

Week Outline



- Week 2.1
 - Traffic flow modeling
 - Developing and testing models
 - Classification of traffic models
 - Microscopic vs Macroscopic modeling
 - How to select the correct model
- Week 2.2
 - LWR Theory
- Week 2.3
 - Examples of LWR Theory

Welcome. My name is Anastasios Kouvelas and this week we are going to discuss about traffic flow modeling. By the end of the week you should have a basic understanding of how we develop the traffic flow models, how we can test them and why they are useful. Afterwards Professor Geroliminis will introduce you to LWR theory which was proposed in the 1950's and constitutes the foundations of traffic flow theory. Let's start with the outline of this week. In this video we are going to discuss about traffic flow modeling, how we develop and how we test models, the classification of traffic flow models. We will discuss microscopic versus macroscopic modeling and how to select the correct simulation model for your application. In the next videos Professor Geroliminis will introduce LWR theory and some examples of LWR theory.

Notes

Summary



0m 05s

What is a model?

A set of mathematical equations (or rules) that tries to describe (i.e. replicate) a physical process (e.g. evolution of traffic congestion over time and space).

Let's start with the definition. What is a model? I will give you my own definition but there are plenty of other definitions available. So a model is a set of mathematical equations or rules that tries to describe or replicate a physical process. In our case the physical process that we are studying is evolution of traffic congestion over time and space. People can come up with mathematical formulations that are partial differential equations, for example or ordinary differential equations but in some cases you come up with models that are much more complicated and they cannot be written in a nice closed loop mathematical form.

Notes

Summary



1m 06s

Why do we need models?

- **Process analysis and understanding**
- **Planning (forecasting)**
 - Introduction of a new mode (multimode)
 - Modification/extension of infrastructure
- **Operations (control)**
 - Design of model-based control strategies
 - Estimation/prediction models
 - Testing the control performance
- **Traffic simulation**

Why do we need models? Why models are important? First of all we can understand and analyze physical processes like traffic flow. Also we can use them for planning and forecasting. Imagine you have a transport system and you want to introduce a new mode, for example, a new metro line or a new bus line. You can use a model to assess the impact of this change in the infrastructure. Also if you want to modify or extend your infrastructure, for example, to build a new highway, again you can use models to do the planning of the new system and assess the performance of the new system. Another use of traffic models is for operations and traffic control. So we can use analytical mathematical models to design model-based control strategies, to build estimation and prediction algorithms and to test the performance of our control algorithms. Last, traffic flow models are important because we can use them for traffic simulation.

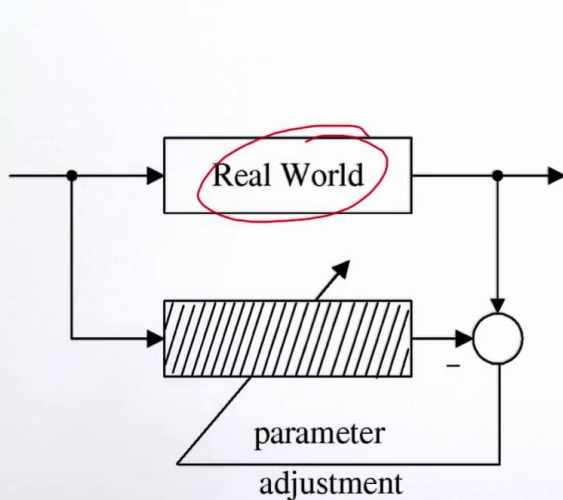
Notes

Summary



1m 56s

How do we develop models?



- Direct application of superb theories (e.g. Newton Physics)
- Black-box Approaches (e.g. ARMAX, polynomial approximations, Neural Nets)
- Mixed Approach: Basic mathematical structure from physical considerations + parameter estimation (e.g. Macroscopic Traffic Flow)

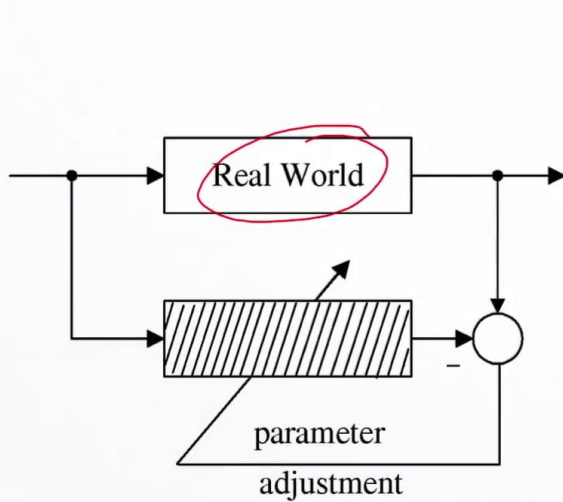
Another interesting question is how do we develop models? So in this video, we have the real world which is the physical process that we want to emulate and we can get real sets of data. So the idea is to develop a model that can replicate this physical process. I have here three different approaches of how to develop a model. First of all, we can use physical laws like superb theories like, for example, Newton Physics. An example of such an equation for traffic flow is the conservation equation. This is an equation that says that the number of vehicles should conserve and we cannot lose vehicles in an intersection. Of course, this is a physical law and no traffic model should violate this law. A second approach is to use what we call Black-box approaches. This is something like neural networks, polynomial approximations or ARMAX and this is purely data driven approach. We can create models just from using the data that we have. And the third approach is mixed between the first two, so where we can use some basic mathematical structure from physical considerations and then we can add to our models some parameters and try to estimate some values for these parameters.

Notes

Summary



How do we develop models?



- Direct application of superb theories (e.g. Newton Physics)
- Black-box Approaches (e.g. ARMAX, polynomial approximations, Neural Nets)
- Mixed Approach: Basic mathematical structure from physical considerations + parameter estimation (e.g. Macroscopic Traffic Flow)

An example for the traffic models is the fundamental diagram. The fundamental diagram which was described in the previous week, it's an empirical relationship. It's not a physical law. It's something that we have observed from real data and we can have some parameters for this fundamental diagram that we can estimate for our model.

Notes

Summary



4m 54s

How do we test models?

- **Logical/mathematical consistency and completeness**
- **Qualitative testing:**
 - All **essential** phenomena reproduced?
 - No contradiction with **essential** phenomena?
- **Quantitative accuracy:**
 - Parameter estimation with real data
 - Transferability?
 - Sensitivity?
- **Computational complexity**

Now how do we test our models? The first test that someone should do is to check for logical and mathematical consistency and completeness. You cannot have a model that is not mathematically consistent. Another test is the qualitative testing. So you should ask yourself does my model reproduce all the essential phenomena of my process. If I have a traffic network and some bottlenecks that create the congestion, my model should be able to reproduce this congestion and equivalently, my model should have no contradiction with the essential phenomenon of the physical process. The third test that we can do is a quantitative test about the accuracy of the model. So we can estimate the parameters of the model with real data and then we can check the transferability of the model. For example, if I develop a model for Switzerland and then I go and test this model in UK, is it consistent with the data that I get from Switzerland and UK and do I need the different set of parameters for my model for different countries? And another key issue is the sensitivity of the model, for example, to weather conditions or driving behavior. And last but not least a key aspect of models is computational complexity. We should be able to develop models that are fast enough and they don't need a lot of computational effort.

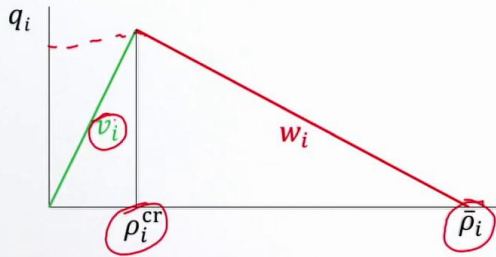
Notes

Summary

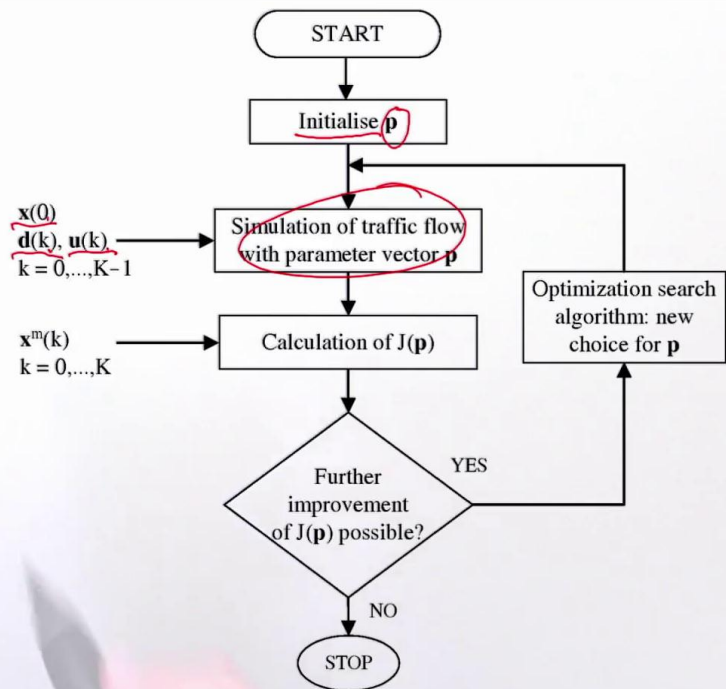


5m 21s

Optimal parameter estimation



k : discrete time index
 $\mathbf{x}$: state vector
 $\mathbf{x}^m$: real measurements
 $\mathbf{u}$: control inputs
 $\mathbf{p}$: vector of model parameters
 $\mathbf{d}$: demands and other exogenous inputs
 $J(\mathbf{p})$: difference between model and real data



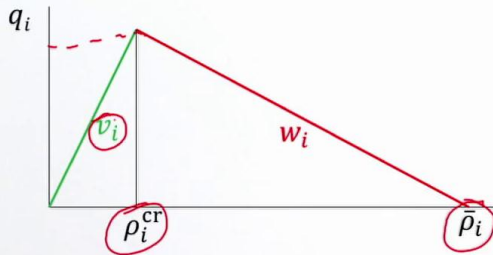
As I said before every model has a set of parameters. I call this parameters with a vector 'p' in this slide and the question here is how do I estimate the optimal values for this parameters. For example, in a traffic model I can have a fundamental diagram and here I have parameters like the jam density, the critical density, the free flow speed and the maximum flow. These are parameters that I can tune for my model with real data, so I have here on the right an iterative process of how do we optimize the values of this parameters. Again the set of parameters is 'p' and I have 'x' for the state of the system; 'x zero' is the initial state. I have 'd' for the demand and any other exogenous inputs that I can have access to and 'u of k' is a vector with the control inputs. Here I assume some control process, so there are some parameters that I can control and change their values and I call this variables with 'u', with a vector 'u'. So what I can do is I can start an iterative process. I initialize the parameters 'p' and then I run my model given the initial state and the vector 'd' and 'u' and I get all the trajectories of the state.

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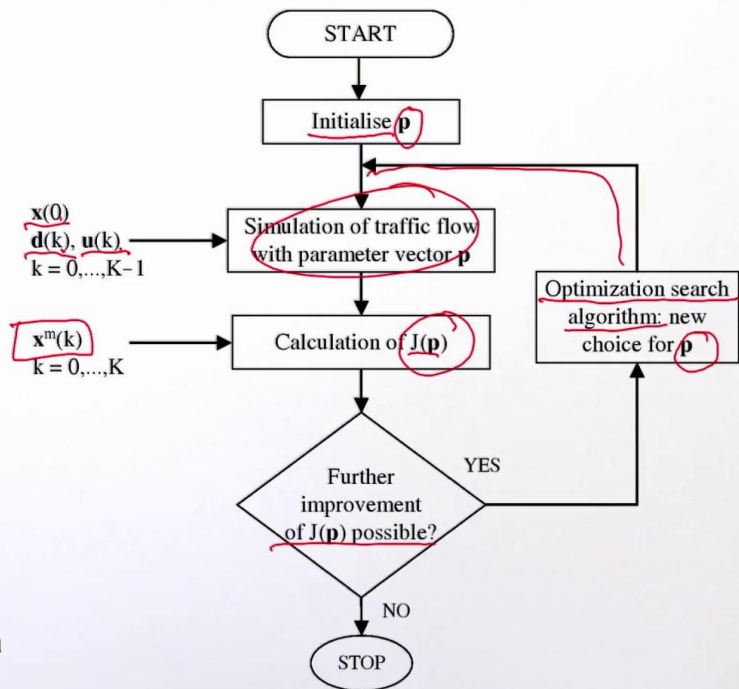
Summary



Optimal parameter estimation



k : discrete time index
 $\mathbf{x}$: state vector
 $\mathbf{x}^m$: real measurements
 $\mathbf{u}$: control inputs
 $\mathbf{p}$: vector of model parameters
 $\mathbf{d}$: demands and other exogenous inputs
 $J(\mathbf{p})$: difference between model and real data



Then I have ' $\mathbf{x}^m$ ' which are real measurements from the field, so I have this vector ' $\mathbf{x}$ ' with real measurement and what I do is I compute the difference between ' $\mathbf{x}^m$ ' and vector ' $\mathbf{x}$ ' and this gives me the value of the function ' J of $\mathbf{p}$ '. Here I check if this ' J of $\mathbf{p}$ ' has converged or if I can improve it further? And if I can further improve it, I have an optimization algorithm here that can change the values of this parameters ' $\mathbf{p}$ ' and go back to the beginning and run my simulation again and get the new output, compare again with the measurements, get the value of $J(\mathbf{p})$ and I can do this iterative process until I get a convergence and I have found the optimal estimation of the parameters.

Notes

Summary



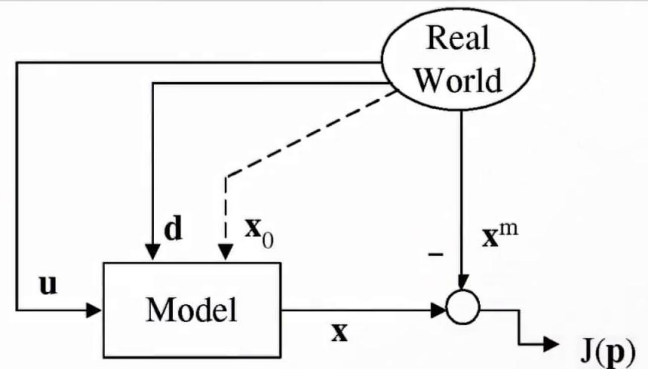
Model validation

$$\mathbf{x} = [\rho_1 v_1 \dots \rho_N v_N]^T$$

$$\mathbf{u} = [r_1 \dots r_N]^T$$

$$\mathbf{d} = [s_1 \dots s_N v_0 q_0 \rho_{N+1}]^T$$

$$\mathbf{p} = [v_f \rho_{cr} \tau v \dots]^T$$



k: discrete time index

$\mathbf{x}$: state vector

$\mathbf{x}^m$: real measurements

$\mathbf{u}$: control inputs

$\mathbf{p}$: vector of model parameters

$\mathbf{d}$: demands and other exogenous inputs

$J(\mathbf{p})$: difference between model and real data

$$\mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k), \mathbf{d}(k), \mathbf{p}], \quad \mathbf{x}(0) = \mathbf{x}_0$$

Optimization problem:

$$J(\mathbf{p}) = \sum_{k=0}^K \|\mathbf{x}(k) - \mathbf{x}^m(k)\|^2 \rightarrow \text{Min}_{\mathbf{p}}$$

Another important aspect is the model validation. How do we validate our models with real data? So in this slide, I have a model. The vector ' $\mathbf{x}$ ' defines the state of the model, the vector ' $\mathbf{u}$ ' is the control input, ' $\mathbf{d}$ ' are demands and other exogenous inputs and ' $\mathbf{p}$ ' are again the parameters. So if I have a set of real data that I get from the real world and I have the control vector ' $\mathbf{u}$ ', the demand and the initial state ' $\mathbf{x}_0$ ' and then I have a model with PDE or an ODE or anything, this ' $\mathbf{f}$ ' can be a linear and non-linear function or whatever and function of ' $\mathbf{x}$ ', ' $\mathbf{u}$ ' ' $\mathbf{d}$ ' and ' $\mathbf{p}$ ', so I can run this model and get the output ' $\mathbf{x}$ ' for every time step and then compare this with the measurements ' $\mathbf{x}^m$ ' that I have and the difference gives me this function ' J of $\mathbf{p}$ '. So when I validate a model, I have to solve this optimization problem where I minimize the square difference between ' $\mathbf{x}$ '; the real measurements of ' $\mathbf{x}$ '. If I minimize this function over the variables ' $\mathbf{p}$ ' the parameters, I get the optimal values for ' $\mathbf{p}$ ' and at the same time the order of magnitude of the accuracy of my model that use real data. The value of $J(\mathbf{p})$ gives me this accuracy of my model.

Notes

Summary



9m 47s

Classification of traffic flow models

- **Microscopic (mainly for simulation)**
car following + lane changing
- **Mesoscopic**
vehicle platoons with similar characteristics
- **Macroscopic**
macroscopic traffic flow variables (in analogy to fluid mechanics)
- **Network (or region) level (MFD)**
macroscopic fundamental diagram (regional accumulations, flows)

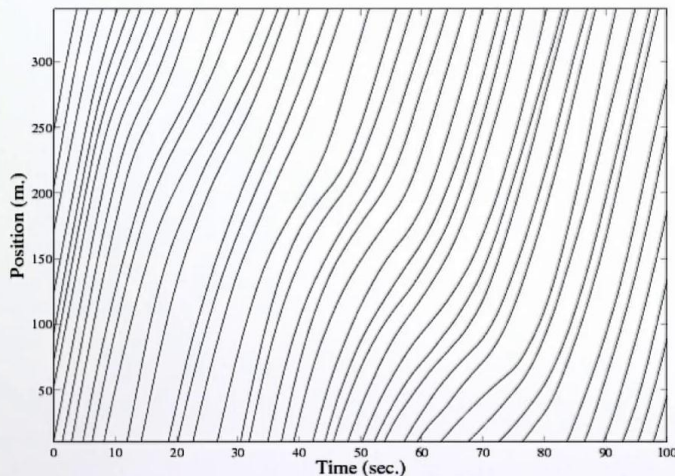
Let's now discuss the classification of traffic flow models. Essentially there are four big categories of traffic models. The first one is microscopic models which are basically used for simulation. These models are based on the car following paradigm and the lane changing paradigm. If you model these two behaviors – the car following and the lane changing – you have a microscopic simulator. The second class is mesoscopic models. These models are sometimes closer to microscopic and sometimes closer to macroscopic models depending on the approach. For example, we can have a model that captures vehicle platoons with similar characteristics. The third category is the macroscopic models. Here in analogy to fluid mechanics, we use macroscopic variables like flow, density and speed to describe the state of the network. And the last category is the network or region level modeling. For this type of modeling, we utilize the macroscopic fundamental diagram and this is for cities that we can cluster them into regions and then we can model the flow of vehicles from one region to the other and the states here are regional accumulations and the regional flows.

Notes

Summary



Microscopic traffic flow models



car following + lane changing

- Detailed **vehicle trajectories** following each other (similar output to **real data!**).
- Many available micro-simulation software.

Gipps, P., 1981. A behavioural car-following model for computer simulation. *Transportation Research Part B*, 15 (2), 105–111.

Gipps, P., 1986. A model for the structure of lane-changing decisions. *Transportation Research Part B*, 20 (5), 403–414.

Let's now have a look at the special characteristics of microscopic models. In this figure, we see the output that we get from a micro-simulation model. What we get is detailed trajectories of all vehicles. So in every point in time we know the exact location of all vehicles. The advantage of such modeling approach is that the output that we get is very close to the real data. So we can compare this output to data that we get from the field. Today there are many available micro-simulation software; commercial and non-commercial that you can use. And here I provide two references; one for the car following model and one for the lane changing model that are some of the first works that appeared in the 80's on these topics.

Notes

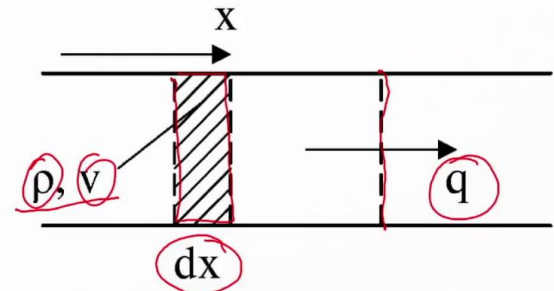
Summary



13m 03s

Macroscopic traffic flow variables (in analogy to fluid-mechanics):

- traffic density $\rho(x, t)$: number of vehicles per unit length (veh/km)
- traffic volume $q(x, t)$: number of vehicles per time unit (veh/h)
- mean speed $v(x, t)$ in km/h



mathematical
abstraction

And let's now look at the special characteristics of macroscopic models. In macroscopic modeling if we get a stretch of a highway like here, we can discretize the space so we get this 'delta x' which is a part of space and we assume that in this space we have some density and some speed. So in analogy to fluid mechanics, we have traffic flowing in this stretch and we define the density which has units vehicle per kilometer, the flow which has units vehicle per hour and some average speed which has units kilometers per hour. So of course, this is a mathematical abstraction and we assume that in all this space we have homogeneous conditions and we have this density and this speed and the variable of flow is not defined for the space but rather for a point. So for this point we have this flow that is measured in vehicles per hour.

Notes

Summary



Microscopic vs Macroscopic modeling

- Macroscopic models are analytical (not just simulators).
- Computation time.
- Required resolution / Application field (e.g. evaluation of incident detection).
- Easier consideration of various control measures (e.g. speed limits) or infrastructure impact in microscopic models.
- Potential accuracy versus validation effort.

Let's now try to do a comparison between microscopic and macroscopic modeling. One key aspect is that macroscopic models are analytical not just simulators. So I can write down the analytical form of a PDE and I can use these, for example, in order to develop model-based control strategies. Another issue that we should look is the computational time. Of course, macroscopic models are much faster than microscopic, so the computational effort is very important and microscopic models are quite complicated and they take a lot of time to run. Another issue that should be discussed is the required resolution or the application that we want to deliver. So depending on what we want to do, for example, if we want to model the evaluation of incident detection we should use macroscopic versus microscopic modeling. Another key issue is the control measures that we want to apply. So if we want to have traffic control we can have traffic lights, we can control the ramps on the highways, we can have speed limit or other different infrastructure for control. So depending on the scope of our project, we could use macroscopic versus microscopic modeling to model all these different controllers.

Notes

Summary



15m 12s

Microscopic vs Macroscopic modeling

- Macroscopic models are analytical (not just simulators).
- Computation time.
- Required resolution / Application field (e.g. evaluation of incident detection).
- Easier consideration of various control measures (e.g. speed limits) or infrastructure impact in microscopic models.
- Potential accuracy versus validation effort.
- Routing models.

Another key issue is the accuracy of the model versus the validation effort. It is well known that microscopic model require a lot of effort for calibration and validation and sometimes depending on the project that we have, this effort may not worth our time. Using a macroscopic model, we can get results much faster and with quite good accuracy. Last we should discuss routing models. Everyone to do some research that involves routing. You should be very careful in selecting a macroscopic versions and microscopic model because the behavior of routing, it is very different in these two categories of models.

Notes

Summary



16m 47s

How to select a simulation model?



**operational outputs /
project complexity**

Let's now discuss the selection of a simulation tool. If I have a project and I want to run a traffic simulation, now a days I have too many options. There are too many software packages that I can choose from. So in order to make a selection someone should look at two key aspects. The one is operational outputs that we need and the second is the project complexity.

Notes

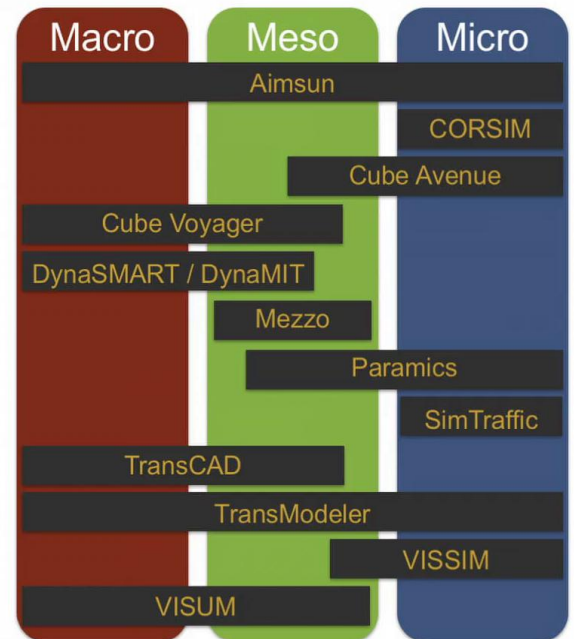
Summary



How to select a simulation model?



**operational outputs /
project complexity**



Again there is a huge variety of available software in all three categories: macro, meso, micro and depending on the project that you have to do and the work that you have to deliver, you have to make a choice. So in the next slide I will try to provide some criteria to help you make this choice.

Notes

Summary



18m 03s

Criteria for selecting a simulation model

- 1) Geographic scope (small network, region, corridor).
- 2) Facility type (highway, arterial, HOV, tolls, bus lanes).
- 3) Travel mode (car, bus, rail, truck, bicycle, pedestrians).
- 4) Management strategy (work zones, emergency management).
- 5) Traveler response (route diversion, mode shift, destination change).
- 6) Performance measures (travel time, density, delay, queue length).

Many students and the researchers have a question of how to select the correct simulation tool. In this slide I will try to identify some key criteria of how to selecting a traffic simulation model. First of all, you should look at the geographic scope of your problem. It is different to model an isolated intersection or a corridor or a city or a big metropolitan area. The facility type that you need to model is also very important. So you may want to model a highway or an arterial network, some tolls, bus lanes or HOV which stands for High-Occupancy Vehicles. Travel mode is another key aspect. If you want to model only cars or you want to model public transport; rail, truck, bicycles, pedestrians, you should choose the right tool. Another component is the management strategy. If you want to model traffic lights or you want to model work zones or emergency management, it's a different type of modeling. You may want to model traveler response or not. In some simulations we may have route diversion, mode shift of passengers or even change of destination. Another important thing is the performance measures. So at the end of the simulation, you want to deliver some metrics.

Notes

Summary



18m 27s

Criteria for selecting a simulation model

- 1) Geographic scope (small network, region, corridor).
- 2) Facility type (highway, arterial, HOV, tolls, bus lanes).
- 3) Travel mode (car, bus, rail, truck, bicycle, pedestrians).
- 4) Management strategy (work zones, emergency management).
- 5) Traveler response (route diversion, mode shift, destination change).
- 6) Performance measures (travel time, density, delay, queue length).
- 7) Cost effectiveness (license, documentation, user support).

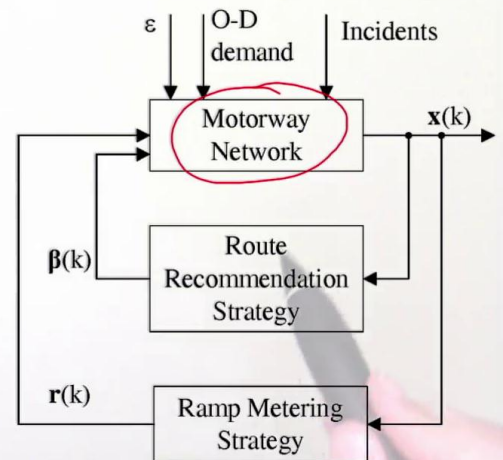
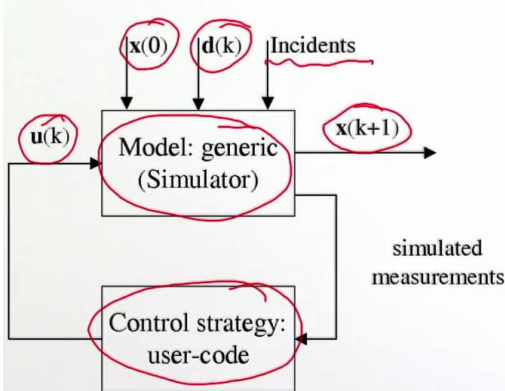
You may want flow, density, delays, queue lengths, travel time, so there are tools that provide some of them and other tools that provide other. And the last but not least, you should check the cost effectiveness of your choice. There are many tools available; commercial or academic, there are tools that you have to pay for the license or tools that are free that are open source, so you have a wide variety of choices and cost effectiveness is one of the factors that you should look at.

Notes

Summary



Testing control strategies (simulation)



In this slide I tried to summarize the key criteria that how someone should select a simulation software for traffic. So next time you have a project and you need to simulate traffic flow, you can ask yourself these seven questions and depending on the answer you will be able to narrow down your options and make a good selection for simulation tool. As I already mentioned traffic models are important because we can use them to test our control strategies. Here I provide two examples of macroscopic traffic flow models. In the example on the left, I have a traffic flow model that I use as a simulator and I have developed a control strategy that controls, for example, the traffic lights of the network. So what I can do here is that I can provide the vector of control inputs and run my simulator and get the state of the network over time. Of course, this simulator also needs an initial state and some demands or exogenous inputs and maybe some incidents. So this is a closed loop system that I can run my controller with feedback and evaluate the performance again. On the right I have another example of a motorway network, so I have the macroscopic model of a motorway network and I have two different controllers.

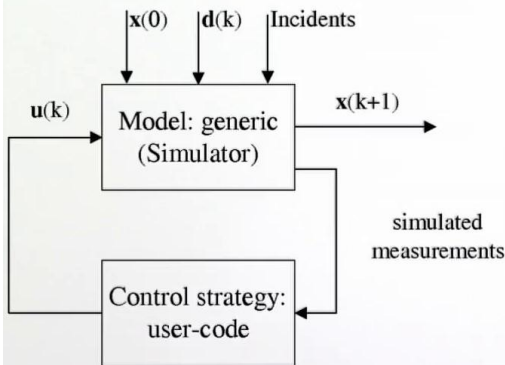
Notes

Summary



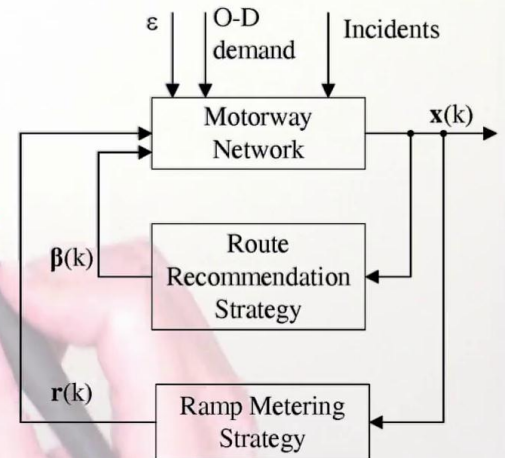
Testing control strategies (simulation)

Metrics



VKT (veh·km):

$$VKT = T \sum_{k=0}^K \sum_i q_i(k) \cdot \Delta_i$$



VKT: Vehicle Kilometers Traveled

VHT: Vehicle Hours Traveled

The first controller is route recommendation strategy. When I run this strategy, I get this parameters beta that defines the route recommendations for the users and I have another strategy that does ramp metering and gives me the optimal rates for the ramps. So if I fit these two values to my model and I simulate the performance, I can get this vector 'x' again and this runs over time. We feedback for the controllers and it gives me the performance of the network. Of course, this model may require some O-D demand, it may have some errors or disturbances and I can include some incidents. These two examples demonstrate how traffic models can work as simulators and give me some performance evaluation. So what are the metrics that I can get from such a traffic simulator? I can basically get VKT and VHT which are Vehicle Kilometers Travelled and Vehicle Hours Travelled. I can get VKT if I sum up all the flows of the network over time and space. So this 'i' sums over all the sections which each of them has a length of 'delta i' and the second summation sums over 'k' which is the discrete time index. And if I multiply this with 'T' which is the sample time of my model, I get the VKT.

Notes

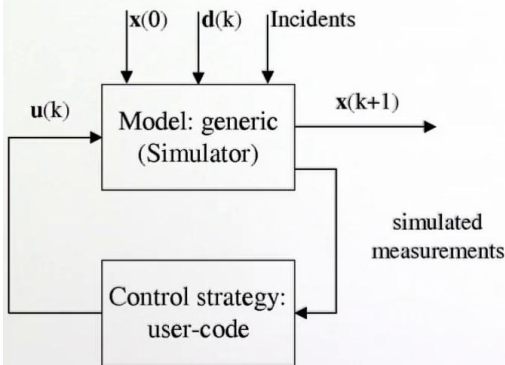
Summary



22m 11s

Testing control strategies (simulation)

Metrics



VKT (veh·km):

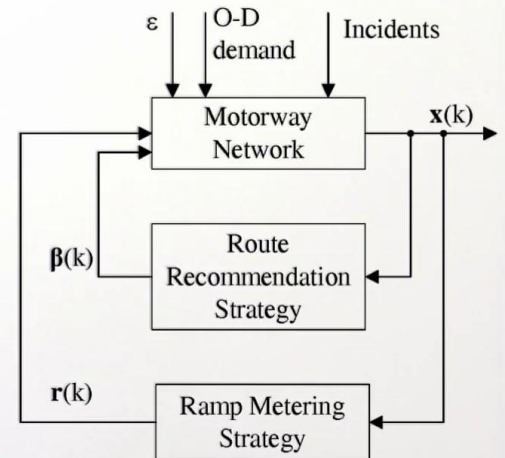
$$VKT = T \sum_{k=0}^K \sum_i q_i(k) \cdot \Delta_i$$

VHT (veh·h):

$$VHT = T \sum_{k=0}^K \sum_i \rho_i(k) \cdot \Delta_i$$

VKT: Vehicle Kilometers Traveled

VHT: Vehicle Hours Traveled



$$v = \frac{VKT}{VHT}$$

Here the flow has the units of vehicles per hour. Equivalently, I can get VHT. If I sum up all the densities of the network over time and space, the one summation again is over space for all the sections, each section has 'delta i' length and the second summation is over time 'k' and again I multiply with 'T' and now if I divide VKT with VHT, I get the space-time mean speed of the network which is a generic performance index for my simulation. This video of week 2.1 was an introduction to traffic flow modeling. We have discussed the basic principles of traffic models, their classification and how to select a software for traffic simulation. Next, we are going to introduce the LWR theory. See you soon.

Notes

Summary



23m 53s