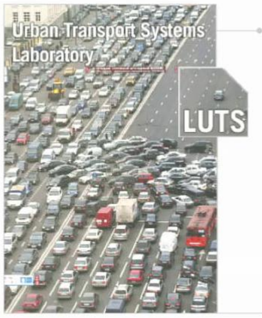
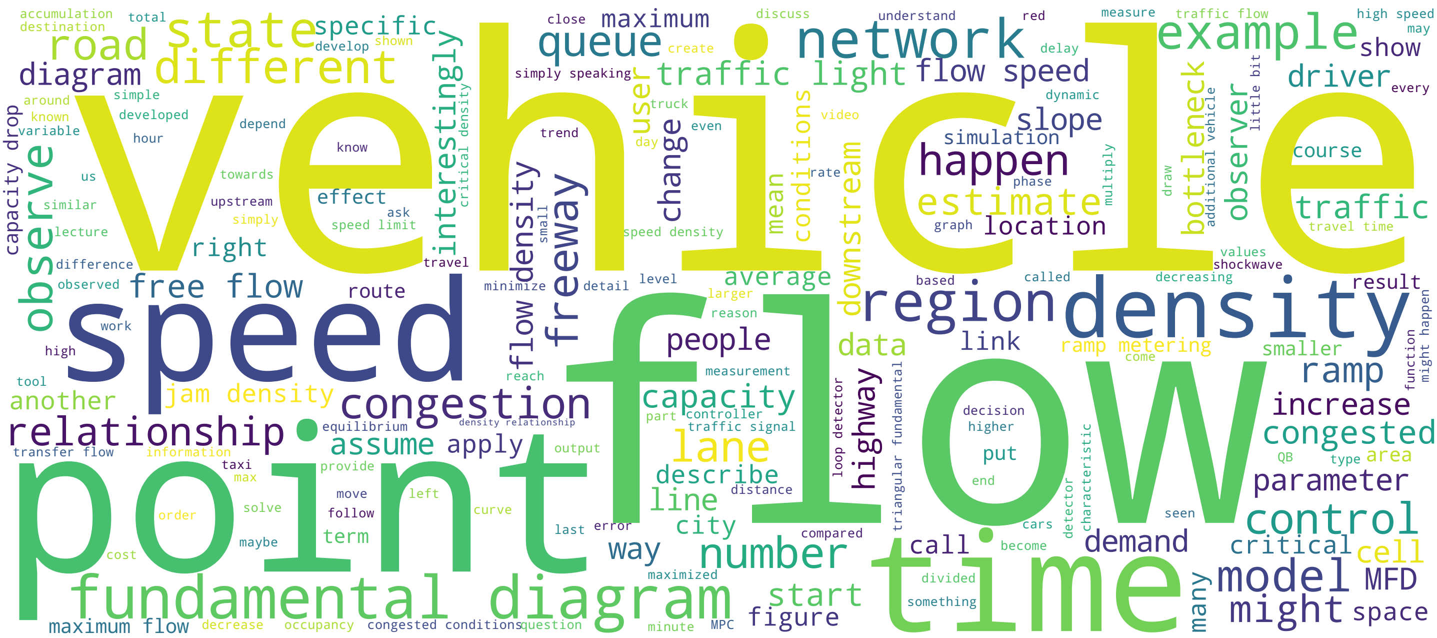
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Fundamental Diagram



Intro to traffic flow modeling and ITS

Prof. Nikolas Geroliminis



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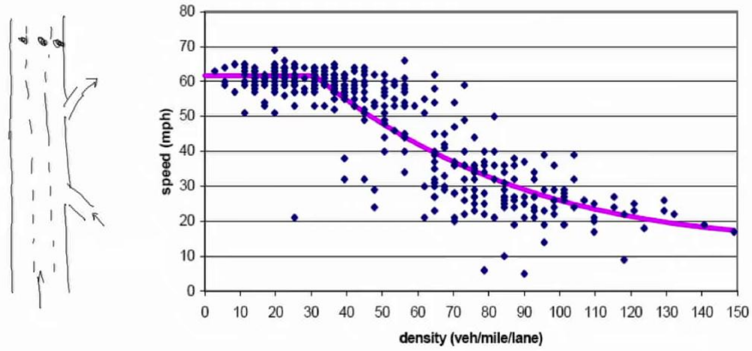
EPFL

Field observations (Freeways)



Scatter is due to:

- Heterogeneity of drivers
- Weather conditions
- Type of vehicles
- Non-stationary conditions

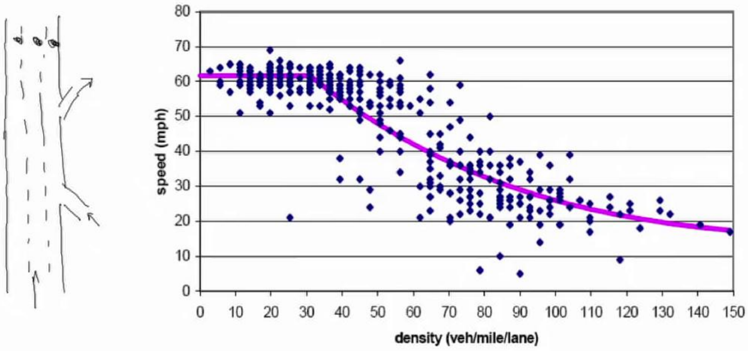
Welcome. The lecture today will focus on the fundamental diagram which is a diagram that describes the relationship between key traffic variables like flow, density and speed. And even if this diagram was first empirically observed more than 80 years ago, still today is a very important tool that helps researchers and practitioners to study traffic flow congestion and how it changes over time and space. As the fundamental diagram is an empirically observed diagram, I would like to start the discussion by directly looking at some data. So let's start with some field observations for data of speed and density that have been collected in a freeway. So what we have, we have a freeway where traffic moves from bottom to the top and let's say this freeway might have some on and some off-ramps for the vehicles to get in and out. And let's assume that we put some loop detectors with some sensors located below the pavement that can measure locally flow, speed and also density of vehicles and each point over here in this graph represents this measurement for a short time period, let's say a couple of minutes.

Notes

Summary

0m 06s

Field observations (Freeways)



Scatter is due to:

- Heterogeneity of drivers
- Weather conditions
- Type of vehicles
- Non-stationary conditions

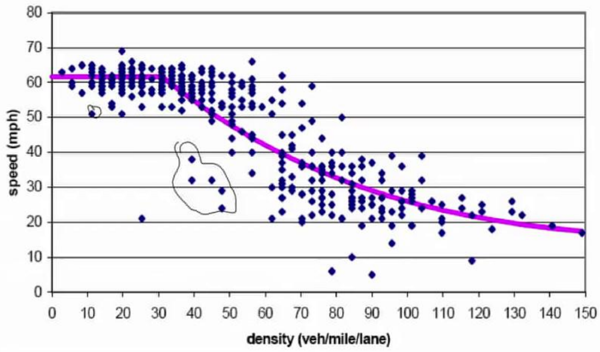
So interestingly what we see, we see that for very low densities, so when the road is almost empty, the speed is very high; close to the speed limit which in this road is 60 miles per hour and as the density increases, let's say as more people want to travel through this freeway, the speed will remain constant but after some point it will start decreasing and as the density gets very high value, we see that we can even reach speeds that are very close to zero. So I would like to spend a little bit of time to describe in more detail this figure. One observation we have is that there is a trend, constant speed and then decreasing and if somebody applies, let's say some best fit using some statistical tools, we can obtain the pink line which is the line that minimizes the errors of all the observed points from this estimated line. So clearly what we see here is that there is significant scatter, so the points deviate from this line. And we would like to provide some explanation for this scatter. First of all the first one is related with the heterogeneity of drivers.

Notes

Summary

1m 42s

Field observations (Freeways)



Scatter is due to:

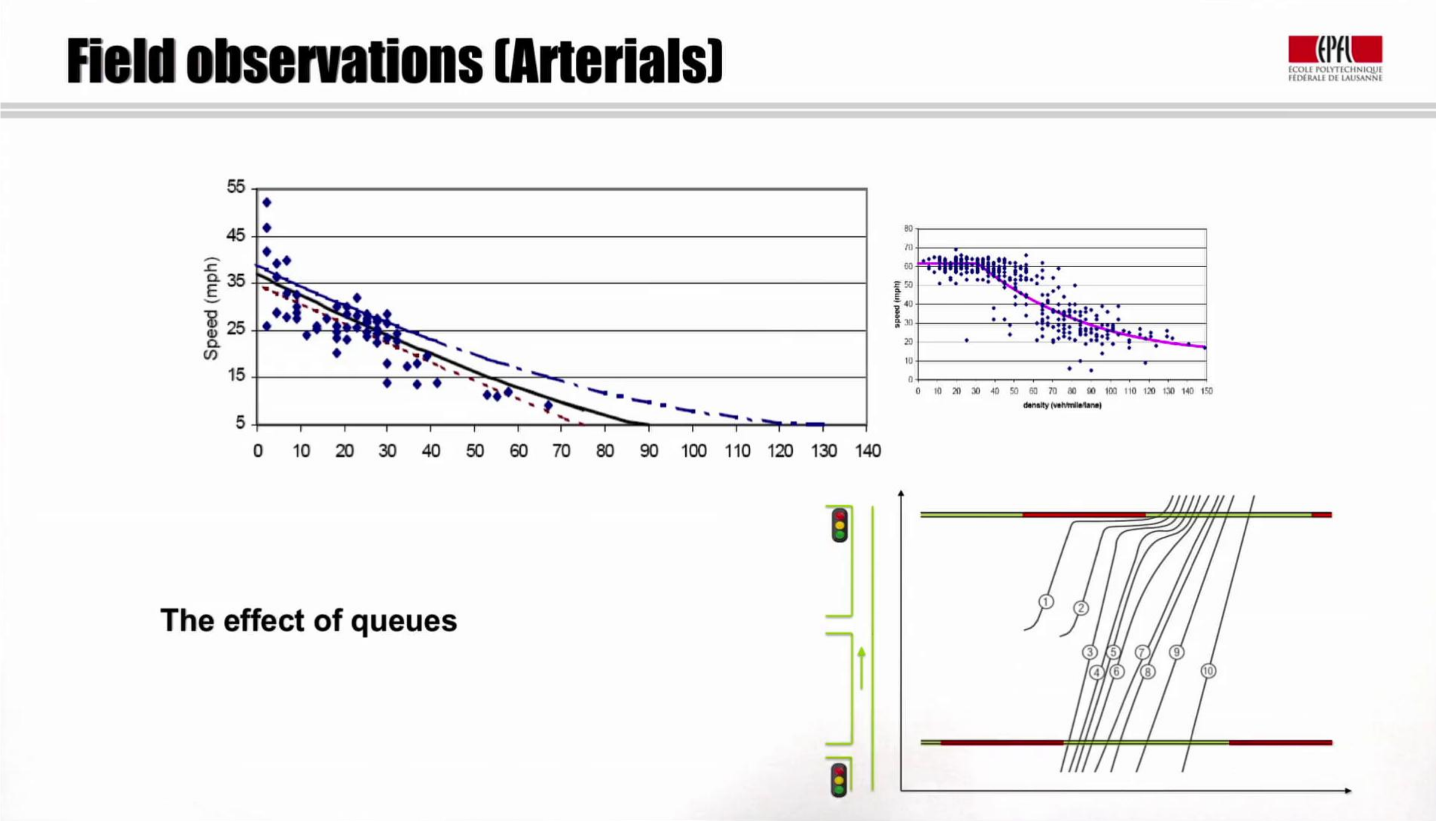
- Heterogeneity of drivers
- Weather conditions
- Type of vehicles
- Non-stationary conditions

So as over here, we take averages over a period of one or two minutes, there are plenty of drivers on this road that they might have different habits, different vehicles and they might also have different behavior; some are more aggressive, some are less aggressive. So when we estimate averages, we might obtain higher or lower values than the other averages. Weather conditions can also play a significant role so usually what we expect that under rainy conditions or under snow or under fog where people and drivers have their worst vision, what might happen is that for the same level of density we might observe the lower speeds. The type of vehicles plays a role so for example, if we have a significant number of trucks on the road travelling to the other with the cars, this might also have an effect and might lower the speed. So maybe these points over here are points of heavy rain or points where the fraction of trucks is much higher than average. The last one is that a fundamental diagram, in that case between speed and density, represent stationary conditions so conditions that are to the maximum extent not saying over time. But as we know traffic is a dynamic process and for example, more vehicles during the morning want to travel through the road and as conditions are non-stationary, we might see these points are deviate from the pink curve.

Notes

Summary

3m 02s

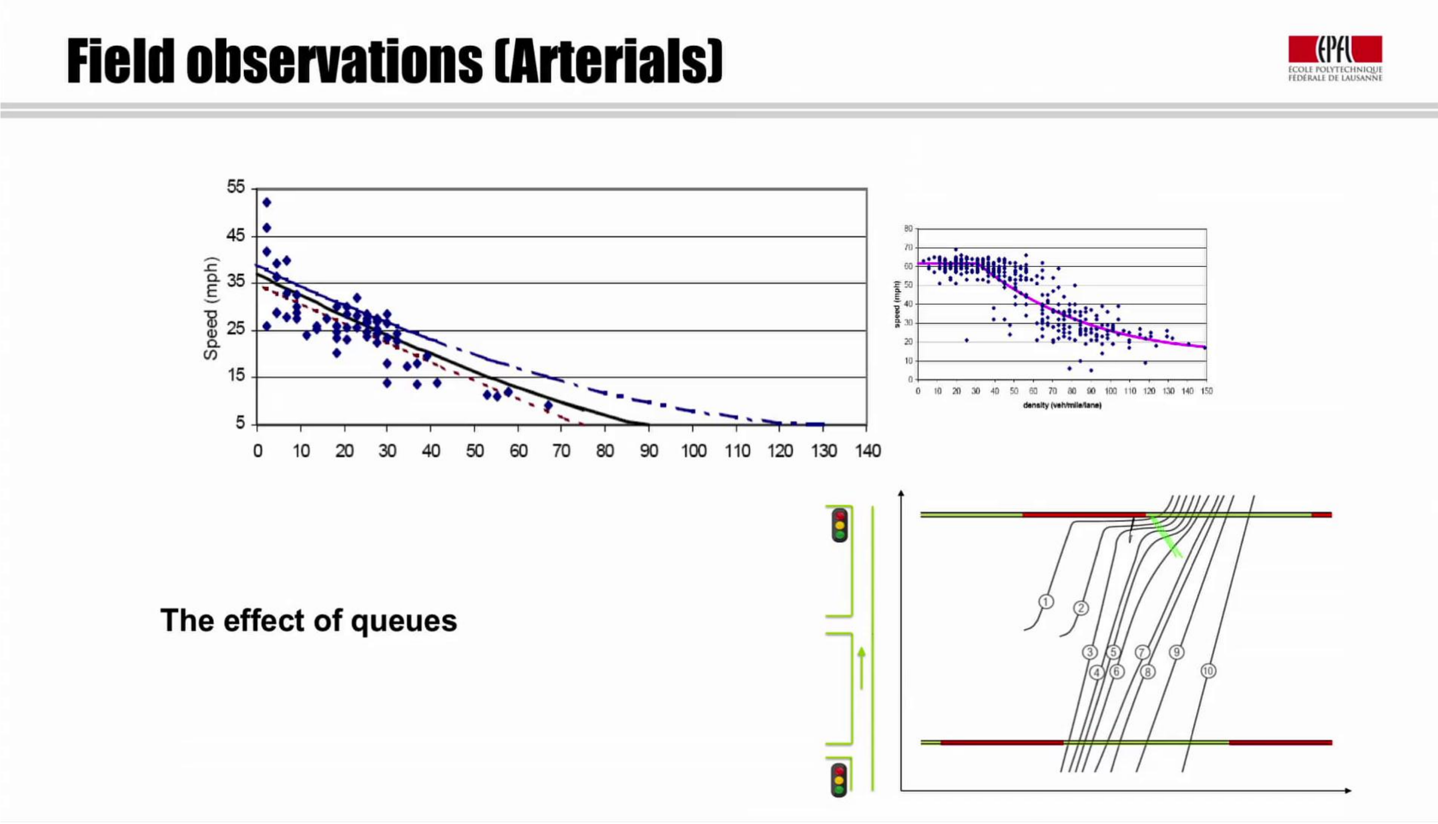


So let's see some other real observations but now from an arterial road, let's say with a traffic signal in its downstream end. And here as a reference we also have the previous figure. So what we see over here, again we see a decreasing part between speed and density 'k' but interestingly there are some difference compared to the freeway fundamental diagram. So the arterial fundamental diagram has more scatter for high speed. This is what we see over here that when density is very low, the speed deviates significantly. So why this might happen? This might happen for example because some drivers might have to wait longer during a red phase where some other drivers might be like here and might advance without stopping at all. Another important observation is what we see; we see that the arterial fundamental diagram is decreasing for low values of density while the freeway remains constant. So this is a significant observation that we would like to explain. The main reason for this is the effect of queues. So if we look at this figure, there is a traffic light and vehicles are joining a queue because the traffic light turns red.

Notes

Summary

4m 37s



So when an additional vehicle joins the queue then the queue length increases as we see over here but also interestingly when the traffic light turns green, vehicles do not start simultaneously but what we see there is a time lag between the time the first vehicle starts and the second and the third and so on and so forth. This is mainly because drivers have some reaction time so there is some delay on how drivers react when the information that a traffic signal turns green propagates backwards and asks them to start their vehicle. So every additional vehicle because of this time lag will have, as an effect, to wait a little bit longer so the average speed if additional vehicles join the queue will be lower. So this is the main difference between arterial and freeway fundamental diagram.

Notes

Summary

5m 57s

First Empirical observations (Greenshields, 1935)

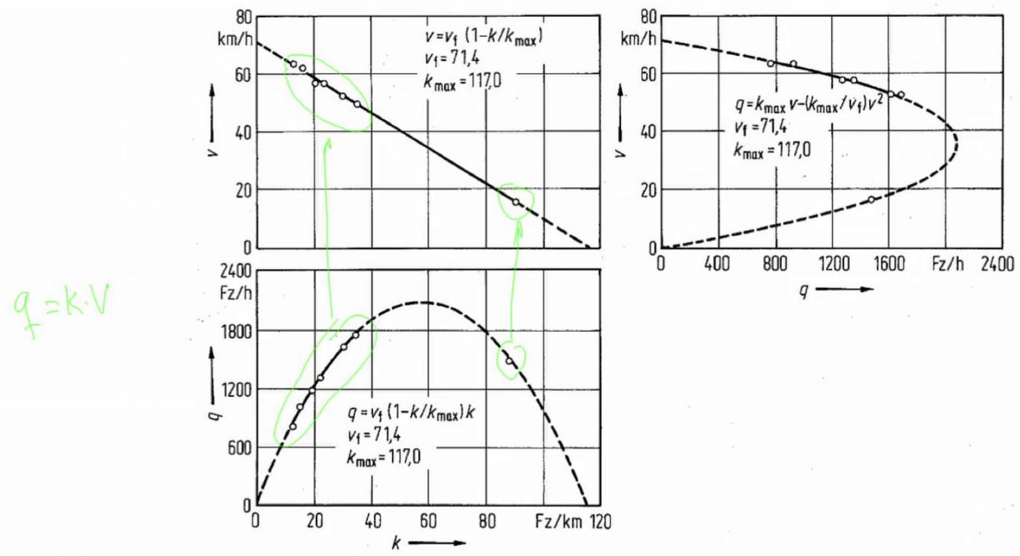
So let's look at the history. So the first empirical observations of fundamental diagram happen more than 80 years ago by a British engineer named Greenshields. So Greenshields, what he did in all of major roads in the UK city, he installed some cameras then these cameras were taking multiple frames per second and after this, he tried to estimate the flow, the density and the speed in this road and then he made related plots.

Notes

Summary

6m 55s

Greenshield's FD

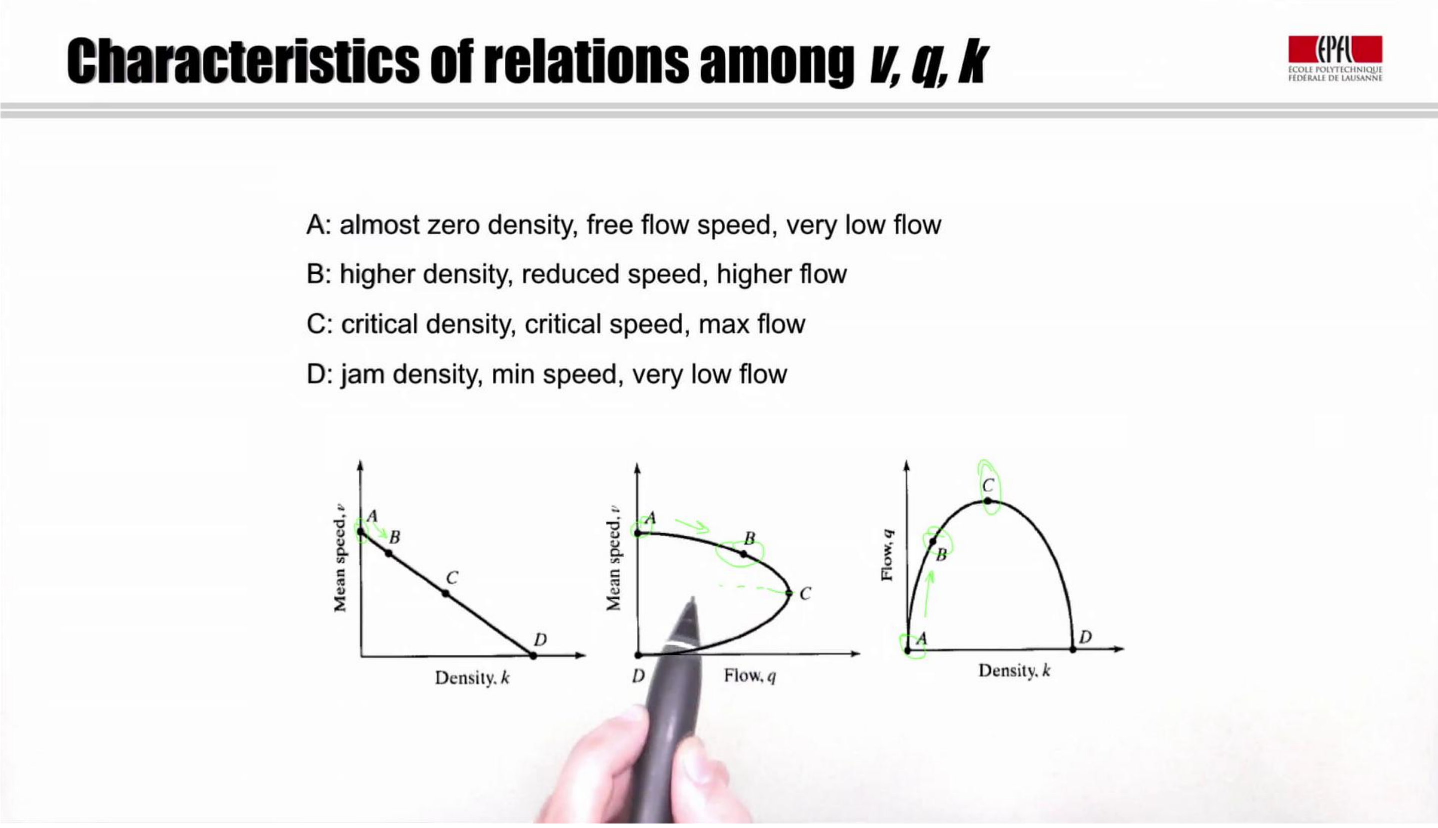


So this what he observed. Over here, we see three diagrams that the Greenshields developed and we see the relationship between speed and density, the relationship between flow and density and the relationship between speed and flow. So interestingly, Greenshields also observed that as density increases, speed will go down but you see that the majority of the points are for low densities where only one point of highly congested conditions occur. So by having these points, Greenshields considered that the relation between speed and density is linear so he did a linear approximation by doing a best fit in these points and he came up with a relationship with some specific parameters and we will define these parameters in the next slide. If we now look at the relationship between flow and density and please keep in mind that what we have, we have the fundamental relations; see that flow is equal to density times speed. What Greenshields observed, observed that flow increases up to maximum value and then decreases. So this point is that point. All these points are that points. In a similar way of thinking we can also construct a relationship between speed and flow that has a shape like this. So let's see in a few more details these three relationships.

Notes

Summary

7m 28s



So here we have some graphical representation of the previous diagrams and over here I have put four different states A, B, C and D. Let's see what each of them represents in each figure. So state A is a state of almost zero density where the road is empty almost and what we have is that vehicles are moving to its maximum speed that we call free flow speed then the flow is extremely small. So you can see that given that we have point A being here in the speed density relationship; high speed low density, high speed low flow and low flow low density. So this is how we see this drawn. If now we look at the case of higher density, what happens is that there is a lower speed and at the same time there is higher flow. So we move at point B from A if we increase the density and here this is how we see the trends. So in the speed-density the trend is decreasing, in the speed-flow the trend is again is decreasing while in the flow-density the trend is increasing. If now the flow further increases what will happen; we will reach a case of maximum flow C and this will happen for, of course, higher density compared to state B but also a smaller speed and this speed is called the critical speed.

Notes

Summary

9m 04s

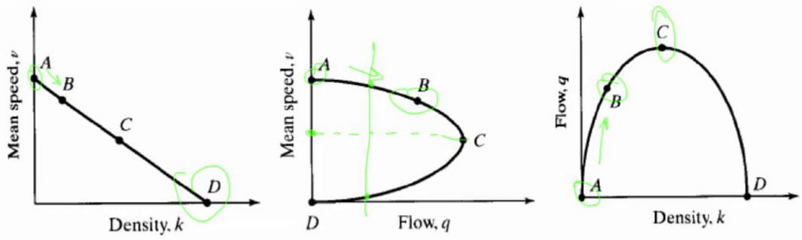
Characteristics of relations among v, q, k

A: almost zero density, free flow speed, very low flow

B: higher density, reduced speed, higher flow

C: critical density, critical speed, max flow

D: jam density, min speed, very low flow



So this is the critical speed for which flow is maximized. And if density continues to increase, so if more cars happen to be on the road, flow cannot increase anymore. Flow will go down and then we reach a state D which is a state of jam density, speed is almost close to zero where flow is also close to zero. So interesting observation is while the speed-density relationship is monotonic, monotonically decreasing, the flow-density relationship is unimodal so it has a well-defined maximum but the speed-flow relationship is not a function because what we observe, we observe for a given flow, we cannot clearly define densities, so for a given flow, we can have low speed or high speed and this is because we can have congested or un-congested conditions for the same flow. Just consider a simple example that when there are very few vehicles on the road or there were a lot of vehicles on the road, the flow of vehicles will be extremely small but the one case the speed is small and the other case the speed is large. So these are the main characteristics we observe among these relationships.

Notes

Summary

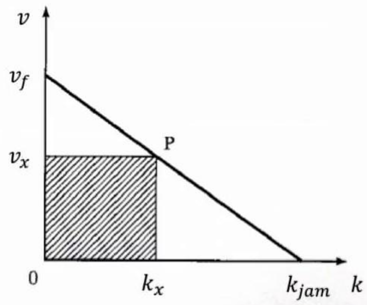
10m 47s

Speed-density FD

How one can estimate flow in the speed-density FD?

What is the speed that maximizes flow?

$$v = v_f(1 - \frac{k}{k_{jam}}) \qquad q = v_f(k - \frac{k^2}{k_{jam}})$$



$$\frac{dq}{dk} = v_f\left(1 - \frac{2k}{k_{jam}}\right) = 0$$

$$k_{cr} = \frac{k_{jam}}{2}$$

$$u_{cr} = 0.5u_f$$

$$q_{max} = u_f k_{jam}/4$$

So now let's have a more careful look and one equation is if the speed-density fundamental diagram is given, how we can estimate the rest of the quantities? So what we need to keep in mind always there is a fundamental relationship between flow, density and speed. So as flow is the product of speed and density, simply speaking, if we have a specific point P then if we would like to estimate the flow at this point, we simply need to create a rectangle and estimate the area of this rectangle. So at point P, the flow at point P is 'vx' times 'kx'. And then a second equation I would like to solve with you; what is the speed that maximizes flow? So as we've seen there is a linear relationship between flow and density. We can write this relationship like that where 'vf' is the maximum speed that can be observed. We call it free flow speed while 'k jam' is the jam density. So here is the free flow speed, here is the jam density. And by using that relationship, we can also estimate what is flow. So we just need to multiply both sides by density 'k' and we come up with this.

Notes

Summary

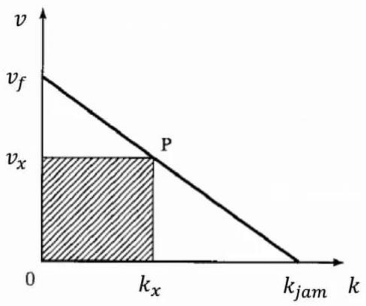
12m 09s

Speed-density FD

How one can estimate flow in the speed-density FD?

What is the speed that maximizes flow?

$$v = v_f(1 - \frac{k}{k_{jam}}) \qquad q = v_f(k - \frac{k^2}{k_{jam}})$$



$$\frac{dq}{dk} = v_f\left(1 - \frac{2k}{k_{jam}}\right) = 0$$

$$k_{cr} = \frac{k_{jam}}{2}$$

$$u_{cr} = 0.5u_f$$

$$q_{max} = u_f k_{jam}/4$$

So now if we would like to estimate the value of density for which flow is maximized, we just need to take this function which is a parabola and take the first derivative equal to zero and if we do some calculations we find that the critical value of density for which flow is maximized is equal to the jam density divided by two. So now if we take this value and we are going to replace it in that equation, we find that the critical value of speed for which flow is maximum is, simply speaking, half of the free flow speed and again if we multiply these two quantities we will obtain that the maximum flow is the one quarter of the product between free flow and jam density.

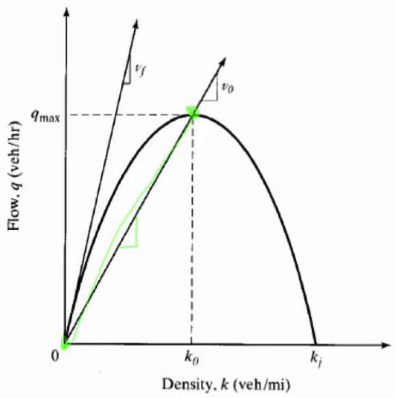
Notes

Summary

13m 31s

Flow - density FD

How one can estimate speed in the flow-density FD?



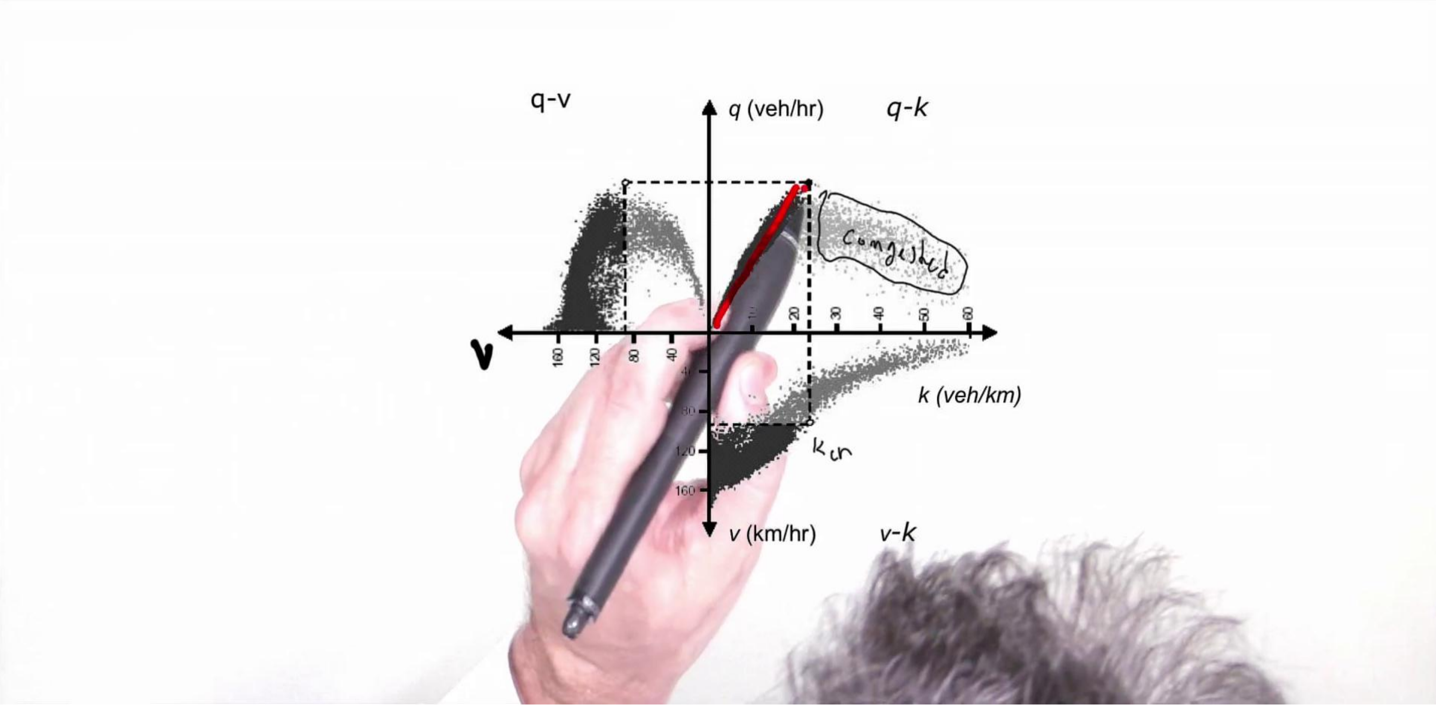
If now the flow-density diagram is given to us and one would like to estimate the speed at a specific point, what it has to do is it simply has to connect this point with the point zero zero where the two axes intersect and this slope as shown is the speed for density ‘k’ zero. So if we ask ourselves what is the free flow speed, what we have to do is that we just need to apply the tangent at the point of the fundamental diagram for which density is close to zero and the slope of this line is equal to the free flow speed.

Notes

Summary

14m 22s

Other empirical data



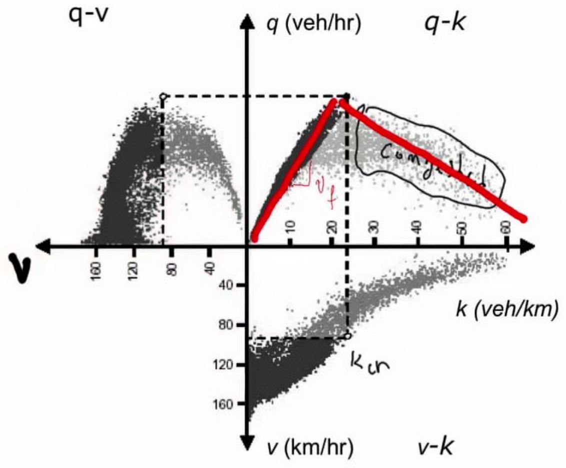
Let's look now at some empirical data from another freeway location and we present this data in the axes zone here, so this axis, the vertical axis is towards the top flow whereas towards the bottom speed and the horizontal axis is towards the right towards the right density where towards the left is speed again. Let's draw our attention on the flow-density diagram. So what we see, we see that there is some critical value of density for which flow is maximized but what we also see, we see that the points to the right of this diagram experience much more significant scatter compared to the points on the left. And actually this happens because these are congested conditions and it is known that under congested conditions traffic operates in a more chaotic and unpredictable way. But also where I would like to draw your attention is that if we try to fit the Greenshields' diagram in this data, probably this will not be a good approximation because what we observe over here, we observe that this diagram is not symmetric. So what we see, we see that the value of critical density over here is much less than the jam density divided by two and what we could also say is that this diagram could possibly be approximated almost with a triangle.

Notes

Summary

14m 59s

Other empirical data



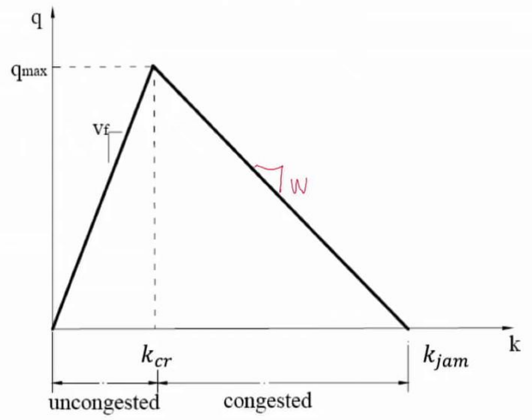
And actually this is one of a very well-defined relationships; a triangular fundamental diagram that will also make our life easier in state of mathematical calculations and this is widely used for freeways. And in case of the fundamental diagram is triangular, what this means, this means that all the states between zero density and critical density they have the same speed.

Notes

Summary

16m 42s

A triangular FD



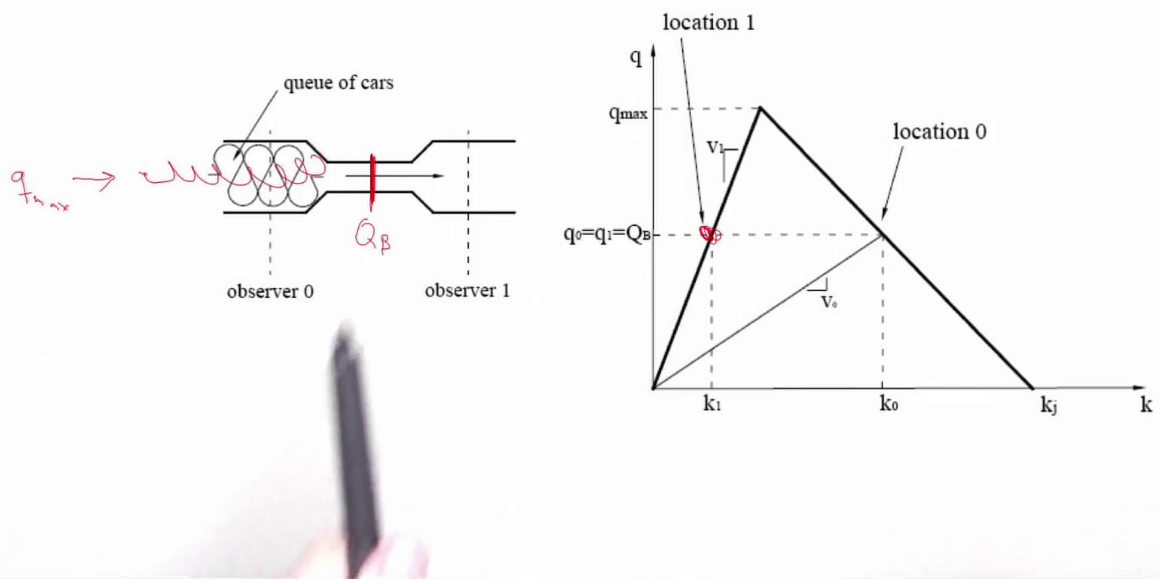
So here how a triangular fundamental diagram looks. We can simply describe it with a few number of parameters. So the maximum flow or capacity, the free flow which is the slope of this line and then the jam density and what we can assume, we can assume that the critical value of density is the one that separates un-congested from congested conditions. I would like also to show you that there is another slope over here that we use only we call this slope ‘w’ and this is the maximum slope of the shockwave that can happen in traffic and we will discuss shockwaves later in this mooc. Here with these parameters, the triangular fundamental diagram is very well-defined.

Notes

Summary

17m 11s

The effect of bottlenecks on FDs



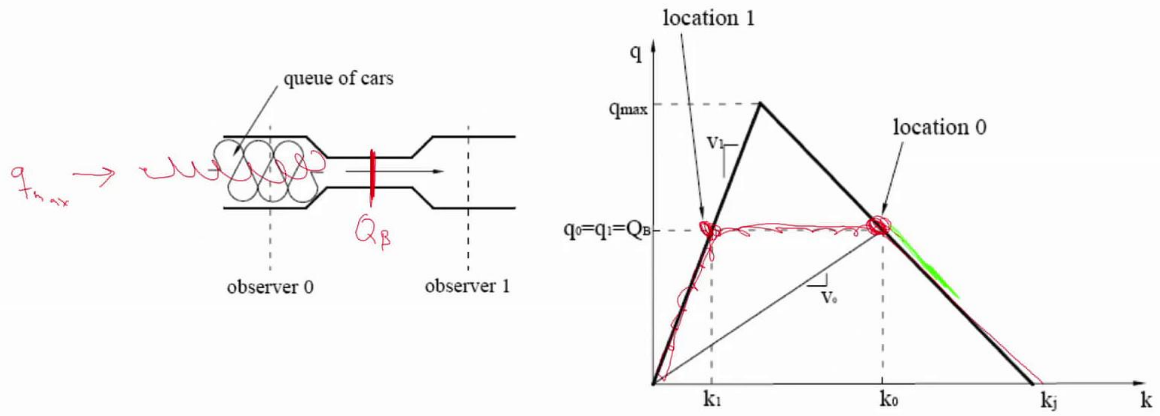
Next topic I would like to discuss with you is the effect of bottlenecks on fundamental diagrams. Let's consider a single case where we have three lanes that go to one lane or to two lanes and then to again three lanes. And actually let's consider that the fundamental diagram at location zero and at location one is like in the figure we have over there where the maximum flow 'q max' is much higher than the capacity of the bottleneck here. So let's assume that the capacity of this bottleneck is 'QB' which is much smaller than 'q max' and it is shown there. So if we ask ourselves, if we let vehicles arrive at a high flow from where the ramp stream, what it will happen so if the flow is larger than 'QB'? So let's say that the vehicles are arriving from here at a flow 'q max', what will happen as this bottleneck has smaller capacity, a queue will be developed and this queue will grow. So if we have an observer that measures flow downstream, this observer will never be able to observe a flow 'q max'. He will only observe a flow equal to 'QB'. And if we ask ourselves at what point of this fundamental diagram this observer will observe points, he will observe at exactly that point.

Notes

Summary

18m 01s

The effect of bottlenecks on FDs



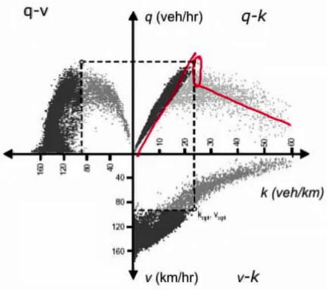
So downstream of a bottleneck, conditions are un-congested where upstream of a bottleneck, conditions are congested. So if we put an observer here, after the time that the queue grows and passes this observer, this observer will also measure flow equal to ‘QB’. But the main difference is that this state will be much more congested than what a person downstream of the bottleneck has, so the observer zero observes that state. So interestingly if, for example, we install some loop detectors in location one and in location zero, we will not be able to observe points in the fundamental diagram up to the ‘q max’ but we will only see points that occur in this part. And actually why that part might arise, that part might arise only if further downstream there is an additional bottleneck that has capacity smaller than ‘QB’. So if there is nothing downstream, the points you will observe they will raise up to here, so what I draw in red while points in green might arise if further restriction occurs at further downstream.

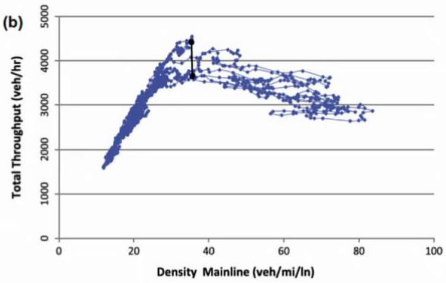
Notes

Summary

19m 31s

Capacity drop phenomena





- Many studies have revealed that there is a stochastic nature of bottleneck capacities.
- Banks (1991) first suggested that discharge flow at bottlenecks diminishes once queues start forming upstream of the location.

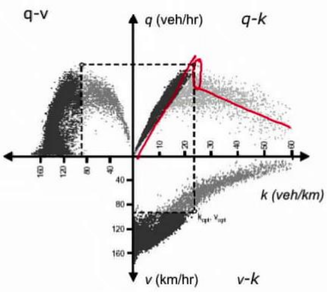
I would like to finish this lecture by discussing with you some additional phenomena we usually observe in freeway locations. So this diagram comes from a freeway and somebody could fit instead of a triangular fundamental diagram, another fundamental diagram that looks like that. So here what we could say is that when conditions are un-congested, we observe that on average we can reach higher flows compared that when conditions are congested. And this phenomenon is known as capacity drop. So this difference between the capacity before the breakdown happens and after the breakdown happens is known as capacity drop. This is a difficult phenomenon because this drop doesn't have a constant value and it can range between 3% and 12% according to different locations. So on the right, we see some empirical data that our team analyzed from a freeway in twin cities in Minneapolis and we were able to observe a capacity drop in the area in the range of about 12%. So Banks was one of the first persons who suggested this capacity drop that the discharge flow at bottleneck decreases once queue start to forming but also there are other researchers like Eddy.

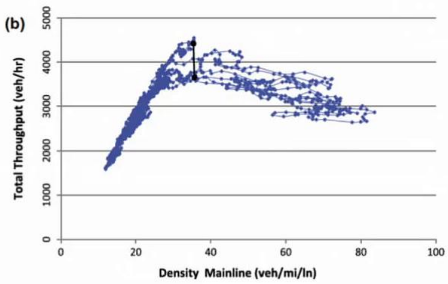
Notes

Summary

20m 49s

Capacity drop phenomena





- Many studies have revealed that there is a stochastic nature of bottleneck capacities.
- Banks (1991) first suggested that discharge flow at bottlenecks diminishes once queues start forming upstream of the location.

Please remember Eddy from the generalized definitions of flow and density that we gave during Quan that many years earlier close 267, he observed some traffic data from another freeway in US and he didn't mention the phenomenon of capacity drop but he was the first person that was able to fit a fundamental diagram which have a gap.

Notes

Summary

22m 11s

Summary

So in this lecture, we introduced the fundamental diagram which is a very important relationship that connects flow, density and speed and this is a tool that we will use in this course many times later as it can really help us to understand how congestion is developed, why queues are growing under a traffic light and also it will help us solve and explain many other traffic phenomena of congestion.

Notes

Summary

22m 34s