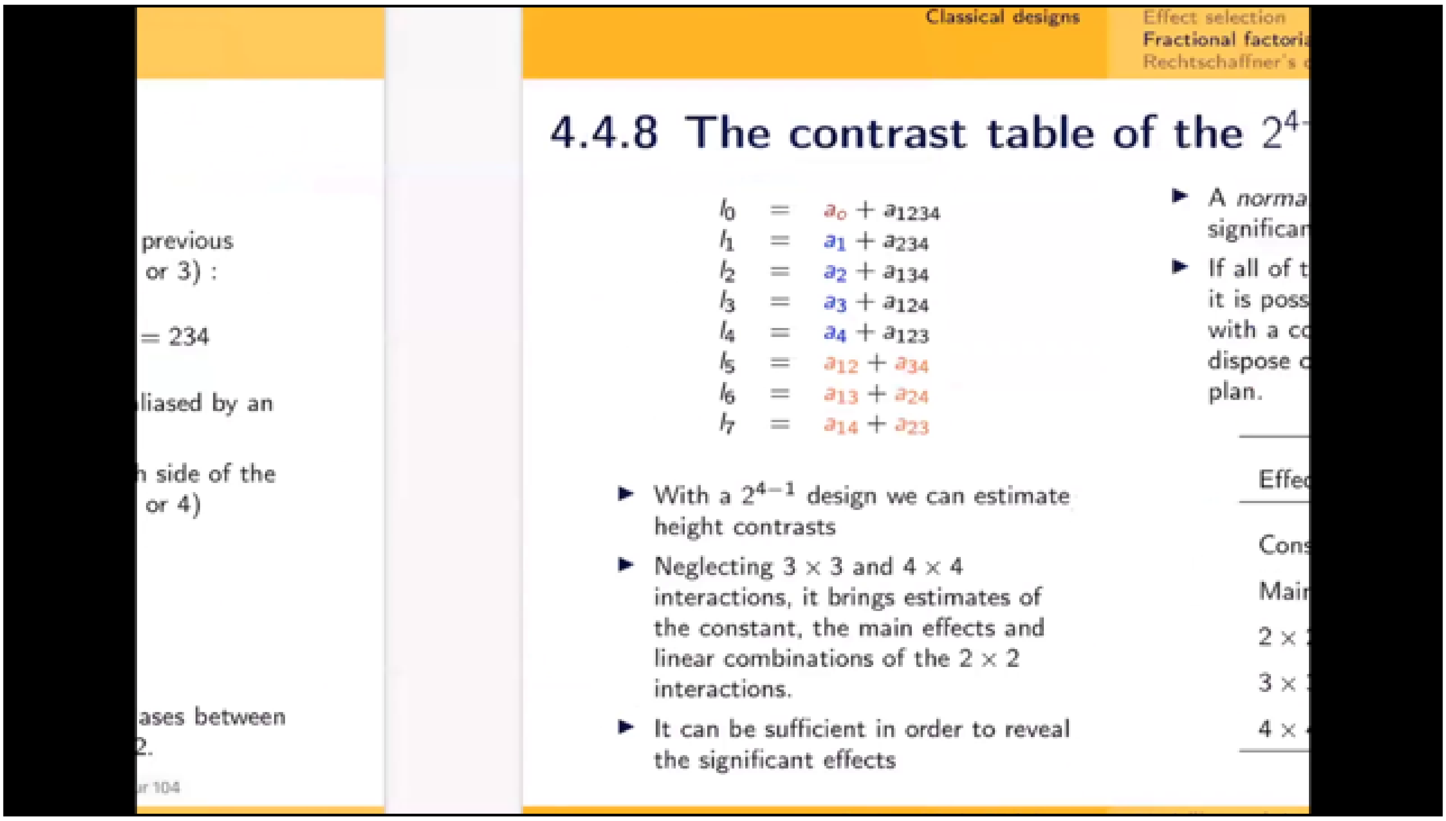


So geometrically, you have observed that I appreciate all the time, make some geometric

notes

summary

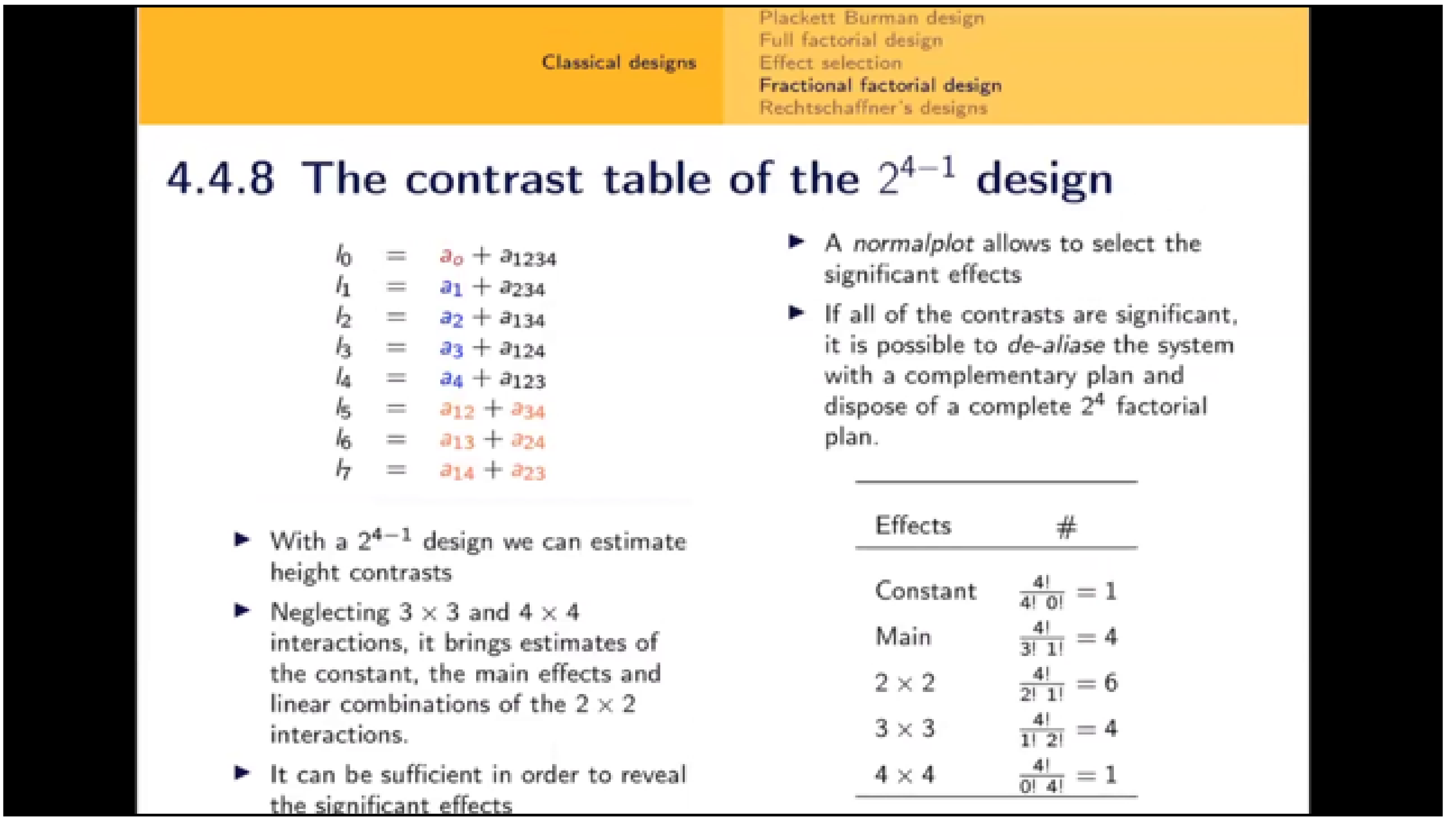
10m 34s



reference, because for my point of view is what exists. And it's easy to understand what we are doing. And algebra sometimes we could invent you seeing that doesn't exist. So in geometry we can. So if you see, we have four dimension. So in four dimension, it's kind of represent my four dimension if I have two level by dimension in two cubes. So the first cube represents the three direction and going from one cube to the other represents a force direction.

notes

summary



Effects	#
Constant	$\frac{4!}{4!\ 0!} = 1$
Main	$\frac{4!}{3!\ 1!} = 4$
2×2	$\frac{4!}{2!\ 1!} = 6$
3×3	$\frac{4!}{1!\ 2!} = 4$
4×4	$\frac{4!}{0!\ 4!} = 1$

So the design I present you in the previous week is represented here by the red dots.

notes

summary

11m 26s

4.4.9 Analyse 5 factors with 8 experiments

- A fifth column of the essay matrix can be introduced in the model matrix, for example :

$$5 = 12$$

- As previously, we can take the canonical form of the generator :

$$I = 1234 = 125$$

- This new generator creates a list of additional aliases :

$$1 = 25$$
$$2 = 15$$

- But also with the higher levels of interaction, for example the 3 × 3 interactions

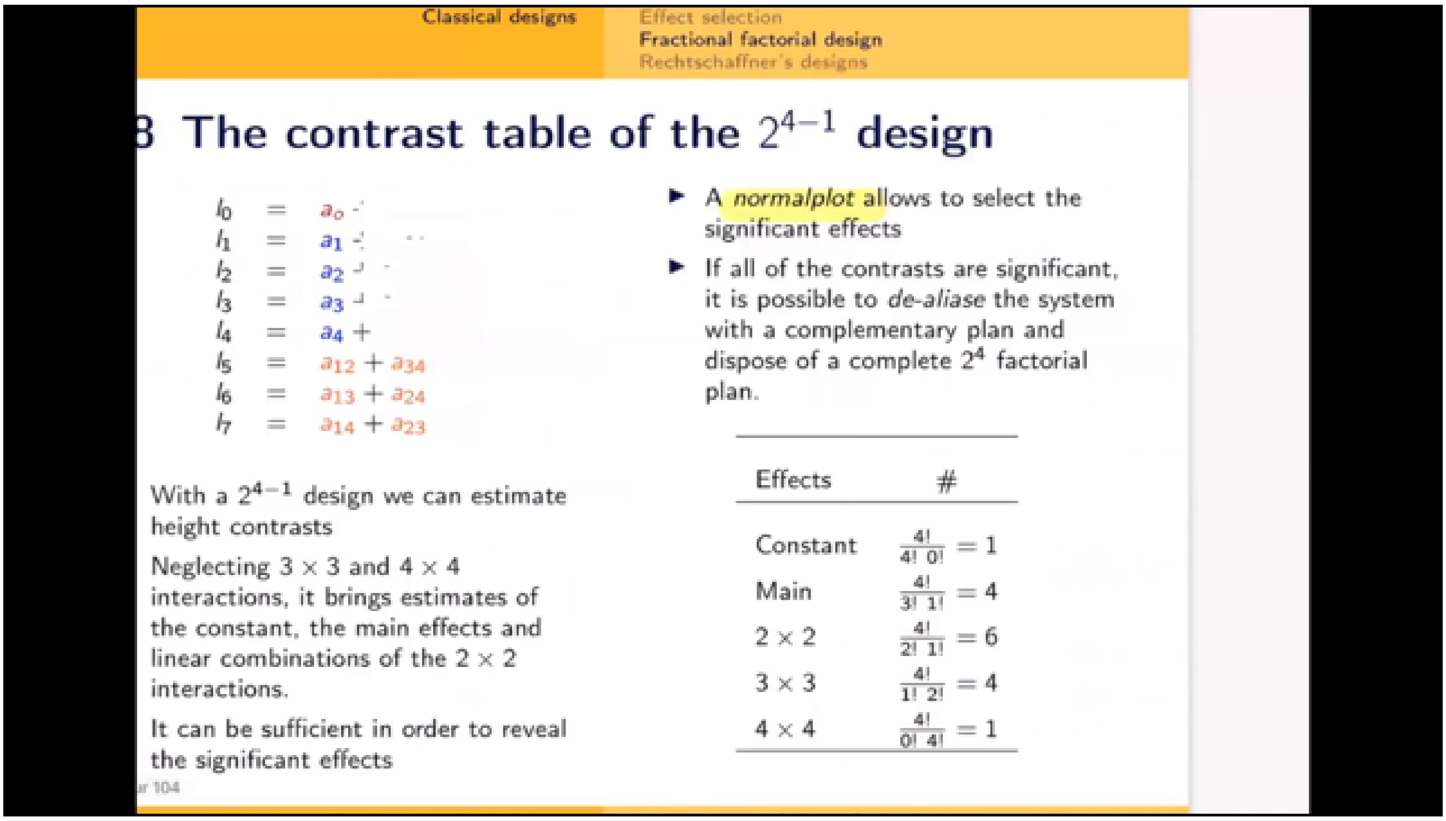
$$135 = 23$$
$$145 = 24$$
$$235 = 13$$
$$245 = 14$$
$$345 = 1234$$

ır 104

And now if you.

notes

summary

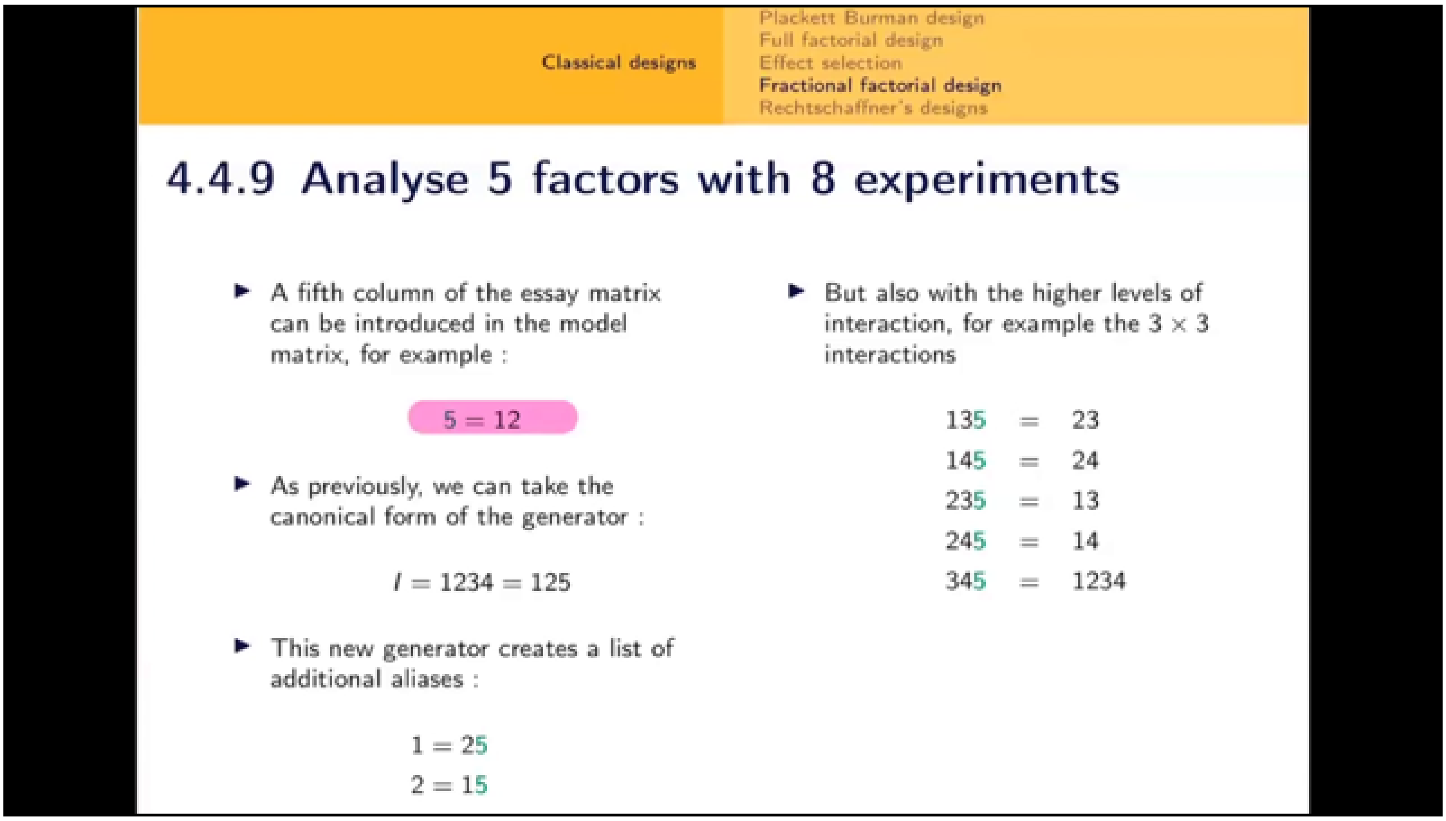


Scratch the force variable, you see that it's constitute a full factorial design.

notes

summary

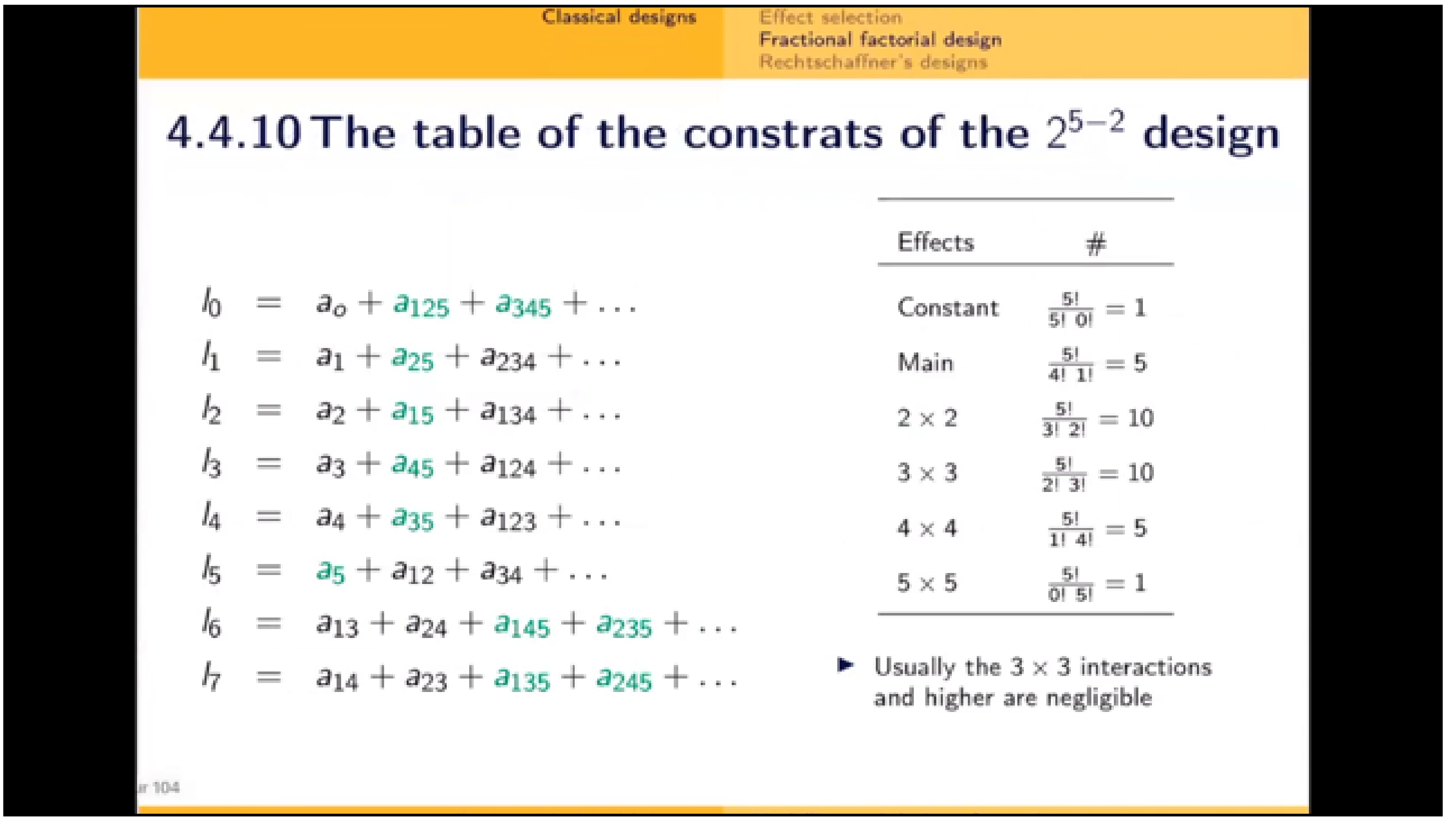
11m 41s



So in the previous. In the previous pages, I have start from this situation. I have created a new dimension, the fourth dimension. And in these slides, I'm doing the reverse or showing you the logic, the geometrical logic, which is behind. So the Cuban is three dimension.

notes

summary



And if you look at the slide before in red, I have start from a full factorial.

notes

summary

12m 10s

4.4.10 The table of the constrats of the 2^{5-2} design

$$
\begin{aligned}
l_0 &= a_0 + \text{[illegible]} + a_{345} + \dots \\
l_1 &= a_1 + a_{25} + a_{234} + \dots \\
l_2 &= a_2 + a_{15} + a_{134} + \dots \\
l_3 &= a_3 + a_{45} + a_{124} + \dots \\
l_4 &= a_4 + a_{35} + a_{123} + \dots \\
l_5 &= a_5 + a_{12} + a_{34} + \dots \\
l_6 &= a_{13} + a_{24} + a_{145} + a_{235} + \dots \\
l_7 &= a_{14} + a_{23} + a_{135} + a_{245} + \dots
\end{aligned}
$$

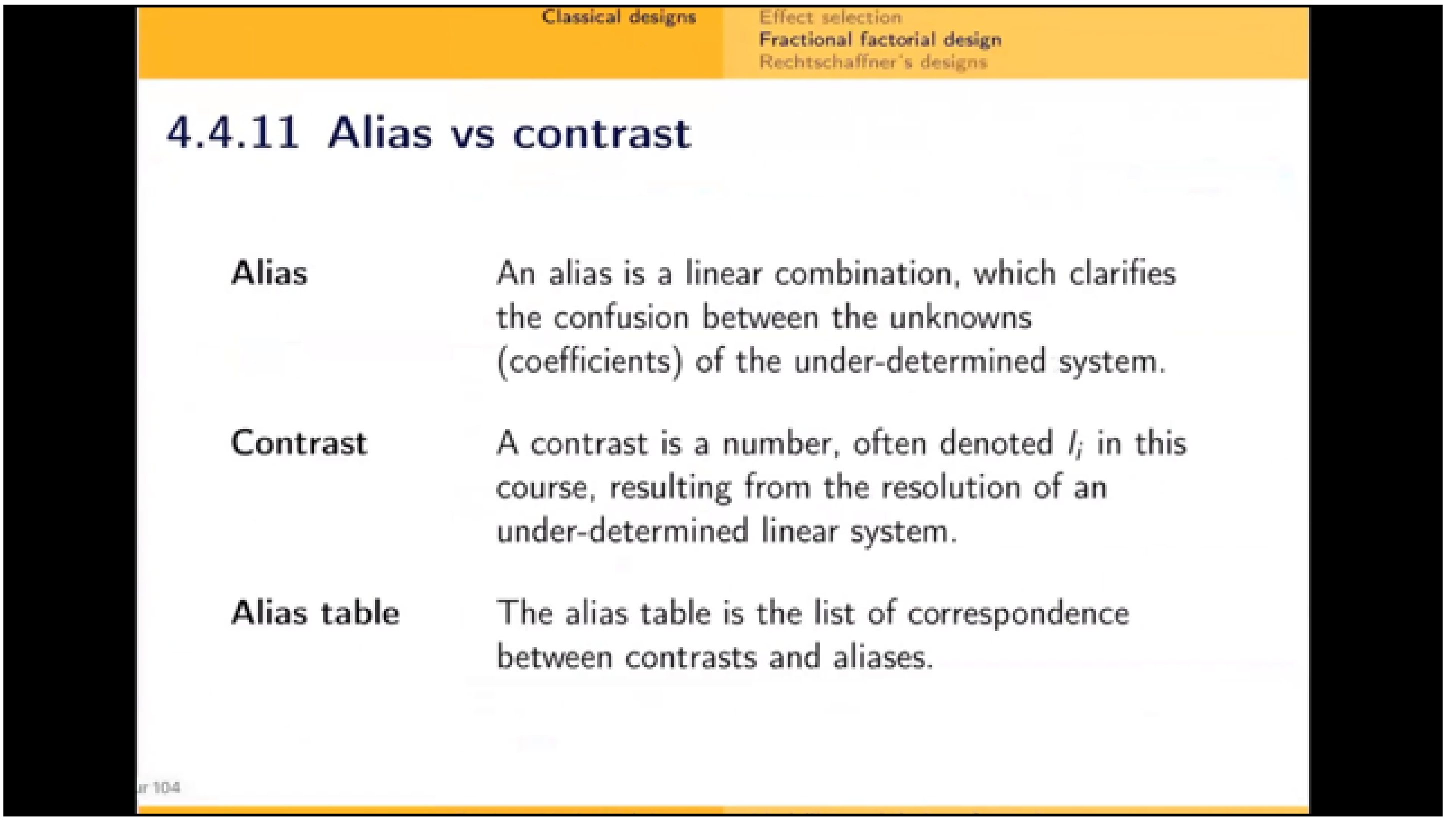
Effects	#
Constant	$\frac{5!}{5!\,0!} = 1$
Main	$\frac{5!}{4!\,1!} = 5$
2×2	$\frac{5!}{3!\,2!} = 10$
3×3	$\frac{5!}{2!\,3!} = 10$
4×4	$\frac{5!}{1!\,4!} = 5$
5×5	$\frac{5!}{0!\,5!} = 1$

- Usually the 3×3 interactions and higher are negligible

Design industry dimension. But now I'm creating a force coordinate.

notes

summary

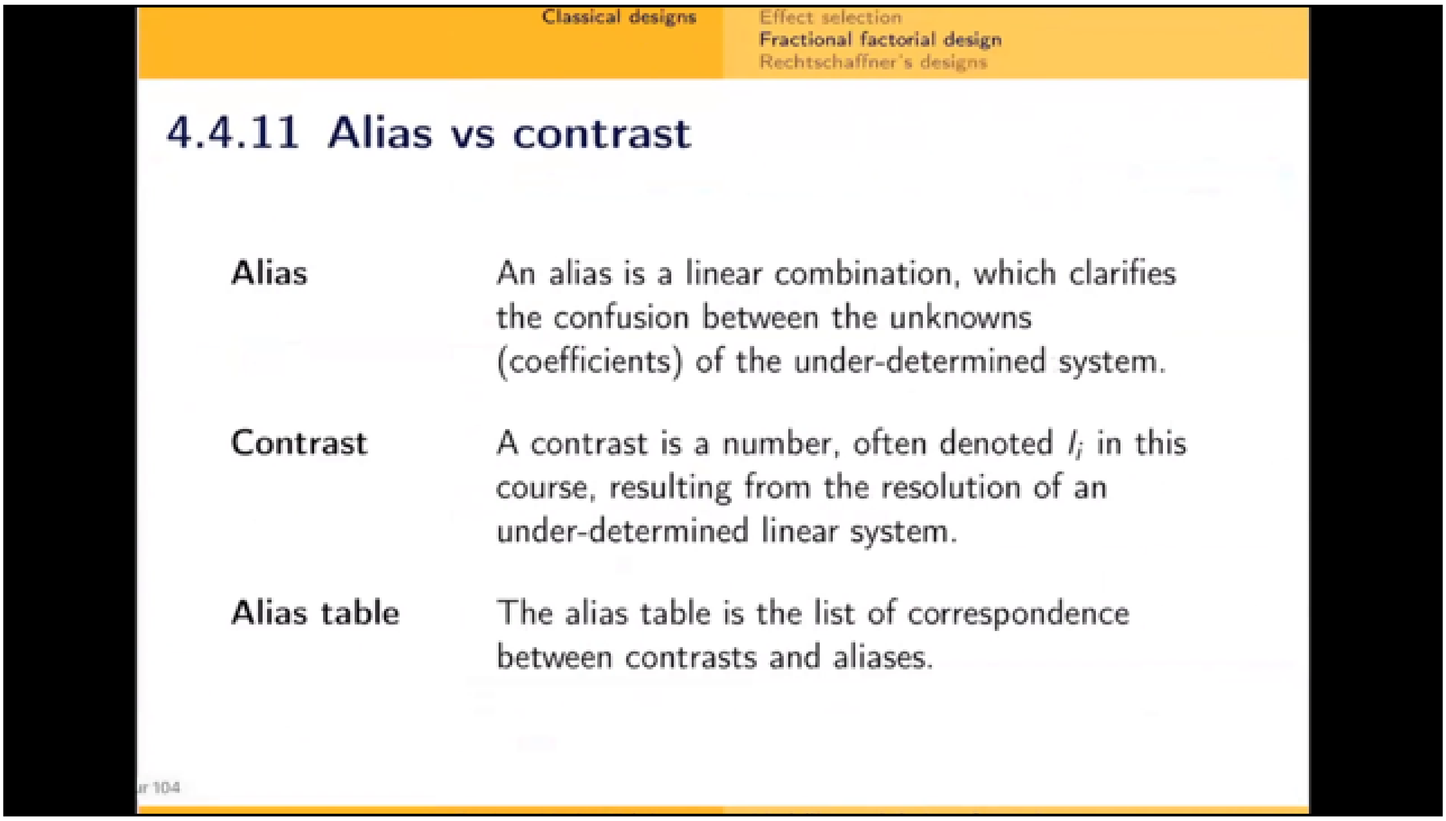


So it's equivalent to say, OK, I do not have one cube. In fact, I have two cubes. So I have to distribute my points in 16 vertices and not eight. So what can I do with such a design? If I'm looking at a polinom for four factors, what do what do I have? I have a constant. I have four main effects and I have six instructions two by two to calculate the numbers of interaction is the number of arrangement. So with four factor, it would be four factorial divided by two factorial divided by two factorial. This makes six. In the three by three, it would be four factorial, three factorial divided by one factorial, which is one. So it would be four. I have four instructions, three by three. So I have 11 coefficients. So with the eight runs, I would not be able to solve all those unknown. I have an under-determined system. So I have aliases between those coefficients. I also have aliases with the three by three and four by four. With this, I can in a first evolution in a very engineering position say, okay, I'm not interested in having those instructions three by three, four by four. Generous picking, it's okay. Specifically, in a case it could be not okay, but a generic picking, it's okay. But what are the aliases that eventually I have in my two by two instructions and my main effects? What is mixed with what? I need absolutely to understand that. It's why just applying fractional factorial design, because I find it in internet without understanding the aliases could be dungeous. You don't know what you do. And a few terrible papers I see on this, what's the case? When people apply without understanding, they don't know what they are doing. It's dungeous. So for playing with that, it's sort of a game. There are

notes

summary

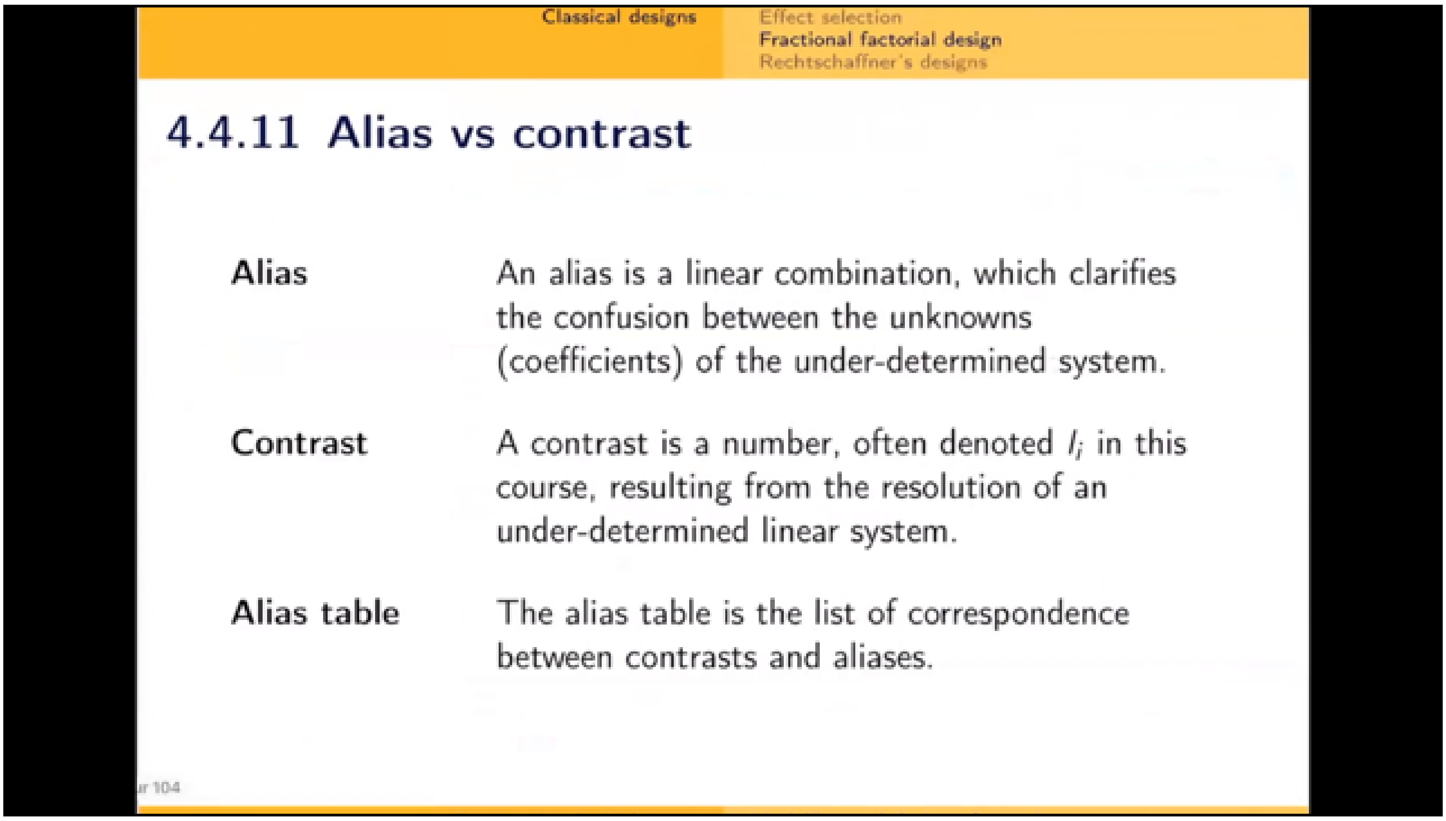
12m 27s



a lot of game on your phone with this type of things. You have to understand the mathematical structure. The mathematical structure, which is behind the column of full factorial design, is a group and is a group of valoir. So for having a group, you need a set. So this is the column of a factorial design, of a full factorial design. The reference is a full factorial design. And for having a group, you need an operation. And the operation is a multiplication term by term of a column. So if it's a group, you remember from high school that group has some very specific properties. One is that you have an identity element. So the identity element is the column of 1111, that I call I. Because if you multiply any column by a column of 1111, you don't change this column. It's a neutral element. After you need an inverse. For each element, you need an inverse in the group of addition with the sets. And the inverse is the opposite. Minus one plus one makes zero, which is the neutral element of the addition. For the multiplication is one under the number. What could be the inverse of a column? The inverse of a column in a full factorial design is itself. If you multiply the column with itself, you get a column of ones. You get the neutral element. It's tricky. So it's why it's a special group. It's called a Galois group. And the group is commutative, which is nice. That means that multiplying one column by another is the same thing as multiplying another by this first column you have in mind. So it's a group. So the fact that it's a group, we can make operations. We understand that any operation you do, any multiplication of columns between them will give you a column of the sets. So a column of

notes

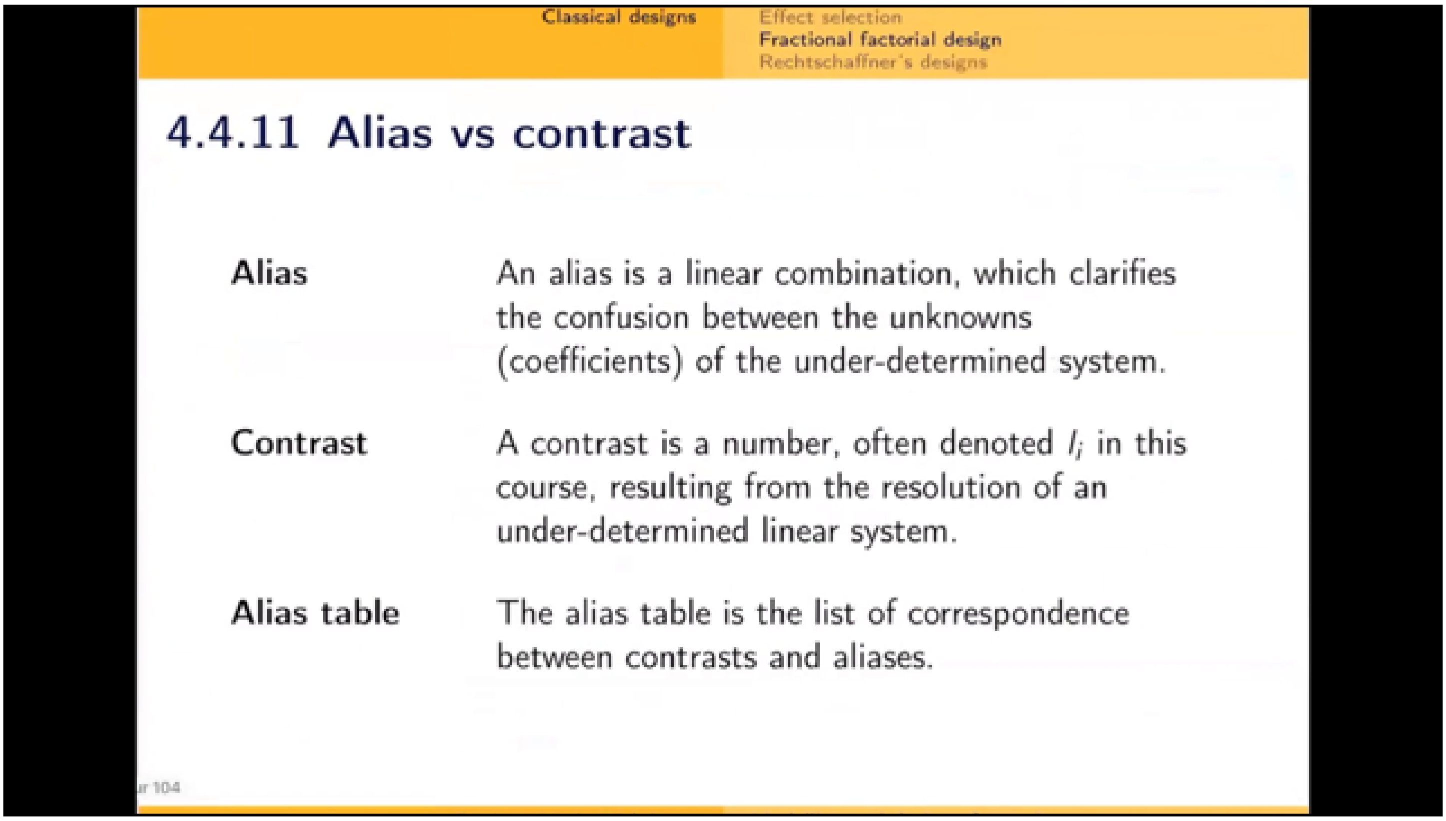
summary



the factorial sets. So there are another way for avoiding to make the calculation with the column of the matrices. You can just mention the column and make the operation. So the ordinal number one, two, three, four represents the first column, the second column, the fourth column. And we will consider that it's represent also the first factor, the second factor, the third factor. There you can mix them. What you call the first factor is perhaps not so tricky or so important in your problem. But it's clear that the mathematical structure must be clear. So first column, second column, etc. So when I was saying that I would like to build a matrix of four columns, multiplying the three first columns, in fact, I was doing that. I was saying my fourth factor, my fourth column of my matrix of S is the result of the multiplication of the three first column. So one, two, three, four. And I write four equal one, two, three, four. And just to make things clear, we put the column as its commutative. We put the smaller number before and the highest number at the end. Like that it's clear because if you have three to one, it's possible. But after you are not sure that you have not two times, one time, read one, two, three, and one time, two, one, three, etc. So each time you write it with the minimum number first and after increasing number when you write it. And so this means a lot of things. This means my factor four is varied as the product of the factor one, two, and three. But it's also indicating something else. Is it indicating the aliases? I'm saying that I have an alias between A1, the coefficient of the factor one, and the coefficient of interactions three by three, one, two, three. So it's in the same time

notes

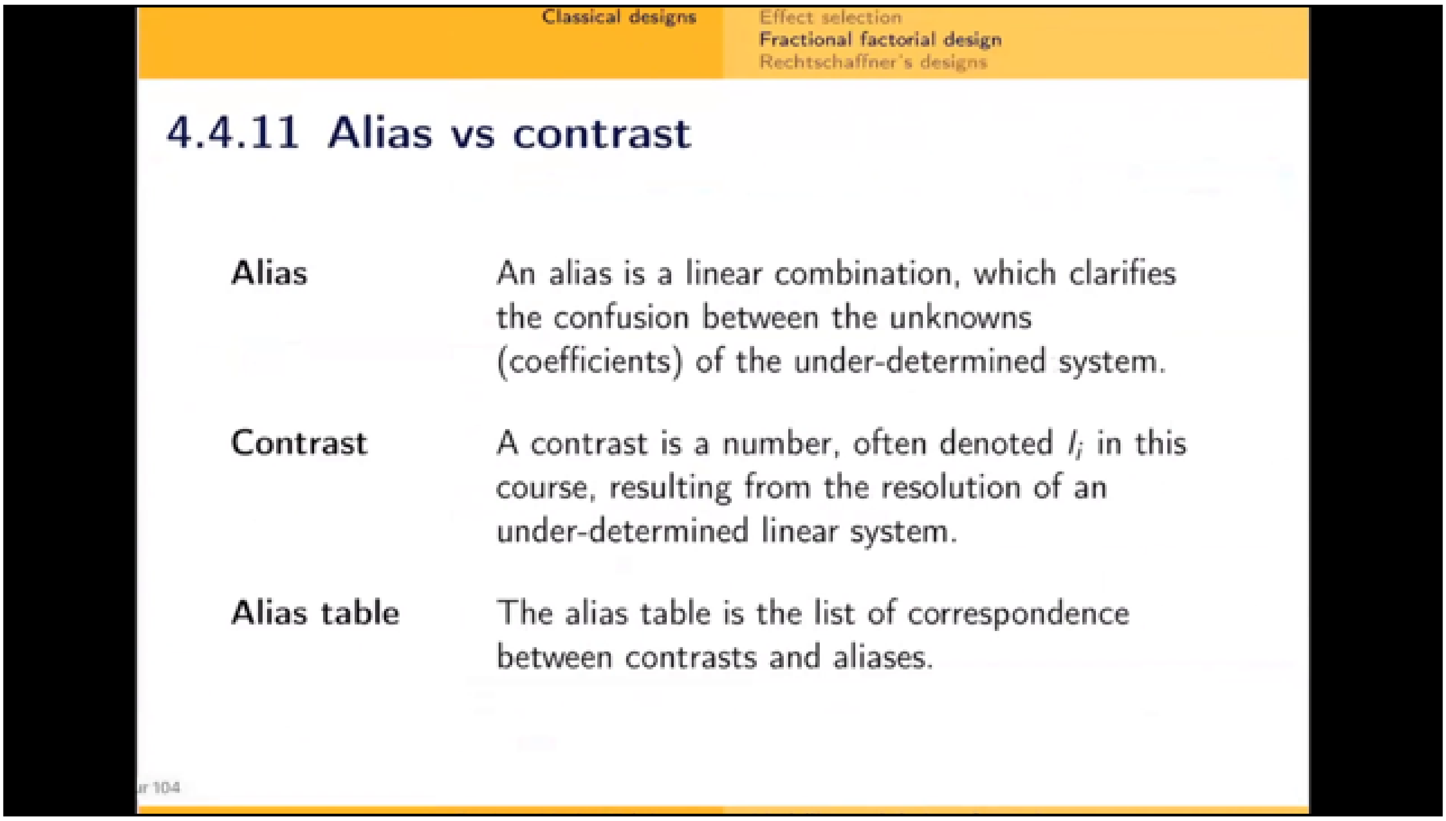
summary



the recipe for calculating my fourth column because I'm starting from a full factorial design for three and I'm adding columns. But it's also the alias structure. It's also telling me that I'm making an alias between four and one, two, three. But I tell you it's a group so I can make some operation. So for example, I can multiply four by itself. So four four equals identity and it's equal to one, two, three, four. Because in an equation you are obliged to make the same operation to the two sides of your equation. I've multiplied it left by four. I must multiply it right by four. That's why I create my one, two, three, four. And again, this is a way of creating this column. I don't need, I know it's one, one, one, but in any case it's a recipe of in this situation the recipe of my column of identity, my column Y. But it's also indicating me that I have an alias between A0 because the coefficient A0 corresponds to the column one, one, one, one, one. And the coefficient for the interaction four by four. For my, I have only one interaction four by four in this situation. And my interaction four by four is aliased with the interaction with the constant. And in this case, the aliases are full. So they are fully aliased. That means that when I believe that I'm calculating A0, in fact, I'm calculating A0 plus A1, 2, 3, 4. This manner of writing Y equal one, two, three, four, we call it the canonical form of the generator. You understand that not when you are creating the relation, I'm creating by the same way also relation because I am in a circle or in a sphere, in hypersphere. But in fact, I have created first an interaction. I'm creating a force factor for the column for force

notes

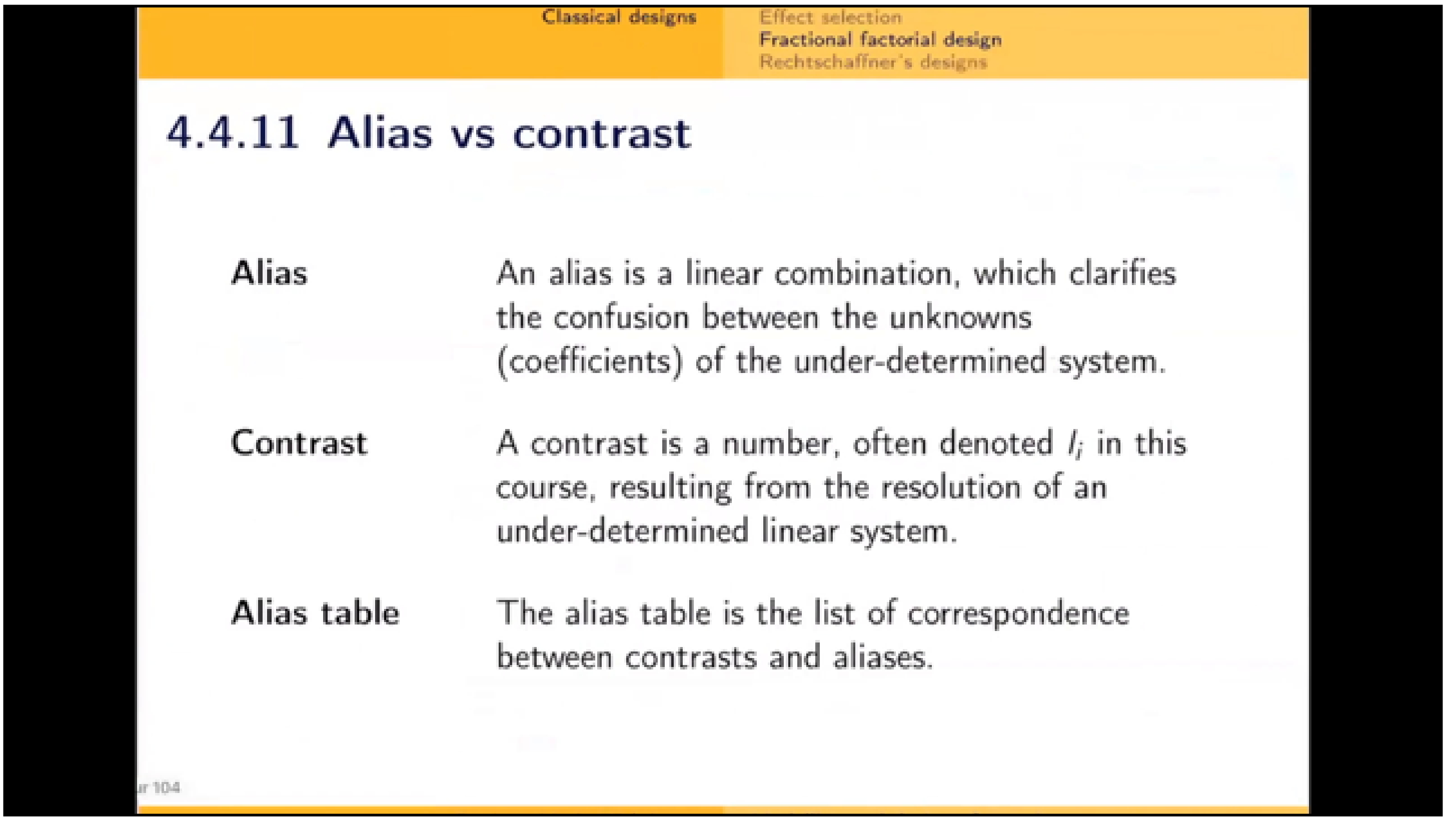
summary



factor. I'm not creating factors, the factors exist, but I'm creating the columns in my asymmetrics for my force factor. And I'm then creating aliases. I'm creating more than just one aliases. I'm not just creating an analysis between four and one, two, three. I'm also creating the same time. I'm also creating the analysis between the constant and the interaction three by three. But I'm also creating the aliases between the coefficient A1 for one equal two, three, four. So I've made one thing, but I'm provoking more aliases. So it's why I absolutely need to understand what I'm doing. And just to be sure what type of aliases I'm creating, all of them. Because what typically I would avoid is mixing A1 and A2, for example, because my main effects will be undifferentiated and it's not good. So I need to do all these calculations, but I can do it quite rapidly. But now I understand I'm starting with just one relation, one generator, four equal one, two, three. And this is provoking the aliases at all the levels. Because in fact, what I'm doing, I'm folding over one part of the design on the other. So that means now every coefficient is aliased with another one. So I can understand that my main effect one is aliased with two, three, four. Also, you understand that my main effect A2, two equal one, three, four, etc. All my main effects are aliased each one with an interaction three by three. In this case, with one, which is not quoting it. So one with two, three, four, two with one, three, four, three with one, two, four. And four with one, two, three. But it's also the case for the interactions two by two. Because with my generator, any one of them, but let's take the first one, four equal one, two, three, I can multiply both sides by

notes

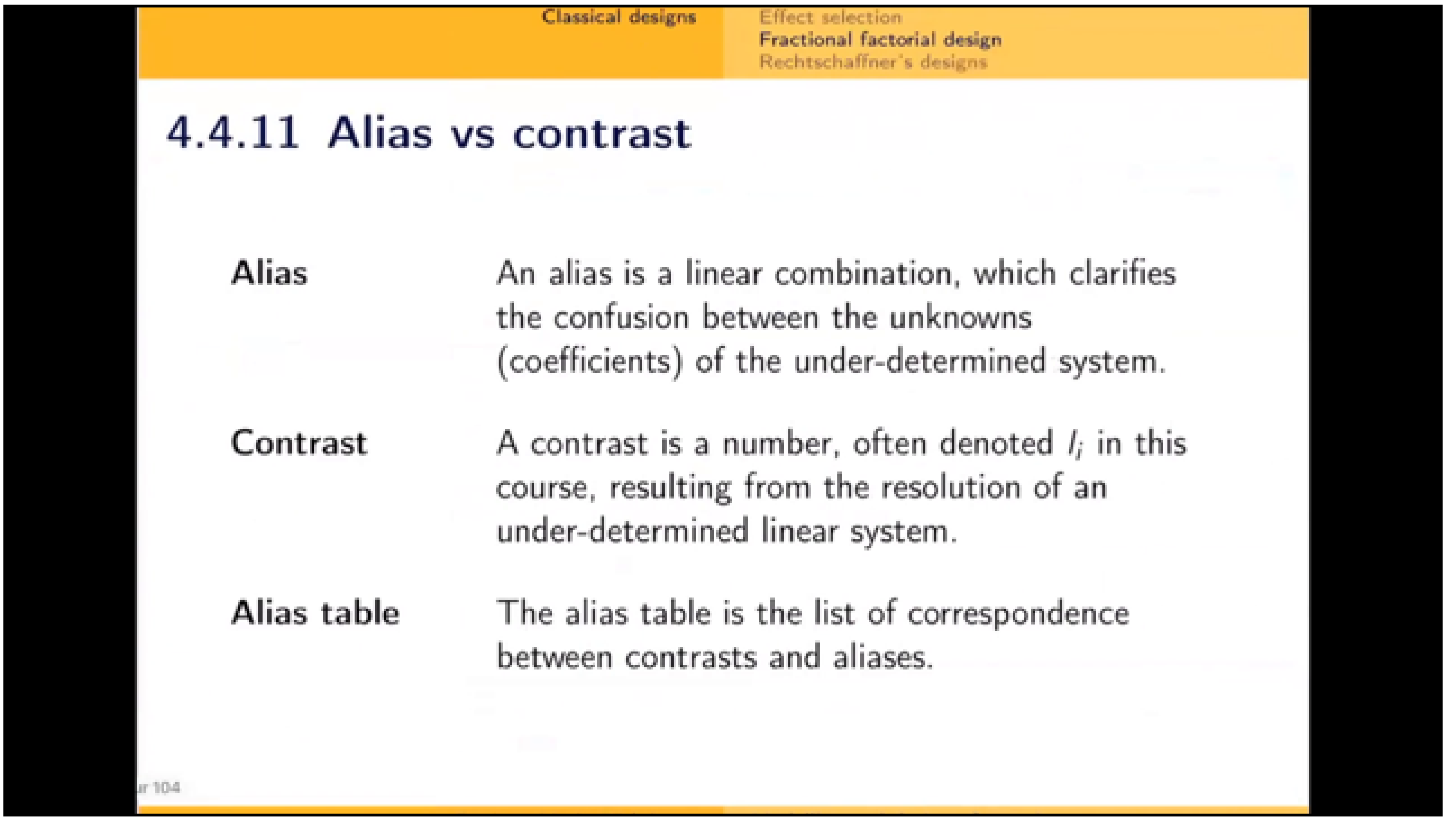
summary



one. And I obtained that one, four is equal one, one, which disappears because it's identity, two, three. So I'm making the aliases between two of the interactions two by two. So each time I forget. So I need first to have this concept of the contrast. So you have the coefficient of your polynomial, the Taylor, the Taylor polynomial, but you also have the numbers that you can get out of your linear system. So if you have eight experiments, you can get eight contrast only. And after the game is to discover what are those contrasts made of. So it's usually I use the letter L for the contrast, use any one. But I use another one that my coefficient of my model for different sheetings, it's two things. In the best case, all contrasts correspond to one coefficient. But if you have aliases, one contrast correspond to the sum or the difference, you will see later, of some of the coefficients. So in the example that I'm presenting you today in those slides, I see that my first contrast, I like to call it L zero. It's not because I'm writing in Python, but it's because it's more easy after for having L one for the first, for having the first main effect. So L zero is a zero plus the interaction four by four. L one is a one plus interaction two, three, four. L two is a two plus one, three, four. And I have at the end, the six, fifths and eights contrasts that are an addition of two by two instructions. So you see that you get more things that with the Placket and Bremen design. In the Placket and Bremen design, we had only the main effects. The contrast was only for the main effect. No, I have a few additional ones. So you see the same theory. It's just another way

notes

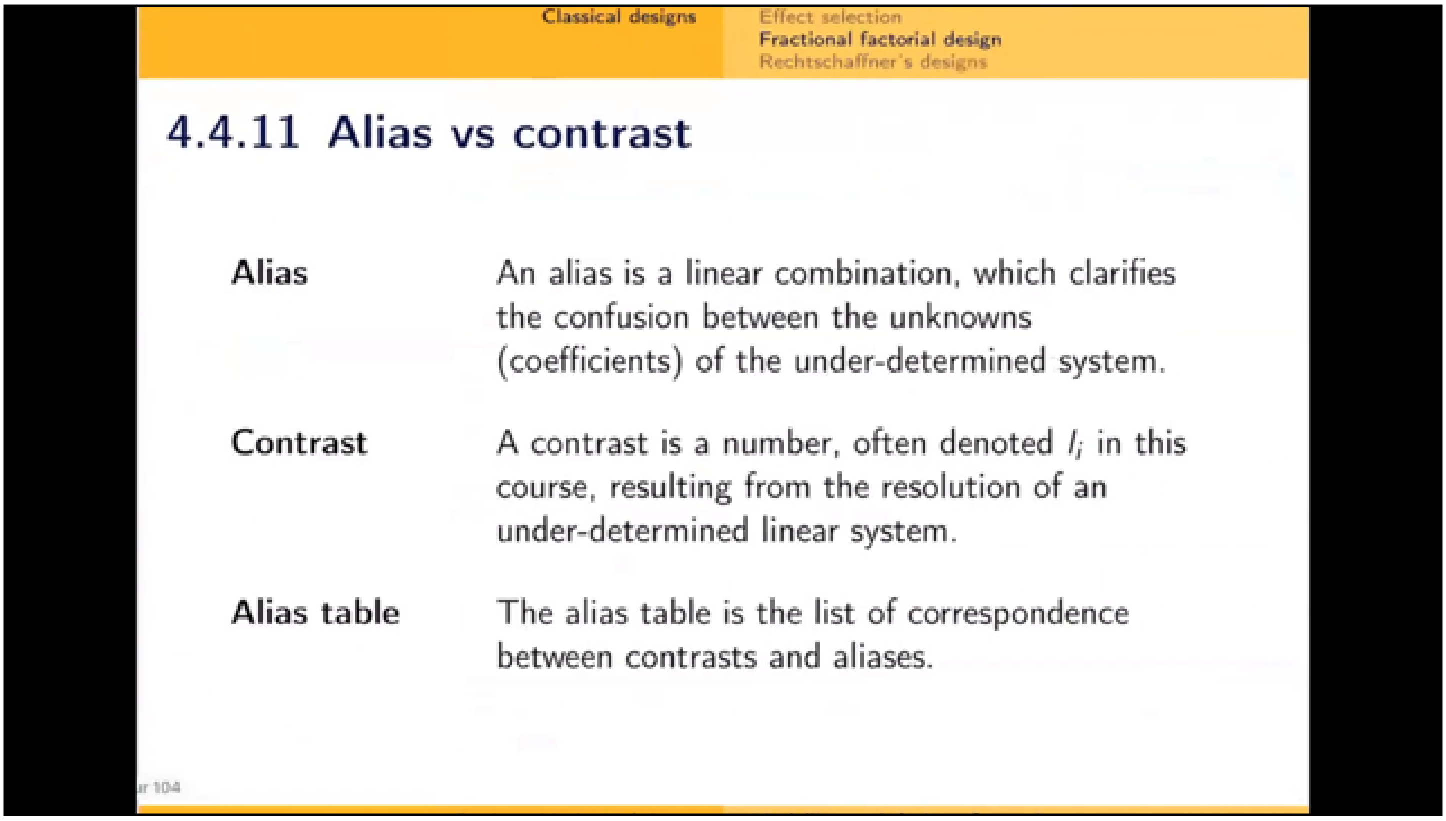
summary



to play. The generator is a little bit different in this case, but it's exactly the same. We are playing with the same structure, which is the structure, the column of the column of the factorial designs that have specific properties. Yes, I said. Yes, exactly. Is the hypothesis that we are using, we could imagine a situation in which we have more factors than it's not the three by three that we are neglecting, but for the three by three, or you can imagine that you know something, that you know that you have some of the three by three instructions that are in fact significant to keep them and you try to understand. You can deal the alias after, and there are a few things you can do. But yes, the standard hypothesis, that's a three by three instruction are neglectable. It's quite logical because the least square feet algorithm have a tendency to shift information towards the main effects. You can try, you make your own model, and only with instructions three by three and four by four, you get results and you try to analyze with the least square feet, and you will see that the algorithm will give you some main effects. It will reinterpret because you have the hypothesis of the noise, you are, as the highest instructions, have a tendency to be smallest, they are also the tendency to be compared with the noise. So, yes, we can consider in most of the situation that the final analysis I have to do, perhaps the final situation will be this one. Nevertheless, you see that you have your main effect alone, but you have three instructions of a siphon, you have a mixture of three. So, in this case, it could be a first step, and after if you see some movement, some values in the instructions, okay, I won't go further. Or perhaps

notes

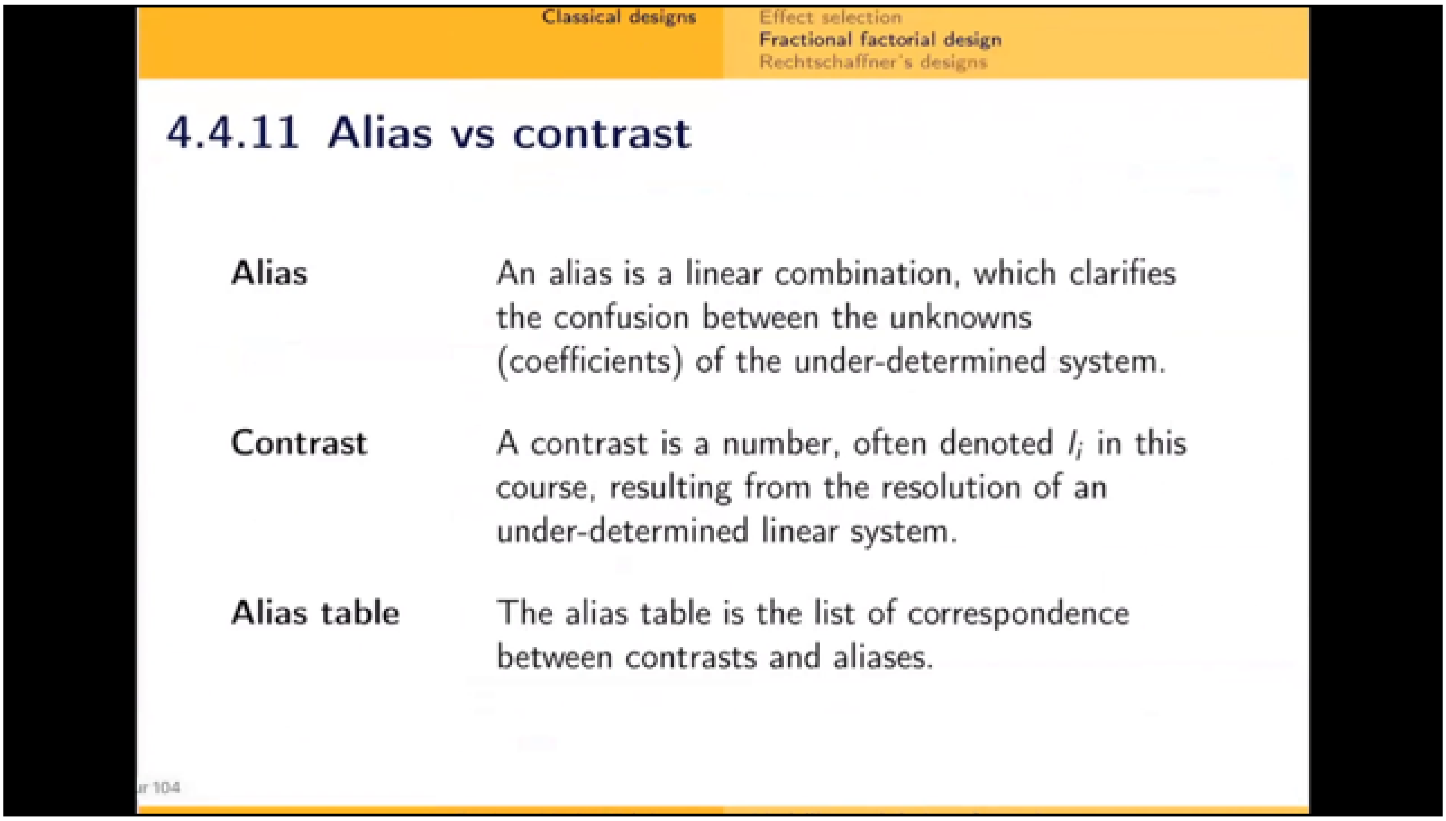
summary



you have information, like in the case of the bicycle that I have treated with Adamar, you know what type of interactions are possible, and you try when you order them, you associate the numbers of the factor with the real factor, which one is the first factor, which one is the second factor, which one is the third one, just to be sure that the instructions that you are expecting are not mixed together. But Murphy's law playing, you will see sometimes it's exactly the ones that you want to separate, they are in any case together, but there are some solutions. Okay, so I believe I have what it's also very important to will see with this example, that at the end, if you are performing a normal plot, you will be eliminating some of those contrasts, and hopefully you will get things that are not too much aliased, or they are aliased with something that doesn't interest you three by three, four by four, five by five, and things like that. So we are in fact, sweeping information towards things that interest us, is the price to pay for diminishing the numbers of experiments. If you are not happy at the end, you have not lost your time, because what you have done is a part, is a quarter, an eighth of a full factorial design, so you can unfold your design, but you can choose which parts you want to unfold for the aliasing what interests you, what was, what you see some movement in some instructions. Okay, here I would like to know what I have, but the positive aspect is usually in nature, the Pareto principle works, that means that there are a minimum of factors that are responsible of the maximum of consequences. It just, how to say that, it's just a trick, that's the fractional factorial design, let you work with this

notes

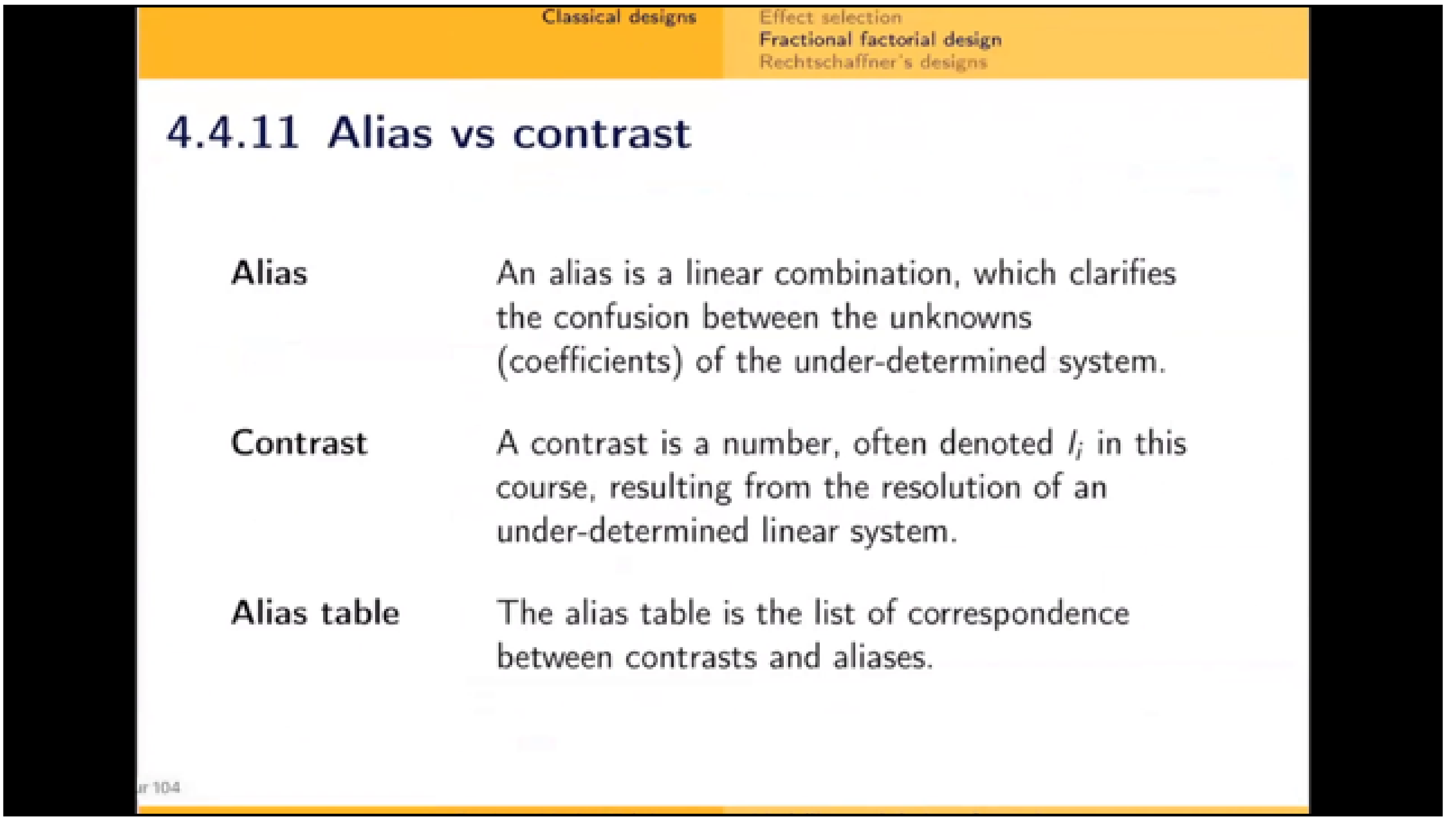
summary



trick without losing time. If it's sufficient, you win time, and if not, you continue and you execute the full factorial design, and you do things step by step, so when you just divide by two, okay, it's okay, or you have to do your full factor, but when you divide by eight, you have a lot of choice of progression of learning things step by step, and it's a lot more powerful than just starting analyzing one factor, and after two factors, and after three factors, and after four factors, this would be a loss of time. I have repeated the number, the calculation of the numbers of interaction effects for having, so graving that in your memory, but I can do more than just adding one factor in the situation, remember I start with three factors, full factorial, I add a fourth, I can also add a fifth, so I need to introduce a second generator, so not work only with one generator, remember what I call one generator was this relation four equal one, two, three, this was the generator, which I start, so I can introduce a new generator, in this case, I have no choice, I cannot use one with four letters, four numbers, I'm obliged to choose one with three, because there are only one with four letters, one, two, three, four, so my new generator is five equal one, two, and I immediately see that the quality is not so good, because I'm making a mixture between the main effect and interaction two by two, because as I tell you, the generator tell you how to build the column, so I'm saying that my fifth column is obtained by the multiplication of one with two, I could have choose one with three or two with three, that make no difference in the structure of my new design, but now I'm making aliases between

notes

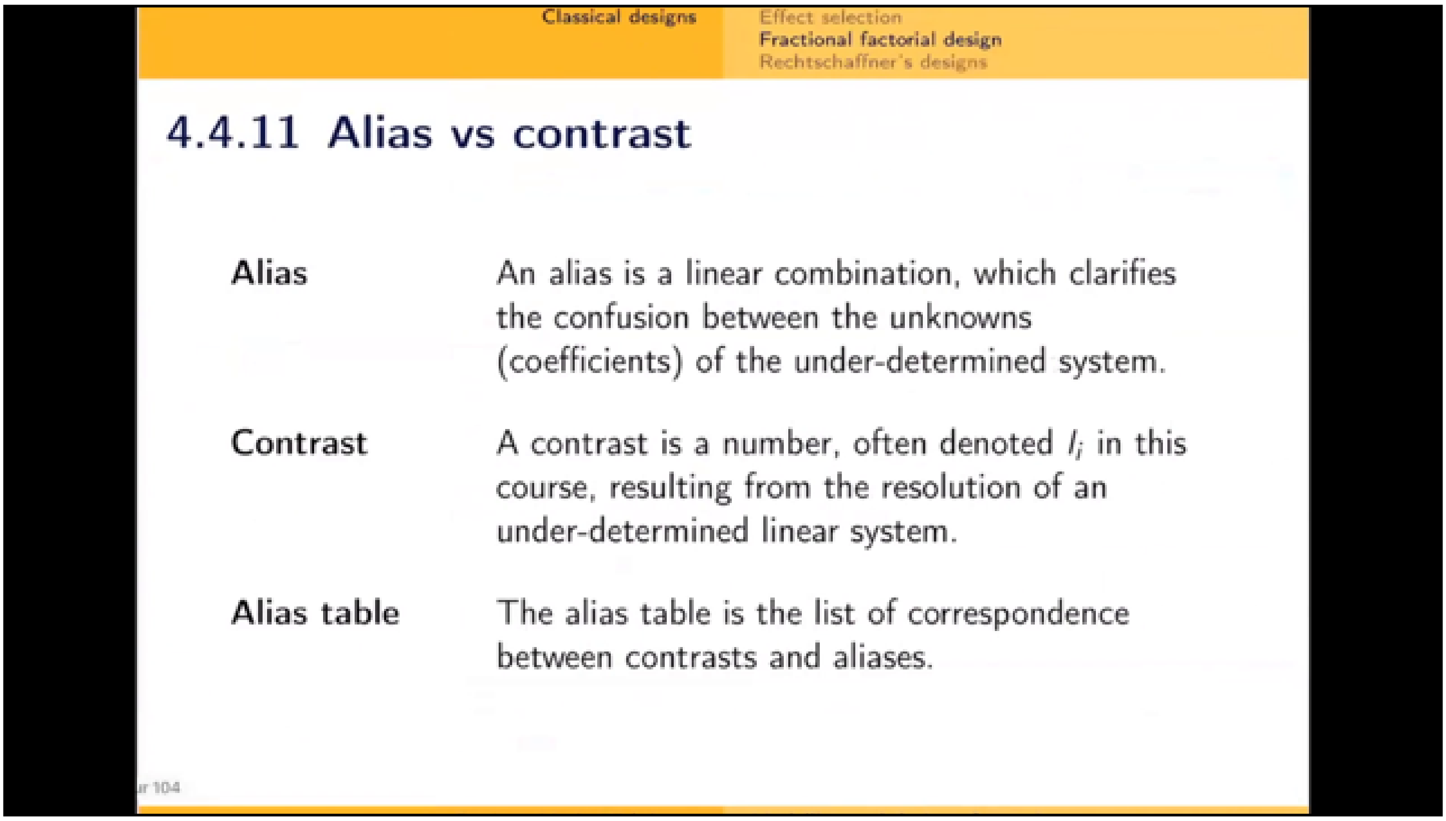
summary



main effect and interaction two by two, before if you remember, I have only two by two with two by two and one with three by three, okay, so I'm cutting me an arm or something like that, nevertheless, as a first step, it could be sometime interesting, it's exactly what we were doing with the placket and Bremann design, it's exactly that that we were being doing, so now I have two generators, so my canonical form of the generator is y equal one, two, three, four, my first generator and one, two, five, with my second canonical generator, and I have to use this generator for determining all the aliases, so when I'm doing that, I'm able to draw a table of contrasts, and in my table of contrasts, I see that my first contrast L0 is mixed with A0 and with some interaction three by three, my contrast L1 is in fact A1, I don't remember why I put some in green and some in black, I have one instructions two by two, A2, five, and more instructions three by three, four by four, five, etc, so you see that the main effects are each time aliases with some interaction two by two, and I also have some instructions two by two mixed together, so if I'm doing the same thing as previously, so interaction, this I can eliminate, so I stay with a group of things that interest me, but they are mixed, so typically I use this situation as a first step, it's a little bit more powerful than a Plackett and Berman design, it's quite equivalent to a Plackett and Berman design, I have two advantages, one I know how to continue directly, to remember with Plackett and Berman I can do a full fold over, but it was not clear what I get about the instruction two by two with that, no it's

notes

summary



evident what I have and I understand that I have divided my first design by four, two times by two, first generator, second generator, each time in the generator divided by two, so I have three more parts of my full factorial design, so I can select, I have three choice for my second step, in this case. So before we can go to pause, just remember a few contrasts, things that a few concepts that I was using in this class, alias, now you understand what mean alias, the thing that I have some coefficient mixed with others, sometimes is mixed with full others, sometimes not in fractional factorial design, but in Plackett and Berman design, it could be the case that is alias with a half or one coefficient and the quarter of another, it's not all the time full alias, in this case it's full alias plus or minus, the contrast is the numbers I get out of a linear system, and the alias table is this relation that I do between my contrast and the different coefficient to understand what is alias with what. With this information, I let you have a pause and we continue in 15 minutes.

notes

summary