



Course material

Course:

ENG606 / PHYS 442

Video:

DOE_lesson9_part2_FractionalFactorialDesign

Concepts (extracted from automatically generated subtitles):

Interesting designs. Main effect. Best thing. Lot of factors. Negative sign. Full factorial matrix. Maximum interactions. Main effects. First slide of the chapter. Much experiment. Good quality. Absolute value. Normal plot. Fractional factor design. Sensitivity analysis.



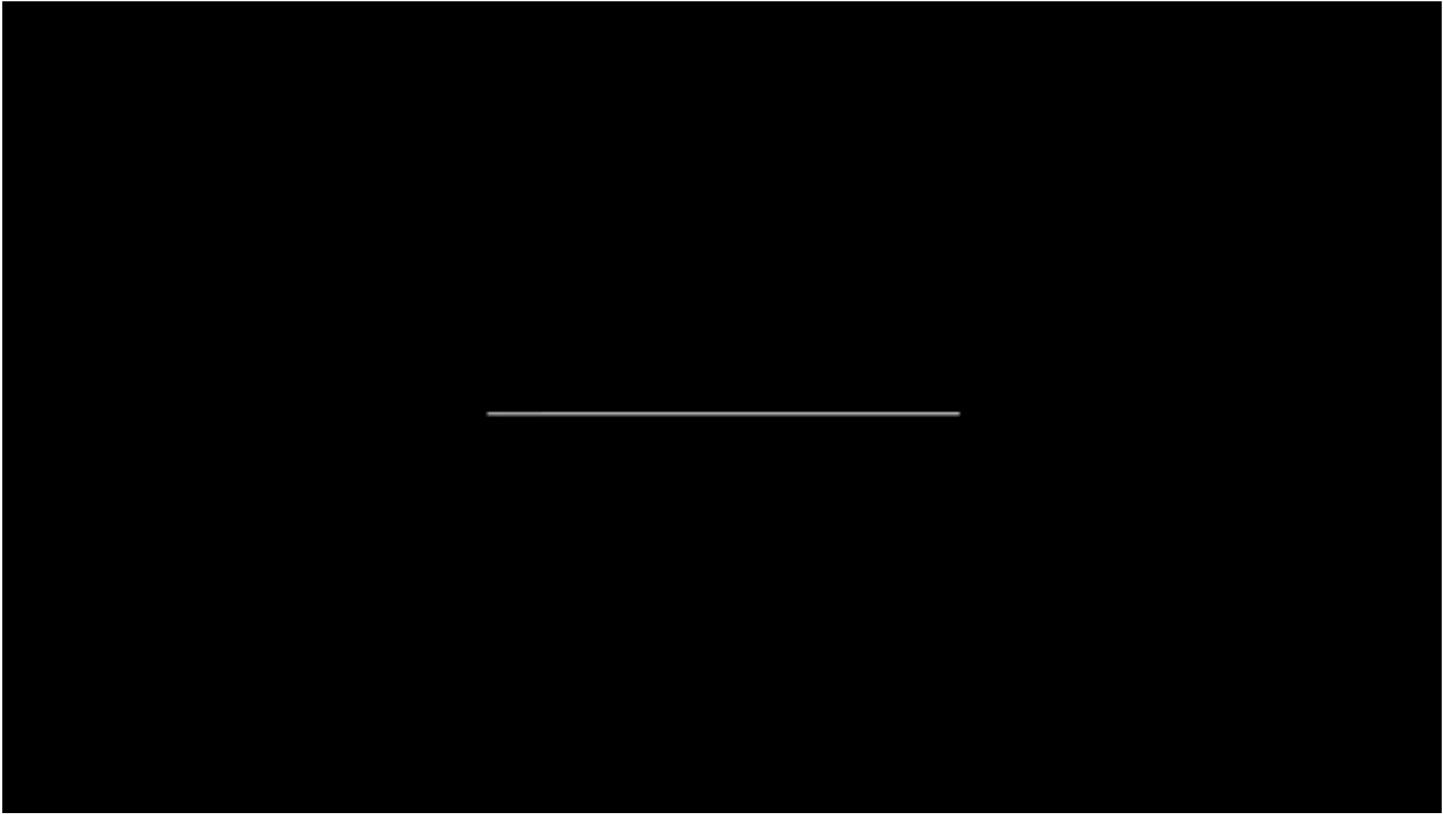
[to video sequence search](#)
(within ENG606 / PHYS 442.)



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notes

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summary

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0m 0s



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4.4.11 Alias vs contrast

Alias	An alias is a linear combination, which clarifies the confusion between the unknowns (coefficients) of the under-determined system.
Contrast	A contrast is a number, often denoted l_i in this course, resulting from the resolution of an under-determined linear system.
Alias table	The alias table is the list of correspondence between contrasts and aliases.

These subtitles have been generated automatically So, let's continue the discovery of those very interesting designs.

notes

summary

0m 1s



4.4.12 Generation of a fractional factorial matrix

MATLAB

- ▶ `[E,conf] = fracfact(gen,'MaxInt',m)`
`E` : runs matrix
`conf` : table of alias
`gen` : generator in the form of a chain of characters such as 'a b c abc'
`m` : maximal level of the interactions to consider in the table of alias
- ▶ `gen = fracfactgen(terms)`
- ▶ `gen = fracfactgen(terms,k)`
- ▶ `gen = fracfactgen(terms,k,R)`
`terms` : factors in the form of a chain of characters such as 'a b c abc'
`k` : exponent defining the number of runs ($N_{exp} = 2^k$)
`R` : resolution of the fractional plan (III, IV ou V, but in Arabic numbers)

And so, this is a summary for the design 2^{5-2} . And so, first you have the table of contrasts and the same thing as before. You can use a normal plot for discriminating between significant and non-significant. You have divided your designs or original design in four. So, you have three parts left and you can de-alias eventually. And it's important to understand now how you get the three other parts. So when you have the generator, usually we consider as a first one, we should do by random, honestly, which one we choose. But when you have one generator, in fact, you have two, the two parts. One is with the positive and one is with the negative sign. So when you have two, you have four possibilities. The two positive, one negative and the two negative. So for generating the other parts, you just change the sign. So it changed the sign of the aliases. I will show you.

notes

summary

0m 5s



4.4.13 The table of 2^{n-p} designs

Factors

Runs

	3	4	5	6	7	8	9	10	11
4	2^{3-1}_{III}								
8	2^{3-1}_{III}	2^{4-1}_{II}	2^{5-1}_{II}	2^{6-1}_{II}	2^{7-1}_{II}				
16	2^{3-1}_{III}	2^{4-1}_{II}	2^{5-1}_{II}	2^{6-1}_{II}	2^{7-1}_{II}	2^{8-1}_{II}	2^{9-1}_{II}	2^{10-1}_{II}	2^{11-1}_{II}
32	2^{3-1}_{III}	2^{4-1}_{II}	2^{5-1}_{II}	2^{6-1}_{II}	2^{7-1}_{II}	2^{8-1}_{II}	2^{9-1}_{II}	2^{10-1}_{II}	2^{11-1}_{II}
64	2^{3-1}_{III}	2^{4-1}_{II}	2^{5-1}_{II}	2^{6-1}_{II}	2^{7-1}_{II}	2^{8-1}_{II}	2^{9-1}_{II}	2^{10-1}_{II}	2^{11-1}_{II}
128	2^{3-1}_{III}	2^{4-1}_{II}	2^{5-1}_{II}	2^{6-1}_{II}	2^{7-1}_{II}	2^{8-1}_{II}	2^{9-1}_{II}	2^{10-1}_{II}	2^{11-1}_{II}

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So in MATLAB, you have a few routines. Unfortunately, I have the impression that Fract fact has changed something. So I forget to change, but I see showing the example to a student last week. I was used at Fract fact. So you give a generator, you give with letters. So it's not one, two, three is A, B, C, D. You can use any letters that you want, but it's just another way of presenting the columns. And so the best thing is to take A for one, B for two, C for three, etc., if you want to remember. And after you have to tell to the program the maximum interactions that you would like to consider. And I was used that it give you the matrix of S, but it's also and you and also the table of aliases. But now the routine have changed and I have to find another way. I will look at it, check it and look at it and make a note in the announcement for telling you. You can also get some generator. So you have a routine called Fract fact gen for generator. And you can say the terms that you want. I don't remember the... Yes, the term, yes, okay. The terms is A, B, C, D and things like that that you can use. And you have, you can tell also the exponents of the run that you want. So the K and you can also indicate the resolution. I will explain you the resolution in a while. So those are, honestly, I'm just using the first one. The rest I'm not using MATLAB for that. I'm using other way of doing it. And I'm sure you find it in Python.

notes

summary

1m 41s



4.4.13 The table of 2^{n-p} designs

Factors

Runs

	3	4	5	6	7	8	9	10	11
4	2^{3-1}_{III}								
8	2^{3-1}_{III}	2^{4-1}_{II}	2^{5-1}_{II}	2^{6-1}_{II}	2^{7-1}_{II}				
16	2^{3-1}_{III}	2^{4-1}_{II}	2^{5-1}_{II}	2^{6-1}_{II}	2^{7-1}_{II}	2^{8-1}_{II}	2^{9-1}_{II}	2^{10-1}_{II}	2^{11-1}_{II}
32	2^{3-1}_{III}	2^{4-1}_{II}	2^{5-1}_{II}	2^{6-1}_{II}	2^{7-1}_{II}	2^{8-1}_{II}	2^{9-1}_{II}	2^{10-1}_{II}	2^{11-1}_{II}
64	2^{3-1}_{III}	2^{4-1}_{II}	2^{5-1}_{II}	2^{6-1}_{II}	2^{7-1}_{II}	2^{8-1}_{II}	2^{9-1}_{II}	2^{10-1}_{II}	2^{11-1}_{II}
128	2^{3-1}_{III}	2^{4-1}_{II}	2^{5-1}_{II}	2^{6-1}_{II}	2^{7-1}_{II}	2^{8-1}_{II}	2^{9-1}_{II}	2^{10-1}_{II}	2^{11-1}_{II}

So now come a very, very famous page of the book by Box. Funny story, really, I used, I was first before my postdoc. I was quite alone doing this type of job here. Not a lot of people were doing that here. And I discovered this page in the book of Box. And I say, oh, this page is very terrible, very important. And I went to a conference in Washington and somebody said, with this page in the pocket, you get a job in a company in five minutes because it's give you the solution for helping company of solving a lot of problems. It's very common that people have a lot of factors and they are lost in discovering what are the weight of each of the factor. With this table, you understand very rapidly what you can do and what will be the price.

notes

summary

4m 0s



← Factors

	1	2	3	4	5	6	7	8	9	10	11
1	2 ¹²										
2	2 ¹²	2 ¹¹									
3	2 ¹²	2 ¹¹	2 ¹⁰								
4	2 ¹²	2 ¹¹	2 ¹⁰	2 ⁹							
5	2 ¹²	2 ¹¹	2 ¹⁰	2 ⁹	2 ⁸						
6	2 ¹²	2 ¹¹	2 ¹⁰	2 ⁹	2 ⁸	2 ⁷					
7	2 ¹²	2 ¹¹	2 ¹⁰	2 ⁹	2 ⁸	2 ⁷	2 ⁶				
8	2 ¹²	2 ¹¹	2 ¹⁰	2 ⁹	2 ⁸	2 ⁷	2 ⁶	2 ⁵			
9	2 ¹²	2 ¹¹	2 ¹⁰	2 ⁹	2 ⁸	2 ⁷	2 ⁶	2 ⁵	2 ⁴		
10	2 ¹²	2 ¹¹	2 ¹⁰	2 ⁹	2 ⁸	2 ⁷	2 ⁶	2 ⁵	2 ⁴	2 ³	
11	2 ¹²	2 ¹¹	2 ¹⁰	2 ⁹	2 ⁸	2 ⁷	2 ⁶	2 ⁵	2 ⁴	2 ³	2 ²
12	2 ¹²	2 ¹¹	2 ¹⁰	2 ⁹	2 ⁸	2 ⁷	2 ⁶	2 ⁵	2 ⁴	2 ³	2 ²

notes

summary

5m 1s



The table of 2^n

	3	4	5	6
4	2^{3-1}_{III}			
8	2^3	2^{4-1}_{IV}	2^{5-2}_{III}	2^{6-3}_{III}
16	2^3	2^4	2^{5-1}_V	2^{6-2}_{IV}

$\pm 3 = 12$
 $\pm 4 = 123$
 $\pm 4 = 12$
 $\pm 5 = 13$

So if you have three factors and you would like to make only four experiments, you can do a two power three minus one of four experiments. And here it's give you the generator.

notes

summary

5m 38s



4.4.13 The table of 2^{n-p} designs

Factors

Runs

	3	4	5	6	7	8	9	10	11
4	2^{3-1} 23								
8	2^{4-1} 123	2^{4-2} 12	2^{4-2} 34	2^{4-2} 13	2^{4-2} 23				
16	2^{5-1}	2^{5-2} 123	2^{5-2} 34	2^{5-2} 13	2^{5-2} 23	2^{5-2} 123			
32	2^{6-1}	2^{6-2} 1234	2^{6-2} 123	2^{6-2} 124	2^{6-2} 134	2^{6-2} 234	2^{6-2} 1234	2^{6-2} 123	2^{6-2} 124
64	2^{7-1}	2^{7-2} 12345	2^{7-2} 1234	2^{7-2} 123	2^{7-2} 124	2^{7-2} 134	2^{7-2} 234	2^{7-2} 1234	2^{7-2} 12345
128	2^{8-1}	2^{8-2} 123456	2^{8-2} 12345	2^{8-2} 1234	2^{8-2} 123	2^{8-2} 124	2^{8-2} 134	2^{8-2} 234	2^{8-2} 123456

This is a generator. It tell you plus minus three equal one, two. That means that you build a full factorial matrix of two column, one, two, and you constrict the third column by having three equal one, two, or three equal minus one, two. You can take the positive of the negative and it creates you the two parts. So you are dividing the you are dividing the full factorial in two. And it gives you the two parts. And you have a Roman number three. So that means that you are in this type of design. If it's a resolution is three, that means that you have words of three letters. So that means all you have the constant with or it's an end, not an or. You have the main effect which is mixed with the three by three instructions. And you have three letters. You can have one letter and two letter and you have the main effects that are mixed with the two by two instructions. It's what means this Roman three. It gives you the quality. It's not a good quality or design because you are mixing main effects with instructions. It's a good step is exactly the same quality as the plaquette and Berman design. It's very nice for estimating main effect. If you think that your interaction probably doesn't exist, but if they exist is not a good design at all.

notes

summary

6m 1s



4.4.13 The table of 2^{n-p} designs

Factors

Runs

	3	4	5	6	7	8	9	10	11
4	2^{3-0} I								
8	2^{4-1} I, II	2^{4-2} I, III	2^{4-2} II, III	2^{4-3} I, II, III					
16	2^{5-1} I, II, III, IV	2^{5-2} I, III, IV	2^{5-2} II, III, IV	2^{5-3} I, II, III, IV	2^{5-4} I, II, III, IV, V				
32	2^{6-1} I, II, III, IV, V, VI	2^{6-2} I, III, IV, V, VI	2^{6-2} II, III, IV, V, VI	2^{6-3} I, II, III, IV, V, VI	2^{6-4} I, II, III, IV, V, VI, VII	2^{6-5} I, II, III, IV, V, VI, VII, VIII			
64	2^{7-1} I, II, III, IV, V, VI, VII, VIII	2^{7-2} I, III, IV, V, VI, VII, VIII	2^{7-2} II, III, IV, V, VI, VII, VIII	2^{7-3} I, II, III, IV, V, VI, VII, VIII	2^{7-4} I, II, III, IV, V, VI, VII, VIII, IX	2^{7-5} I, II, III, IV, V, VI, VII, VIII, IX, X	2^{7-6} I, II, III, IV, V, VI, VII, VIII, IX, X, XI		
128	2^{8-1} I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII	2^{8-2} I, III, IV, V, VI, VII, VIII, IX, X, XI, XII	2^{8-2} II, III, IV, V, VI, VII, VIII, IX, X, XI, XII	2^{8-3} I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII	2^{8-4} I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII, XIII	2^{8-5} I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII, XIII, XIV	2^{8-6} I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII, XIII, XIV, XV	2^{8-7} I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII, XIII, XIV, XV, XVI	

But eventually you are not interested by only four experiments.

notes

summary

7m 46s



	6	7	8	9	10
12	2^{6-3}_{III}	2^{7-4}_{III}			
13	$\pm 4=12$ $\pm 5=13$ $\pm 6=23$	$\pm 4=12$ $\pm 5=13$ $\pm 6=23$ $\pm 7=123$			
	2^{6-2}_{IV}	2^{7-3}_{IV}	2^{8-4}_{IV}	2^{9-5}_{III}	2^{10-6}_{III}
		$\pm 5=123$ $\pm 6=234$	$\pm 5=234$ $\pm 6=134$ $\pm 7=123$	$\pm 5=123$ $\pm 6=234$ $\pm 7=134$ $\pm 8=124$	$\pm 5=123$ $\pm 6=234$ $\pm 7=134$ $\pm 8=124$ $\pm 9=1234$

You have money for making eight. So with three factor, you can make a full factorial design. So you have a two power three or you are rich or your director is rich. And you are money for making 16 experiment. And you can make two time a full factorial design and three time and four time, etc. So for the first column is not the most interesting. It became quite interesting if we go. Let's see, for example, for seven factors. Most even your boss probably doesn't know how to solve this problem. For most of people, seven factors, they will tell it's a lot. We don't know what to do with seven factor was the fractional factor design. You can manage a situation with seven factor. So the first choice you have with eight experiments. So you have a design which is not a very good quality. You see the the the the Roman three indicating that it's a resolution three, not very high resolution. It's a two seven minus four. Is really a comparable to an Adam are eight run. And it give you the generator. So you have four generator minus four. That means that you have four generators. It's not the only one that exists, but this are OK. So four equal one, two, five equal one, three, six equal two, three and seven equal one, two, three. For any reason, if you don't like it, if you mixed what is it left of the equal sign to right to equal sign is also OK.

notes

summary

7m 52s



4.4.13 The table of 2^{n-p} designs

Factors

Runs

	1	2	3	4	5	6	7	8	9	10	11
4	2^{3-1} 2 ³										
8	2^{7-4} 2 ³	2^{7-5} 2 ²	2^{7-6} 2 ¹	2^{7-7} 2 ⁰	2^{7-8} 2 ⁰	2^{7-9} 2 ⁰	2^{7-10} 2 ⁰	2^{7-11} 2 ⁰			
16	2^{15-10} 2 ⁵	2^{15-11} 2 ⁴	2^{15-12} 2 ³	2^{15-13} 2 ²	2^{15-14} 2 ¹	2^{15-15} 2 ⁰	2^{15-16} 2 ⁰	2^{15-17} 2 ⁰	2^{15-18} 2 ⁰	2^{15-19} 2 ⁰	2^{15-20} 2 ⁰
32	2^{31-20} 2 ¹¹	2^{31-21} 2 ¹⁰	2^{31-22} 2 ⁹	2^{31-23} 2 ⁸	2^{31-24} 2 ⁷	2^{31-25} 2 ⁶	2^{31-26} 2 ⁵	2^{31-27} 2 ⁴	2^{31-28} 2 ³	2^{31-29} 2 ²	2^{31-30} 2 ¹
64	2^{63-40} 2 ²³	2^{63-41} 2 ²²	2^{63-42} 2 ²¹	2^{63-43} 2 ²⁰	2^{63-44} 2 ¹⁹	2^{63-45} 2 ¹⁸	2^{63-46} 2 ¹⁷	2^{63-47} 2 ¹⁶	2^{63-48} 2 ¹⁵	2^{63-49} 2 ¹⁴	2^{63-50} 2 ¹³
128	2^{127-80} 2 ⁴⁷	2^{127-81} 2 ⁴⁶	2^{127-82} 2 ⁴⁵	2^{127-83} 2 ⁴⁴	2^{127-84} 2 ⁴³	2^{127-85} 2 ⁴²	2^{127-86} 2 ⁴¹	2^{127-87} 2 ⁴⁰	2^{127-88} 2 ³⁹	2^{127-89} 2 ³⁸	2^{127-90} 2 ³⁷

You can also say that four equal one, two, three and you can change. But what is important that your your different. Product here are the things that you want and you associate them to the four additional column that you want. What you call what you call four, what you call five or six have no importance is is the combinator, which is the important things.

notes

summary

10m 1s



4.4.13 The table of 2^{n-p} designs

Factors

Runs

	1	2	3	4	5	6	7	8	9	10	11
1	2^{11-1}	2^{11-2}	2^{11-3}	2^{11-4}	2^{11-5}	2^{11-6}	2^{11-7}	2^{11-8}	2^{11-9}	2^{11-10}	2^{11-11}
2	2^{11-1}	2^{11-2}	2^{11-3}	2^{11-4}	2^{11-5}	2^{11-6}	2^{11-7}	2^{11-8}	2^{11-9}	2^{11-10}	2^{11-11}
3	2^{11-1}	2^{11-2}	2^{11-3}	2^{11-4}	2^{11-5}	2^{11-6}	2^{11-7}	2^{11-8}	2^{11-9}	2^{11-10}	2^{11-11}
4	2^{11-1}	2^{11-2}	2^{11-3}	2^{11-4}	2^{11-5}	2^{11-6}	2^{11-7}	2^{11-8}	2^{11-9}	2^{11-10}	2^{11-11}
5	2^{11-1}	2^{11-2}	2^{11-3}	2^{11-4}	2^{11-5}	2^{11-6}	2^{11-7}	2^{11-8}	2^{11-9}	2^{11-10}	2^{11-11}
6	2^{11-1}	2^{11-2}	2^{11-3}	2^{11-4}	2^{11-5}	2^{11-6}	2^{11-7}	2^{11-8}	2^{11-9}	2^{11-10}	2^{11-11}
7	2^{11-1}	2^{11-2}	2^{11-3}	2^{11-4}	2^{11-5}	2^{11-6}	2^{11-7}	2^{11-8}	2^{11-9}	2^{11-10}	2^{11-11}
8	2^{11-1}	2^{11-2}	2^{11-3}	2^{11-4}	2^{11-5}	2^{11-6}	2^{11-7}	2^{11-8}	2^{11-9}	2^{11-10}	2^{11-11}
9	2^{11-1}	2^{11-2}	2^{11-3}	2^{11-4}	2^{11-5}	2^{11-6}	2^{11-7}	2^{11-8}	2^{11-9}	2^{11-10}	2^{11-11}
10	2^{11-1}	2^{11-2}	2^{11-3}	2^{11-4}	2^{11-5}	2^{11-6}	2^{11-7}	2^{11-8}	2^{11-9}	2^{11-10}	2^{11-11}
11	2^{11-1}	2^{11-2}	2^{11-3}	2^{11-4}	2^{11-5}	2^{11-6}	2^{11-7}	2^{11-8}	2^{11-9}	2^{11-10}	2^{11-11}

It's just a choice. Again, you are dividing your design in sixteenths set subsets. And so the plus minus signs let you decide which subset you would like to use. A priori, they are all equivalent. In reality, perhaps some could be better. Typically, you are making alias. If you are making alias between adding alias between something that are all positive or all negative, it's OK. You are increasing your effects. The risk that you are making alias with something which is positive and negative. The risk that two effects are compensated, but you don't know this before. Except if you have some information about your instructions, but I never see people taking this care before. Again, you have choice between 16 subsets. You choose the one that you want. Most of people usually is a plus plus plus one by sake of simplicity, by sake of not thinking to something. So this was not so interesting with resolution three. But in fact, eventually you have sufficient money for making 16 experiments from scratch. So you take the other design. You can take the seven minus three. You see it's of resolution four. You see this for the roman four. So that means that you have four letters. If you have four letters, that means that you have the constant which is mixed with the instruction four by four. That means that you have the main effects that are mixed with instructions three by three. And that means that instructions two by two are mixed with the twin term. So it's interesting. You have the instruction two by two mixed. If you don't have so many, eventually you would be able to guess which one is which one. But in any case, you have the main effects that are mixed only with three by three. That's usually you neglect. So the four are usually the first one we consider

notes

summary

10m 30s



4.4.13 The table of 2^{n-p} designs

Factors

Runs

	1	2	3	4	5	6	7	8	9	10	11
4	2^{11-4} (1,2,3,4)										
8	2^{11-5} (1,2,3,4,5)										
16	2^{11-6} (1,2,3,4,5,6)										
32	2^{11-7} (1,2,3,4,5,6,7)										
64	2^{11-8} (1,2,3,4,5,6,7,8)										
128	2^{11-9} (1,2,3,4,5,6,7,8,9)										

when we made fractional factorial design. The Rolls Royce is the fifth. You see here the next one is also a four. Less mixture between the two by two, but still mixture between the two by two. And after it makes 64 run, it's a resolution seven, which is that makes no problem. You have all your main effect, all your instruction two by two and seven. So that means that the three by three are mixed with the four by four. So even in this case, you have all the three by three. It's quite an expensive one. A very good design is a resolution five. This is a really good design. So you see one. You don't have so many, but you see one. One here. Yeah, you don't see so many. You see one here. Why? Because you have the constant, which is mixed with five by five, not a problem. The main effects, which makes with four by four, not a problem. The two by two mixed with three by three. Perfect. So we neglect those three by three. That means you have everything you usually want. They are really good. They are the optimal design that we appreciate it. So resolution five, it's the best tradeoff between not making too much experiment and getting everything that I would like without aliases that I don't want. And it's also interesting to work with this table horizontally, not only vertically.

notes

summary

← Factors

Runs

notes

summary

15m 1s



4.4.14 The resolution of a design

- R=III** No alias between the main effects a_i ,
Aliases of the main effects a_i with first order interactions a_{ij} .
- R=IV** No alias between the main effects a_i ,
No alias between the main effects a_i and first order interactions a_{ij} ,
Aliases between the first order interactions a_{ij} .
- R=V** No alias between the main effects a_i ,
No alias between the main effects a_i and first order interactions a_{ij} ,
No alias between the first order interactions a_{ij} .
Aliases of the first order interactions a_{ij} with second order interactions a_{ijk} .

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But in experiments, usually we don't work with so many factors. But in sensitivity analysis, numerical experiments, sometimes it's interesting to go further than 11 factors.

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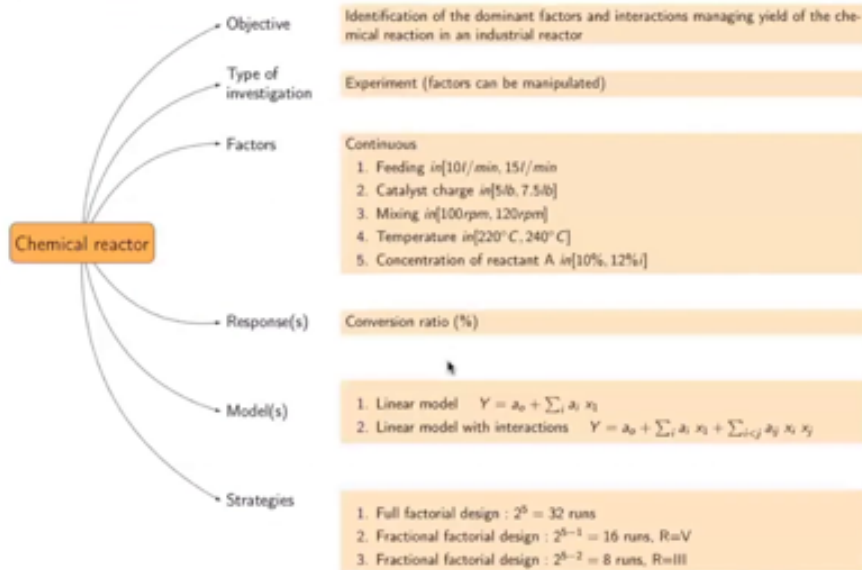
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17m 11s



4.4.15 Chemical reactor with 5 factors



Dr Jean-Marie Fürbringer

Modelling and design of experiments

OK, so this is what I already explained you is the explanation of the resolution. Usually we work with this three level of resolution, resolution three, like blackhead Berman design, it's a good start. Or if you know that you don't have instruction, probably you have only a few instructions, it could be interesting. Resolution four, it could be an interesting start. And reasons five, it's really very practical. The good balance between economy of your other numbers of experiments and the quality of your contrast.

notes

summary

17m 25s



4.4.16 Runs and aliases

- ▶ We choose the 2^{5-1}_V plan
- ▶ We choose as generator 5=1234
- ▶ The plan is
- ▶ The plan is of resolution V and allows thus to evaluate the 5 main effects and the 10 first order interactions

Matlab

```
>> [X,conf]=fracfact('bcde b c d e','MaxInt',2)
```

X =

1	-1	-1	-1	-1
-1	-1	-1	-1	1
-1	-1	-1	1	-1
1	-1	-1	1	1
-1	-1	1	-1	-1
1	-1	1	-1	1
1	-1	1	1	-1
-1	-1	1	1	1
-1	1	-1	-1	-1
1	1	-1	-1	1
1	1	-1	1	-1
-1	1	-1	1	1
1	1	1	-1	-1
-1	1	1	-1	1
-1	1	1	1	-1
1	1	1	1	1

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conf =

16x3 cell array

Term	Generator	Confounding
'X1'	'bcde'	'X1'
'X2'	'b'	'X2'
'X3'	'c'	'X3'
'X4'	'd'	'X4'
'X5'	'e'	'X5'
'X1*X2'	'cde'	'X1*X2'
'X1*X3'	'bde'	'X1*X3'
'X1*X4'	'bce'	'X1*X4'
'X1*X5'	'bcd'	'X1*X5'
'X2*X3'	'bc'	'X2*X3'
'X2*X4'	'bd'	'X2*X4'
'X2*X5'	'be'	'X2*X5'
'X3*X4'	'cd'	'X3*X4'
'X3*X5'	'ce'	'X3*X5'
'X4*X5'	'de'	'X4*X5'

So let's look at a case. We come back to the chemical reactor we have used in the full factorial experiment. So I don't spend time explaining the. My mind map is quite the same thing, but we are interesting in the strategy. Now we have more strategy. We have made an experiment with. The previous chapter with 32 experiment, we can eventually divided by two or divided by four. We have divided by two.

notes

summary

18m 3s



- ▶ We choose as generator 5=1234
- ▶ The plan is

matlab

```
[X,conf]=fracfact('bcde b c d e','MaxInt',2)
```

conf

16x

1	-1	-1	-1	-1
-1	-1	-1	-1	1
-1	-1	-1	1	-1
1	-1	-1	1	1
-1	-1	1	-1	-1
1	-1	1	-1	1
1	-1	1	1	-1
-1	-1	1	1	1
1	1	1	1	1

So I would like to choose two power. Five minutes one, and it is a region five. So very good design. I don't need to care too much about the aliases. In fact, I don't care at all. My generator, I find it in the table of box or I find it in my routines in Matlab. Five equal one, two, three, four. It was not very difficult to guess. And I have used the routine of FractFact. So see how I have used the routine FractFact. Because I use things published by box, I wanted to have exactly the same design as you have. So it's why I have not used five. It's the same one. I didn't have used five equal one, two, three, five. I've used one equal two, three, four, five, which is exactly the same generator. So I tell Matlab that my generator is BCDE, which is the same thing as two, three, four, five. And I say that the maximum interaction that I would like to consider is interaction two by two. And I have given it's the old fashion.

notes

summary

18m 44s



4.4.16 Runs and aliases

- ▶ We choose the 2^{5-1}_V plan
- ▶ We choose as generator 5=1234
- ▶ The plan is

- ▶ The plan is of resolution V allows thus to evaluate the effects and the 10 first order interactions

Matlab

```
>> [X,conf]=fracfact('bcde b c d e','MaxInt',2)
```

X =

1	-1	-1	-1	-1
-1	-1	-1	-1	1
-1	-1	-1	1	-1
1	-1	-1	1	1
-1	-1	1	-1	-1
1	-1	1	-1	1
1	-1	1	1	-1
-1	-1	1	1	1
-1	1	-1	-1	-1
1	1	-1	-1	1
1	1	-1	1	-1
-1	1	-1	1	1
1	1	1	-1	-1
-1	1	1	-1	1
-1	1	1	1	-1
1	1	1	1	1

conf =

16x3 cell array

'Term'	'Generator'	'Confounding'
'X1'	'bcde'	'X1'
'X2'	'b'	'X2'
'X3'	'c'	'X3'
'X4'	'd'	'X4'
'X5'	'e'	'X5'
'X1*X2'	'cde'	'X1*X2'
'X1*X3'	'bde'	'X1*X3'
'X1*X4'	'bce'	'X1*X4'
'X1*X5'	'bcd'	'X1*X5'
'X2*X3'	'bc'	'X2*X3'
'X2*X4'	'bd'	'X2*X4'
'X2*X5'	'be'	'X2*X5'
'X3*X4'	'cd'	'X3*X4'
'X3*X5'	'ce'	'X3*X5'
'X4*X5'	'de'	'X4*X5'

I have to change the slide. I wanted the E-matrix, not the X-matrix, is the E-matrix that is giving me. And the conf is the contrast.

notes

summary

20m 13s



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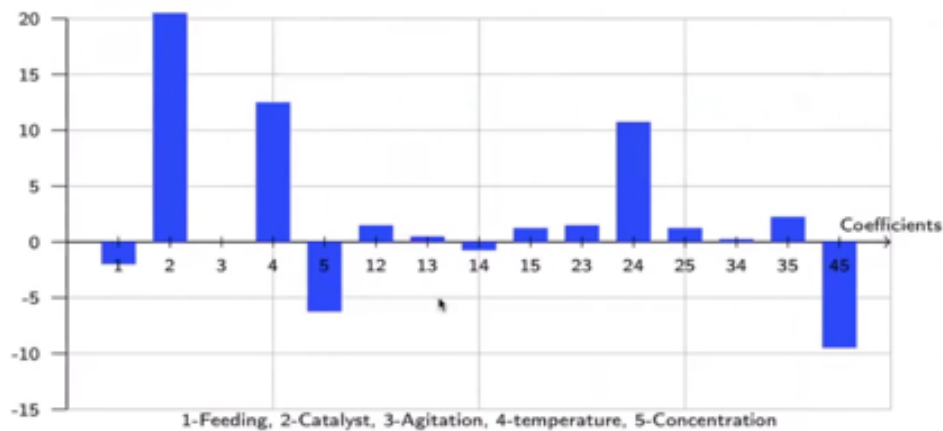
Dr Jean-Marie Fürbringer Modelling and design of experiments

notes

20m 29s



4.4.18 Standardized effects



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So I have my design. So you have to understand what means plus minus. So minus means the minimum and plus means the maximum of my different variable. You remember in this experiment, I have the feeding of the catalyst, the quantity of feeding, the catalyst, the agitation, the temperature, the concentration. It's treated in the case, the variable are not exactly the same in the previous case. If you check the previous case, they have changed a little bit. The fact is not exactly the same factor. So you have your matrix of experiment. Remember, we realize those designs not in the order that give you an algorithm. You realize them in a random order. And after I have my rate as a solution.

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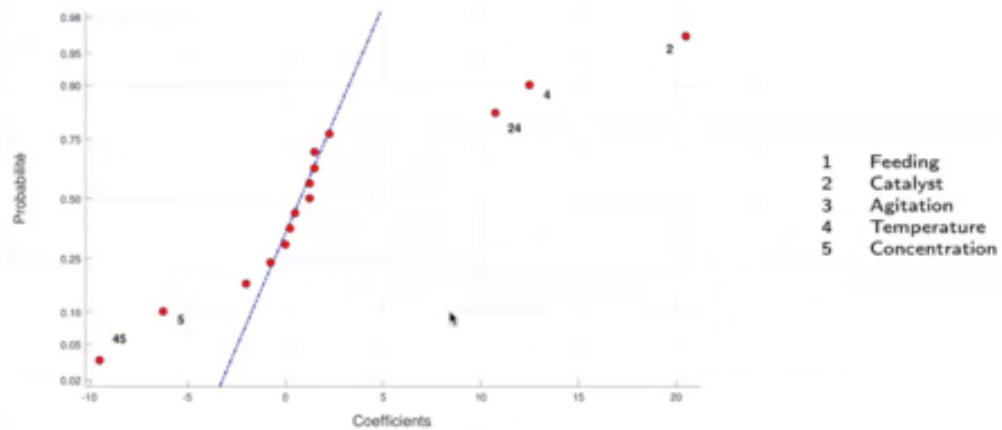
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21m 29s



4.4.19 Normal plot of the 2^{5-1} design



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Modelling and design of experiments

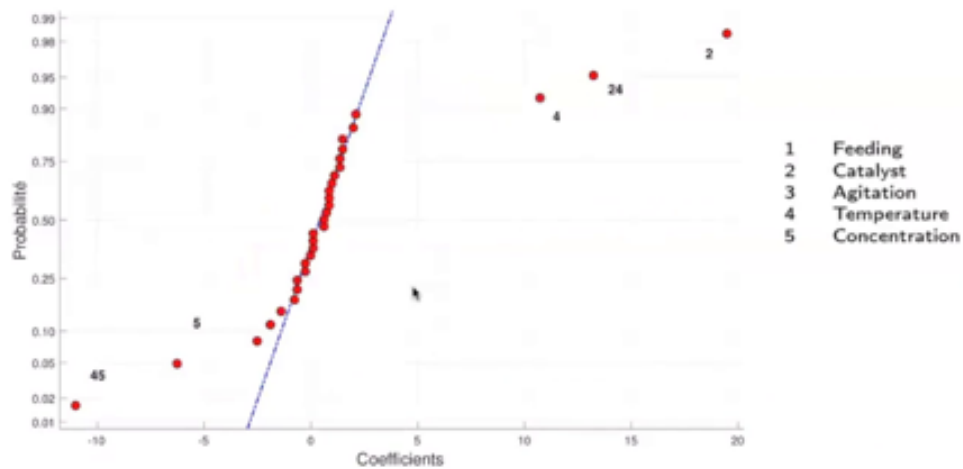
So when I have that, I'm able to make a graphic of my different effect as I have good aliasing. I'm not afraid of the aliasing. And you see what is important and what is not.

notes

summary

22m 21s



4.4.20 Normalplot of the 2^5 

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And then I can make a normal plot and you see that I have a few coefficients that are significant. There's a few ones that are not significant, that are all in the center. So you see that with 16 experiments, in fact, I just have five coefficients that are significant. It's the concentration of the catalyst, the temperature and is the mixture between the temperature and the concentration and between the catalyst and the temperature. So this explains why it works. Because I'm mixing things in this case, not too much, but even if I interact with a resolution for this case, it could have worked. Because a lot of things, all the time, the Pareto principle are in the noise. So what interests you have to come out of the noise and it will be a minimum of coefficients. So look, this is what I get with the fractional factorial design.

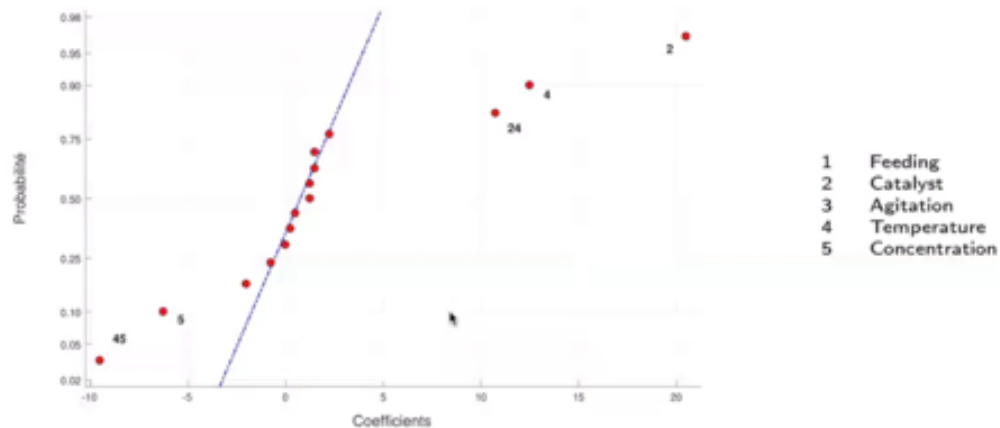
notes

summary

22m 37s



4.4.19 Normal plot of the 2^{5-1} design



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Modelling and design of experiments

And this is what I would get with the full factorial experiment. I would have a better precision on my noise and not on my factors. So this is the key point of fractional factorial design. Don't lose time presizing what is not important. Stick on presizing what is important. If it would not have been the case that in fact every factor would have been important, no problem. No problem. Your normal plot reveals that in fact you cannot eliminate any coefficients. All coefficients are outside of the noise. And that means that if you have aliases, well, you have to deal with them and continue. You are not losing time. It's just that you are taking steps with the risk and the probability that in fact just one step will be necessary and not the second, the four steps. This is really the philosophy of design of experiments. Why I tell you that this chapter is really the core of design of experiments.

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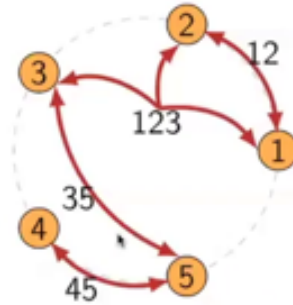
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23m 51s



4.4.20 A smart Pareto analysis

How DOE helps to determine interactions



This is the good news. The bad news that it doesn't work all the time exactly like that. And the case in which really the situation, the real situation in which you can really apply, you have your factor like that, they are not so common. Each time you have very specific factors. One is quantitative, the other is qualitative. One is really important to have three levels. Two level doesn't work, etc. So I'm conscious that the good recipe is not applying all the time, but at least it could inspire you and they are part of your research that could be really stick to the good recipe.

notes

summary

25m 13s



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4.4.20 Case presentation

Factors
width 1 : x_1 →
radius 1 : x_2 →
volume 1 : x_3 →
volume 2 : x_4 →
temperature 2 : x_5 →

► During a technology validation of production is analysed (and their potential interactions) of a medical device.
► There are 5 factors, x_i
► There is one answer, y , to

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So when you have interactions, it's and you have a lot of factors. It could be quite complicated to put order in that very rapidly. Even if you eventually you have also determined some three by three interaction, it became really a nightmare to see what is important, what is not important and how to manage that. So I propose you something I call a smart Pareto principle.

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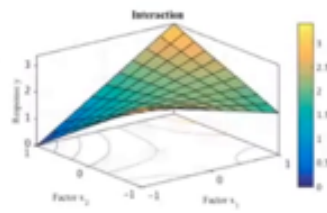
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26m 0s



4.4.20 Empirical model

$$y(\vec{x}) = a_0 + a_1x_1 + a_2x_2 + a_{12}x_1x_2 + \epsilon$$



- In this case, the target model has 16 coefficients

$$y = a_0 + \sum_{i=1}^5 a_i x_i + \sum_{i>j}^5 a_{ij} x_i x_j$$

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So this was I take out of one of my conferences. I imagine that you have a production process and of a medical device and you are interested in. Increasing the life duration of your of your device. So and you when you make the analysis, you see that you have five interesting factors. So this is the system that you have.

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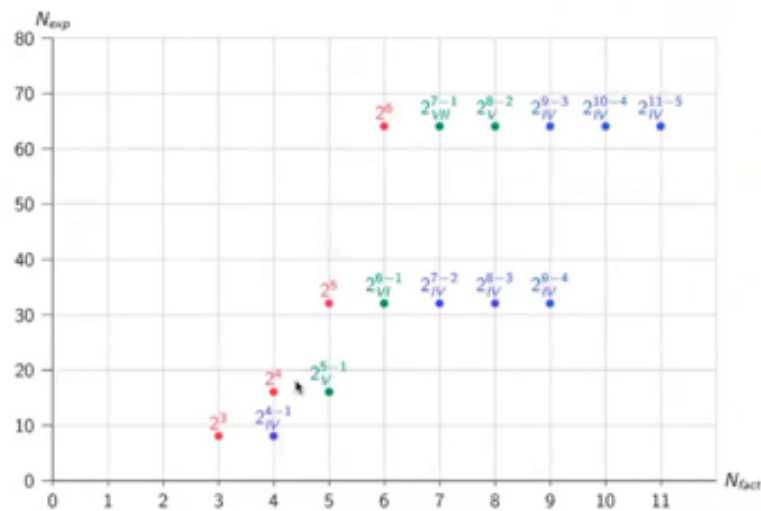
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26m 27s



4.4.20 Fractional factorial designs



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So you use an empirical model for looking at the data. So I just write two factor. A model because I wanted to simplify, but in this case we have five five five coefficients. So the the model for our situation is this one. Quite complicated model, a lot of coefficients.

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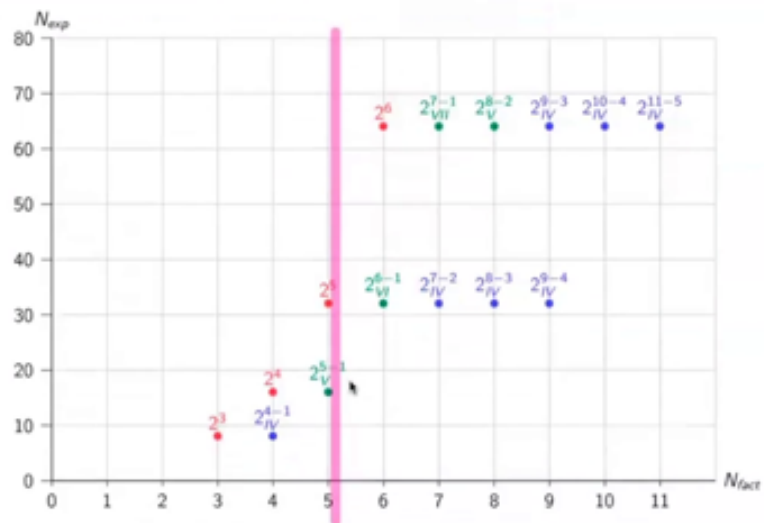
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27m 9s



4.4.20 Fractional factorial designs



So what are the choice that you have? So if you have five coefficient, then you have the choice between in this case a full factorial design or a fractional factorial design.

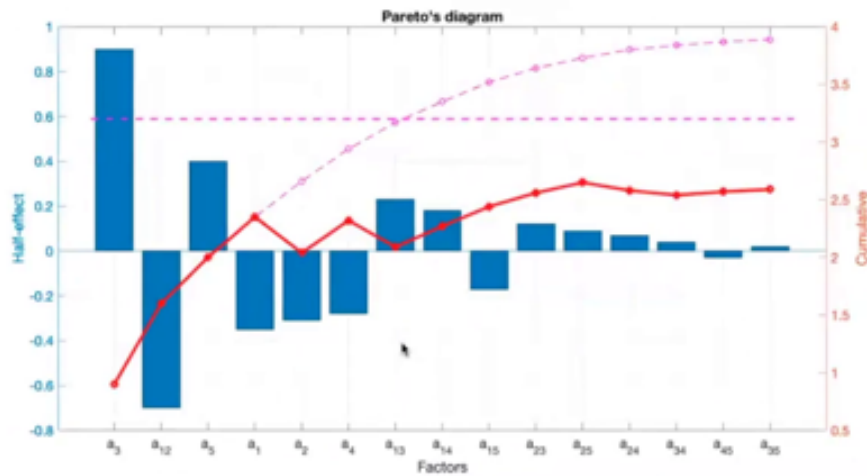
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4.4.20 Pareto Diagram



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Five minutes one after we'll show you another design and additional design also exists. So those are the different experiments that you can do.

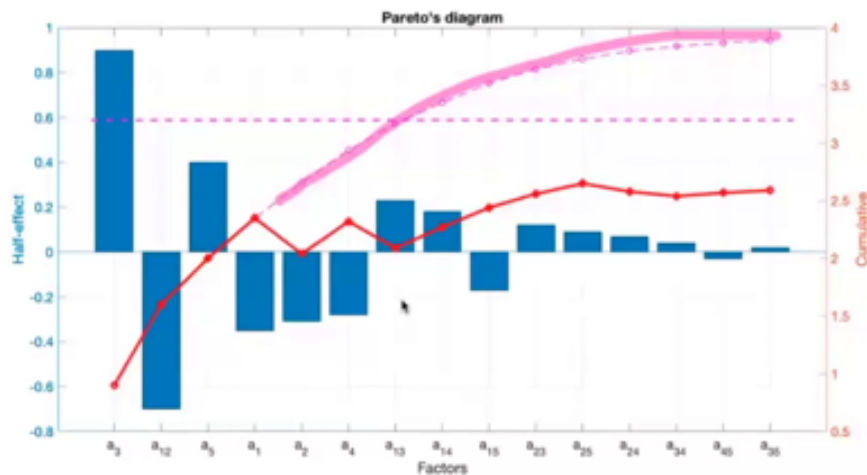
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28m 7s



4.4.20 Pareto Diagram



And then you get your effects and is where this smart Pareto principle apply. So I have ordered my effect. If you look by order of importance in the absolute value, they go from the highest absolute value. I'm not considering the a zero because a zero appear all the time. So I'm just playing with the effects with the coefficients. So the most important coefficient in absolute value was the third one a three after it was the interaction one two after became the effect of five. The effect of one of two of four of one three of one four one five, etc. You see they are becoming smaller and smaller. OK. The first line interesting to check is the cumulative. So this would be what you can get as improvement of your life duration. So you're interesting of improving. I hope you are good engineers. You don't want to diminish the life duration, but you want to increase the life duration of your devices. So this is a this line is a theoretical line. Just the addition of the effects that could bring you to the maximum duration life. But this can will not work just like that because you have instructions and detractions. You have counter effects. You have some instructions. It could be of different signs of the main effect. And this what I'm presenting to you is exactly for putting order in that and very easily understand what is practical.

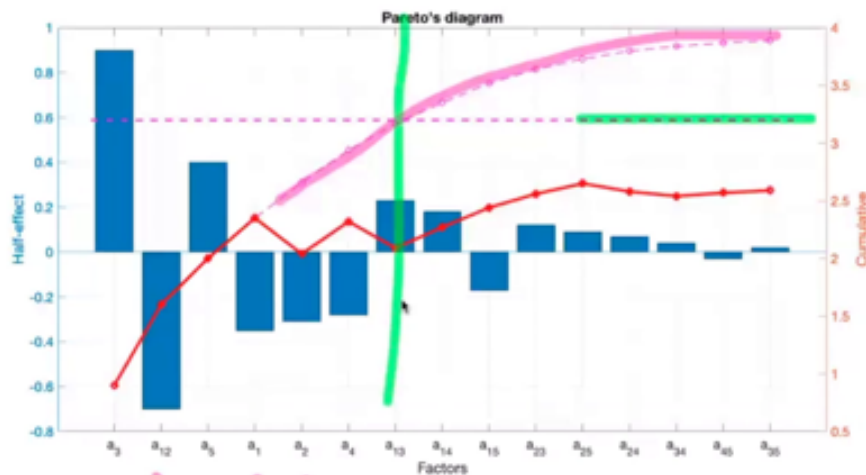
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28m 16s



4.4.20 Pareto Diagram



Or you can also use an optimization algorithm. But this is for my point of view is more didactic than an optimization algorithm. So what you have to do, you have to consider things from left to right. So first you have the factors three. The factor the coefficient three is positive is first with other interaction involving factors three. So if you want to increase the life duration, you have to take the second value that you have taken because the positive effect means that when you are going from minus one to plus one, you are increasing your answer. After you have the instruction one, two. So the instruction one, two is before the effect one and before the effect two. And it's negative. That means that if you want to take profit of this negative value, you need to have the product of X one and X two, which is negative because negative with negative will make positive. So if you want to have the sign X one X two negative, that means that both have to have different signs. So you need to go to see. So you have to check this with one and two and one you observe is that the effect of X one and X two are negative. So that means that if you take both the negative, the minus one for them, the product will be positive. And that means that you will have the interaction that will be negative. And you wanted positive. That means that you have to sacrifice one of the main effects for getting the positive aspect of the interaction. So you understand that you have to sacrifice the last one. A two. Because it's the smallest one. And in any case, it will be smaller than the addition of A one, two and A two. So it's what you see in this red line, which is

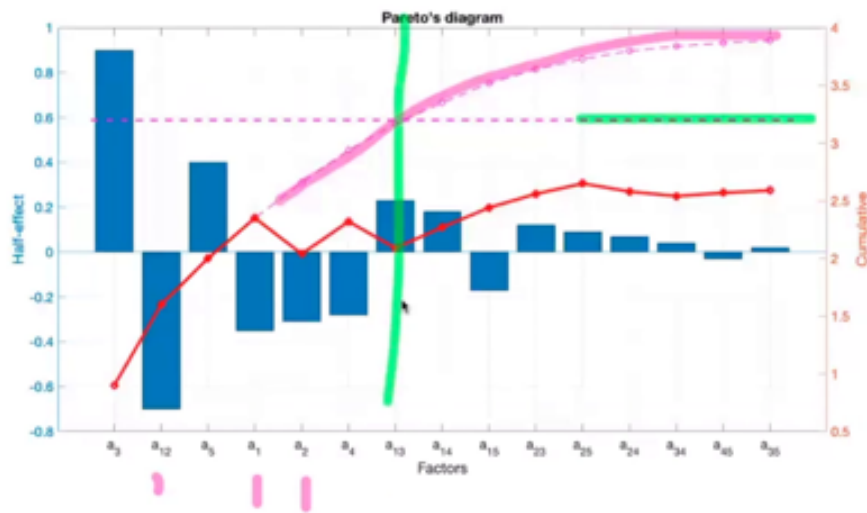
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30m 13s



4.4.20 Pareto Diagram



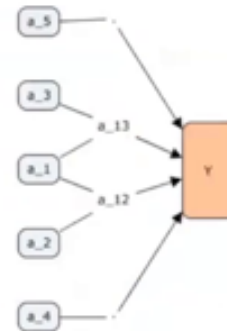
trying to take the maximum profit, considering prioritizing what's come from left in consideration of what could come at right later. Did you understand the mechanism? Quite not in the eyes. I'm not constantly retake it by yourself. So you have first the purple line that was a theoretical improve with my effect, but I cannot do that. Do that. Is the red line that is in fact my real improvement. And you see also that usually when you make Pareto, you cut at 80%. So this green, this line was 80% of the maximum improvement that I could get. But in fact, it's no value. The Pareto principle in this case is not so interesting.

notes

summary

4.4.20 Conclusion for this case

- Interactions are key information for managing a process
- In this case, the use of a DOE technique brings efficiency in term of determining the interactions and permitting a smart Pareto analysis.
- The choice of the adequate design let's us minimizing the number of experiments
- DOE expertise allows the experimenters to determine before the experiments which coefficients are calculable and also the size of their confidence interval in function of the precision of the measurement technique



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If I would apply the Pareto principle, I will cut my management at that level and say, OK, the rest is not so much, so much important. In this case, what you see that you have interest to manage things still here after the change have not influence. So it's what I call it smart Pareto. It's a diversion from the Pareto principle, which is usually good. But when you have interactions, you cannot just apply blindly because the interaction could play against you.

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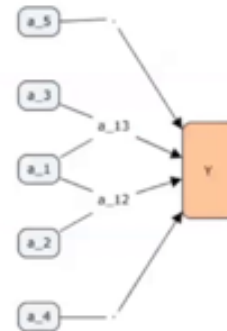
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33m 57s



4.4.20 Conclusion for this case

- Interactions are key information for managing a process
- In this case, the use of a DOE technique brings efficiency in term of determining the interactions and permitting a smart Pareto analysis.
- The choice of the adequate design let's us minimizing the number of experiments
- DOE expertise allows the experimenters to determine before the experiments which coefficients are calculable and also the size of their confidence interval in function of the precision of the measurement technique



And this let us finish the example with a model explaining what is important. And in this case, it was possible to show that what is important in this slide is what is right. So I have five factors. Some factors have a direct influence on the life duration of my device. And I see that I have three factors that have interactions that manage in fact the device.

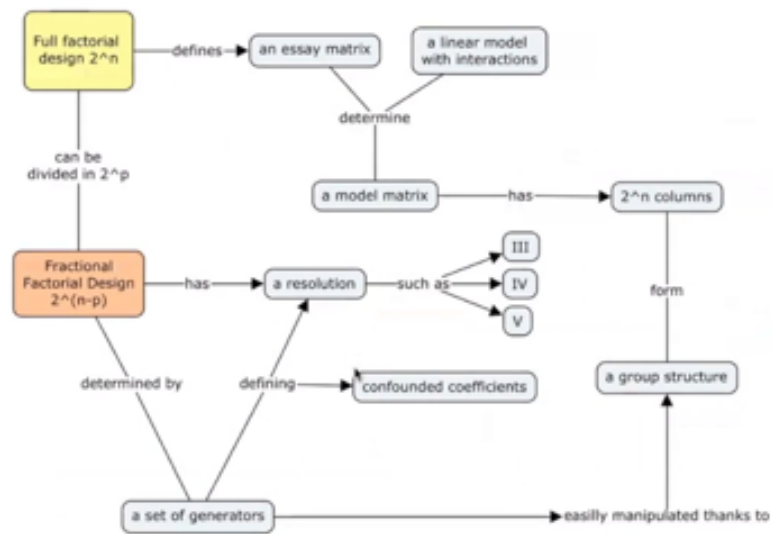
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34m 37s



4.4.21 Summary fractional factorial design



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So it's let make you a model which is more robust.

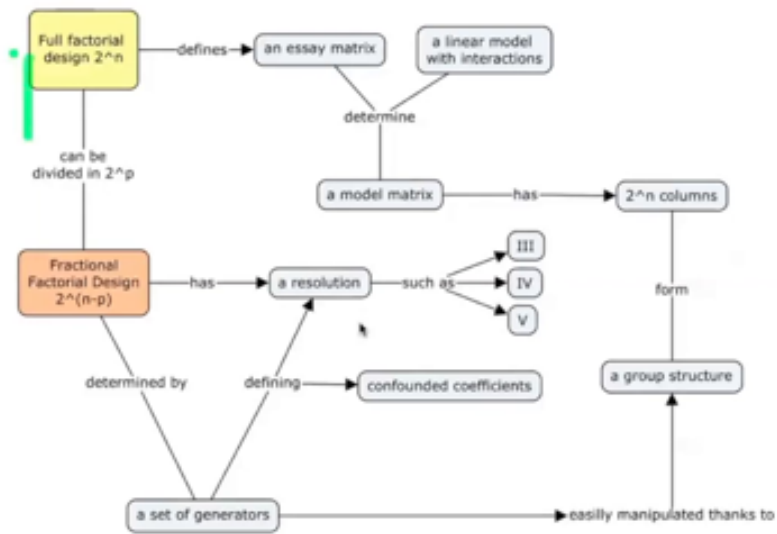
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summary

35m 13s



4.4.21 Summary fractional factorial design



This is a summary of what we have done. So we have starts with the full factorial design from the full factorial design.

notes

summary

35m 18s



4.4.22 Summary fractional factorial design

- ▶ **Definition** : A fractional factorial design is a statistical technique used to study the effect of a subset of factors on a response variable. Unlike full factorial designs, fractional factorial designs do not test all possible combinations of the factors, but instead test a carefully chosen subset of them.
- ▶ **Design** : In a fractional factorial design, the number of experimental runs required is a fraction of the total number of runs required for a full factorial design. The design is carefully constructed to minimize the confounding of higher-order interactions between the independent variables.
- ▶ **Analysis** : The data from a fractional factorial design is typically analyzed using an ANOVA to determine the main effects of each factor and their interactions.
- ▶ **Advantages** : Fractional factorial designs are more efficient than full factorial designs, as they require fewer experimental runs while controlling for confounding between higher-order interactions.
- ▶ **Limitations** : fractional factorial designs may not be able to identify higher-order interactions between the independent variables, which may be important in some applications. Finally, the choice of the fraction used in the design may impact the accuracy of the estimates of the effects of the independent variables.

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Modelling and design of experiments

We have been able to create fractional factorial design. When we work with full factorial design, we are able to create an essay matrix. And if we have a model, we can build a model matrix. And in this case, we have two ends columns, but these columns form a group. So when we use fractional factorial design, we are using this group structure. For getting a resolution which is smaller than the full resolution of the full factorial resolution. And we can manage the level of contrast. Now is like a picture and depending on your objective, depending on the time of exposure, you can have more or less contrast in your picture is quite the same image that we are using in this case. We have typically resolution three, four and five, haven't really hired, but what we manage is this level. Three is for Placket and Burman approach, just having your main effect or it could be just a step. When you know you will make further, but you want just to check a few things and check what could be the most important factors making the screening. Four and five. And so, but when you have a resolution, that means that you have confounded coefficient and you really need to understand what is confounded and be really ready to answer that and check what is confounded or not making counter sense interpretation of your data. But when you have that, you also need to manage a set of generator and you have that in this famous slide, the page by box of all the generator.

notes

summary

35m 33s



4.4.23 Learning outcomes of lesson on fractional factorial design

- ▶ At the end of this lesson, you must be able to select, use and analyze a fractional factorial design → table by Box
- ▶ to choose a generator
- ▶ to determine the aliases
- ▶ to build the essay matrix
- ▶ to perform the analysis of the results → Normalplot

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Modelling and design of experiments

This is a summary of the fractional factorial design. A fractional factorial design is a statistical technique used to study the effect of subset of factors on a respond variable and like full factorial design. Fractional factorial design do not test all possible combination of the factors, but instead test a carefully chosen subset of them. Design in a fractional factorial design, the numbers of experimental runs require is a fraction of the total numbers of run required in a full factorial design. The design is carefully constructed to minimize the confounding of a higher order interaction between the independent variable. For the analysis, the data from fractional factorial design is typically analyzed by an ANOVA to determine the main effects, but you need sometimes to use the normal plot because you to free some degrees of freedom when I say free for putting them in noise and not in your model. Limitation fractional factorial design may not be able to identify higher order interaction between the independent variable, which may be important in some application. Finally, the choice of the fraction using the design may impact the accuracy of the estimate effect of the independent variable again on the concept of resolution. Okay, so that was my objective. At the end of this lesson, you must be able to select, use and analyze fractional factorial design, the table by box, choose a generator, determine the aliases and build an SM matrix. Perform the analysis of the results, a normal plot. Yeah, this chapter in the exam, you have a lot of chance to have a question related to this chapter is a very key chapter of this theory. So I'm finished for today. I hope you enjoy. This chapter that I find very interesting because we play with the different elements. It's very interesting. I really encourage you to make the exercise because the theory is not, but this

notes

summary

37m 39s



4.4.23 Learning outcomes of lesson on fractional factorial design

- ▶ At the end of this lesson, you must be able to select, use and analyze a fractional factorial design → table by Box
- ▶ to choose a generator
- ▶ to determine the aliases
- ▶ to build the essay matrix
- ▶ to perform the analysis of the results → Normalplot

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Modelling and design of experiments

is very typically a chapter. If you don't do exercise in two weeks, you don't remember nothing. So I really encourage you to make exercise.

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