Course material

Course:

ENG606 / PHYS 442

Video:

DOE_lesson10_part1_RechtschaffnerDesign

Concepts (extracted from automatically generated subtitles):

Presentation of the design. Good idea. Full factorial designs. Fractional factorial design. Main effect. Map of the different designs. Factorial fractional design. Effect of factors. Interesting intermediate situation. Best dispersion matrix. Sort of screening factors. Less expensive experiment. Second place. Linear linear increase. Types of lines.

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notes

summary

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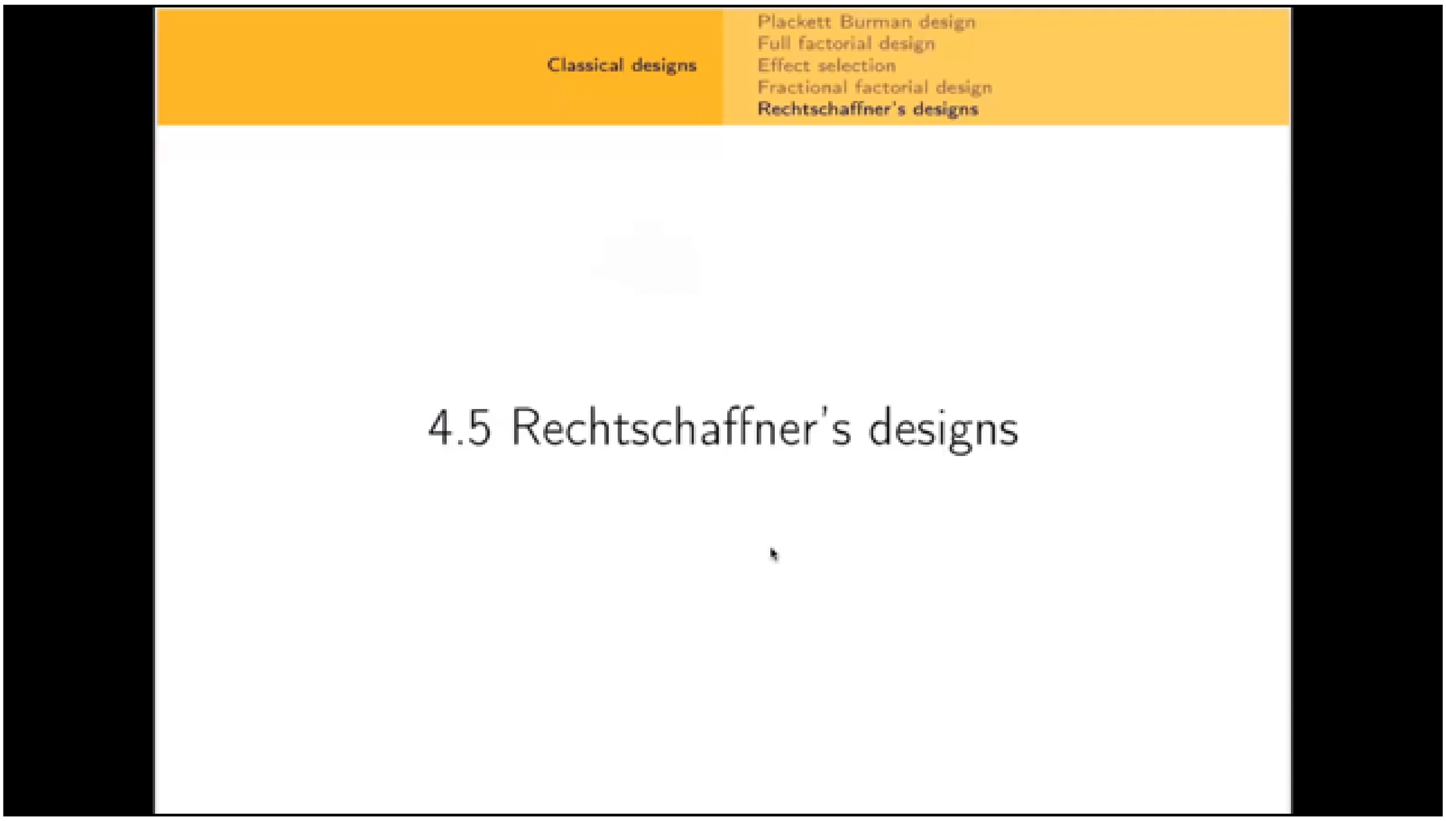
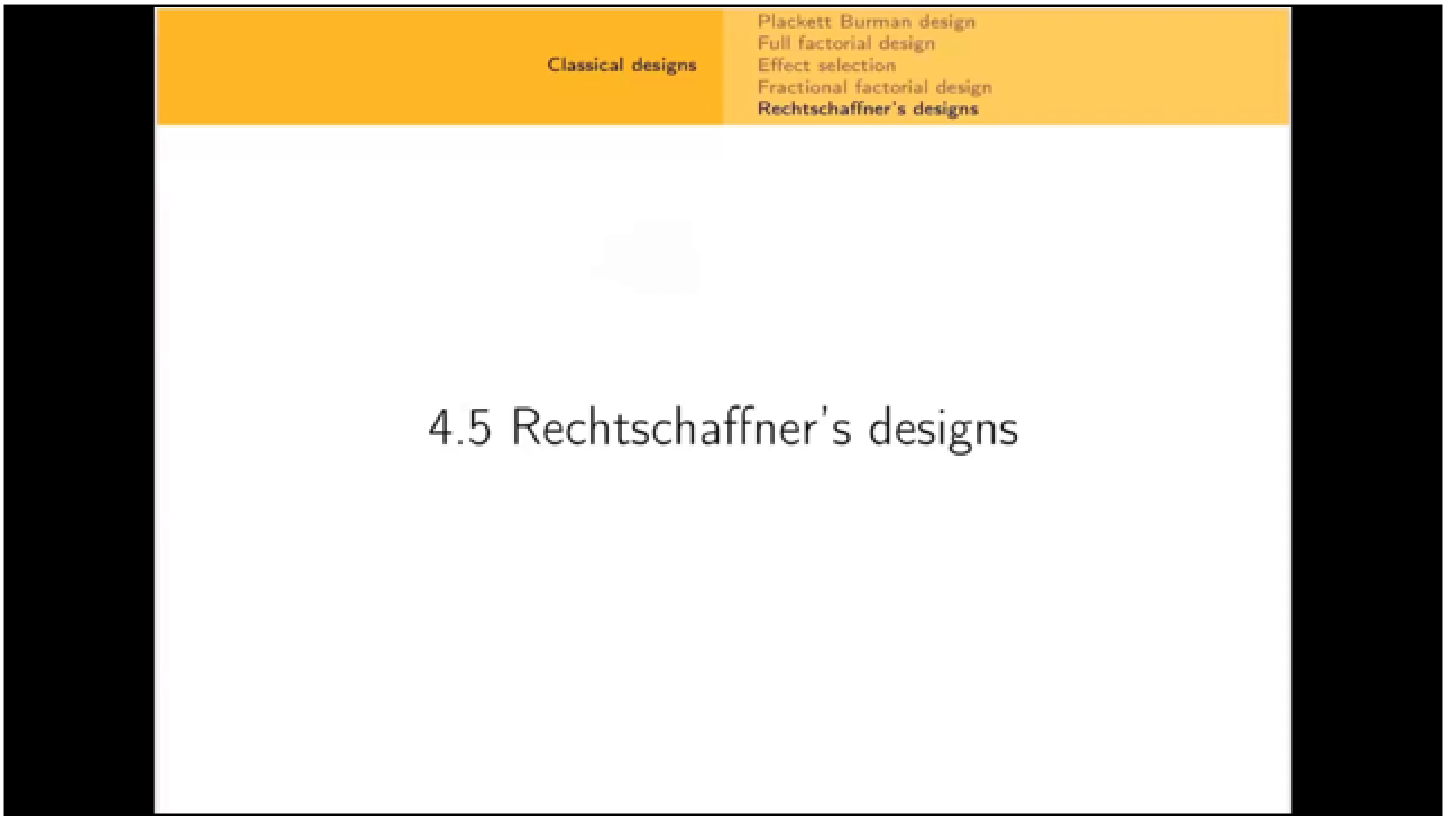


These subtitles have been generated automatically So today we have two points in the program.

notes

summary

0m 1s

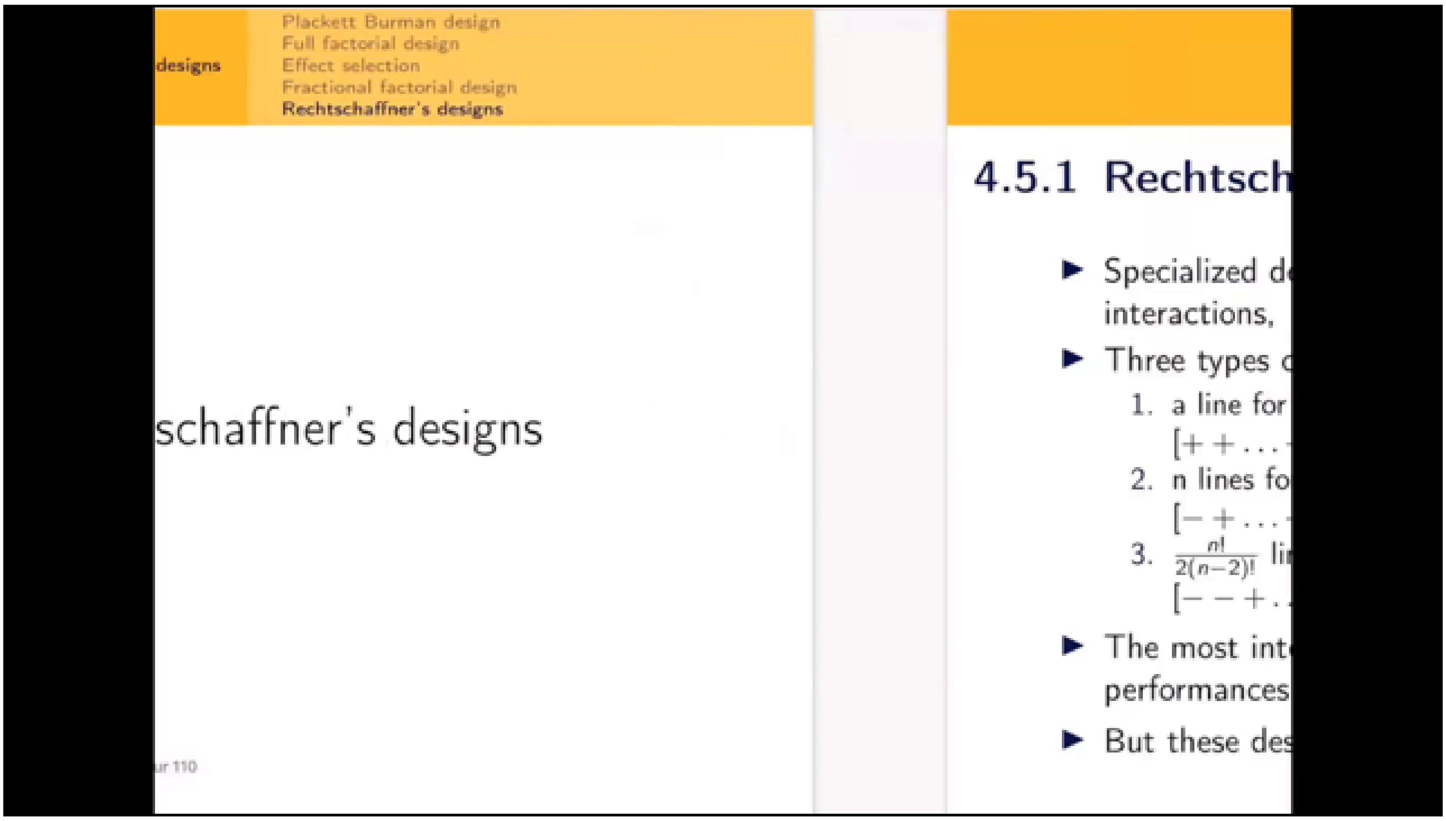


I will finish the presentation of the design that are adequate for factors for determining effect of factors and the two by two instructions. So if you remember, we start with Placket Berman that were very adequate for main effect only so when you do not have instructions, after we jump to the father of all of this or the mother of all of these designs that are the full factorial designs that are efficient for determining all the instructions with the big disadvantage of increasing the numbers of run increasing exponentially. After I show you very interesting intermediate situation with the fractional factorial design. In the fact factorial fractional design, we have seen that the resolution determine what we can really determine if we can determine interaction two by two with or without aliases. So now we will finish this chapter with two more designs. One is reschafter design and the other is a three quarter design. There are specific designs for determining main effect and interaction two by two. So they are let's say in the same family than Adamar design. There are specific lines of a full factorial designs. They have the advantage of being very efficient to that to determining main effect and interaction. So it's still a sort of screening factors but taking into account the fact that we have interactions. This is the advantage. The disadvantage is that they are not orthogonal designs.

notes

summary

0m 5s

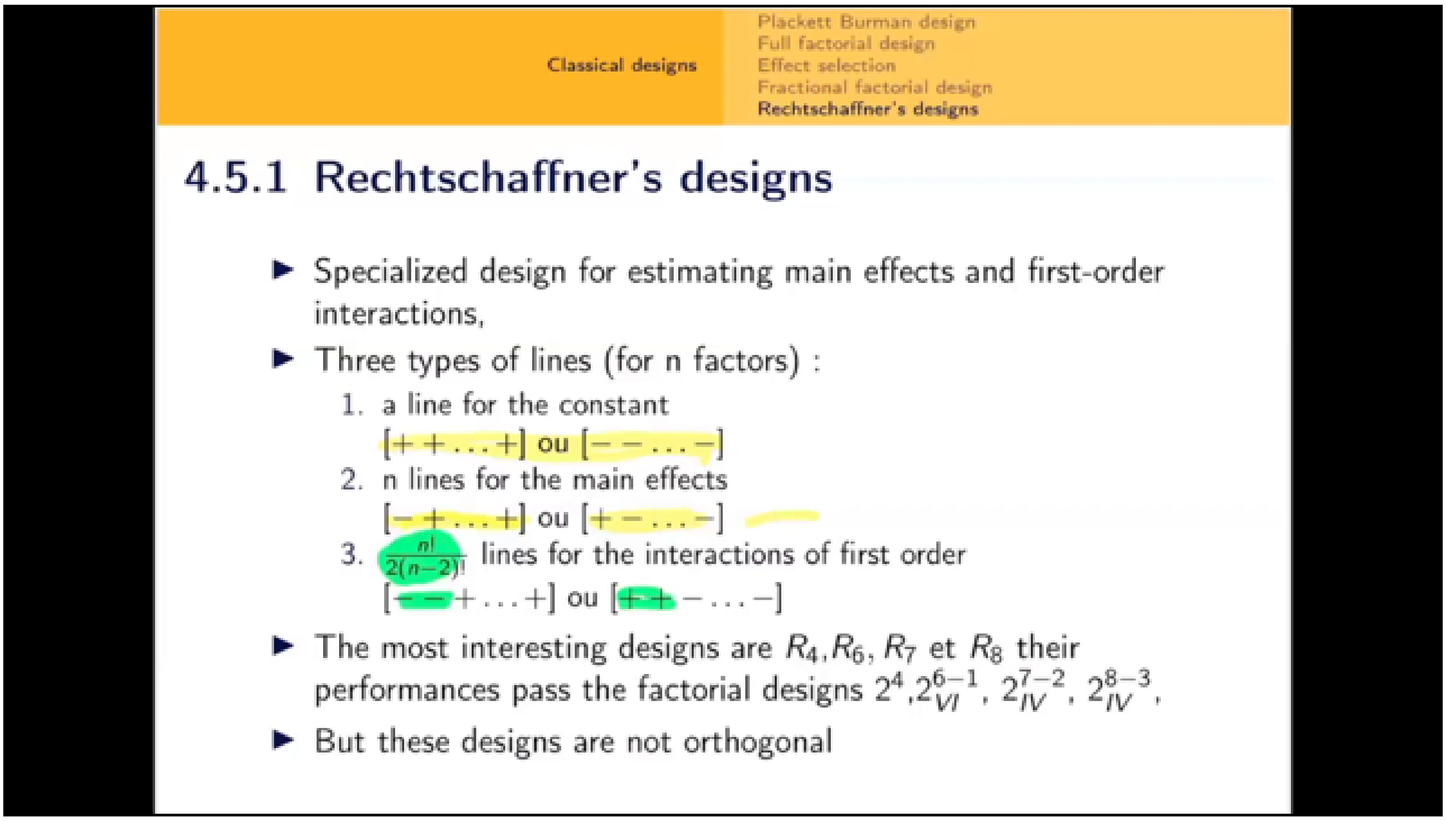


So if you are in a situation where you have very unprecise, it could be a problem.

notes

summary

2m 25s

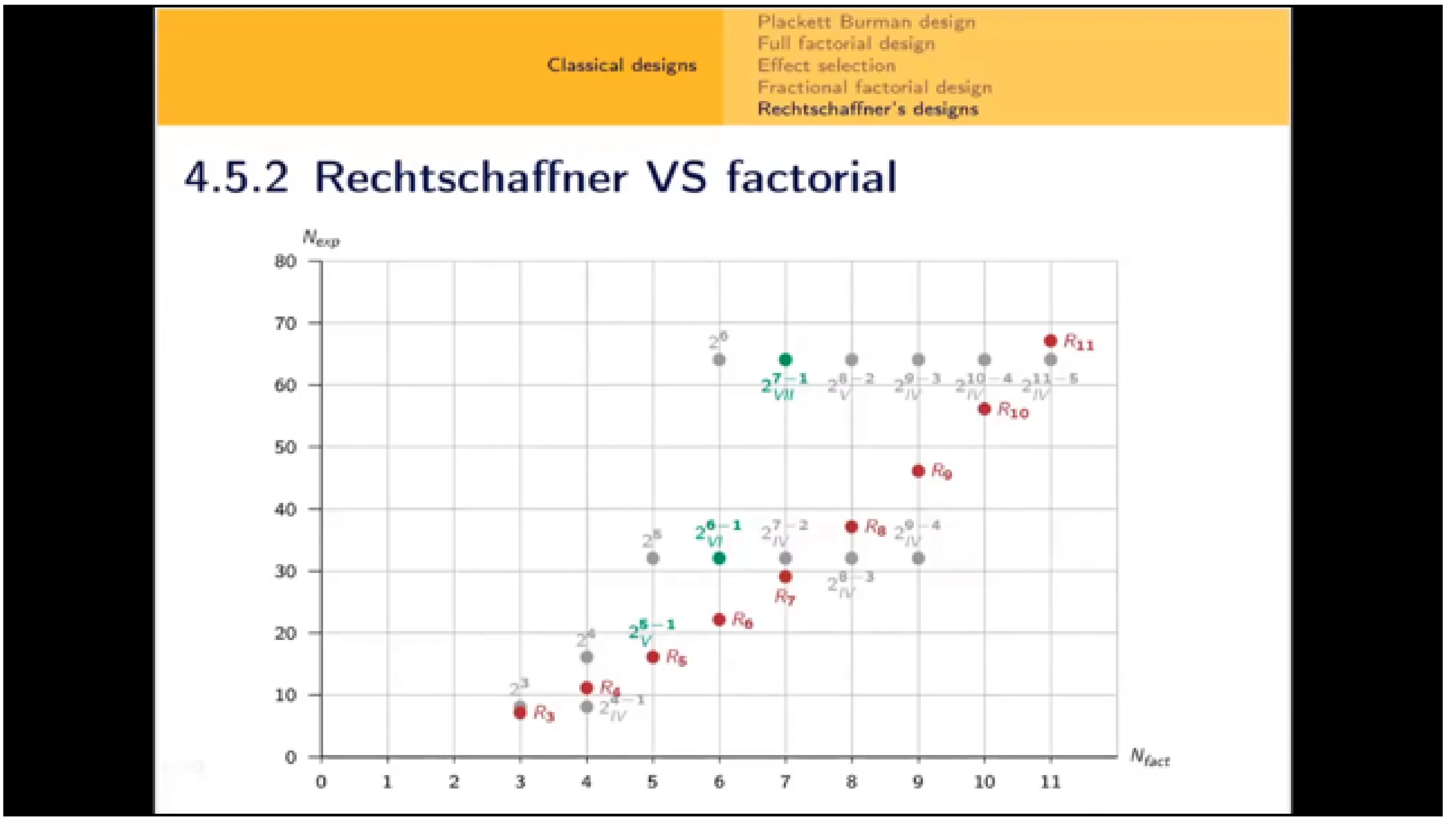


So first one called reschafter. So these reschafter designs are made of three types of lines. So you build them by yourself. As far as I know, it doesn't exist a routine in MATLAB building them. So you have to build them by yourself or if you're in Python. So you first have one row with full of ones or full of minus one. One of both. It's equal after you can perhaps check the quality of your final design if you prefer one situation or the other. Eventually you can do the check if the experiments one one one is more expensive than the expensive minus minus minus. You can also choose a less expensive experiment. After we have a few lines with one value different than the other. So it could be a minus one, so a minimum and all the rest are maximum. Or it could be one is the maximum or all at the minimum. This is for detecting the main effects. And after so you do also permutation of this but the permutation is very easy. You just have the different one which is one time in the first place. One time in the second place, one time in the third place. So you understand. After you are interested of detecting the interaction two by two. So the lines could be the permutation of the possibility of having two factors different than the other. So the two first and after the one and the third one is the force and etc. So all the permutations you can use the permutation the perms function in MATLAB for building them. How many of those lines do you have? You have the arrangement of the n, the numbers of factor and two. So this is a way of building those designs.

notes

summary

2m 34s

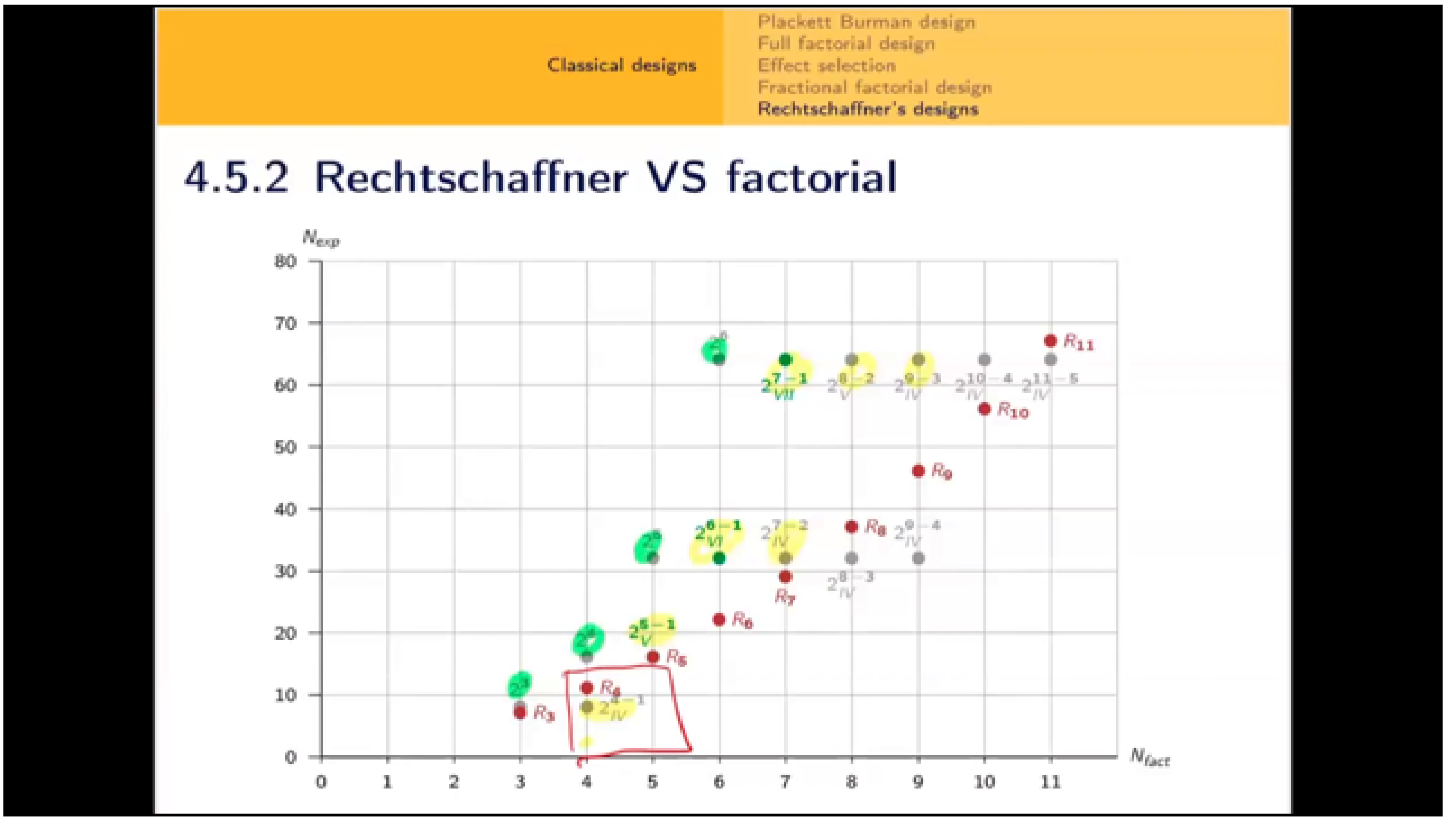


The literature quotes some of the designs that are better than the equivalent for the fractional factorial design. Sometimes it's better to take a fractional factorial design. Sometimes it could be better to take a reshapen. So in the literature you could check that our force or reshapen for four factors seems to be interesting for six, for seven, for eight. But remember that those designs are not orthogonal. It means two things. So it means that you are not allowed to use the permutation. But it would be not a good idea of using the simplified algorithm. You remember when you just take the transpose of the matrix of the model and divide it by the numbers of experiment after you multiply by the vector of. So don't use this algorithm X transpose Y divided by n. This doesn't work for those designs because they are not orthogonal. And understand that when you have a situation where the orthogonality would be very important because the quality of your measurement is very bad. And you really are afraid of having too much covariance between your measurements. It's it's could be dangerous to use them. So good one, but not of perfect quality.

notes

summary

5m 1s

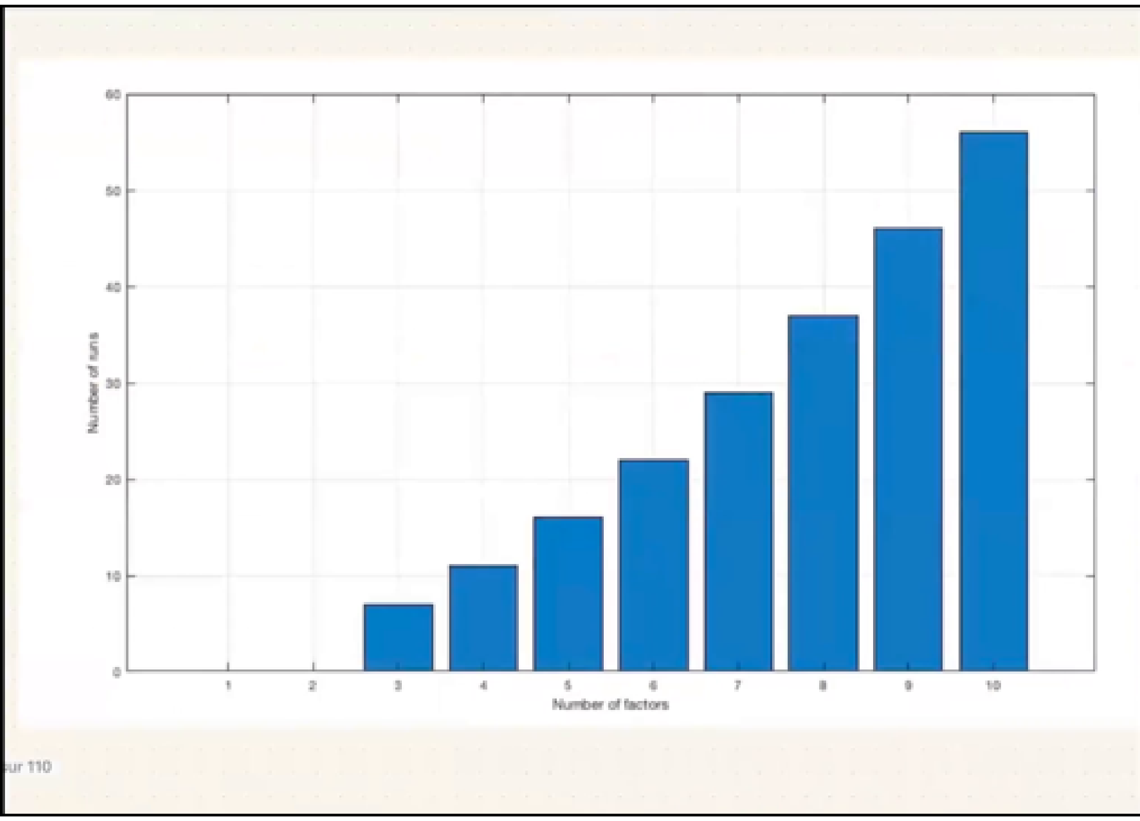


So in here you see a map of the different designs that we have now for the situation, a multifactorial analysis. You see all the full factorial design. The increases exponential after you see the fractional factorial design. You find one here. So in this case, you see that it seems to be better than the rest shaft. After explain you better, you have here two that are quite the same numbers of experiments. And after so, those are the fractional factorial designs. And in red, you see the rest shaft design. So what is the interest of the rest shaft design?

notes

summary

6m 34s

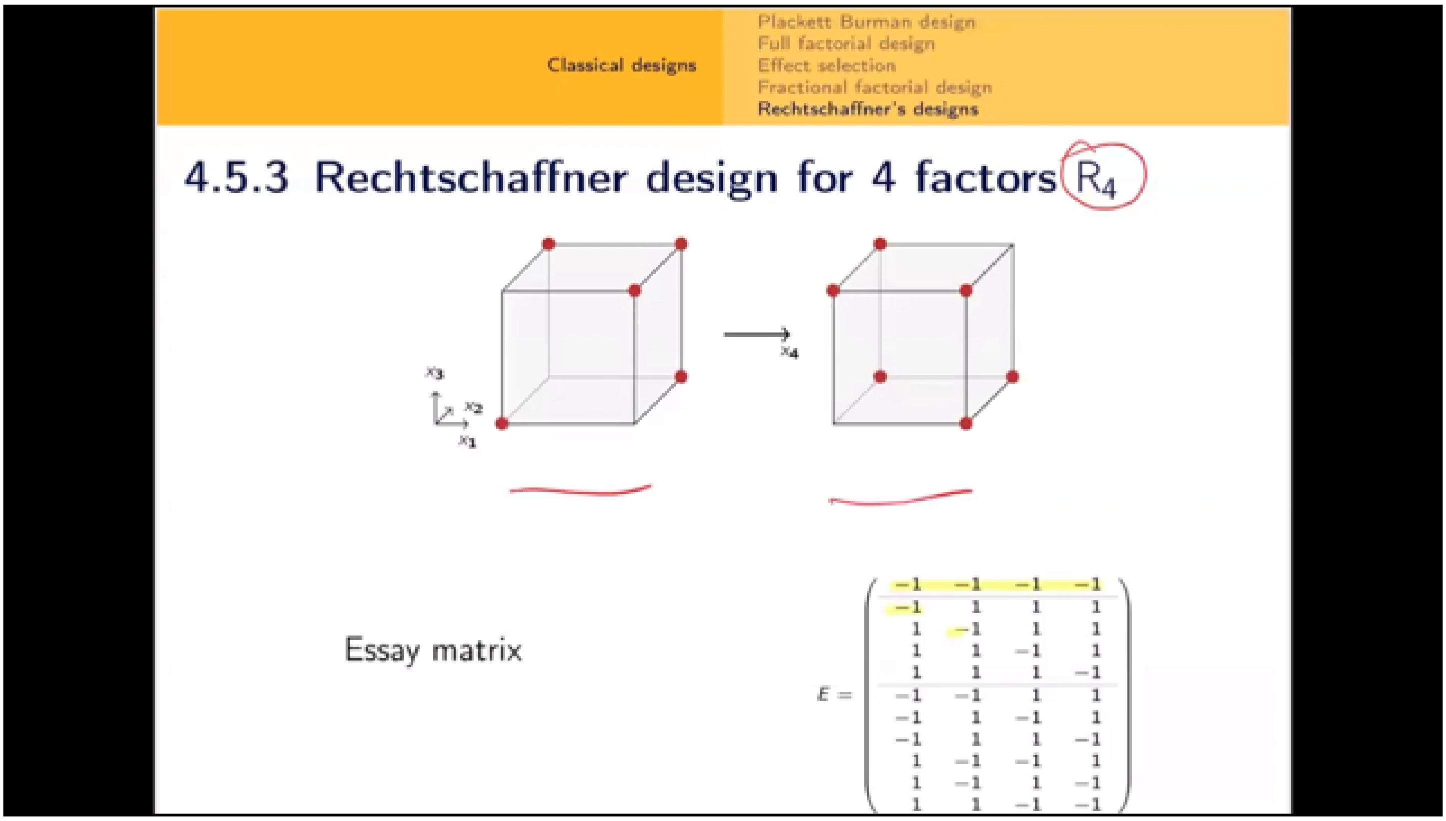


For example, if you look here, even if the rest shaft design is proposing probably 11 run when the fractional factorial design is proposing eight runs. That's the fractional factorial design is a resolution four. That is, is mixing the thumb interactions two by two. And that was three runs more. You can avoid this. So that's why the rest shaft number four is interesting in this case. It's not more economic than the fractional factorial design, but it's more economic than the full factorial design. So each time you have to check, I do not have general rules. So you understand, no, that in your choice, you have all the time to balance between the cost of making a lot of experiments and the accuracy of what you will get at the end, the orthogonality or the resolution. So it's a balance each time between the cost, the orthogonality and the what was my third resolution, the fact that you can separate well your different factors.

notes

summary

7m 37s



I just have you don't have in your slide just calculated now. So increase of the number of experiments was the rest shaft design. So you see it's quite a linear linear increase, a little bit more curve than a linear increase, but it's something as linear increase. So at four, you are at 11. After you are at, I don't know, 15 after 22, et cetera. So you see this increase. So it's an interesting design with some drawbacks. If you want to see it geometrically, so let's see the resolution four. I call it R4 is my way of calling them. It's not standard to call them R4. So you see in the four dimension, so it's still the representation where I have two cubes for representing my four dimension. So you see that it's some of the points, not the logic is not, is not evident. So you see each time the same structure.

notes

summary

8m 54s

4.5.4 Dispersion matrix (Rechtschaffner-4)

$$
\begin{array}{c}
\begin{array}{ccccccccccc} a_0 & a_1 & 2 & 3 & 4 & 12 & 13 & 14 & 23 & 24 & 34 \end{array} \\
D = \frac{1}{144} \begin{pmatrix}
14 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\
-1 & 20 & 2 & 2 & 2 & -2 & -2 & -2 & 7 & 7 & 7 \\
-1 & 2 & 20 & 2 & 2 & -2 & 7 & 7 & -2 & -2 & 7 \\
-1 & 2 & 2 & 20 & 2 & 7 & -2 & 7 & -2 & 7 & -2 \\
-1 & 2 & 2 & 2 & 20 & 7 & 7 & -2 & 7 & -2 & -2 \\
1 & -2 & -2 & 7 & 7 & 20 & 2 & 2 & 2 & 2 & -7 \\
1 & -2 & 7 & -2 & 7 & 2 & 20 & 2 & 2 & -7 & 2 \\
1 & -2 & 7 & 7 & -2 & 2 & 2 & 20 & -7 & 2 & 2 \\
1 & 7 & -2 & -2 & 7 & 2 & 2 & -7 & 20 & 2 & 2 \\
1 & 7 & -2 & 7 & -2 & 2 & -7 & 2 & 2 & 20 & 2 \\
1 & 7 & 7 & -2 & -2 & -7 & 2 & 2 & 2 & 2 & 20
\end{pmatrix}
\end{array}
$$

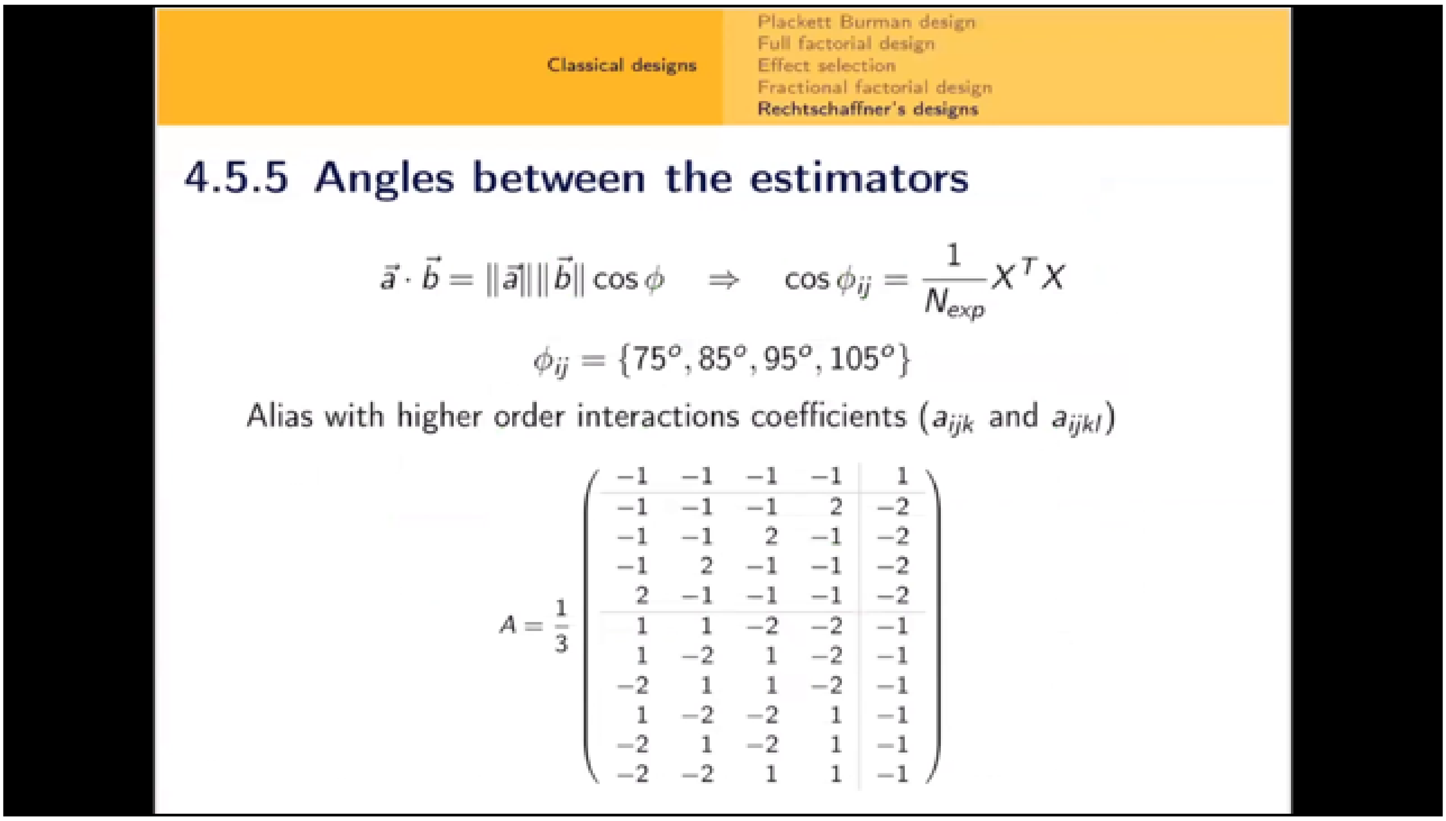
- Transfer of the experimental variance : 10%(a_o) and 14% (a_i, a_{ij}) to be compared with 6.25% for a 2^4 design that counts 5 more experiments,
- VIF : 1.1(a_o) and 1.5 (a_i, a_{ij}) to be compared to 1 for a 2^4 design.

In this case, it was a line of minus one after a matrix with one minus one and the rest. And after you have two at the minimum and two at the maximum, I have checked in this case, it was not clear which one to choose for each one. So plus, plus, plus or minus, minus. I have checked the different possibilities and checked the one with the best dispersion matrix. So this would be the dispersion matrix for the model. So with main, main effect. So here would be the A0, A1, 2, 3, 4. And after the interaction, 1, 2, 1, 3, 1, 4, 2, 3, 2, 4, and 3, 4. So you can observe that you have, it's not, as I said, it's not an orthogonal design. So the dispersion matrix is not orthogonal. But you see that the values of the non-diagonal element are an order of magnitude smaller than on the diagonal. So it's, it's, it's okay. So sorry, the values are to be divided by one, 144. But the matrix of dispersion is all-time symmetric. And on the diagonal, you see the value, you have quite a good evaluation. So one tenth, no, 14 divided by 144. It makes something as one tenth, so 10% of the accuracy. So you, we have made 11 experiments and we are around one tenth, a little bit higher than one. Now a little bit smaller than one tenth. So it's quite good for the constant that doesn't interest us so much. And after you see that the value is 20 divided by 144. So you have something as 10% of accuracy for the constant and something as, in this case, 14% of accuracy. It's not bad. It's not the best possible, but it's not bad. You see also if you want to check the variance inflation factor, again for the constant is very good.

notes

summary

10m 13s



And that it's 1.5 for the other coefficient, which again is not bad at all. It's not perfect. It's not theoretical optimum, but it's not, it's not bad. So it's quite an interesting design.

notes

summary

12m 49s

4.5.7 Orthogonality vs. Resolution

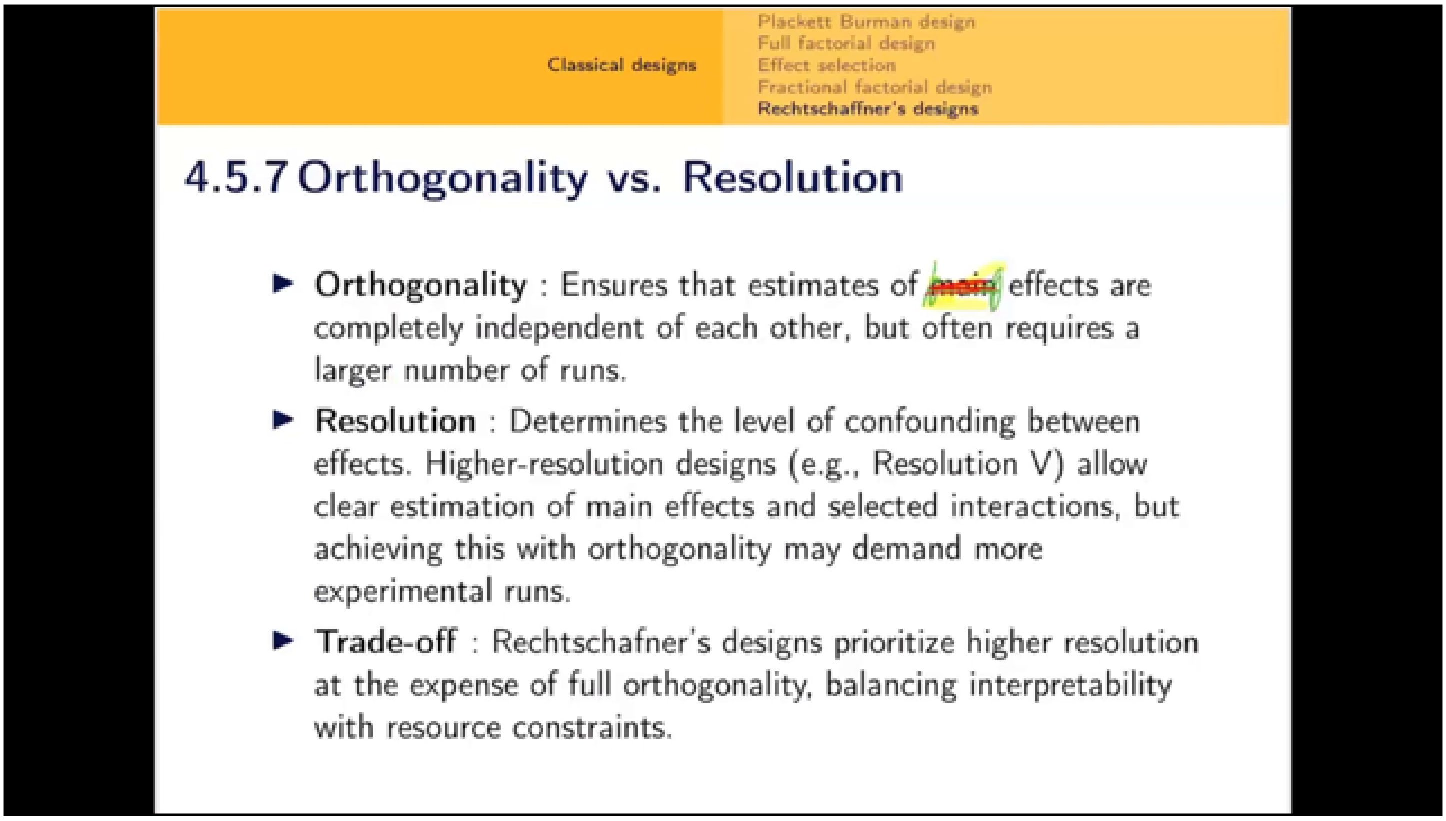
- **Orthogonality** : Ensures that estimates of main effects are completely independent of each other, but often requires a larger number of runs.
- **Resolution** : Determines the level of confounding between effects. Higher-resolution designs (e.g., Resolution V) allow clear estimation of main effects and selected interactions, but achieving this with orthogonality may demand more experimental runs.
- **Trade-off** : Rechtschafner's designs prioritize higher resolution at the expense of full orthogonality, balancing interpretability with resource constraints.

I have also checked the situation where the, check the angle between the estimator for showing you how much it is not orthogonal. It's not a critical point. I like to calculate the angle, but it's not something that usually people do. So you see that the angle between the different estimator are 75, 85, 95, 105. So they are not orthogonal, but not too far away of orthogonality. And this is what it must be, the matrix of alias. Here is the situation for six factor. In this case, we see that the accuracy for the, either around 5% for quite all the coefficients. And we see that we have variance inflation factor that are very close to one. So it seems also that this rest shaft new designs are improving when you increase the numbers of factors.

notes

summary

13m 5s

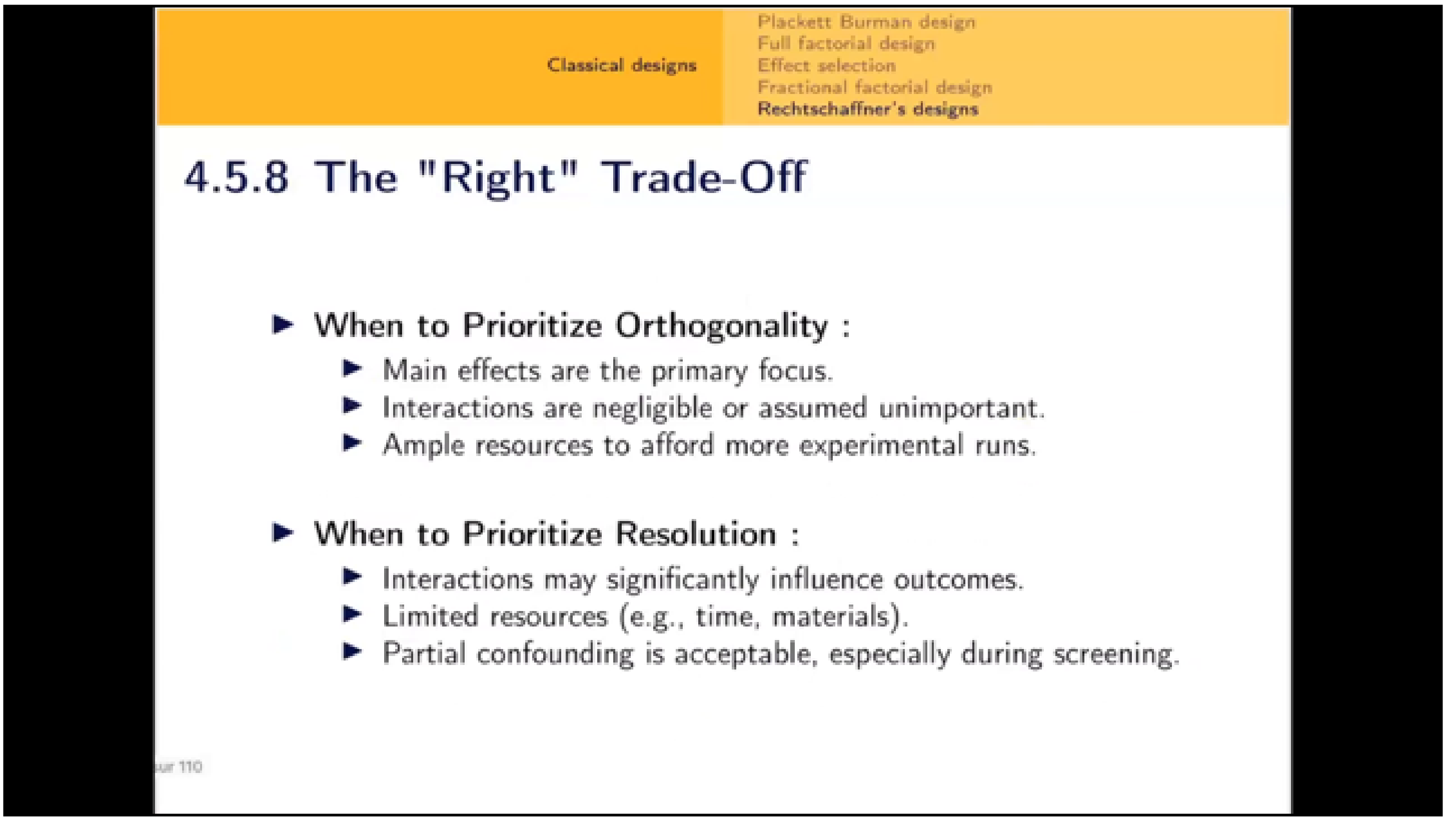


A few slides that I made with the help of chat GPT for making the comparison. So the importance of the orthogonality, so the orthogonality ensure that estimates of main effects are completely independent each other, but often require a larger numbers of run. Or they say main effect, not only the main effect, in fact is all the effects. Yesterday when I play with chat GPT, he tell me horrors. So you better check every time what you give you because if you ask something in the subject, you don't know there are risks that what they give you. It's not so good. So it's not only main effect is all the effects are. This is what is the meaning of orthogonality. So the resolution, you remember, it's determined what are the type of coefficients that are

notes

summary

14m 15s

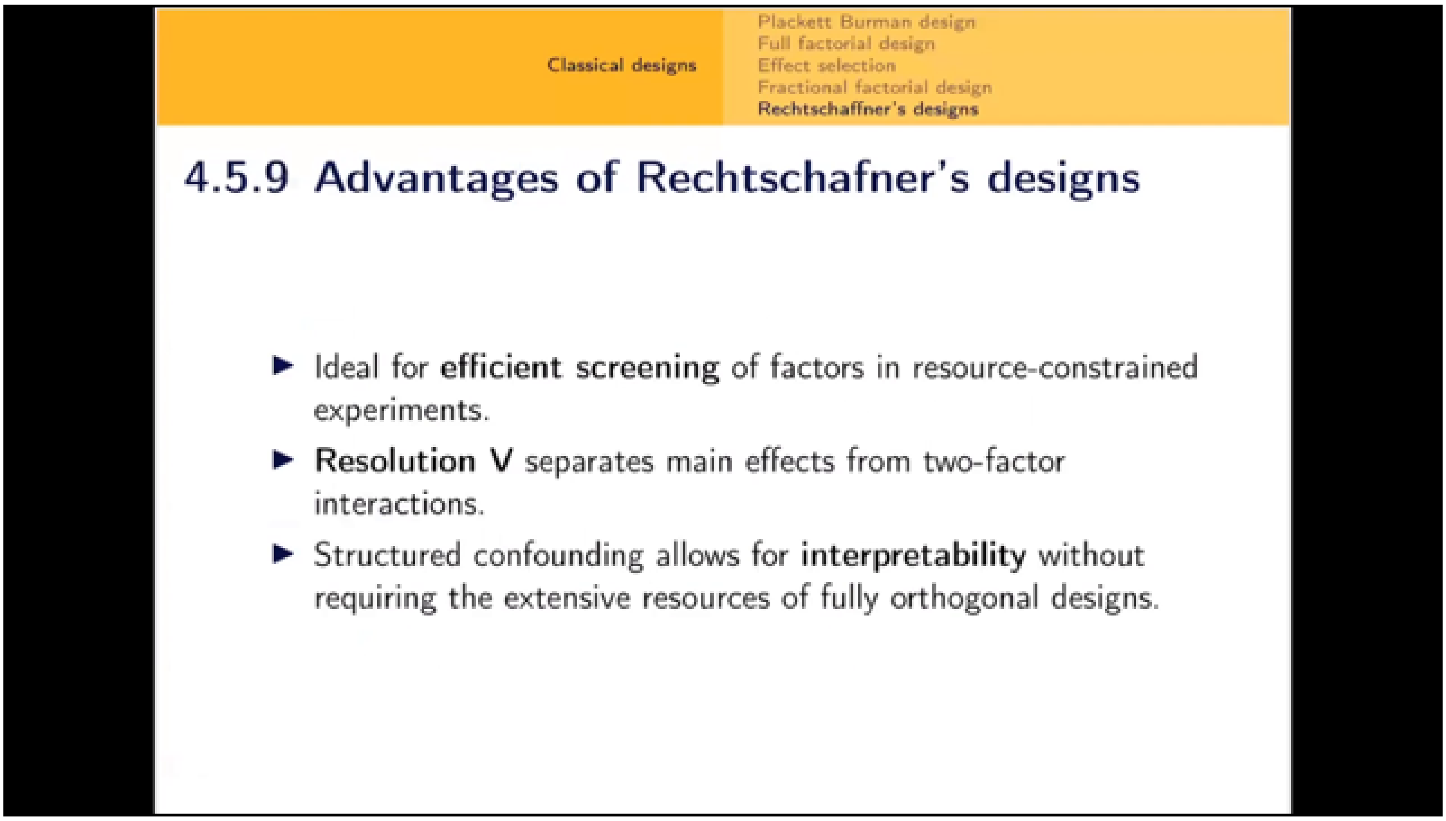


confounded that are aliased and the rest after design as a resolution five because it's not mixing two by two interaction between them is this advantage. The question is the tradeoff between the orthogonality and the resolution.

notes

summary

15m 25s

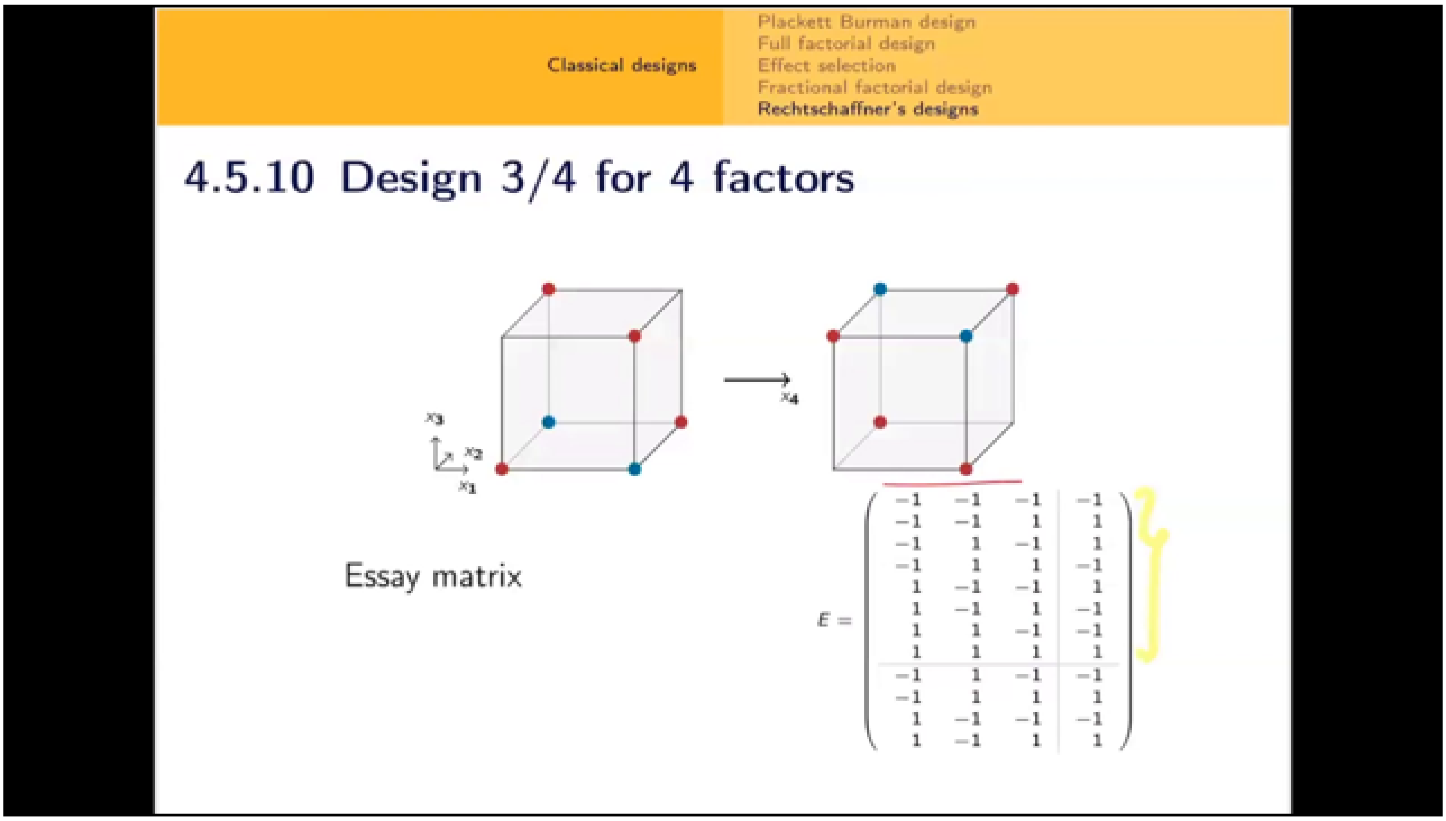


So when you would like to prioritize orthogonality, when the main effects are the primary focus and interactions are negligible or not so much important, the ample resources to afford for your experiment is sufficient. You have sufficient money and time for making the experiment. So you would like to prioritize the resolution when the instructions probably are more important. I check for the example of the bicycle that I presented in the first part of this chapter, but it was too much experiment. So we have made the experiment with eight plus eight. We were able to select things. So Reischafner design for seven experiment is was 21. So in this case, the numbers of experiment was critical because you didn't want to arrive tired to the race. So you see Reischafner design are interesting. They are not all the time solution.

notes

summary

15m 42s



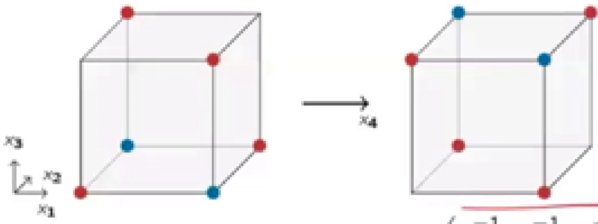
So they are nice for making a screening when you would like to take into account the fact that you have interactions two by two, but they are more expensive than Adamer designs in any case. And they are sometimes even more expensive than fractional factorial design. You really have to check each time step by step. The second design I would like to present is a very strange way of building it. You take a fractional factorial design and after you repeat half, half, not half of it, but half of the part you do not have done. So I have four factors.

notes

summary

17m 4s

4.5.10 Design 3/4 for 4 factors



Essay matrix

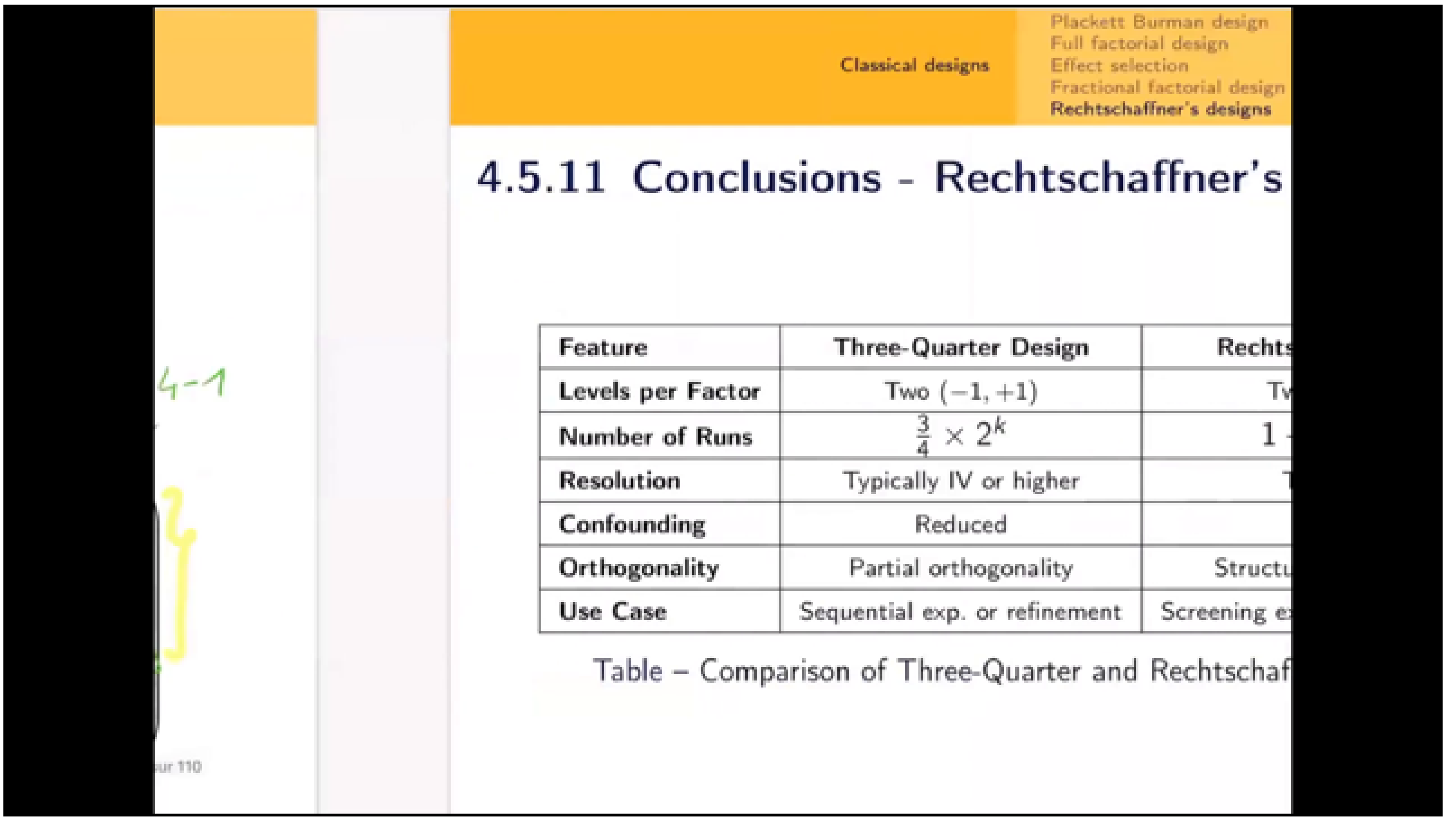
$$E = \left(\begin{array}{rrr|r} -1 & -1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ \hline -1 & 1 & -1 & -1 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 \end{array}\right)$$

I have take the full factorial design for three factors.

notes

summary

18m 1s



I have added a fourth factor. So now I have two power four minus one. I have divided one design, my full factorial design for four factor by two, but I belt it from the three and just creates the additional column. And after I take those lines. So look, this one, this one was the same as this one, but the value here was one. I have taken the opposite here. I've taken the minus one, this column one minus minus plus plus minus plus plus was this one. But here I have a minus one. So here I take a plus one. So I'm taking three quarter of a full factorial design. So I take a full factorial design. I divided by two. I keep one half and after the other half that stay, I divided by two. And it's make a three quarter design. What is interest? It's better than the rest shaffner design for the orthogonality. So we talk about quasi orthogonality for the three quarter design. Again, we start, we enter in this type of subtilities when we are really running after money or after time because you really want the maximum and your numbers of experiments that you have to do are really critical. I would say if not, don't lose your time. If one or more experiment is not, it's not a problem. Stay in fractional factorial design. Full factorial design. Okay. So this is the end of this chapter.

notes

summary

18m 2s