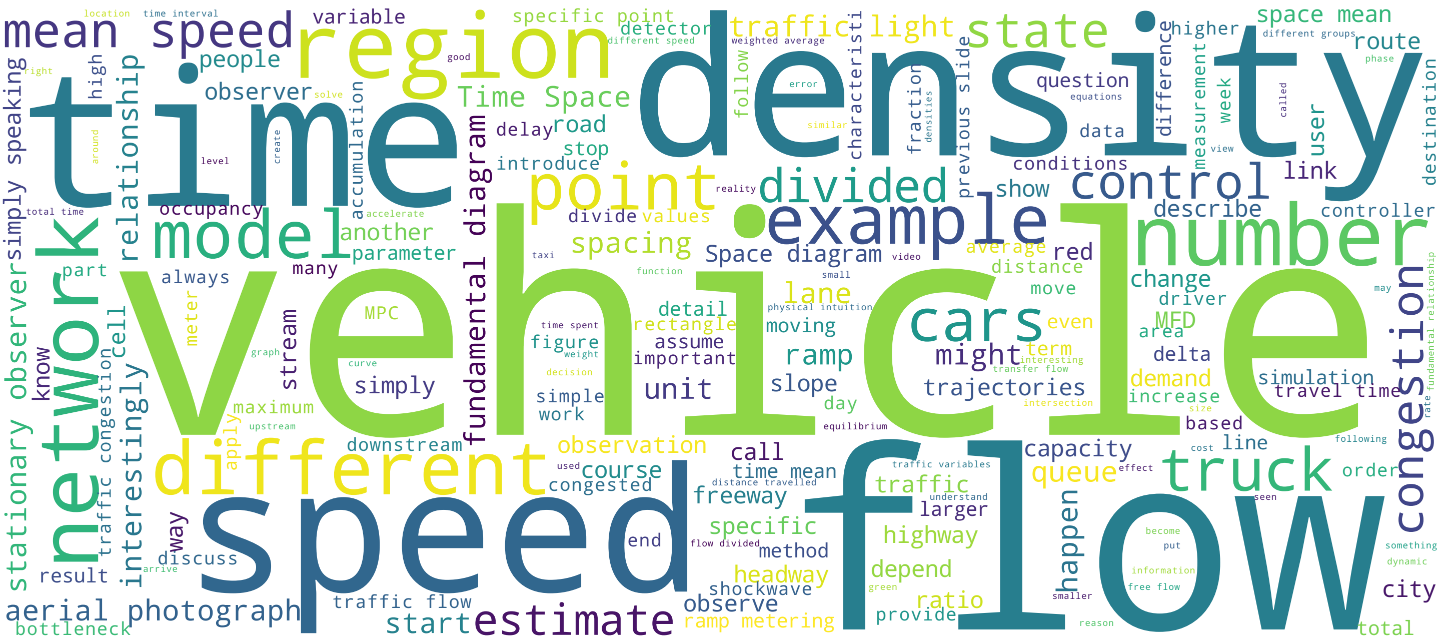
Traffic flow variables

Intro to traffic flow modeling and ITS

Prof. Nikolas Geroliminis



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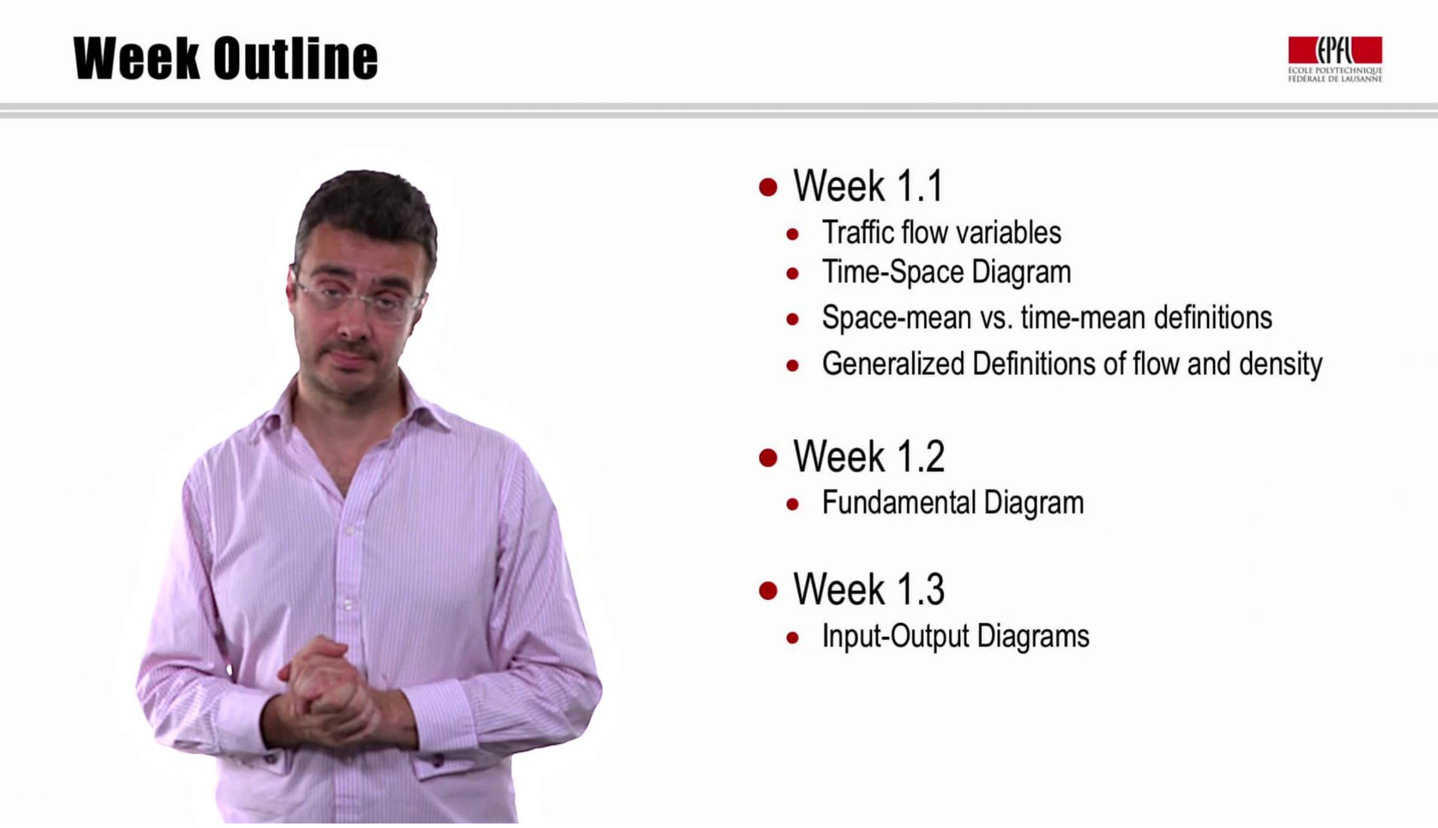


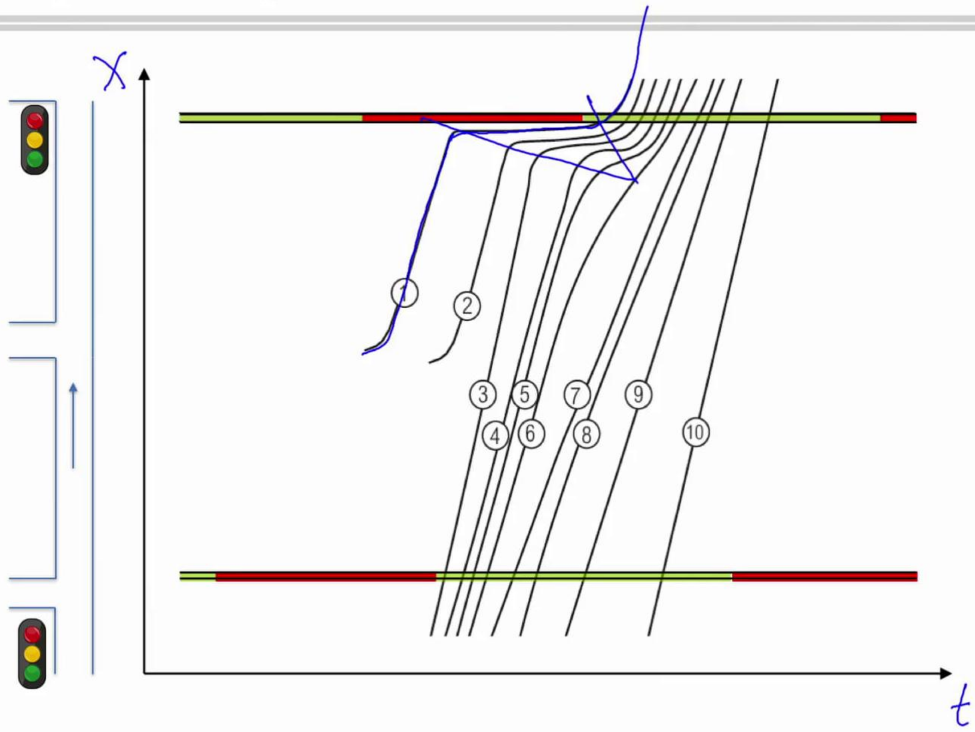
Welcome. My name is Nikolas Geroliminis. This course is about traffic congestion. Why traffic congestion occurs? Is it because you have too many cars on the road? Not only. Traffic congestion is a complex process and to describe it we need to develop models. The models are abstractions of the real world that are based on some mathematical equations but most importantly physical intuition. And to develop models the first thing we need is traffic variables. So during this first week we will start from the basics. We will introduce some fundamental variables that are necessary to describe these models. We will also talk about Time-Space diagrams which is a graphical tool necessary to explain many traffic phenomena and then we will also show how we can estimate known quantities like speed in a rigorous way that also depends on the method of observation we use. Later in this week, we will see the relation among these traffic variables and we will also describe an very interesting way to estimate the effect of congestion which is called Input-Output diagram.

Notes

Summary

0m 05s

Time-Space Diagram



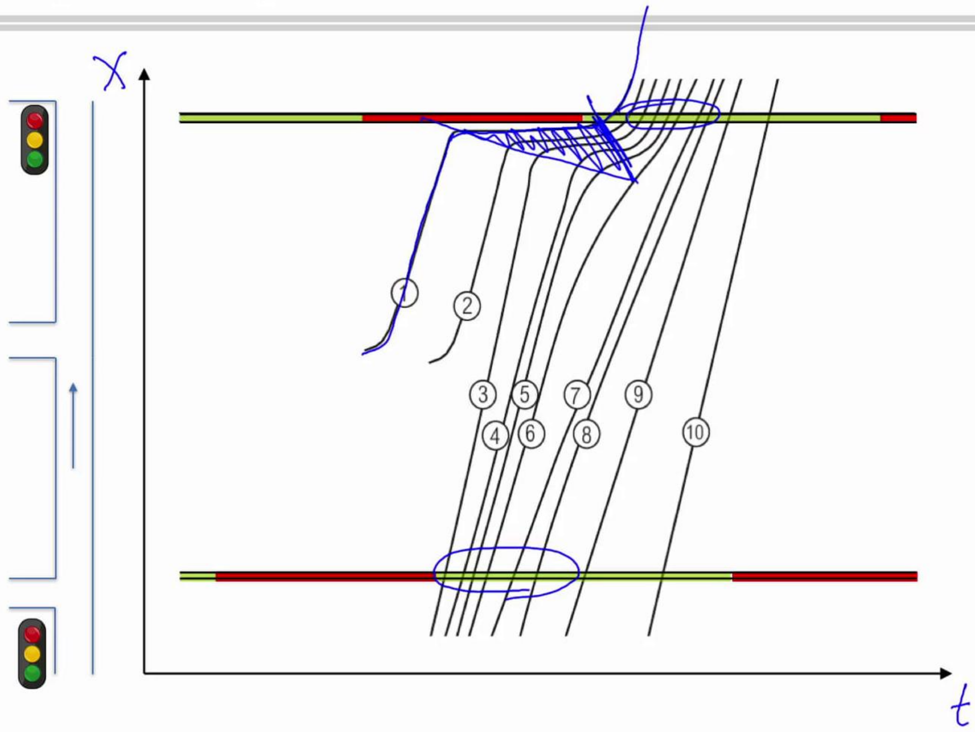
So let's start with a simple example to introduce the Time-Space diagram. Time-Space diagram is a two-dimensional diagram where the vertical axis is distance travelled and the horizontal axis is time. What we have here, we have an one-dimensional road where vehicles move from bottom to top and what we have, we have two signalized intersections with traffic lights while in the middle we have an un-signalized intersection. So in this Time-Space diagram, what we see, we see trajectories of many vehicles that are moving in this road. And if we look at the trajectory of vehicle number one, this vehicle starts from the minor road in the middle, it accelerates then it reaches its maximum speed which remains constant and at some point it reaches a traffic light which is red. So it needs to decelerate and then stop and it stops until the traffic light becomes green and then accelerates and departs again. The vehicles that follow him also have to stop and interestingly they stop with a time lag. So what we see, we see that the stopping happens by following approximately some straight line and when the signal turns green, vehicles do not start all simultaneously like the wagons of a train but again there is another time lag that follows another straight line.

Notes

Summary

1m 38s

Time-Space Diagram



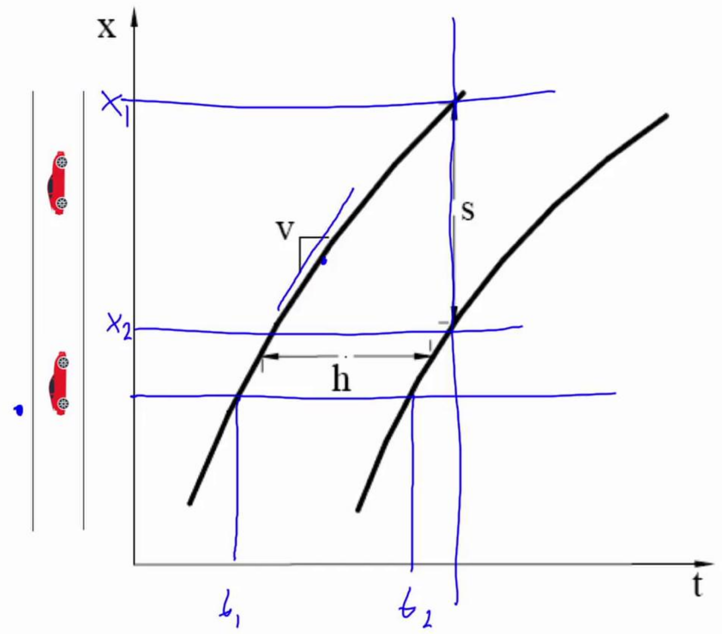
These lines we call them shockwaves and we will describe them in more details next week. What else we see in this figure? Interestingly, let's compare the stream of vehicles in the upstream and in the downstream intersection. What we can see, we can see that the trajectories in the downstream in the intersection are closer to each other and the reason that this happens is because these vehicles are discharging from a queue which is created when the traffic light is red. And this light we introduce what is a Time-Space diagram and actually we introduce many aspects of traffic flow that we will see in details as we move forward.

Notes

Summary

3m 06s

Headway and Spacing definitions



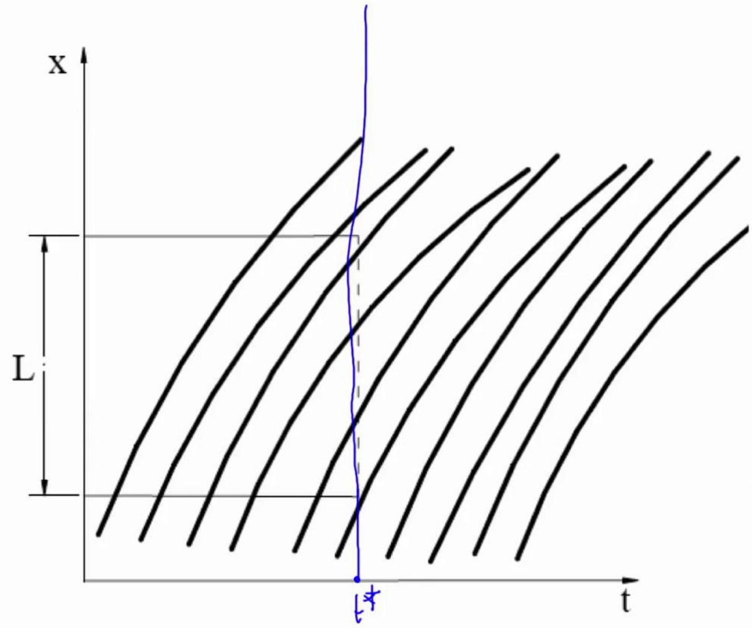
Let's start now by introducing simple traffic variables that are necessary to describe traffic congestion. Let's consider again a Time-Space diagram and two vehicles that are moving. So what we see here, we see the trajectories of the vehicles and actually if we want to estimate the speed of the vehicle, we just need to look at the slope of the tangent at a specific point. So this is the speed 'v' of the leading vehicle at a specific point in time. To introduce Headway and Spacing, what we have to look at, we have to look at different ways observing these trajectories. So if we can see there stationary observer which is a person standing outside the route at a specific point, so this person will see the two vehicles passing in front of him at times 't1' and 't2'. So this difference between 't1' and 't2' is the Headway. If now we consider a different method of observation which is an aerial photograph. This aerial photograph happens at a specific point in time and in this aerial photograph what we can see, we can see that the front end of its vehicle appears in the thought out locations 'X1' and 'X2', so the spacing between these two vehicles is the distance between 'X1' and 'X2'.

Notes

Summary

3m 54s

Density definition



- **Aerial Picture at a given time *t****

$K = \frac{m}{L}$

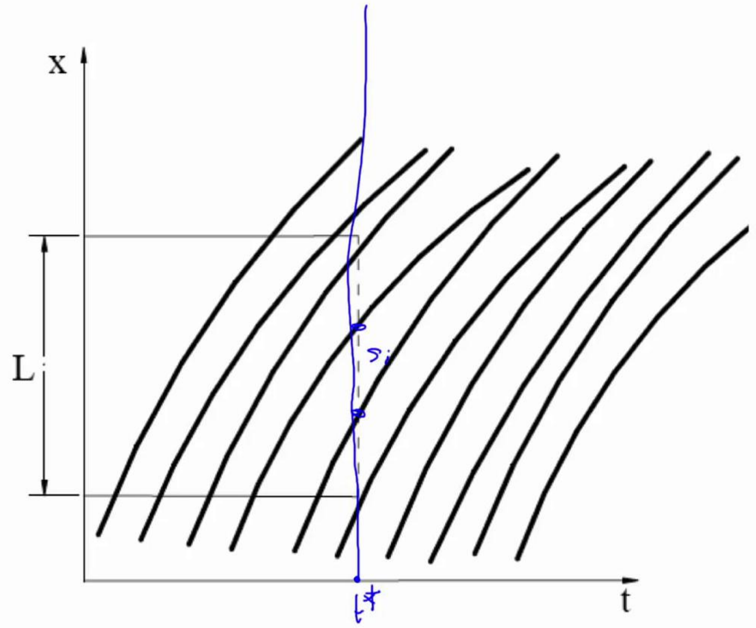
Let’s now look at the definition of flow. So let’s now consider a stream of vehicles that are moving and again we have a stationary observer at some location, let’s call it ‘X star’. And this observer, during some time interval ‘t’, observes a number of vehicles ‘n’. So if we divide the number of vehicles by time ‘t’, what we have, we have flow. The units of flow are vehicles per second or vehicles per unit time. So now we would like to connect flow with headway and in this figure we can see that, for example, between these two consecutive vehicles this headway is ‘Xi’. We can simply say that time ‘t’ is the sum of the headways for vehicles from ‘i’ to ‘n’. So if we replace this interval ‘t’ in the above equation of flow, what we will have, we’ll have that flow is ‘n’ divided by sigma of ‘hi’. This is simply one over the average headway. Let’s now define density. Again we change our method of observation and what we use, we use an aerial picture which is taken at a specific point in time ‘t star’. So in this picture the way we can define density is that during some distance ‘L’, we can observe ‘m’ vehicles. So if I divide the number of ‘m’ vehicles by this distance ‘L’, then we estimate density.

Notes

Summary

5m 30s

Density definition



- **Aerial Picture at a given time *t****

$$k = \frac{m}{L} \quad (\text{veh}/\text{m})$$

$$L = \sum_{i=1}^{m} s_i$$

$$k = \frac{m}{\sum s_i} = \frac{1}{\bar{s}}$$

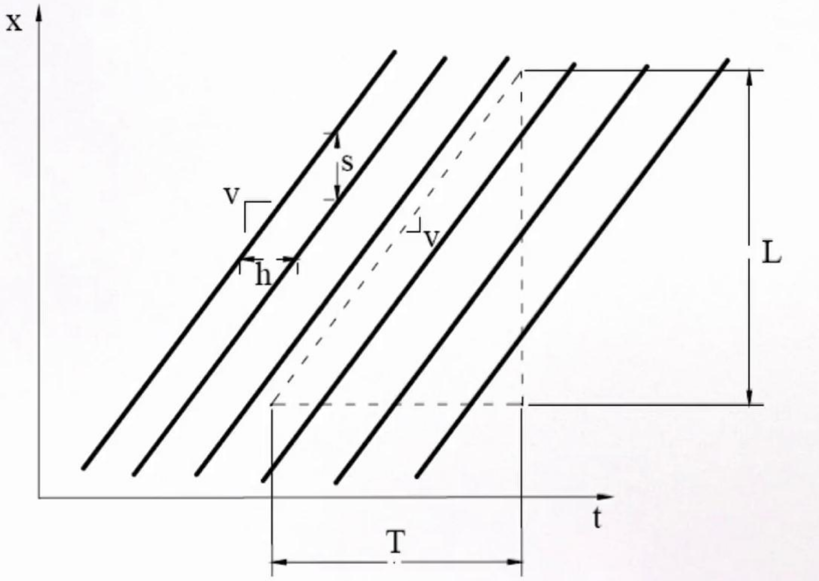
What are the units of density? The units of density is vehicles per meter. And again as in the previous slide, I would like to connect density with spacing now. So what we can write down is that this distance 'L' is equal to the sum of the spacings for all the 'm' vehicles that we see in this aerial picture. So this is, for example, one of the spacings between two consecutive vehicles. So again if we replace 'L' by this equation, we will have that density is 'm' divided by the sum of spacings which is, simply speaking, one over average spacing. So interestingly what we see, we see that we are able to create relationship between some microscopic variables, between two individual vehicles like spacing and headway and some more aggregated variables like flow and density that describe stream of vehicles.

Notes

Summary

7m 35s

Relation between flow, density and speed



$$V = \frac{s}{h}$$

$$V = \frac{\frac{1}{k}}{\frac{1}{q}} = \frac{q}{k}$$

$$\frac{m}{sec} \sim \frac{\frac{veh}{sec}}{\frac{veh}{m}} = \frac{m}{sec}$$

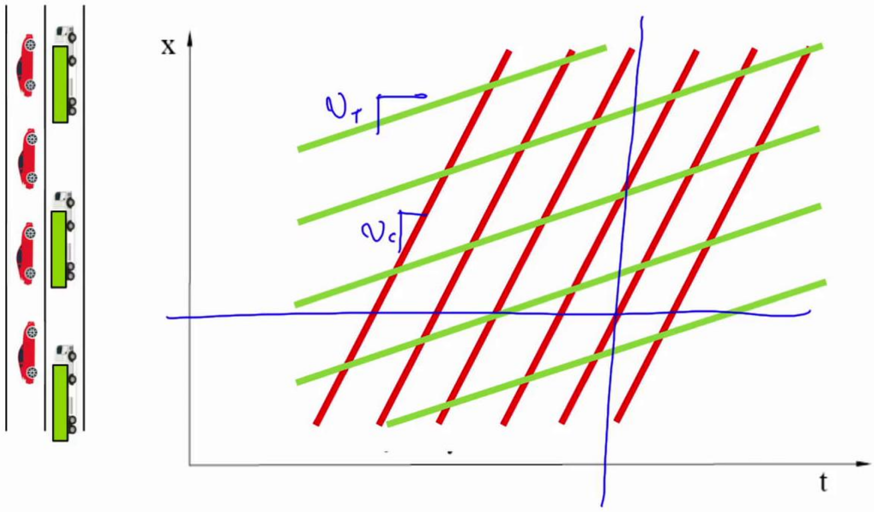
An interesting question is what is the relationship between all of them. So now let's use again a Time-Space diagram and let's now consider that we have vehicles that have always same speed 'v' and actually they also have equal spacing and equal headway. So in this case, very easy to say that speed is spacing divided by headway but if we think what we discuss in the previous slides, we can replace spacing by one over density and headway by one over flow. So in that case what we will have, we will have that speed is flow divided by density. And this is a fundamental relationship of traffic flow that we will see many times in this course. And this relationship is very intuitive and we can even understand it by looking at the units of these variables. So speed has units of meters per second while flow has units of vehicles per second and density has units of vehicles per meter. So interestingly here if we make some simple calculations we see that our relationship is correct from a dimensionless analysis point of view.

Notes

Summary

8m 49s

Groups of vehicles – Methods of observations



- **Stationary observer**
  - **Fraction of trucks is**
- **Aerial Photograph**
  - **Fraction of trucks is**

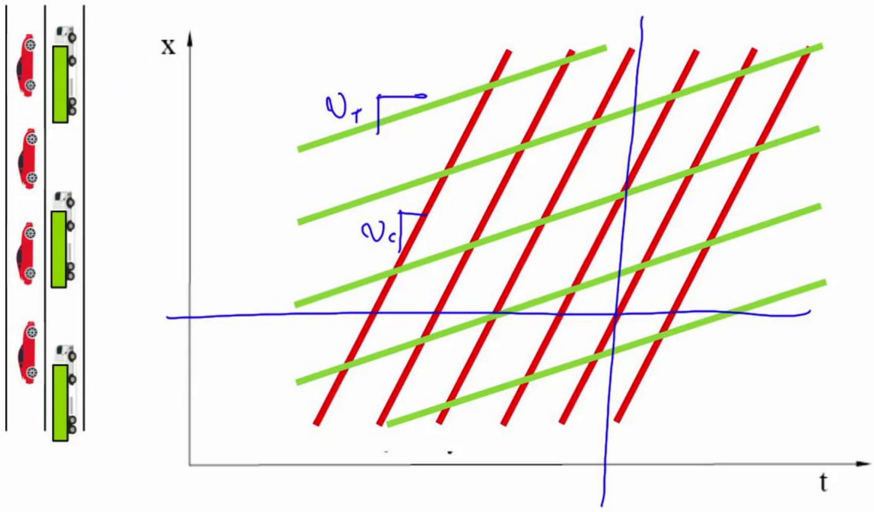
So far we talked about one stream of vehicles that have the same characteristics or same speed, spacing and density but what we know, we know that in reality we don't have always these homogeneous conditions. So let's consider an example where we have a road with two lanes where the left lane is used by fast cars while the right lane is used by slow trucks. And this is what we see in this Time-Space diagram. So we have some green trajectories with slope which is equal to the speed of the truck 'Vt' and we have some red trajectories which are with a slope that is the speed of the cars. So here the question is how can we estimate the average speed for this stream that consist of different groups of vehicles. So this question does not have a unique answer and it depends on the method of observation. So it depends if we want to estimate the average speed from an aerial photograph or from a stationary observer. And let's look this in more details. The difference is the following. If we can see there a stationary observer or an aerial photograph, what we will see, we will see that a fraction of trucks and cars in these two methods of observation are quite different.

Notes

Summary

10m 30s

Groups of vehicles – Methods of observations



- **Stationary observer** (So)
  - Fraction of trucks is

$$q_T/(q_T+q_c)$$

- **Aerial Photograph** (Ap)
  - Fraction of trucks is

$$k_T/(k_T+k_c)$$

So the fraction of trucks in a stationary observer is simply speaking the flow of the trucks divided by the total flows; the flow of the trucks plus the flow of the cars. While for an aerial photograph, this fraction is the density because an aerial photograph measures density, divided by the total density which is the density of the trucks plus the density of the cars. And for simplicity let's call a stationary observer 'So' and an aerial photograph 'Ap'. So now we are ready to define two different types of speeds that depends on these two methods of observation.

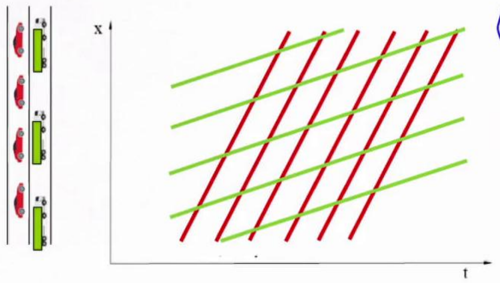
Notes

Summary

12m 03s

Space-mean and time-mean speed

$q_i = k_i v_i$



$$(AP)\quad v_{SMS} = \frac{k_T v_T + k_C v_C}{k_T + k_C} = \frac{\sum_i k_i v_i}{\sum_i k_i}$$

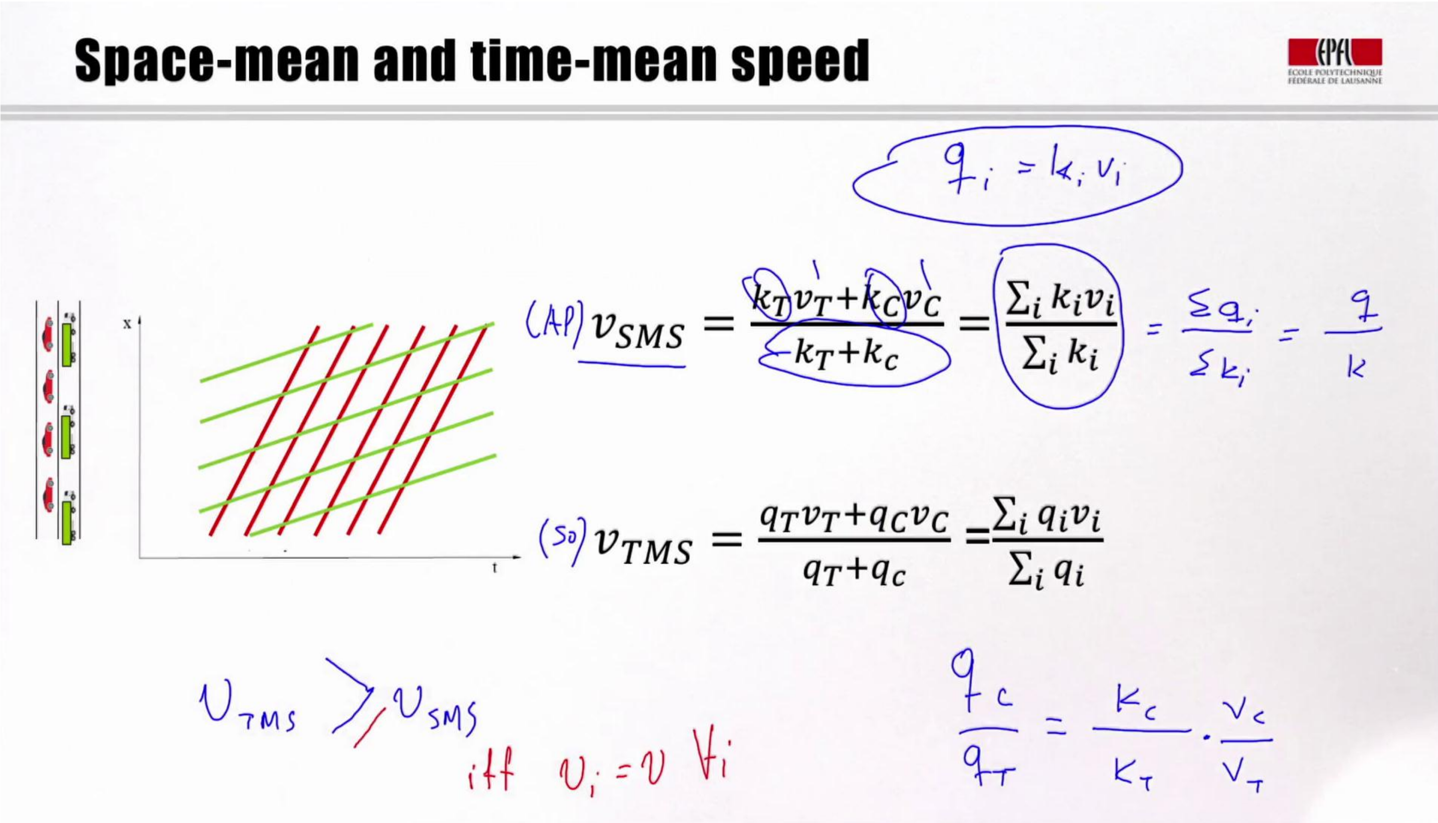
$$(SO)\quad v_{TMS} = \frac{q_T v_T + q_C v_C}{q_T + q_C} = \frac{\sum_i q_i v_i}{\sum_i q_i}$$

So first we have the space-mean speed with space-mean speed is from an aerial photograph and a time-mean speed which is based on a stationary observer. So here what we have, we have two groups of vehicles with two different speeds. So when we estimate the space-mean speed, this is a weighted average of the speed of trucks and the speed of cars and the weights we use are what we observe in an aerial photograph. So it is the density of trucks and the density of cars and of course, as this is the weighted average, we need to divide by the total density. While in the time-mean speed, we have to follow a same procedure but now the weights we use are the flows of the trucks and the flow of the cars. Actually these two equations are very very general and we can generalize them even if we have a large number of different groups. So we write here that simply speaking the space-mean speed is the weighted average of speeds of different groups of cars and the weight is density while the time-mean speed is the weighted average of speeds of different groups of cars where the weight now is the flow. What we said before is that for each stream, for each group we have the fundamental relationship between flow and density, so ‘i’ can be car or truck.

Notes

Summary

12m 50s



So interestingly this equation becomes the sum flows divided by the sum of densities. So this is the total flow divided by the total density. So what we see, we see that we have another fundamental relationship as in the case of vehicles that have all the same speed that connects speed, flow and density. So flow divided by density is equal to space-mean speed and this is always the case independently if we have vehicles at the same speed or a different speeds. But what is interesting is, what is the relationship between time-mean and space-mean speed? So what we can say is that actually the time-mean speed is always larger than the space-mean speed. And actually they are equal only under one condition. This is the condition if and only if all the vehicles in our stream have the same speed. And even if I will not provide a formal proof for this inequality, I would like to give some physical intuition and to give this physical intuition, I would like that we look at the ratio of the flow of cars versus the flow of trucks. This ratio, if we use again this relationship, is equal to density of cars multiplied by the speed of cars divided by the density of trucks by the speed of trucks.

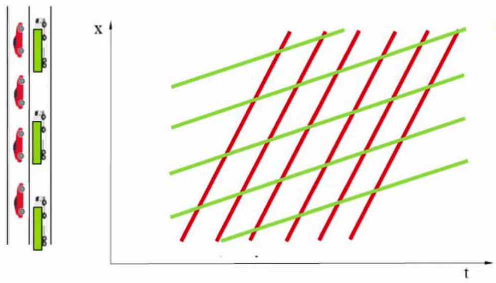
Notes

Summary

14m 28s

Space-mean and time-mean speed

$q_i = k_i v_i$



(AP) $v_{SMS} = \frac{k_T v_T + k_C v_C}{k_T + k_c} = \frac{\sum_i k_i v_i}{\sum_i k_i} = \frac{\sum q_i}{\sum k_i} = \frac{q}{k}$

(SO) $v_{TMS} = \frac{q_T v_T + q_C v_C}{q_T + q_C} = \frac{\sum_i q_i v_i}{\sum_i q_i}$

$v_{TMS} \geq v_{SMS}$

iff $v_i = v \ \forall i$

$\frac{q_c}{q_T} = \frac{k_c}{k_T} \cdot \frac{v_c}{v_T} > 1$

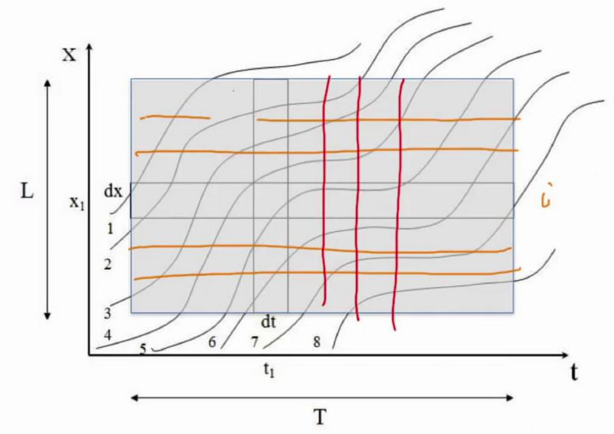
But interestingly cars are faster than trucks. So this ratio is larger than one which also means that the ratio of flows between cars and trucks is larger than the ratio of densities between cars and trucks. So this means that a stationary observer sees more frequently faster vehicles and this is the main reason that the time-mean speed is larger than the space-mean speed.

Notes

Summary

16m 18s

Generalized definitions of flow and density



$$q_i = \frac{n_i}{T}$$

$$n_{ob} = \frac{L}{dx} \qquad n_p = \frac{T}{dt}$$

VKT: **Vehicle Km Travelled**
VHT: **Vehicle Hr Travelled**

Based on Edie (1963)

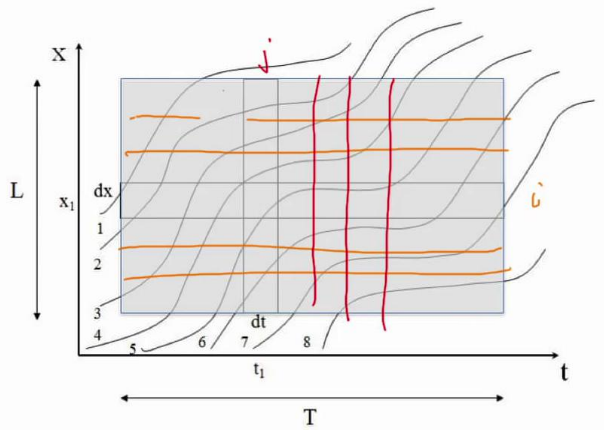
So what if we are not under homogeneous conditions and we have some vehicles that are moving in a specific row then each vehicle has its own trajectory and different speed over time and space. How can we define flow and density in this case? So let's consider that we have a rectangle of time and space, so some duration 'T' and some distance 'L' and we have a number of vehicles that are crossing this rectangle. So what we do, we discretize both time and space and let's say that we discretize space with 'delta X' space intervals and we discretize time with 'delta T' time intervals. This means that we put one stationary observer every distance 'delta X' and we take one aerial photograph every time 'delta T'. So if ask ourselves how many photographs and how many stationary observers we have in this experiment; the number of observers is simply speaking 'L' divided by 'dX' and the number of photographs is equal to 'T' divided by 'delta T'. So each of these stationary observers, let's call them 'i', they measure flow at a specific location. So the flow measured by observer 'i', simply speaking, the number of vehicles it sees divided by time.

Notes

Summary

16m 53s

Generalized definitions of flow and density



$q_i = \frac{n_i}{T}$ $\qquad k_j = \frac{m_j}{L}$

$$Q = \frac{\sum_{i=1}^{n_{ob}} q_i}{n_{ob}} = \frac{\sum n_i\, dx}{L \cdot T} = \frac{VKT}{L \cdot T}$$

$$K = \frac{\sum_{j=1}^{n_p} k_j}{n_p} = \frac{\sum m_j\, dt}{L \cdot T} = \frac{VHT}{L \cdot T}$$

$n_{ob} = \frac{L}{dx}$ $\qquad n_p = \frac{T}{dt}$

VKT: **Vehicle Km Travelled**
VHT: **Vehicle Hr Travelled**

Based on Edie (1963)

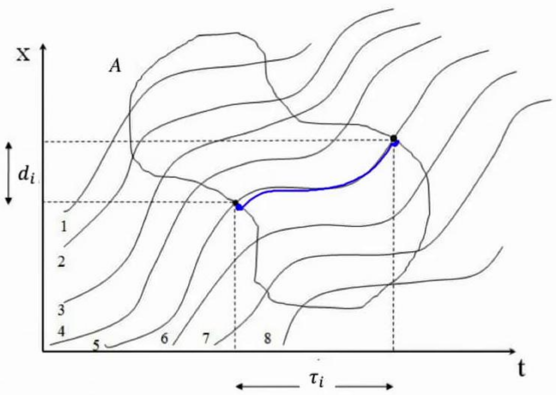
At the same time the density measured by a specific aerial photograph ‘j’ is ‘kj’, the number of vehicles seen in this photograph; let’s call them ‘mj’, divided by distance ‘L’. So now we are ready to define the generalized flow and density which is simply the average flow estimated by all these observers or photographs. The average flow is simply speaking the sum of the flows from ‘i’ equal to one to ‘n’ observer divided by the number of observers while the average density is the sum of the densities from ‘j’ equal to one to number of photographs divided by number of photographs. So let’s do some more calculations here. So what we have, we have sigma times delta ‘dX’ divided by ‘L’ times ‘T’ and for the density we have sigma ‘mj dt’ again divided by ‘L’ over ‘T’. So the question is what is sigma ‘ni delta X’? Actually this is the total vehicle kilometers travelled, is the total distance travelled by all these trajectories in this Time-Space domain. While the sum of ‘mj delta T’ is the total time spent, the vehicle hours travelled by all the vehicles that moved again in this Time-Space domain. So these are the generalized definitions of flow and density and actually we can see this also from a different point of view.

Notes

Summary

18m 43s

Generalized definitions of flow and density



$$Q = \frac{\sum_i d_i}{|A|} = \frac{d(A)}{|A|} = \frac{VKT}{Area}$$

$$K = \frac{\sum_i \tau_i}{|A|} = \frac{\tau(A)}{|A|} = \frac{VHT}{Area}$$

$$V_{SMS} = \frac{d(A)}{\tau(A)} = \frac{VKT}{VHT}$$

VKT: **Vehicle Km Travelled**
VHT: **Vehicle Hr Travelled**

Based on Edie (1963)

And this different point of view is that if we consider that each of these individual trajectories, let's for example grab this specific trajectory, this specific vehicle spends time 'tau i' and it travels distance 'di'. You might question yourself that now this region A is not a rectangle but it has a different shape. Yes because actually we can define flow and density with the generalized definitions for any shape; it doesn't need to be a rectangle. So there the total distance travelled is simply speaking the sum of 'di' while the total time spent is the sum of 'tau i' that we have for all these trajectories available. So the equations are the same as in the previous slide and the flow is VKT divided by the size of area A while density is VHT divided by again the same size. And interestingly, if we divide the vehicles kilometers by the vehicles hours travelled, what we get, we get the space-mean speed and you can easily see that this relationship also is consistent with the units of the variables we use.

Notes

Summary

21m 04s

Traffic flow variables - Summary

We introduced what is a Time-Space diagram and we saw many transparent phenomena in aspects of traffic flow that we will see in detail as we move forward in this course.

Notes

Summary

22m 27s