



Course material

Course:

**ENG606 / PHYS 442**

Video:

**DOE-Lesson14\_part1-Mixture**

Concepts (extracted from automatically generated subtitles):

**Quantity of glass. Type of model. Different ideas. Fraction of glass. Different ratio of wall. Surface of wall. Building physics. Straight line. Mixture problem. Part of your winter garden of your greenhouse. Temperature capture. Model of the greenhouse. Lot of situations. Fact of the sum of the level. Different types of greenhouse.**



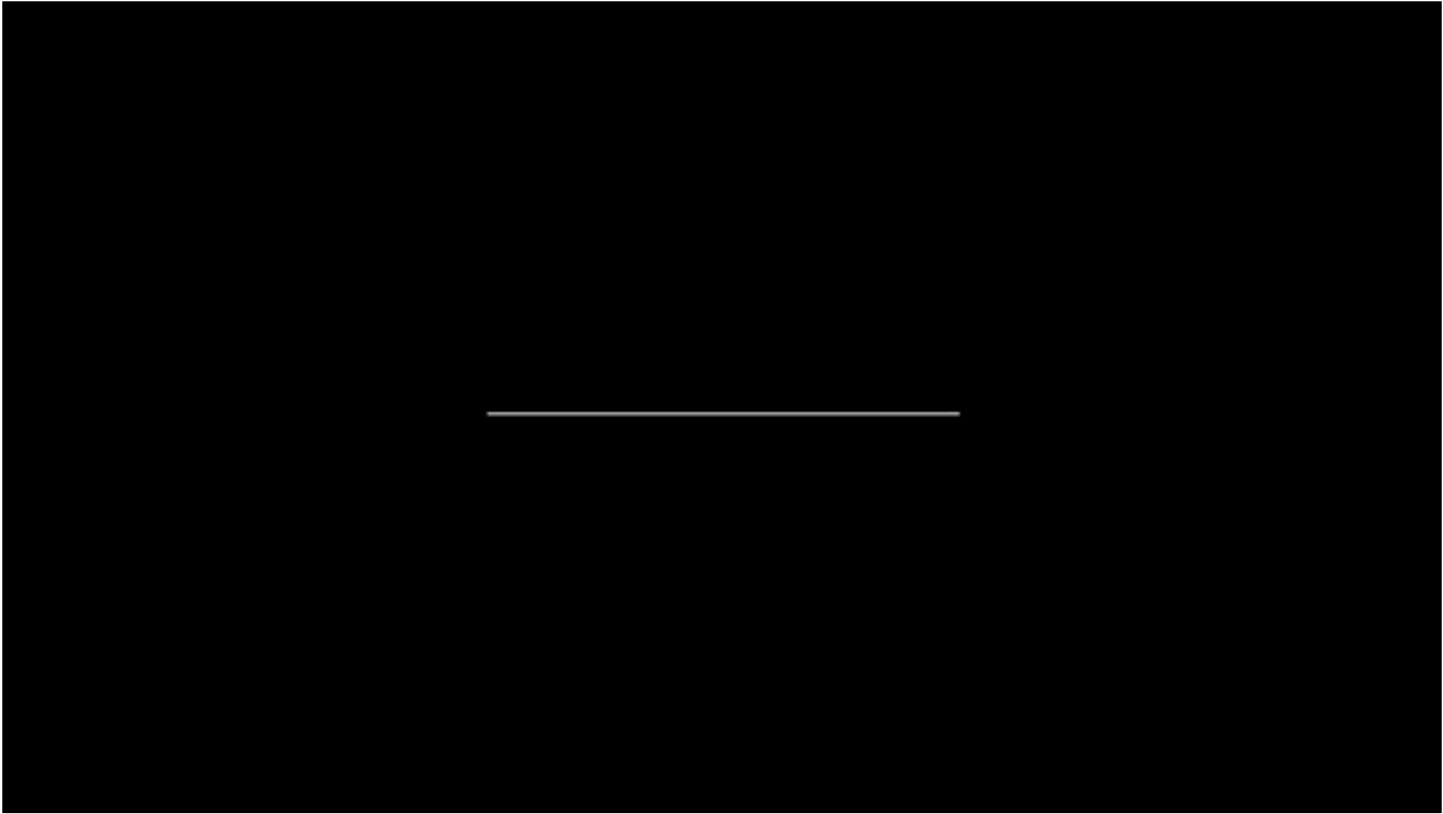
[to video sequence search](#)  
(within ENG606 / PHYS 442.)



[to video](#)

Center for Digital Education. More educational support material here:

<https://www.epfl.ch/education/educational-initiatives/cede/educational-technologies-gallery/boocs-en/>  
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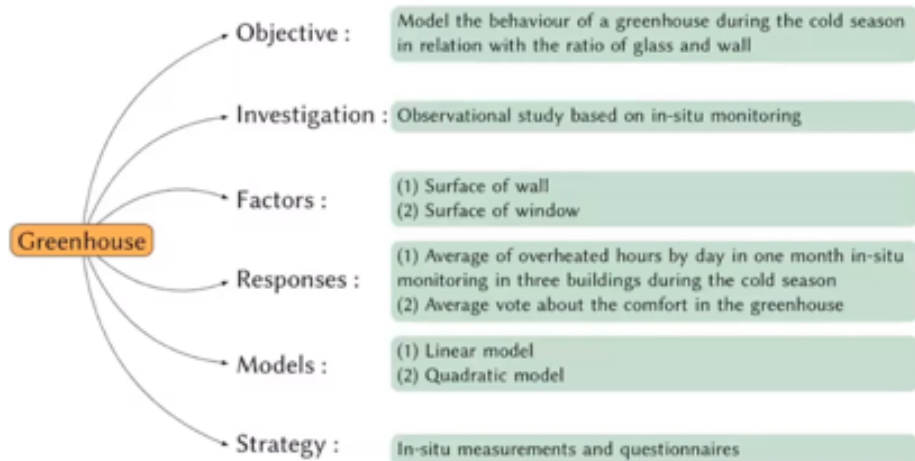
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## 7.1.8 Optimization of a greenhouse : mindmap



These subtitles have been generated automatically Okay, so the objective of today is discovering few examples of mixture. You can see that mixture appears also in situations where it's not a chemical or cooking mixture. It's useful in chemistry but it's also very useful in physics and engineering. There are a lot of situations where we have parts of the factors that have to correspond to a fixed sum. So it's when we have this mixture problem. After I will show you a few of the designs that are interesting in those situations. And also the type of model because with the situation of mixture we can make a few changes in the model. So I will present to you what we call the Sheffey model. And it's an interesting way of looking at problems of mixture that simplify a little bit. This new equation that we have all the time that the fact of the sum of the level of the factors is one. With this we can have a change in the model. And I will also show you a few designs. I will also show you what are the consequences when we have limitation in the mixture. Sometimes we say this product must be limited to 10 percent, this product must be limited to 80 percent. And so we have a few elements that do not appear in the standard situation. After I would like to make a summary of the course. I have extracted a few slides that I find important and I will just recall you. Okay, so when I was making my master project, when I was at your place, also when I was making my PhD, I was working in the laboratory which is the solar energy laboratory and building physics. So a few of the examples I have know are coming from this field. So if you would like when you build

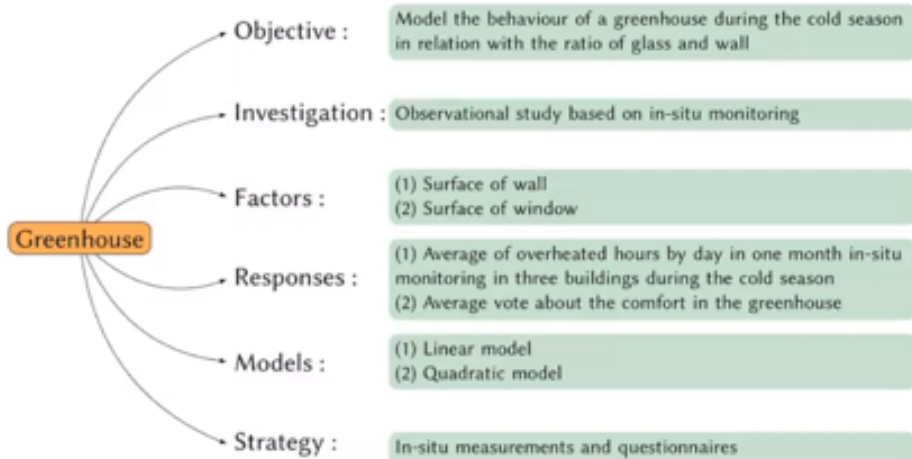
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### summary

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## 7.1.8 Optimization of a greenhouse : mindmap

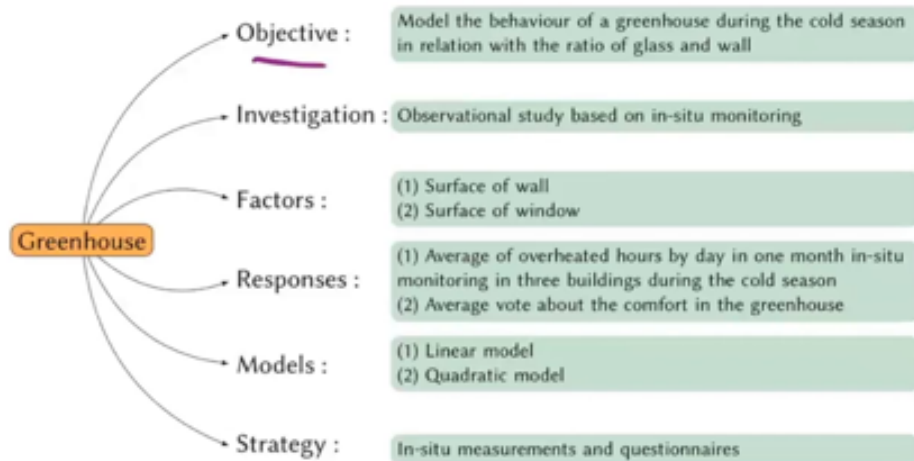


a greenhouse in in the house, so there are different ideas. The idea is to live in this greenhouse, to profit of a space, but it's also interesting for capturing part of the energy and eventually warming the rest

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## 7.1.8 Optimization of a greenhouse : mindmap



of the of the building. So when you are in this situation, what you would like to do is to make a model of the greenhouse and typically decide what should be the quantity of glass in reference of the quantity of concrete or bricks that you put. So what is the part of your winter garden of your greenhouse that is transparent and what is the part of your greenhouse which is not transparent? You understand that if you have a big part with this transparent to kept a lot of energy, but eventually it could be after too hot to take profit of this space because when it's too hot it's not very agreeable to be there. So they are very probably a balance between the fraction in your greenhouse, the fraction of glass and the fraction of concrete. So the objective in this case would be to model the behavior of the greenhouse during the cold season during in summer. Usually you open the door of your greenhouse and you are not interested in capturing energy for your house. Usually it's a reverse you would like to protect and it's another problem with greenhouse you have to make solar protection, but we are tackling the problem of the winter behavior of the greenhouse

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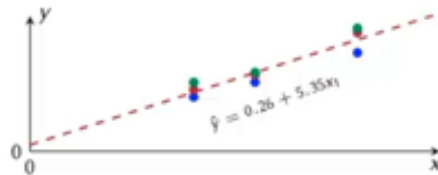
2m 44s



## 7.1.9 Optimization of a greenhouse : data

Table – Average overheating hours ( $T_{in} \geq 25^{\circ}C$ )

Building	A	B	C
Window	40%	55%	80%
D	2.2 hrs/D	2.8 hrs/D	4 hrs/D
J	2.5 hrs/D	3.1 hrs/D	4.8 hrs/D
F	2.8 hrs/D	3.2 hrs/D	5 hrs/D



and we would like to model it in relation with the fraction of glass and wall. So in this case, you see it's a case of operational, we do not have, you cannot change so easily your building. Sometimes it happens, there are laboratories of building physics where you build no five different greenhouses and you test them. The building in which I was making my PhD was a building like that, we can change the facade and change a few things and make different types of greenhouse, but usually what you do in this type of field, like in medicine, you look building that have different ratio of wall and of glasses and you try to see if you can observe something, you make some measurement around the winter and so like that you can have an evaluation. So it's an observational, usually it's something you have to be very careful with observational because if you have correlation between factors, it could be more tricky than when you make an experiment. So we have two factors, the surface of wall and the surface of window and it's evident that the sum of these two surface make your greenhouse. We could have eventually different types of, what would it first, what would be the response. So in this type of measurement, it's typical, you ask or you make temperature capture and you decide that when it's more than 25 degrees too hot for people and they will not take profit of the place. So you say it's not the right moment that you would like to avoid the period of time in which the temperature is very high. You can also ask people to vote, they have to go every day at a precise time in the greenhouse and vote if it was agreeable too cold, too hot and etc. This type of measurements that we do in this type

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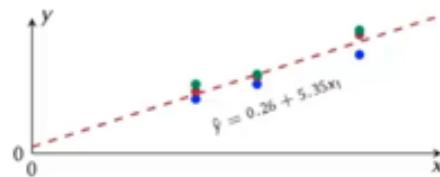
4m 13s



## 7.1.9 Optimization of a greenhouse : data

Table – Average overheating hours ( $T_{in} \geq 25^{\circ}C$ )

Building	A	B	C
Window	40%	55%	80%
D	2.2 hrs/D	2.8 hrs/D	4 hrs/D
J	2.5 hrs/D	3.1 hrs/D	4.8 hrs/D
F	2.8 hrs/D	3.2 hrs/D	5 hrs/D

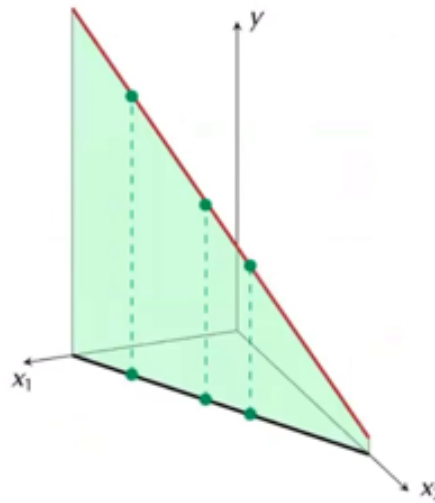


of thing, it's also when you measure temperature in this situation, you have also very special simmer meters, sometimes it's just the temperature of the air or you can make a mixture of the temperature of the air plus the temperature of the wall, what you feel of the temperature of the wall and it exists on very specific probes for evaluating the comfort of people. So there are different types of measurements that we will do and so the idea is to make a relation between the situation and what you measure and what could be the use of these elements. And so in this strategy, we do what we say in situ measurement and questionnaire, we do not have a special design and it's also the fact that it's depend on the meteor, you can have sunny days, you can have cloudy days, so it's very complicated sometimes to make very precise measurements. So in this situation, you put instruments, you let your instrument during all the winter and you have an analysis and an answer at the end of the winter season. So imagine the situation, we have three buildings, A, B and C and in this building, one have the surface of glass is 40%, another has 55% and another have 80% of surface. And we have the months, so December, January, February and it's very typical in this field, you make measurements, months, measurements and so it's like three different measurements. The climate is not exactly the same but it's not, it's winter so it's not also not completely different than they are considered as average. And you see that if you make, so we have the

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Here is the solution in the plane  $x_1 + x_2 = 1$



### Modelling and design of experiments

notes

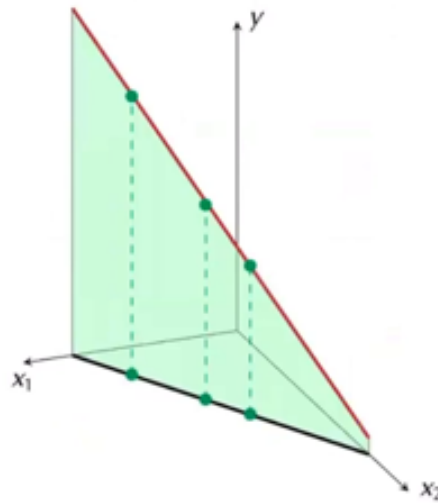
8m 25s





### 7.1.10 Optimization of a greenhouse : model

Here is the solution in the plane  $x_1 + x_2 = 1$



make greenhouse with no glasses on the wall, so you are interested in different situations and even 80% of glasses is quite high but okay and you know that you can make a model, can make a fit of this data and you have here the value of the fit so you see that so why was the the numbers of hours that you cannot use when your greenhouse is not gradable and it depends on x which is the fraction of your glass in reference to your thing but eventually this very short as data again it's clear that you can have a big and a small greenhouse, it can have an effect, you can have

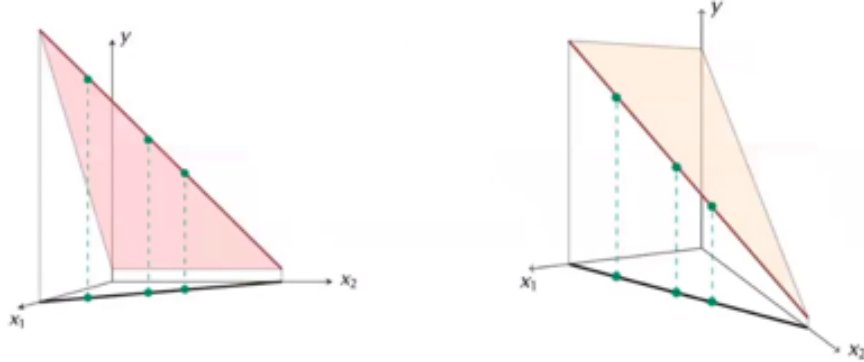
#### notes

#### summary

### 7.1.11 There are an infinity of solutions

$$\hat{y} = 0.26 + 5.35x_1 = 0.26(x_1 + x_2) + 5.35x_1 = 5.61x_1 + 0.26x_2$$

$$\hat{y} = 5 + 5.35x_1 - 4.74(x_1 + x_2) = 5 + 0.61x_1 - 4.74x_2$$



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Modelling and design of experiments

other facts but I just wanted to make a very simple case. Nevertheless if another lab would make the same measurement we are not sure everybody will get the same results because in fact we are in a space of three dimensions, we have two factors and an answer and there are different ways of representing the results, we can represent the results just as I presented to you and in this situation it's not a problem, my case is really limited for what I want to prove but so it could be in the plane you see this plane which is over the relation representing the 100% of surface between the glass and the wall and you see the model which is the red line here and you see the three measurement points and it could be the model that we have in fact our model is this straight line but as we know that  $x_1$  plus  $x_2$  is the ratio of the surface of glass and the surface of wall

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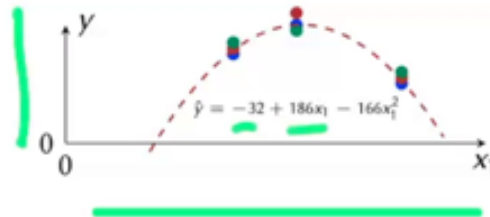
summary

11m 37s



## 7.1.12 Average votes about the comfort

Building	A	B	C
Window	40%	55%	80%
P1	15	20	10
P2	16	22	11
P3	17	19	12



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is the surface, the total surface, we can replace the constant by something so the model that I have that was one quarter is a constant plus five times the ratio of the glass could be replaced by another model when you see that the quarter of the I know here the quarter of the constant could be considered as the sum the 100% of wall and of glasses and now I'm able to rewrite this function saying my model is not a constant plus the ratio of the glass is the ratio of the glass 5.6 times plus a quarter of my ratio of I believe  $x_1$  is what's the glass so  $x_2$  was the wall and we can also propose another so we have an infinity in fact we have any model represented in the planes that are integrating these red lines that represent my model and all those models are equivalent so it will be necessary to be agree between researchers to see when we do this type of measurement what do we decide do we make a constant plus the fraction of wall constant plus the fraction of glass or just the fraction of both those models are equivalent but we just have to decide just for publishing things just for for for being agree of what what we call what what when we ask people also the comforts you understand that in the greenhouse you understand that when it's too cold people don't appreciate this is also the problem of the greenhouse it could be too cold in winter because you are losing a lot of energy from the greenhouse so perhaps during the night or late in the in the evening it's too cold and you want to don't want don't want to stay there for reading or eating and sometimes it's too hot and you don't want to stay there so now if

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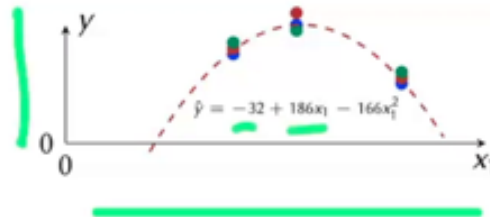
summary

12m 49s



## 7.1.12 Average votes about the comfort

Building	A	B	C
Window	40%	55%	80%
P1	15	20	10
P2	16	22	11
P3	17	19	12

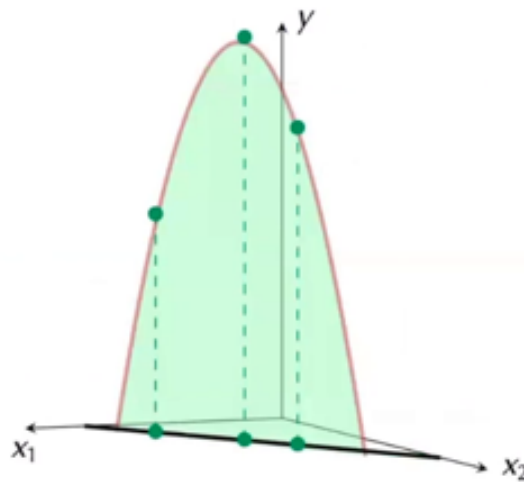


you are talking about not only the numbers of hours it was a statistical result if we are putting as an answer some things that could be too much or not sufficient we have something which could represent a sort of second degree model so you see here when we have these three situations the building ABC again with the same type of ratio of glass in relation to the total surface and we have asked people to vote sorry I don't remember what was the vote perhaps it was perhaps 20 was was very good and after they could be more or less if it's not comfortable and it's like a mark in the course etc and you see that if you represent again the data that we have with horizontally the ratio of glass in our greenhouse and vertically the vote of people in this case we can observe something that corresponds to the the comfort appreciated by the people that have been tested with this model of comfort again it could be represented with second degree curves so here you have a second degree curve with a constant minus 32 with a linear influence of the ratio of glasses of the greenhouse something as 190 and a quadratic coefficient negative because the the comfort is decreasing when we are outside of the

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### 7.1.13 Solution in the plane $x_1 + x_2 = 1$



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optimum and we have something as 166 the coefficient of the quadratic the quadratic term

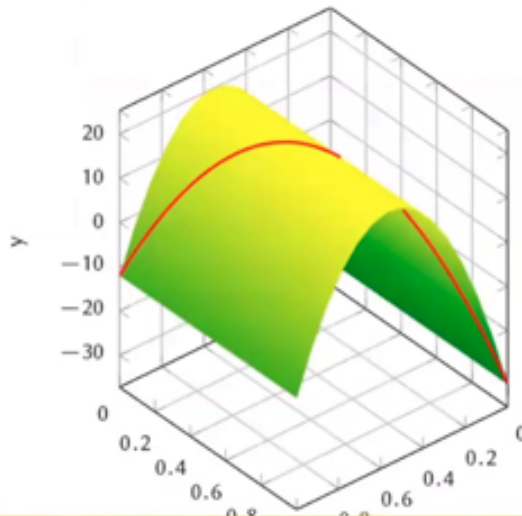
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summary

17m 1s



- ▶ The function  $y = -32 + 18x_1 - 166x_1^2$  can be interpreted as a paraboloid in the 3D space  $x_1 x_2 y$
- ▶ But an infinity of functions have the same intersection with the vertical plane  $x_1 + x_2 = 1$



### Modelling and design of experiments

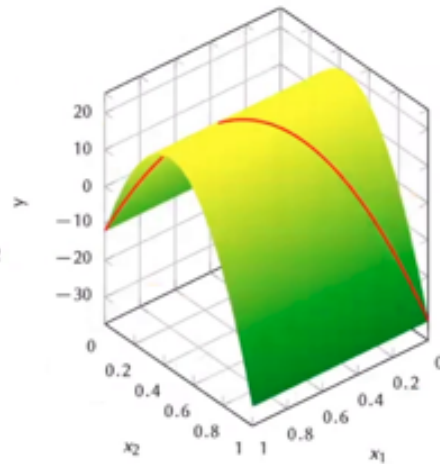
notes

17m 13s



## 7.1.15 Here is another possible model

$$\begin{aligned}
 y &= -32 + 18x_1 - 166x_1^2 \\
 &= -32 + 186(1 - x_2) - 166(1 - x_2)^2 \\
 &= -12 + 146x_2 - 166x_2^2
 \end{aligned}$$



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between wall and glass but in fact I can change it because I can replace my constants by some elements by the the sum of the two elements because it makes one when I make the sum of my two elements and so it could be this this picture which is a tunnel and you see the model which is a

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17m 42s



## 7.2.1 Scheffé's and slack models

- ▶ The reduction to a  $(N_{fact} - 1)$  dimension experiment space, due to the additional equation  $\sum_{i=1}^q x_i = 1$ , has the consequence that several models allows to model a given response.
- ▶ The imposed correlation between the factors reduces the rank of the essay matrix  $E$  of one unit.
- ▶ A *canonical* model is needed : a unique representation for the different models that represent the same response
- ▶ There are several possible choices
  - ▶ Scheffé's models (models without an intercept)
  - ▶ Slack models (models with an intercept)

red line which is well represented here but it could be any of the different tunnel that can fit with this arc that finally models that can be accepted so again all those models are equivalent all those models are good it's just to be agreed between people what do we do so there are

notes

summary

18m 9s





## 7.2.1 Scheffé's and slack models

- ▶ The reduction to a  $(N_{fact} - 1)$  dimension experiment space, due to the additional equation  $\sum_{i=1}^q x_i = 1$ , has the consequence that several models allows to model a given response.
- ▶ The imposed correlation between the factors reduces the rank of the essay matrix  $E$  of one unit.
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- ▶ There are several possible choices
  - ▶ Scheffé's models (models without an intercept)
  - ▶ Slack models (models with an intercept)

in fact two different ways of doing things so the first things I present you without I think was a slack model I was just taking into account one of the factors I have two factors that sum up to one so I just take one the ratio after we have to decide which one it could be the ratio of glass of the ratio of wall but we can but it exists another model which is called the Sheffey model that in fact is usually preferred because I will have the ratio of each one and I see my factors because I have this new equation saying that the sum of my factors is one I have a rank or I have a degree of freedom is synonym which is one unity smaller so if I have two factors I just have one degree of freedom if I have three I just have two degrees of freedom if I have four I just have three degrees of freedom etc so it was also what give us the possibility with three factors to represent these ternary graphs that we have last week because with three factors we can play on two degrees of freedom that means that we can play on a surface and we are not obliged to play in the volume which is also a little bit more simpler to play on the surface and to play on the volume so we need something we call the canonical model we have to decide which of those different possible model we can do and one way is to make a Sheffey model and the Sheffey model I don't have a constant so I will represent the model just with the ratio of the first factor the ratio of the second factor even though the ratio of the third factor but not a constant and this will represent a model

### notes

### summary

18m 30s



## 7.2.1 Scheffé's and slack models

- ▶ The reduction to a  $(N_{fact} - 1)$  dimension experiment space, due to the additional equation  $\sum_{i=1}^q x_i = 1$ , has the consequence that several models allows to model a given response.
- ▶ The imposed correlation between the factors reduces the rank of the essay matrix  $E$  of one unit.
- ▶ A canonical model is needed : a unique representation for the different models that represent the same response
- ▶ There are several possible choices
  - ▶ Scheffé's models (models without an intercept)
  - ▶ Slack models (models with an intercept)

the Sheffey model it's what we call the Sheffey model and when we have an intercept that means that we need to have one variable less because we are missing one degree of freedom so we call that slack model it's work too it's less appreciated because in fact you have a hidden factor it's factors that you never see and it's quite nice to see your factors and a cognitive aspect epistemology aspect that you don't appreciate having a model in which you have one factors that is eventually important even the most important and you don't see it in the equation so it's usually

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## 7.2.2 Scheffé's linear mixture model

- ▶ Let's start with a standard linear model of rank  $(q + 1)$  :

$$y = a_0 + \sum_{i=1}^q a_i x_i$$

- ▶ Let's introduce, at the level of the constant  $a_0$ , the proportionality constraint  $\sum_{i=1}^q x_i = 1$

- ▶ The Scheffé's model (of rank  $q$ ) is then :

$$y = \sum_{i=1}^q (a_0 + a_i) x_i = \sum_{i=1}^q \beta_i x_i$$

- ▶ With the Scheffé's linear coefficients  $\beta_i = (a_0 + a_i)$

these two types of canonical models that we use the Sheffey model and the slack model

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21m 13s

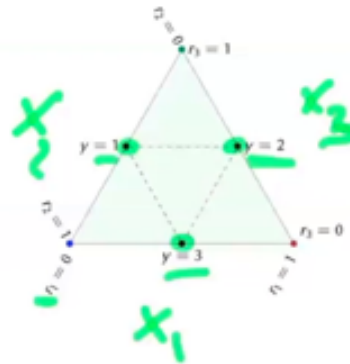


## 7.2.3 Example of a Scheffé's linear model

1. A recipe is made with three ingredients  $x_1$ ,  $x_2$  and  $x_3$  so that  $x_1 + x_2 + x_3 = 1$
2. Experiments have been made at the middles of the vertices of the ternary scheme. The output  $y$  is a KPI of the process.

$x_1$	$x_2$	$x_3$	$y$
0.5	0.5	0	2
0.5	0	0.5	3
0	0.5	0.5	1

3. Determine the coefficients of the Scheffé's linear model



personally I really prefer Scheffey model so let's see to what what is the the figure of a linear Scheffey model and after I will show you the quadratic model and I also work you the third degree model so when you are working with Scheffey model for a mixture so that means that we are looking for some planes we are representing so we are looking for some slopes when I'm integrating this constraint of having the sum of my factors that sum up to one I then can make disappear the constant and now I have a model that will be the constant plus the I don't know how to call it the standard coefficient or the classical least square feet coefficient I now make one new coefficient that we called beta we appreciate having when we have sometimes we jump from one representation to the other so usually I called alpha the classical one when we are a standard space with  $n$  degrees of freedom and when we are in a space with one degrees of freedom less we call them beta so now our linear coefficients are beta and you see that if you have for some reason for your calculation you have to go first by a standard model the beta coefficient of the Scheffey model is just for each coefficient the sum of the coefficient plus the constant so this is a Scheffey linear model here small example for illustrating that imagine we are in a situation where we have three products in our recipe or three elements they have to sum up in the I don't know it could be also a greenhouse but you have I don't know very transparent type of glass and the type of another glass and wall you have different type of wall in any case you have three factors  $x_1$   $x_2$   $x_3$  and so here you have

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### summary

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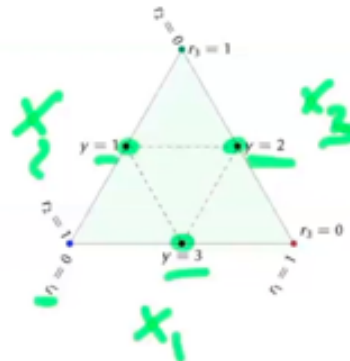


### 7.2.3 Example of a Scheffé's linear model

1. A recipe is made with three ingredients  $x_1$ ,  $x_2$  and  $x_3$  so that  $x_1 + x_2 + x_3 = 1$
2. Experiments have been made at the middles of the vertices of the ternary scheme. The output  $y$  is a KPI of the process.

$x_1$	$x_2$	$x_3$	$y$
0.5	0.5	0	2
0.5	0	0.5	3
0	0.5	0.5	1

3. Determine the coefficients of the Scheffé's linear model



a name I don't know if you were so capable key performance indicators canonical word for expressing when you are making measurement so you see how here is our data I have made a measurement where I have my two first factor at 50% each and 0% for the last one I have a measurement where I have 50% of the first one and the third one and 0% for the second one and I have another experiment so in fact it represents those points the points that are at the center of the sides of the of my ternary plot and so the first one was this one the answer was two the second one the answer was three and the last one the answer was one and so you see how I have represented you remember in ternary so check the order if you have not  $x_1$   $x_2$   $x_3$  is not important because you have the name and you see well where are the things and the fact that the fact is the first one and second one should not have an important that in this case we have  $x_1$  which is here we have  $x_2$  which is here and we have not so sorry we have  $x_3$  which is here and  $x_2$  which is here and I have also the minimum of  $x_1$  at left and the maximum as right you see here

#### notes

#### summary

## 7.2.5 Quadratic Scheffé's model

- The standard quadratic model of rank  $(q+1)(q+2)/2$  is :

$$y = a_0 + \sum_{i=1}^q a_i x_i + \sum_{i \leq j}^q a_{ij} x_i x_j$$

- The constraint of proportionality allows to write :

$$\begin{cases} a_0 = a_0 \cdot 1 = a_0 \sum_{i=1}^q x_i = \sum_{i=1}^q a_0 x_i \\ a_{ii} x_i^2 = a_{ii} x_i \left( 1 - \sum_{j \neq i}^q x_j \right) \end{cases}$$

I call sorry I call it r and not x but so the first ratio is 0 the blue the blue point and 0 at left and it's one at right so let's determine the chef's coefficient of this situation and so as I'm interested by a linear model it would be a plane over this triangle you have the triangle and imagine just a plane but you need to understand the orientation of the plane and it's it's clear that usually we represent a plane in two dimension find the third dimension but the plane over a surface as one one altitude and after two orientation but we can also represent it with three orientation it's also possible it was the same type of one because the cheffy model would be three orientation so in this case my model the model I'm trying to guess will be y equal beta one x1 plus beta 2 x2 plus beta 3 x3 plus a residue this will be my model my cheffy model so I will like in the sorry if it's very evident but I just wanted to show you that we are playing the same game as before so I have here my my responses 2 3 and 1 it was my vector of responses my y now I have my my system of equation so I represented I put in evidence one half do you remember each point was at zero or at the center the minimum and maximum ratio so I have in evidence one half and after for the first equation as well at one half one half and zero so when I put in one half in evidence it was one one and zero the second one was one half zero one half so now it's one zero one and one half in evidence etc and so I have my system I can also rewrite

### notes

### summary

25m 37s



## 7.2.5 Quadratic Scheffé's model

- The standard quadratic model of rank  $(q+1)(q+2)/2$  is :

$$y = a_0 + \sum_{i=1}^q a_i x_i + \sum_{i \leq j}^q a_{ij} x_i x_j$$

- The constraint of proportionality allows to write :

$$\begin{cases} a_0 = a_0 \cdot 1 = a_0 \sum_{i=1}^q x_i = \sum_{i=1}^q a_0 x_i \\ a_{ii} x_i^2 = a_{ii} x_i \left( 1 - \sum_{j \neq i}^q x_j \right) \end{cases}$$

my system is a standard linear system I just do it just for showing it but in fact you just put all the data in your algorithm and everything is solved so we have the sum of beta one plus beta two which is four the sum of beta one beta three which is six and sum of beta two and beta three which is two if you solve the system you arrive to the solution beta one sorry beta three equal two beta one is four and beta two is zero so I have in this case a model which is  $\hat{y}$  is four times  $x_1$  plus two times  $x_3$  and I'm interested to draw is a line so I'm interested to see in which direction in my concentration I have values that are the same so I have you see that I have  $x_1$  equal  $x_2$  so it's make vertical vertical lines and it's correspond for the other  $x_2$  equal  $x_1$  minus  $y$  divided by two plus one and  $x_3$  equal  $y$  divided by two minus two  $x_1$  and you see here I have draw a few iso lines so I have a plane you see that it's I don't know if you're going to visualize it so you see the values so the maximum is here the minimum is here medium is here so I have the triangle the front of me I have a plane which is like that if we have linear model we can work with quadratic model and you will see what is funny that in a quadratic cheffet model we do not have quadratic terms so we start from the same thing we start from the standard model so it should be a  $\hat{y}$  when I write it like that it's a constant plus linear coefficient mainly what I call main effect plus

### notes

### summary

## 7.2.5 Quadratic Scheffé's model

- The standard quadratic model of rank  $(q+1)(q+2)/2$  is :

$$y = a_0 + \sum_{i=1}^q a_i x_i + \sum_{i \leq j}^q a_{ij} x_i x_j$$

- The constraint of proportionality allows to write :

$$\begin{cases} a_0 = a_0 \cdot 1 = a_0 \sum_{i=1}^q x_i = \sum_{i=1}^q a_0 x_i \\ a_{ii} x_i^2 = a_{ii} x_i \left( 1 - \sum_{j \neq i}^q x_j \right) \end{cases}$$

interaction effect and also quadratic terms because you have here i smaller or equal to j so I have quadratic terms and when I'm integrating the constant in in in this I have that a zero equal a zero multiplied by one so I have a zero multiplying by the sum of the factors it's equal to the sum of the factors multiplied by a zero and so I can also rewrite the quadratic terms and the quadratic terms become the equivalent multiplying the linear term and one minus the sum of the other interaction terms so when I fill everything I arrive to a model that have linear terms of the cheffet model the beta and I have interaction terms beta ij but I do not have beta ii I do not have pure quadratic terms nevertheless it's corresponding in the situation not as it was sorry for the one that's in the videos they perhaps will not see me well but so the interaction in the standard model was something like that we have quite a shifted shifted plane but not a pure second-degree surface with hills and valleys but now with this type of model with the cheffet model yes we have we have a hill or we have a valley it's really a typical quadratic surface and not just a model with interaction it's what makes special of the cheffet model so I have represented what I could obtain with these different functions for the charnary plots that I present you if you're last week if you have more than three factors okay you must represent things within tetragone or if you have more factors it became quite complicated to represent inventory you will make projection and different subspace for trying to understand but you see the equivalent of the color a color surface so you see that you have if the clear the yellow represents

### notes

### summary



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the highest surface so you see a model where you have the surface which becoming high at the at the two corner here and quite low at the last corner you see better but the minimum is not at the corner in this case the minimum is somewhere in one of the vertices you see better what's happened was these surface plots which perhaps the best one to see things where you see what is your response is a really a clear parabolic and here we have the contour line the equivalent of the contour line and you can also with following the line or going perpendicular to the line arrive to the maximum that are here and here or try to arrive to the minimum which is by here okay so it's 10 o'clock so I will make a small pause and I continue in 15 minutes

notes

summary