



Course material

Course:

**ENG606 / PHYS 442**

Video:

**DOE\_lesson11\_part1\_ResponseSurface**

Concepts (extracted from automatically generated subtitles):

**Second-degree shape. Less experiments. First designs. Composite design. Very interesting design. Measurement points. Factorial design. Second-degree function. Center of the faces. Possible experiments. Real number. Designs. Crystalline structure. More clean design. Dehler design.**



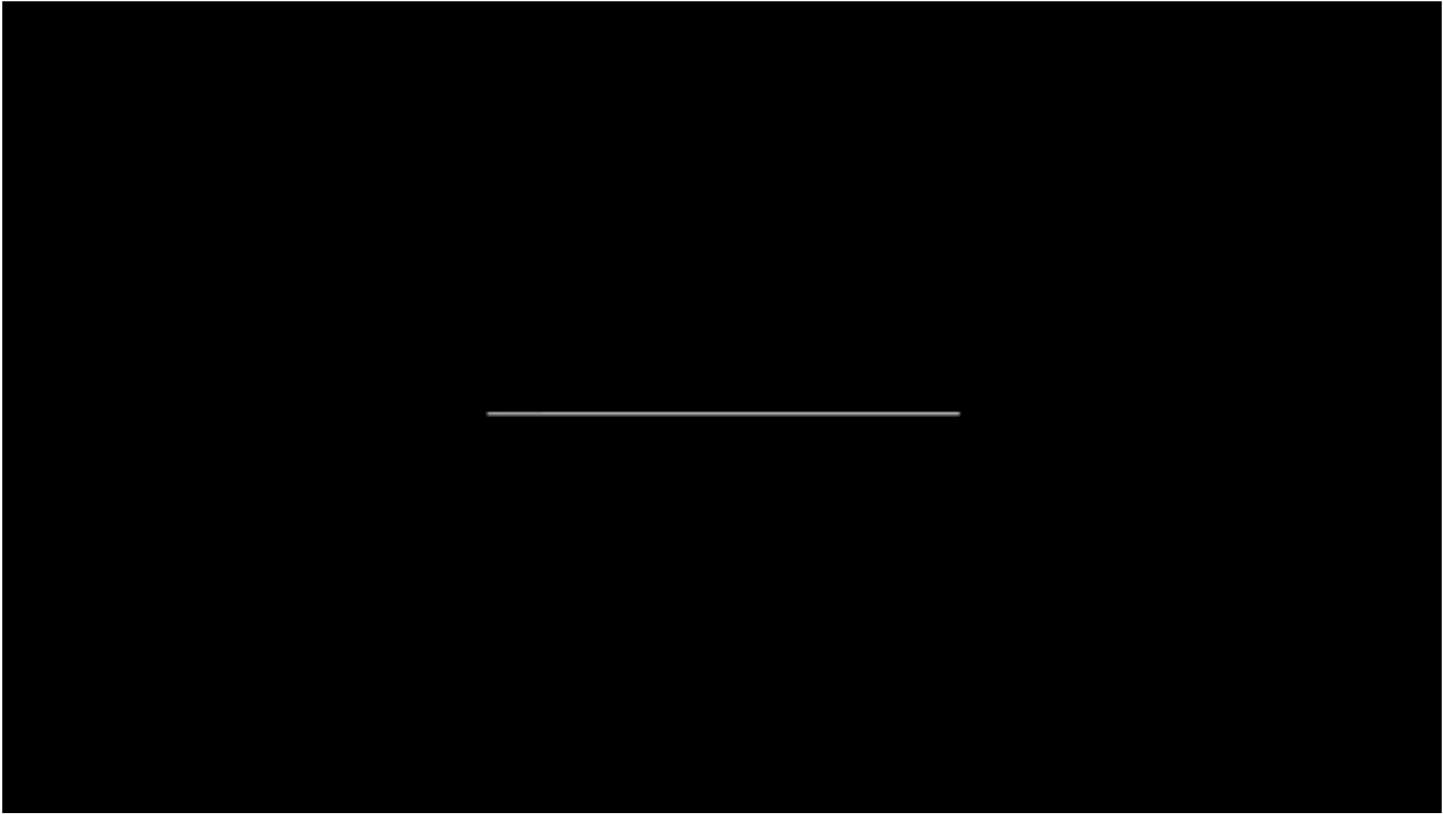
[to video sequence search](#)  
(within ENG606 / PHYS 442.)



[to video](#)

Center for Digital Education. More educational support material here:

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notes

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summary

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## 5.2 Classical designs

- ▶ Factorial  $3^k$  designs
- ▶ Composite design
- ▶ Doehlert design
- ▶ Box-Behnken

These subtitles have been generated automatically So today we will finish this chapter about the designs that are dedicated to what we

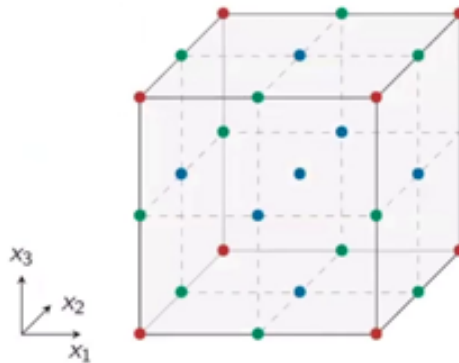
notes

summary

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## 5.2.1 The $3^k$ designs



3 factors : 27 measurement points , 3 levels per factor

call response surface or second-degree shape when you would like to see some heels or some valleys for understanding where you have maximum, where you have minimums. As already mentioned, we work with second-degree function as a Lego blocks, but it's clear that sometimes the relations are not regular, but with a second-degree function it's possible to understand where you have minimum and maximum, so it's why they are very useful. I've already seen the two first designs, so the factorial 3 power k, the composite design, I will go deeper in the composite design, it was quite short last week, and after we will

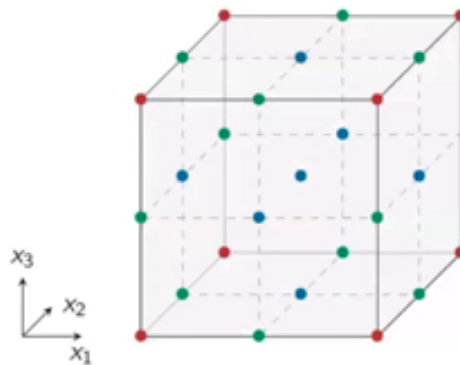
notes

summary

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## 5.2.1 The $3^k$ designs



3 factors : 27 measurement points , 3 levels per factor

discover the Dehler design and the Box-Benken design. So you remember that the  $3^k$  power  $k$ , that means that for each factor, for the whole possibility, you have three values and not two, so when you have two values, it's very practical for determining all the instructions, the main effect and the instructions. When you have three values, you can begin to evaluate the second-degree coefficients. It's a quite expensive design, you will see in the case for three factors, for example, you need 27 measurement points, when we can have quite half of it, we can have the same result with half and twice. It's never exactly the same result. I mentioned that as a job, but there are no free meals in mass and in statistics, so that means that if you make less experiments, you are losing something in any case, and you will see that at the end of the chapter. And in this case, three levels per factor, you understand that the word factorial design, there's a word factorial, means that you make all the possible experiments you can do with

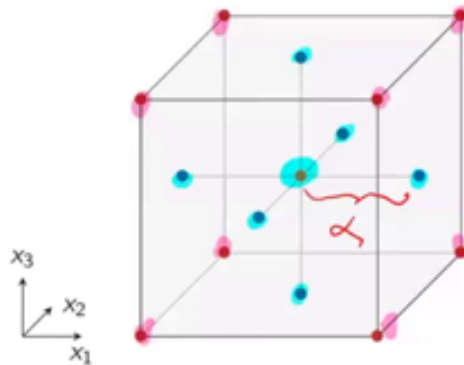
notes

summary

1m 8s



## 5.2.2 The central composite design ( $\alpha = 1$ )



3 factors : 15 measurement points , 3 levels per factors

values you have chosen. The second design I present to you was the composite design, it's a little bit cheaper, it's very good, it was the favorite of box composite because it's made of a two-power  $k$  factorial design, so the corner of the domain and some points at the center of the faces. So you see here the corner, and after you can make an experiment at the center, discover that you need to go for a second degree, test a lack of fit, and realize some experiments at the center. So it's a cubic face, I believe, no, if you talk about crystalline structure, I believe it's the structure of the carbon, if I remember some course of crystallography, so you have some atoms on the corner and some atoms at the center of the faces and one in the center. So it's very interesting because it's a step-by-step strategy, you start making a factorial design, you can also eventually make a fractional factorial design if you have a lot of factors. Well, with these steps of two-power  $k$  factorial design, eventually you eliminate some factors that are not important, and after you go for a second degree, it's very, a very good strategy because going for a second degree for five, seven factors, it's a nightmare, you will make so many experiments, it will be a loss of time, as probably not all factors have a curvature, so it's really interesting to do that way. After for this design, they are, let's say, a trick or a detail that can be adapted. So this value of  $\alpha$  is, in fact, the ratio of the distance between the point at the center of the faces and the center of the domain,  $\alpha$ . So in this case, in this drawing,  $\alpha$  equals one, you have to understand that the quality of a design is defined by the orthogonality, and

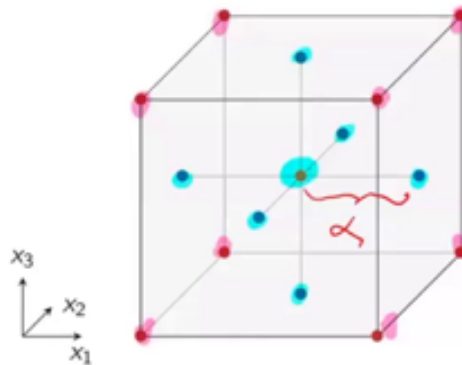
notes

summary

2m 23s



## 5.2.2 The central composite design ( $\alpha = 1$ )



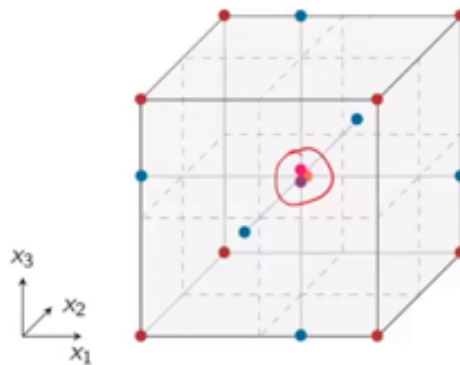
3 factors : 15 measurement points , 3 levels per factors

there are another property which is important, is what we called is iso variance per rotation. The fact that the variance, the evolution of the variance in your domain, so the quality of your model in your domain should be the same in any direction, a sort of iso variance. And this is very interesting because at the start, you don't know how is your phenomena organized. If you have important direction in your phenomena, where you have eigenvalue, you will see the end of the chapter, there are iterations with eigenvalue. If you have big eigenvalue, that means you have an increase very, very quick in some direction, and if you are not iso variance, eventually you make the wrong choice, and it's where you need more points that you have less points at the end. So it's why it's interesting, these two properties.

notes

summary

## 5.2.4 Central composite design ( $N_o = 3, \alpha = 1.353$ )



3 factors : 15 measurement points , 3 levels per factors

So a first way of optimizing that and having this iso variance per rotation, and so we never have a perfect orthogonal design for the second degree, because the second degree coefficient cannot be orthogonal to the first degree coefficient, it's impossible. But what you try is to having what we call the quasi orthogonality for being the most closer to orthogonality, is this for ensuring that the covariance are the smallest possible, that means the confidence interval are independent from one factor to another. It's make a more clean design. So if you would like to have this property, so after some time a trade-off, and if you look at the routine in MATLAB, you can have different trade-offs and you can see what you prefer. But if you increase your alpha value of 22%, it seems that it's make a better design. So that means that this range that we have defined, this normalization between minus one and one, if you think to go for a second degree, you have to keep a little bit of space for this quadratic model, eventually, and not being at the limit, not making your initial factorial design at the real limit of your phenomena. So keep 20, 40% of space on all directions for making eventually more dimensions. So it's why it's nice to have, if you want to go to the second degree, plan it from the start. Perhaps you go, you don't go, but for letting. And a question that arises, we do not do a new normalization. We keep the normalization as we have it from the start, and we keep the plus one, minus one for the original factorial design we have done. We do not change the scale during the experiment. It will not bring a lot of advantage, and it will bring a lot of complication after you have two normalizations. You have to remember

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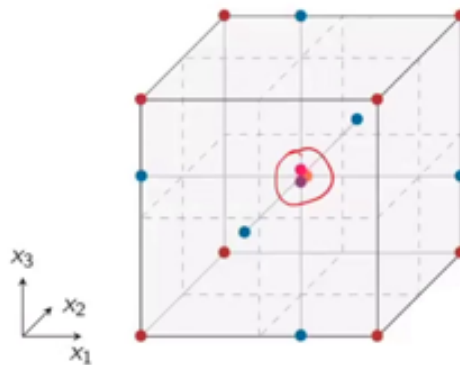
summary

5m 59s





## 5.2.4 Central composite design ( $N_o = 3, \alpha = 1.353$ )



3 factors : 15 measurement points , 3 levels per factors

what you have done, and you really want to compare what you have obtained with the linear model, the linear response model, and what you get with the quadratic model. So it's why we do not change. But eventually, because you are interested of making a lack of fit, you would like to have a measurement of the pure error. So you are interested to make more than one measurement at the center. So in this graphic, you see three points. In fact, there are one above the other, but I just put them a little bit aside for showing that there are three points. In this case, you can even increase your alpha value of 35%. So the real number proposed is 30, 50, 3053. We are just talking approximately, and we are not making things so precisely as the third digit. So if you make more points at the center, you can increase your domain. And you will see

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summary

## 5.2.5 Optimisation of the radius $\alpha$

Trade off between *isovariance per rotation* and *pseudo-orthogonality*

- ▶ The isovariance per rotation (rotability) :  $var_y(x_1, x_2, \dots) = var_y(\sqrt{x_1^2 + x_2^2 + \dots})$
- ▶ The pseudo-orthogonality : property limiting the number of terms in the matrix of dispersion (improve the accuracy of the estimated coefficients).
- ▶ Matlab :  $E = ccdesign(n)$

Nb of factors ( $n$ )	2	3	4	5	5	6	6
Factorial design	<del>2</del> <sup>2</sup>	$2^3$	$2^4$	$2^{5-1}$	$2^5$	$2^{6-1}$	$2^6$
Nb fact exp ( $2^{n-k}$ )	4	8	16	16	32	32	64
Nbr star pts ( $2n$ )	4	6	8	10	10	12	12
Nbr central pts ( $n_0$ )	1-3	1-3	1-3	1-3	1-3	1-3	1-3
Total ( $2^{n-k} + 2n + N_0$ )	9-11	15-17	25-27	27-29	43-45	45-45	77-79
$\alpha$ si $n_0 = 1$	1	1.22	1.41	1.55	1.60	1.72	1.76
$\alpha$ si $n_0 = 2$	1.08	1.29	1.48	1.61	1.66	1.78	1.82
$\alpha$ si $n_0 = 3$	1.15	1.35	1.55	1.66	1.72	1.83	1.89

in the next slides that if we change the dimension, we also have other types of increases. So understand that if you remember the information function. So you remember the variance function that was the function for representing what will be the quality of the model in your domain when you have finished your analysis. So when you calculate the information function, it is exactly the reverse. So where you have low value of variance, you have high value of information. So it's like a tant. So imagine the measurement like the stick. If you want to make a tant, what you're interested is having a volume under your tant. So you won't have sticks. So you can have sticks on the border of your fabric, or you can also have one stick at the center. So it's why if you have three sticks at the center, in fact, you can go more away in your domain. Okay, so you can have this as an image. So this is the table explaining how you can increase your domain depending on the factorial design you have executed first, and also the numbers of points at the center. It's clear that this table is just a part of space. You can also, if you're interested, if you have to optimize, you can go in the formulas, calculate the isovirons' rotation, calculate the orthogonality, and try to optimize things by yourself. It's what do more or less the routine of MATLAB. It's called CC design, central composite design, and you can tell him how many measurements you wanted to center, and you have a few parameters for adapting the estimation of the design. When I mentioned the routine of MATLAB, I really expect that after you go to read the help, I don't want to present them. But they have a few parameters, so don't hesitate when you use them to spend time

notes

summary

9m 37s



## 5.2.5 Optimisation of the radius $\alpha$

Trade off between *isovariance per rotation* and *pseudo-orthogonality*

- ▶ The isovariance per rotation (rotability) :  $\text{var}_y(x_1, x_2, \dots) = \text{var}_y(\sqrt{x_1^2 + x_2^2 + \dots})$
- ▶ The pseudo-orthogonality : property limiting the number of terms in the matrix of dispersion (improve the accuracy of the estimated coefficients).
- ▶ Matlab :  $E = \text{ccdesign}(n)$

Nb of factors ( $n$ )	2	3	4	5	5	6	6
Factorial design	<del>2<sup>2</sup></del>	2 <sup>3</sup>	2 <sup>4</sup>	2 <sup>5-1</sup>	2 <sup>5</sup>	2 <sup>6-1</sup>	2 <sup>6</sup>
Nb fact exp ( $2^{n-k}$ )	4	8	16	16	32	32	64
Nbr star pts ( $2n$ )	4	6	8	10	10	12	12
Nbr central pts ( $n_0$ )	1-3	1-3	1-3	1-3	1-3	1-3	1-3
Total ( $2^{n-k} + 2n + N_0$ )	9-11	15-17	25-27	27-29	43-45	45-45	77-79
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$\alpha$ si $n_0 = 3$	1.15	1.35	1.55	1.66	1.72	1.83	1.89

to read them correctly. Sometimes the help is nice. You can also use outside help like internet asking questions, for things sometimes the explanation are a little bit better. The help is really small. So, okay, how to analyze this table? So, you see here N as the number of factors. So, I'm proposing for two, three, four, five, or six factors. After I indicate the factorial design that we have executed as a first step. So, this should be 2 power 2 and not 22. 2 power 3, 2 power 4. With five, it could be interesting to use fractional factorial design, not too small because you don't want too much aliases. So, typically, you would like to use one, which is of resolution 5 because you can even try to use a reschaffner design as the first design. So, it's why here we are proposing the situation with 5 minus 1 and 5, 6 minus 1 and 6. So, after I have represented the numbers of experiments that you have. So, the numbers of factorial experiments, so you have 4, 8, 16, 32, etc. The number of points that you have this, what I call the star point. So, they are not anymore at the center of the faces. If you have alpha, which have increased, but you understand, I call those points the star point. And you have two points per dimension and after you have the center point in addition. So, here if you have two dimensions, you have four star points, you have six if you have three dimensions, etc. And after you can decide how many points you are interested to do at the center. So, this point at the center has two interests. One is to calculate the pure error in case you are interested to make a lack of fit. The other is to control your process. If imagine that you start

notes

summary

## 5.2.5 Optimisation of the radius $\alpha$

Trade off between *isovariance per rotation* and *pseudo-orthogonality*

- ▶ The isovariance per rotation (rotability) :  $var_y(x_1, x_2, \dots) = var_y(\sqrt{x_1 + x_2 + \dots})$
- ▶ The pseudo-orthogonality : property limiting the number of terms in the matrix of dispersion (improve the accuracy of the estimated coefficients).
- ▶ Matlab :  $E = ccdesign(n)$

Nb of factors ( $n$ )	2	3	4	5	5	6	6
Factorial design	<del>2</del> $2^2$	$2^3$	$2^4$	$2^{5-1}$	$2^5$	$2^{6-1}$	$2^6$
Nb fact exp ( $2^{n-k}$ )	4	8	16	16	32	32	64
Nbr star pts ( $2n$ )	4	6	8	10	10	12	12
Nbr central pts ( $n_0$ )	1-3	1-3	1-3	1-3	1-3	1-3	1-3
Total ( $2^{n-k} + 2n + N_0$ )	9-11	15-17	25-27	27-29	43-45	45-45	77-79
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$\alpha$ si $n_0 = 3$	1.15	1.35	1.55	1.66	1.72	1.83	1.89

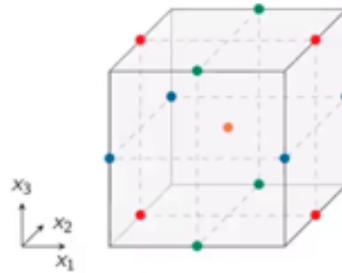
making an experiment at the center, when you start making your experiment, you make your factorial experiment, you make a new experiment at the center, you get the value, you can compare the first and the

notes

summary

## 5.2.6 Box-Behnken design

$$E = \begin{pmatrix} 0 & -1 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \\ -1 & -1 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



- ▶ 3 levels per factor
- ▶ No experiments at the vertices
- ▶ Isovariant per rotation : 4 ou 7 factors
- ▶ Blocking : factorial design  $2^2$
- ▶ Matlab :  $E = \text{bbdesign}(n)$

Factors	Coefficients	Run
3	10	13
4	15	25
5	21	41

second value, you make your star design and you execute again point at the center. Like that, you can see if your process is stable and eventually detect some problems. After in one line, I have a representation of the total number of experiments. So, between 9 and 11, depending the numbers of experiments at the center. This is the table. So, you see that if you just do one measurement, if you have two dimensions and you make one measurement at the center, it seems to be the optimum for having something with quite okay. So, is variance and orthogonality. And after you see the values increasing, increasing with the number of factors and increasing with the numbers of points that you do at the center. So, don't take those numbers too strictly. It's an indication and in any case, it could be nice after to calculate the variance function and to calculate the orthogonality and eventually make a few change. It's also possible not having the alpha the same on all the direction. It's for some, probably it's not a statistical interest. Probably you will decrease the quality of your design, but it could be an experimental interest of being close to those things. So, typically I understand this table as an indication and not as a strict thing to apply. So, in that aspect, the composite design is a very interesting design. Again, sorry if I'm repeating things, but so it's the interest of this step-by-step strategy. You start analyzing the first degree, eventually eliminating factors, and after you go for a second degree and for the second degree, you can really adapt yourself to the specificity of your experimental space. Eventually, you can increase in some dimension, not in other, you have the possibility to adapt you. That's the two things that you have to check is the iso-variant per rotation. Look, you have also another

### notes

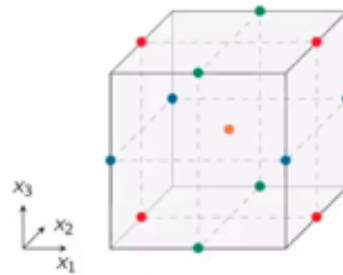
### summary

14m 37s



## 5.2.6 Box-Behnken design

$$E = \begin{pmatrix} 0 & -1 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \\ -1 & -1 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



- ▶ 3 levels per factor
- ▶ No experiments at the vertices
- ▶ Isovariant per rotation : 4 ou 7 factors
- ▶ Blocking : factorial design  $2^2$
- ▶ Matlab :  $E = \text{bbdesign}(n)$

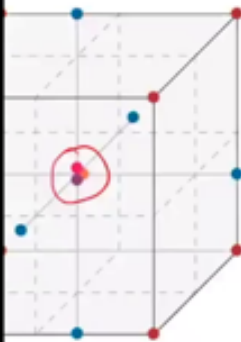
Factors	Coefficients	Run
3	10	13
4	15	25
5	21	41

name for that, the rotatability of your design. The fact that your design is equivalent independently of the direction you are looking at and we are also interested in having an absurdo-octagonality. So, to be as close as possible as the octagonality, that means that having regresses that are the most possible, orthogonal, if not the highest possible angle between your regresses.

notes

summary

## Design ( $N_o = 3, \alpha = 1.353$ )



38 points, 3 levels per factors

## 5.2.5 Optimisation

Trade off between *isovariance*

- The isovariance per rotation
- The pseudo-orthogonality of dispersion (improve t)
- Matlab :  $E = \text{ccdesign}()$

Nb of factors ( $n$ )	2
Factorial design	$2^n$
Nb fact exp ( $2^{n-k}$ )	4
Nbr star pts ( $2n$ )	4
Nbr central pts ( $n_o$ )	1-3
Total ( $2^{n-k} + 2n + N_o$ )	9-11
$\alpha$ si $n_o = 1$	1
$\alpha$ si $n_o = 2$	1.08
$\alpha$ si $n_o = 3$	1.15

Another interesting design, and honestly, I tell you my life, but I didn't appreciate it at the start, I find it complicated, etc. But in fact, it's quite a very, very interesting design.

notes

summary

17m 37s



## radius $\alpha$

and *pseudo-orthogonality*

ability) :  $var_y(x_1, x_2, \dots) = var_y(\sqrt{x_1 + x_2 + \dots})$

limiting the number of terms in the matrix  
(of the estimated coefficients).

	5	5	6	6
	$2^5-1$	$2^5$	$2^6-1$	$2^6$
2	16	32	32	64
10	10	10	12	12
1-3	1-3	1-3	1-3	1-3
27	27-29	43-45	45-45	77-79
1	1.55	1.60	1.72	1.76
8	1.61	1.66	1.78	1.82
5	1.66	1.72	1.83	1.89

38

## 5.2.6 Box-Behnken

$$E = \begin{pmatrix} 0 & -1 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \\ -1 & -1 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- ▶ 3 levels per factor
- ▶ No experiments at the vertices
- ▶ Isovariant per rotation : 4 factors
- ▶ Blocking : factorial design
- ▶ Matlab :  $E = \text{bbdesign}(n)$

You can check, yes, I didn't make this mention before. With the composite design, remember we are at 27 for the 3-power k, 3-power 3 maker, 27 experiment, at 15 experiment for the central

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summary

17m 54s

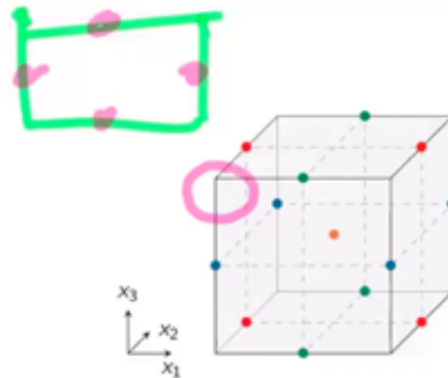




## 5.2.6 Box-Behnken design

$$E = \begin{pmatrix} 0 & -1 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \\ -1 & -1 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- ▶ 3 levels per factor
- ▶ No experiments at the vertices
- ▶ Isovariant per rotation : 4 ou 7 factors
- ▶ Blocking : factorial design  $2^2$
- ▶ Matlab :  $E = \text{bbdesign}(n)$



Factors	Coefficients	Run
3	10	13
4	15	25
5	21	41

composite. And now, we are at 13, even less runs. And you will see later that the quality of this design is quite good. This design exists only from three dimensions. At two dimensions, it would be the star design. And you know, it would not be a very good design, because I call it a fat one-factor at a time. And this is a bad design in two dimensions. But it became a quite interesting design if you have more than three dimensions, no, three and three and more than two dimensions. So you have three level perfectors, as the two, the composite you can have three, or you can have five, depending if you are alpha bigger than one. What is interesting that you do not have experiment at the extreme? That means that you are, well, it's a question also of your

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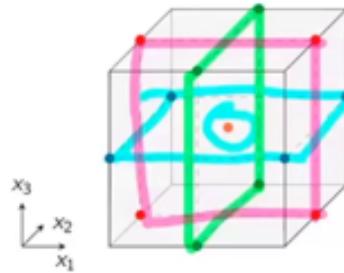
summary

18m 10s



## 5.2.6 Box-Behnken design

$$E = \begin{pmatrix} 0 & -1 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \\ -1 & -1 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



- ▶ 3 levels per factor
- ▶ No experiments at the vertices
- ▶ Isovariant per rotation : 4 ou 7 factors
- ▶ Blocking : factorial design  $2^2$
- ▶ Matlab : `E = bbdesign(n)`

Factors	Coefficients	Run
3	10	13
4	15	25
5	21	41

radius, no? But that means that you do, this experiment is a factorial experiment. Sometimes is the risk. Those are the experiments where usually you are the extreme of your factors, and it's where it explodes. It costs a lot. It takes a lot of time, etc. So those experiments at the extreme are perhaps a priori, could be the most complicated experiment to realize. So it's interesting avoiding them and having something of good quality without, well, you understand that it makes a sphere, or an hypersphere if you are in more dimension, and that is just a problem of the radius. After there are some things I do not realize fully, but if you are at the same radius of the center, it's not the same thing that being really at the extreme. So your problem is managed by variable, and they are sometimes a multi-dimensional aspect of your problem, but you also have the simple dimension problem to be at the extreme of the range of one. Sorry, my discourse is a little bit fuzzy. I have myself not a fully understanding of this relation between the multi-dimensional problem and the individual problem, but I have observed that something happened this. Another interest of this design is its blocking quality. See the color of the different points that I have represented. So if I take the red one, you see the red one is making, in fact, a factorial design at two dimensions. If I take the green points, it's also constituting a factorial design at two dimensions and the same thing with the blue points. What is different is the center. So it could be interesting to use this design if you have to make a blocking. You are separating your experiment between, let's say, three groups for having the numbers arriving correctly. That means that you give a color to each group, and everyone is

notes

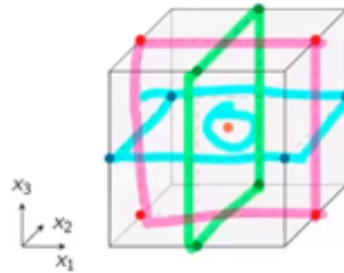
summary

19m 25s



## 5.2.6 Box-Behnken design

$$E = \begin{pmatrix} 0 & -1 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \\ -1 & -1 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



- ▶ 3 levels per factor
- ▶ No experiments at the vertices
- ▶ Isovariant per rotation : 4 ou 7 factors
- ▶ Blocking : factorial design  $2^2$
- ▶ Matlab : `E = bbdesign(n)`

Factors	Coefficients	Run
3	10	13
4	15	25
5	21	41

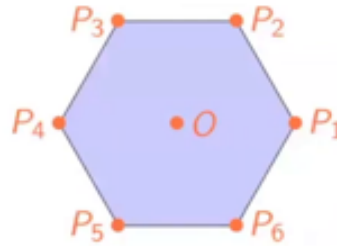
making an experiment at the center. So you have the center experiment for comparing the groups, and you have the different block. And so also you have an experiment manipulating, in this case, we have three factors in total. So each team is manipulating two factors, but in total we are manipulating three factors. So the same thing could be organized with more dimension. So this is quite an interesting design to apply when you have to make a blocking, when you have to separate your experiment in different blocks and you want to be protected of this aspect. You have a MATLAB function with a very funny name, BB Design. As I remember, there are no parameters, just like that, because the point is this design is perfectly defined. You do not have parameters for varying it. You see here the increase, so you have 13 for three dimensions, 25 for four dimensions, 41 for five dimensions. You see that with the second degree, when you are increasing the numbers of factors, the increase in the number of experiments is also time exponential, not sometime not too quick exponential, but in nevertheless it's sort of exponential. So when you go to a second degree, think well the numbers of factor you want, because if not, you are really a big, big numbers of experiments. And this again, a good strategy for going to, for the composite design. Sorry, I make a lot of advertisement for this design. If you have no other aspect, could be the first one to choose,

notes

summary

## 5.2.7 2D Doehlert design

$$E = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -1 & 0 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$$



- ▶ 2 factors : 7 measurement points, 3 and 5 levels per factor,
- ▶ No experiments at the vertices
- ▶ Isovariant per rotation
- ▶ Matlab : `E = doehlert(n)` to download from Moodle

and after to see others could be interesting for very specific situations. Box-Benken Design. I suppose, I don't know, perhaps it's my bad spirit, but I suppose Benken was a PhD student and Box was a professor, and I don't know who arrived with that.

notes

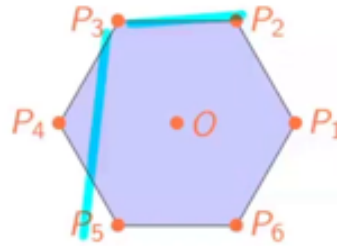
summary

24m 13s



## 5.2.7 2D Doehlert design

$$E = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -1 & 0 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$$



- ▶ 2 factors : 7 measurement points, 3 and 5 levels per factor,
- ▶ No experiments at the vertices
- ▶ Isovariant per rotation
- ▶ Matlab : `E = doehlert(n)` to download from Moodle

The last, but not the least, is the Dehler design. Dehler was also a statistician, not a German one, an American one. And it's very, it's a very interesting design. Its quality is not so high, it's a cheap design, but it's versatility, the fact that you can use in so many, with so many options that could be interesting. So it's very cheap, because with in two dimension, with seven points, you can have a second degree. So you can also consider it could be an extension of a factorial design. After, be careful of your, of your axis, but it could be more or less the extension with three more points of a factorial design, a two dimension, you can get the second degree. After, also observe that you do not have the same numbers of value for the axis,

notes

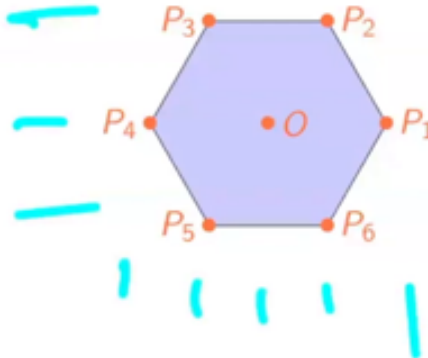
summary

24m 35s



## 5.2.7 2D Doehlert design

$$E = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -1 & 0 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$$



- ▶ 2 factors : 7 measurement points, 3 and 5 levels per factor,
- ▶ No experiments at the vertices
- ▶ Isovariant per rotation
- ▶ Matlab : `E = doehlert(n)` to download from Moodle

it could be also interesting sometime. You have, in this case, the horizontal axis that have five values, when the horizontal one has only three values. And after, it's also changed, we will go for more dimension, you will see also this difference of numbers of level perfected. The small difficulty is that the position of the points you have to remember your trigonometry with angles of 60 degrees. And remember that the sinus and the cossinus of 60 degrees are root of three divided by two. So it's what you have to remember. You don't have in MATLAB routine that I was in the same office when I make my PhD with Raphael Compagnon, and he makes the routine, and I'm using it for now, or more than 30 years. And next week, I will go for his retirements, his last lesson is in Fribourg, in the University of Fribourg, and I will go, so he's retiring, so we were many years together. And so you can download this routine from Moodle, and if you are in Python, you can copy it, open it and copy it for making it in Python. So what is the interest of this de Lerth design? So the first advantage is its price, very cheap.

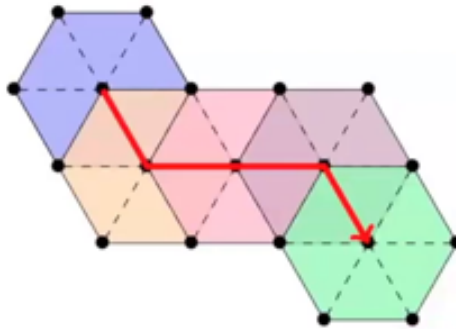
### notes

### summary

25m 53s



## 5.2.9 Hexagonal simplex



► Sequential exploration of a domain ( $7+3+3+3+3=19$  exp)

In two dimensions, with seven, in three dimensions, is 13 experiments, so quite the same price of of a box bank. But the interest is that when you are making an experimental optimization, you can rapidly move your center of interest. So consider the Violet hexagon as your start, you have made the seven orange points, and you discover that in this case, P2 is the maximum of your points, but it's really a maximum, or you are going up the hill. So with only three more points, you can move all your center of interest in one direction, but you can do in other direction depending on your results. And this is a property, which is also in three, four, five dimensions. Five dimensions, you have to think where to move and how to move, so good luck. But in three dimensions, probably you can also make it with not too much, too much work. But honestly, I never see a situation with second degree, with more than four, even five, is very, very seldom. Usually when you are with a second degree, you are with two or three dimensions, you're trying to optimize something, you're already well advanced in your problem, and usually you optimizing not so many, so many factors. So you can also use this as a simplex, is also belonging to what we call the simplex design. That means it's a simplex, is a geometrical form with which you can fill the space. So it's why we call it the simplex. So it's not the cheapest simplex, but it's in any case a simplex. You can like that follow routes to a maximum or to a minimum. Each time was three more measurements for understanding where to take your direction. So

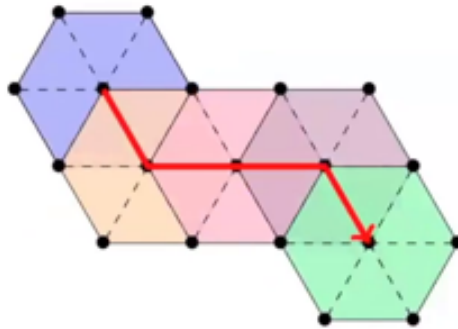
notes

summary

27m 29s



## 5.2.9 Hexagonal simplex



- Sequential exploration of a domain ( $7+3+3+3+3=19$  exp)

38

in this case, with 19 experiments, I was able to move. So a most interesting simplex would be triangular. This is a very efficient simplex for finding where is the maximum, where is the minimum. But if you are in Lausanne and you try to go up to the Matterhorn, it will take a lot of triangles to arrive there. But nevertheless, it's a method. It's not a quick method, but it's a method. The

notes

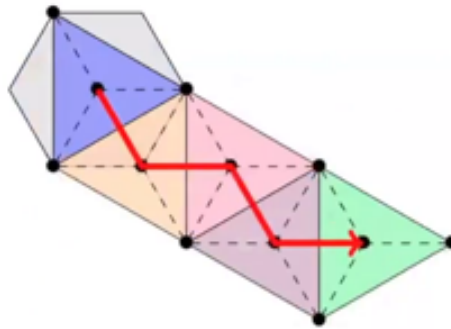
summary

29m 47s





## 5.2.10 Triangular simplex



- Exploration séquentielle du domaine ( $3+1+1+1+1=7$  exp)

advantage of the dollar design that eventually you can perhaps make a map, a second degree map, and eventually discover where is your maximum minimum, go there and do the same job and advance a little bit. We call that steepest ascent approach. And it's a way of working with the design, making a design in one place, discover where you have to go and to go in this direction and make new experiments in the direction where you have discovered the things.

notes

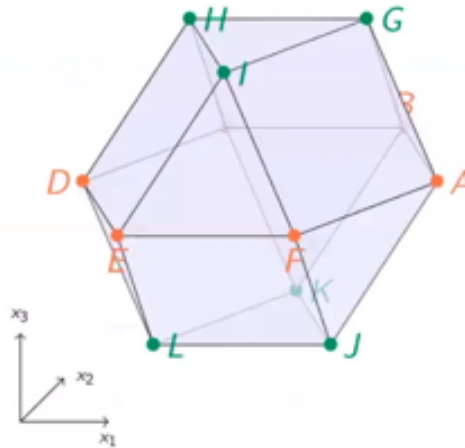
summary

30m 13s



## 5.2.11 3D Doehlert design

$$E = \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{2}\sqrt{3}}{3} \\ \frac{1}{2} & \frac{\sqrt{3}}{6} & \frac{\sqrt{2}\sqrt{3}}{3} \\ -\frac{1}{2} & \frac{\sqrt{3}}{6} & \frac{\sqrt{2}\sqrt{3}}{3} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{6} & -\frac{\sqrt{2}\sqrt{3}}{3} \\ 0 & -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}\sqrt{3}}{3} \\ 0 & \frac{\sqrt{3}}{3} & -\frac{\sqrt{2}\sqrt{3}}{3} \end{pmatrix}$$



So look at the difference. 19 experiments for advancing with a delert and 7 for advancing with a triangle. The triangle is better for that. But nevertheless, you can apply it. It could be interesting if you are close to your maximum. If you are very far away from your maximum,

notes

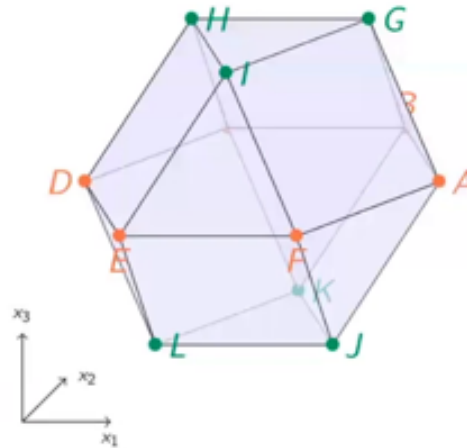
summary

30m 48s



## 5.2.11 3D Doehlert design

$$E = \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{2}\sqrt{3}}{3} \\ \frac{1}{2} & \frac{\sqrt{3}}{6} & \frac{\sqrt{2}\sqrt{3}}{3} \\ -\frac{1}{2} & \frac{\sqrt{3}}{6} & \frac{\sqrt{2}\sqrt{3}}{3} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{6} & -\frac{\sqrt{2}\sqrt{3}}{3} \\ 0 & -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}\sqrt{3}}{3} \\ 0 & \frac{\sqrt{3}}{3} & -\frac{\sqrt{2}\sqrt{3}}{3} \end{pmatrix}$$



perhaps it's not a so interesting, simplex design. So we can go for a third dimension. And the first

notes

summary

31m 11s



## 5.2.12 Sequentiality with Doehlert designs

$$E = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{\sqrt{2}}{3} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{\sqrt{2}}{3} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{\sqrt{2}}{3} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{\sqrt{2}}{3} & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{3} & 0 & 0 \\ 0 & \frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{3} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{\sqrt{2}}{12} & -\frac{\sqrt{2}}{12} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{\sqrt{2}}{12} & -\frac{\sqrt{2}}{12} & 0 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{\sqrt{2}}{12} & \frac{\sqrt{2}}{12} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{\sqrt{2}}{12} & \frac{\sqrt{2}}{12} & 0 \\ 0 & -\frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{12} & \frac{\sqrt{2}}{12} & 0 \\ 0 & \frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{12} & \frac{\sqrt{2}}{12} & 0 \\ 0 & -\frac{\sqrt{2}}{3} & -\frac{\sqrt{2}}{12} & \frac{\sqrt{2}}{12} & 0 \\ 0 & \frac{\sqrt{2}}{3} & -\frac{\sqrt{2}}{12} & \frac{\sqrt{2}}{12} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & 0 \\ 0 & 0 & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & 0 \\ -\frac{1}{\sqrt{2}} & -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{12} & -\frac{\sqrt{2}}{12} & -\frac{\sqrt{2}}{20} \\ \frac{1}{\sqrt{2}} & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{12} & -\frac{\sqrt{2}}{12} & -\frac{\sqrt{2}}{20} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{\sqrt{2}}{12} & \frac{\sqrt{2}}{12} & \frac{\sqrt{2}}{20} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{\sqrt{2}}{12} & \frac{\sqrt{2}}{12} & \frac{\sqrt{2}}{20} \end{pmatrix}$$

thing is to observe that you can work with two factors and add afterwards a third factor. And this also the interest of this design. Sometimes your screening has been not emotional, but to read in the literature, you have experienced that, okay, you have, perhaps I don't know, five interesting factors, but you know that you have two or three that are the most important one, you know, this a priori. So you can start making your campaign on those factors and afterwards adding the other factors. It's clear that you can do that also with other, but with a very higher cost. So it's very interesting for that. So you see it in the matrix, because you see that for the first seven experiments, the third factor was not valid, zeros. And this structure will continue when you increase the dimension. So it's interesting for that. It's also why this design is very appreciated in industry. You have to think to the time to the market. So you want rapidly improve something. So okay, you decide what you want to improve, you work with those those factors. And after when you continue in the next step, you don't need to start again from scratch. You have your experiment and you continue to amplify the range of the factors that you are controlling that you are manipulating. But that's implied that the the factor that you have not used yet have to be to its middle value. Geometrically, you see you have this as the two dimension problem. And after for going to the third dimension, you just have had

notes

summary

31m 18s



$$E = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 \\ -\frac{1}{2\sqrt{3}} & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 \\ -\frac{1}{2\sqrt{3}} & -\frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{3}\sqrt{3} & 0 & 0 & 0 \\ -\frac{1}{2\sqrt{3}} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{3}\sqrt{3} & 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{3}\sqrt{3} & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{3}\sqrt{3} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{3}\sqrt{3} & 0 & 0 & 0 \\ -\frac{1}{2\sqrt{3}} & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{12} & -\frac{\sqrt{3}}{3}\sqrt{3} & 0 & 0 \\ -\frac{1}{2\sqrt{3}} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{12} & \frac{\sqrt{3}}{3}\sqrt{3} & 0 & 0 \\ -\frac{1}{2\sqrt{3}} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{12} & \frac{\sqrt{3}}{3}\sqrt{3} & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{12} & -\frac{\sqrt{3}}{3}\sqrt{3} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{12} & -\frac{\sqrt{3}}{3}\sqrt{3} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{4} & -\frac{\sqrt{3}}{3}\sqrt{3} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{3}}{4} & -\frac{\sqrt{3}}{3}\sqrt{3} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{3}}{4} & \frac{\sqrt{3}}{3}\sqrt{3} & 0 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{12} & -\frac{\sqrt{3}}{3}\sqrt{3} & -\frac{\sqrt{3}}{5}\sqrt{3} & 0 \\ -\frac{1}{2\sqrt{3}} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{12} & \frac{\sqrt{3}}{3}\sqrt{3} & \frac{\sqrt{3}}{5}\sqrt{3} & 0 \\ -\frac{1}{2\sqrt{3}} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{12} & \frac{\sqrt{3}}{3}\sqrt{3} & \frac{\sqrt{3}}{5}\sqrt{3} & 0 \end{pmatrix}$$

notes

33m 25s



## 5.2.12 Sequentiality with Doehlert designs

$$E =$$

0	0	0	0	0
-1	0	0	0	0
1	0	0	0	0
$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	0	0
$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	0	0
$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	0	0
$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	0	0
$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0
$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0
$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0
$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0
0	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	0
0	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	0
0	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	0
0	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	0
$-\frac{1}{\sqrt{2}}$	0	0	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$-\frac{1}{\sqrt{2}}$	0	0	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
$-\frac{1}{\sqrt{2}}$	0	0	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$-\frac{1}{\sqrt{2}}$	0	0	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
0	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	0
0	$\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	0
0	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	0
0	$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	0
$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	0	0
$\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	0	0
$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	0	0
$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	0	0
$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	0	$\frac{1}{\sqrt{2}}$
$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	0	$-\frac{1}{\sqrt{2}}$
$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	0	$\frac{1}{\sqrt{2}}$
$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	0	$-\frac{1}{\sqrt{2}}$
$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	0	$\frac{1}{\sqrt{2}}$
$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	0	$-\frac{1}{\sqrt{2}}$
$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	0	$\frac{1}{\sqrt{2}}$
$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	0	$-\frac{1}{\sqrt{2}}$

So you see each time. So here, here you have your design for two dimension. Here you have your design for three dimension. And after my page was too small, you can continue like that. And it's a structure which continue like that with dimension. But I don't think you will make a quadratic model with seven factors. If you do it right, right me, and I want to see the paper, and I would change my mind. But usually when you are in the second degree, three, four factors. It's already a quite complicated geometry. It's incredible, just a second degree, but just a second degree is not so easy to manage. So it's why I will show you a manner of managing

notes

summary

33m 41s

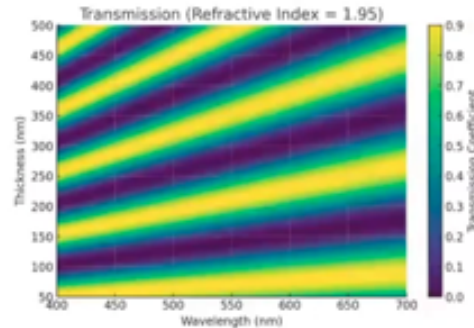


## 5.2.13 Application : light transmission

- The transmission or reflection of light by a thin film is a complex phenomenon.
- Based on Snell and Fresnel equations, for a perpendicular non-polarized beam, the transmission coefficient  $T$  giving the fraction of intensity which is transmitted is function of the wave length  $\lambda$ , of the refractive index  $n$  and the film thickness  $t$  such as

$$T = \left( 1 - \left( \frac{n-1}{n+1} \right)^2 \right) \sin^2 \left( \frac{2\pi n t}{\lambda} \right)$$

- The objective is to experimentally determine  $R$  around the point ( $\lambda = 475 \text{ nm}$  (blue),  $t = 425 \text{ nm}$ ,  $n = 1.95$ )



those dimensions better. So yesterday, as I mentioned, I made a small example. I asked Chagy Pity to proposing me three examples and I chose one. He proposed me one of chemistry. But I don't know if you are a chemist among you, but there are a lot of physicists that didn't want to have problem. And two problems of physics, one with light and one with magnetic. And I feel better explaining things in optics and in magnetism. So I chose that. So when you are interested to understand the quantity of energy, the light energy going through a thin film, first you usually start to understand what's happened when your beam is perpendicular to your thin film. And if the film is sufficiently thin and in comparison to the wavelengths, a few phenomena appear. I don't want to enter too much in that. It's related to the Fresnel law. I don't know if you see that, but there are after different problems with the polarized and the phase of the light, etc. But in any case, the transmission, so  $T$  represents the quantity of intensity of energy, which is going through. And this problem is quite interesting because it's all you can define if you have a mirror or if you are able to have lights going through and it could depend on the wavelength. So it's something that you better manage correctly. And you see that it's quite a complicated model.  $N$  is the index of refraction,  $T$  is the thickness of your microfilm,

notes

summary

34m 33s

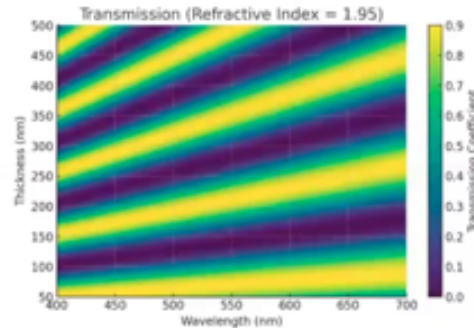


## 5.2.13 Application : light transmission

- The transmission or reflection of light by a thin film is a complex phenomenon.
- Based on Snell and Fresnel equations, for a perpendicular non-polarized beam, the transmission coefficient  $T$  giving the fraction of intensity which is transmitted is function of the wave length  $\lambda$ , of the refractive index  $n$  and the film thickness  $t$  such as

$$T = \left( 1 - \left( \frac{n-1}{n+1} \right)^2 \right) \sin^2 \left( \frac{2\pi n t}{\lambda} \right)$$

- The objective is to experimentally determine  $R$  around the point ( $\lambda = 475 \text{ nm}$  (blue),  $t = 425 \text{ nm}$ ,  $n = 1.95$ )



and lambda is the wavelength of the light. So if you check this function over the space, vary just two factors. You see it's, I don't know if it's in English, but like the roads, I don't know in Chile or in Peru, when you have dust and it's make, in French, we say Toll on Dule. I have no idea how you say that in English. In French, we say Toll on Dule. So you see the color represents the level of transmission. And so where it's yellow, you have quite a lot of transmission. And when you have blue, quite no transmission. And it's depend on the wavelength horizontally. And it's depend on the thickness. And after you have also the N value of the material, we will make the change. It's quite complicated. So this, but usually you are not interested to make a map or all the domain. This is theoretical physics will help you back to experimentally. You could be interested to determine what's happened in a small domain. Eventually, you are interested for looking for a maximum or eventually you will be interested to looking for a minimum. So in this place, you are expecting a second degree model. So I propose that we investigate, numerically in this case, but as if it would be experiment, we investigate that for three parameters for the wavelength around 475. So it's the sort of blue was the size, which is quite the same, a little bit smaller than the wavelengths. It could be the same wavelength or a multiple of. So it's work also with multiple of not 200 multiple, but one time, two times, three times, four times. And with a material which have an N, which is close to, but I will do that after, suppose.

notes

summary

36m 33s

