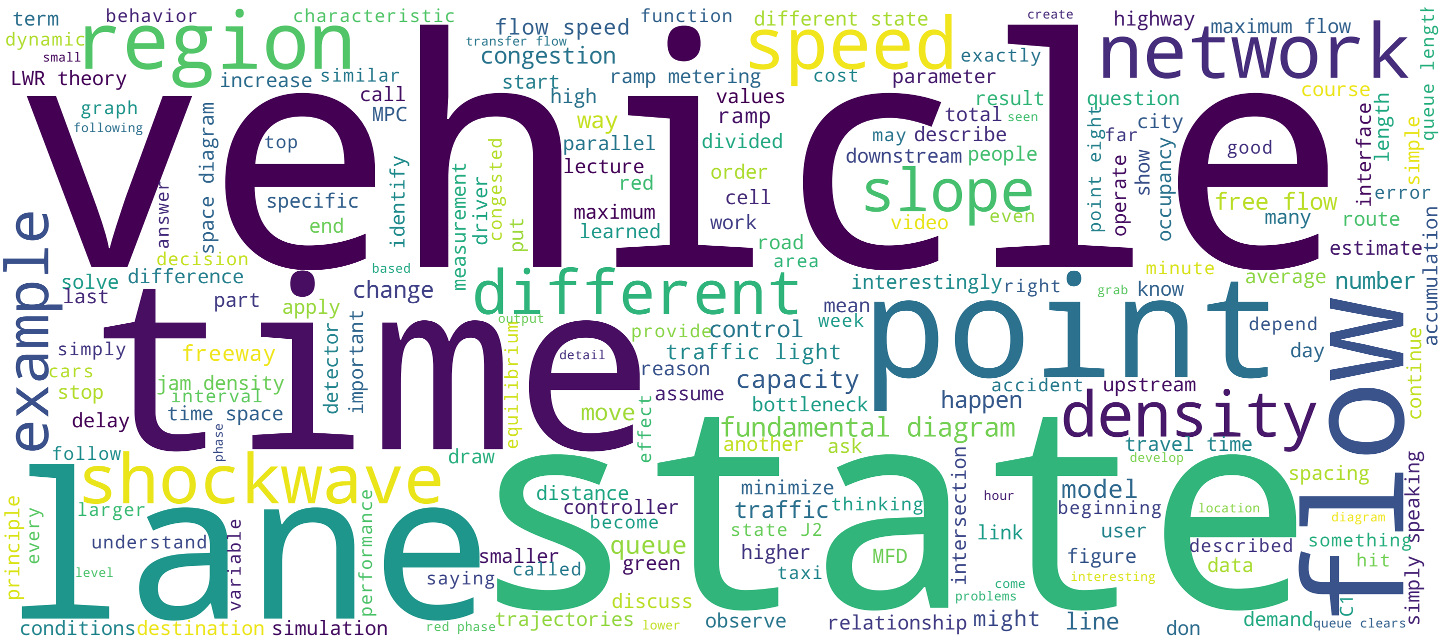
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Traffic flow modeling

Examples of LWR Theory

Intro to traffic flow modeling and ITS

Prof. Nikolas Geroliminis



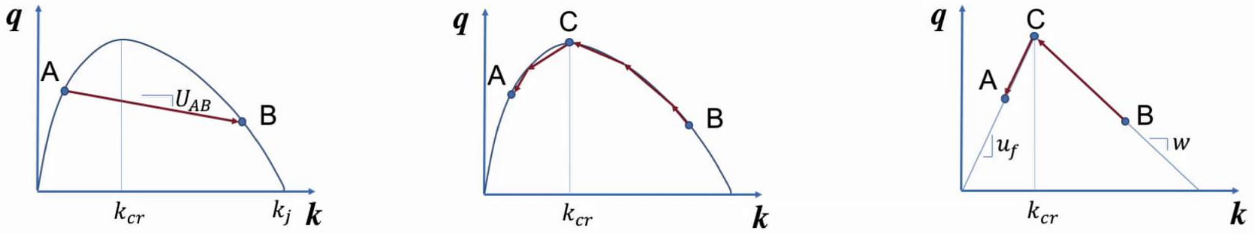
Search MOOC

Video

EPFL

Principles of LWR theory

- All possible states that can be observed in a road segment should belong in a FD (stationary conditions)
- When a traffic stream is moving from A→B with $k_A < k_B$ it follows a linear interface between A and B in the FD
- When a traffic stream is moving from B→A with $k_A < k_B$ follows a path **on the FD** (entropy conditions)
- If additionally $k_A < k_{cr} < k_B$ then the path passes through capacity state C (queues at active bottlenecks discharge at capacity)
- The speed of a shockwave between two states is $U_{AB} = \frac{q_B - q_A}{k_B - k_A}$
- If FD is triangular, then from B→A, there are only characteristics with speeds u_f and/or w



We have learned so far the basic principles of LWR theory which is a continuous model that describes the evolution of traffic congestion over time and space. In this lecture, we will see some concrete examples on how we can apply this theory to solve simple but also more difficult problems. I would like to refresh your mind with the major principles of LWR theory that we described in the previous lecture. This is a slide that we used before. I will not describe it again. I have it here as a reference and you can refer in the previous video for the description.

Notes

Summary

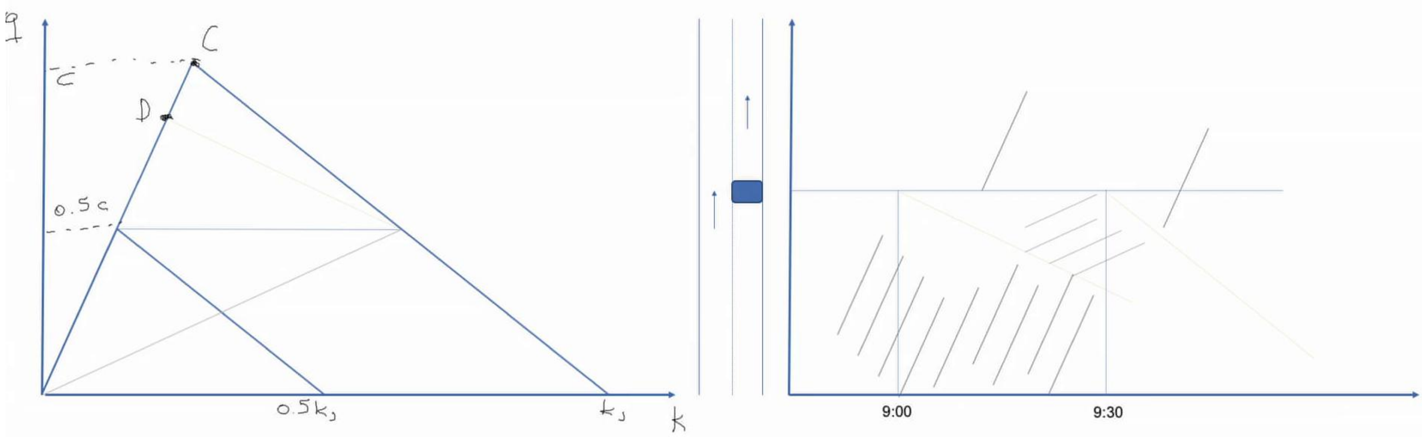
0m 05s

Example I: A freeway accident

Consider a two-lane freeway with a triangular FD with the characteristics below. Traffic is in the uncongested regime with flow q=0.8c. At 9am an accident happens that blocks one lane for 30min.

Estimate the maximum queue length.

What is the clearance time of the accident?



So let's consider a two-lane freeway where vehicles move from bottom to the top at a specific constant flow equal to point eight c; c we will be defining in a minute. At 9 am an accident happens that blocks one of the two lanes for a period of 30 minutes. So we want to understand what is the effect of this accident in the development of congestion and in the movements of the vehicles in these trends? So we start by first defining the fundamental diagram for these two lanes of freeway. The fundamental diagram between flow and density for the one and for the two lanes are shown in this figure. So 'c' is the maximum flow observed for the two lanes while for the one lane is point five c. In the same way we have the jam density of two lanes and the jam density of the one lane is half of that. Flow arrives from bottom to the top at point eight c which is this point over here, so let's call this state D, let's call the state of maximum flow C and what will happen is that as vehicles are arriving and the accident happens, they have to slow down. So there is a rapid change in the density from lower densities to higher densities because of congestion.

Notes

Summary

0m 48s

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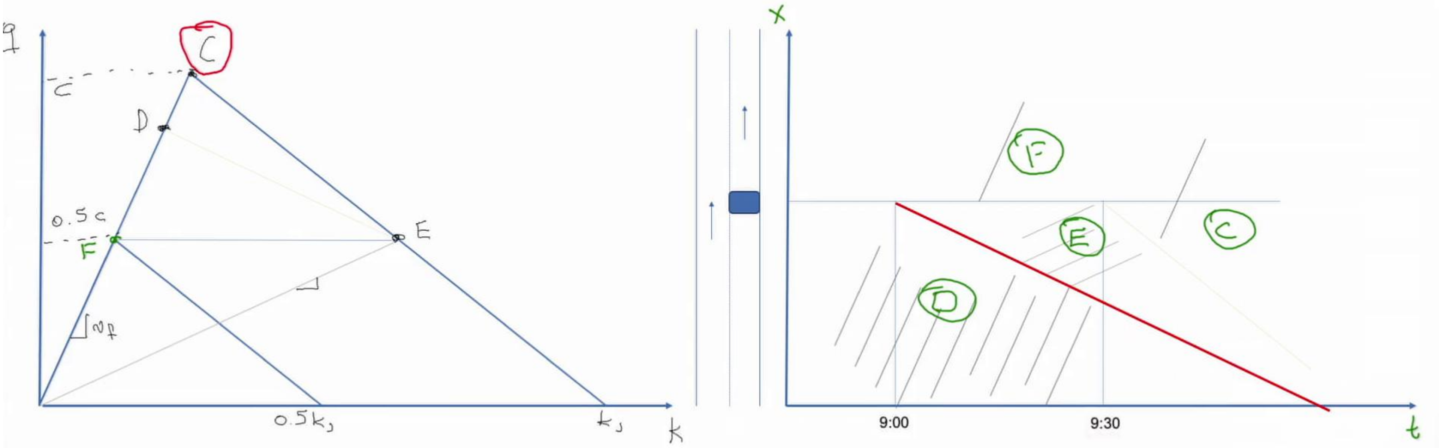

And in that case the speed of the vehicles when the accident happens will go down from the free flow speed which is the slope of this lines to the speed of the state E and this is that specific speed. So the trajectories we see here have speed equal to that while the trajectories we see in this part have speed equal to ‘vf’. So what we learn in one of the principles of the LWR theory is when we go from a lower to a high density profile, we will have a shockwave and this shockwave has slope equal to ‘DE’ which is the change in the flow divided by the change in the density between states D and E. So this shockwave will start from this point at 9 o’clock and it will propagate all the way down and it has the same slope with ‘DE’. So at 9:30, the accident will disappear so vehicles will return to higher speeds. And what is the state that we will be observed afterwards, according to the entropy condition we described these are the states where flow is maximized so is state C. So if we write with green color the different state of the fundamental diagram that we see in this time-space diagram, we have here state D, we have here state E and we have over there state C while over there we have state F which is the capacity of the one lane.

Notes

Summary

2m 39s

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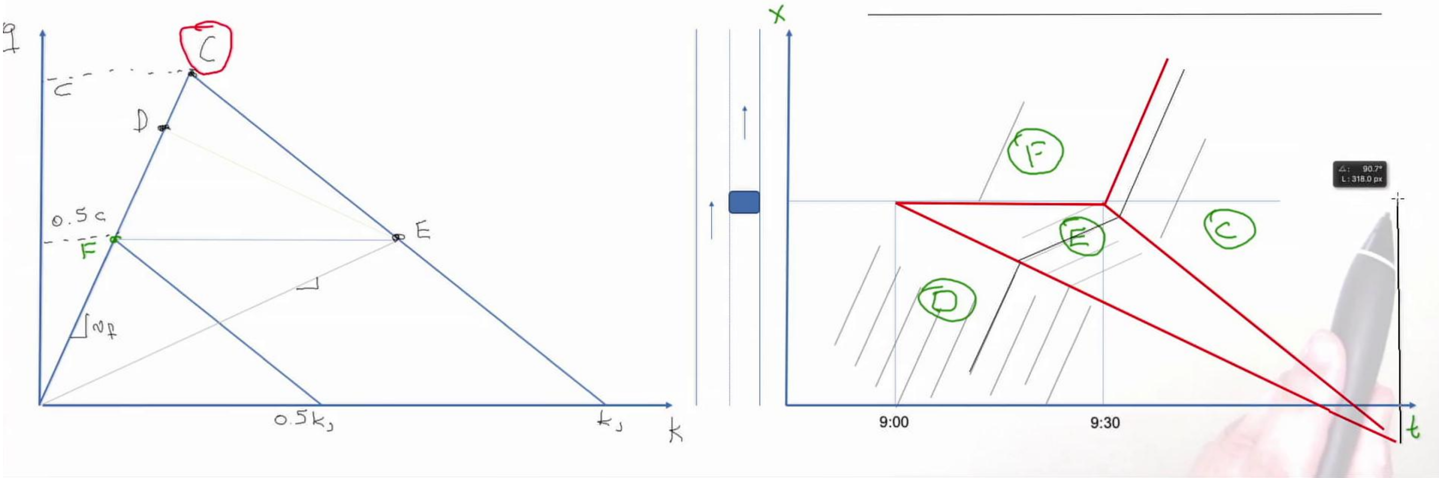


Ok. Very good. So what we are missing over here, we have to put lines that separate these different states, so between state E and C the speed of the characteristics is equal to this slope and that point we will go all the way down and interestingly this is that will be a point where these two lines meet. This one and that one and this actually will describe when the queue finishes. And also we need to put another line that separates state F from state E which is the interface that has actually zero speed and also we need to put a line that separates states F from C and this line has the slope equal to the free flow speed. So over here, we describe all the different states we see in the time-space diagram and now as we also have the states, we can also draw the trajectories of the vehicles. So for example, if I grab this vehicle over here, this vehicle will go at free flow speed up to this point then it will slow down up to there and after that it will go again at free flow speed. And if we ask ourselves what is clearance time of the accident? This is the time between here and there while the maximum queue length is the distance between the intersection of these two lines up to there.

Notes

Summary

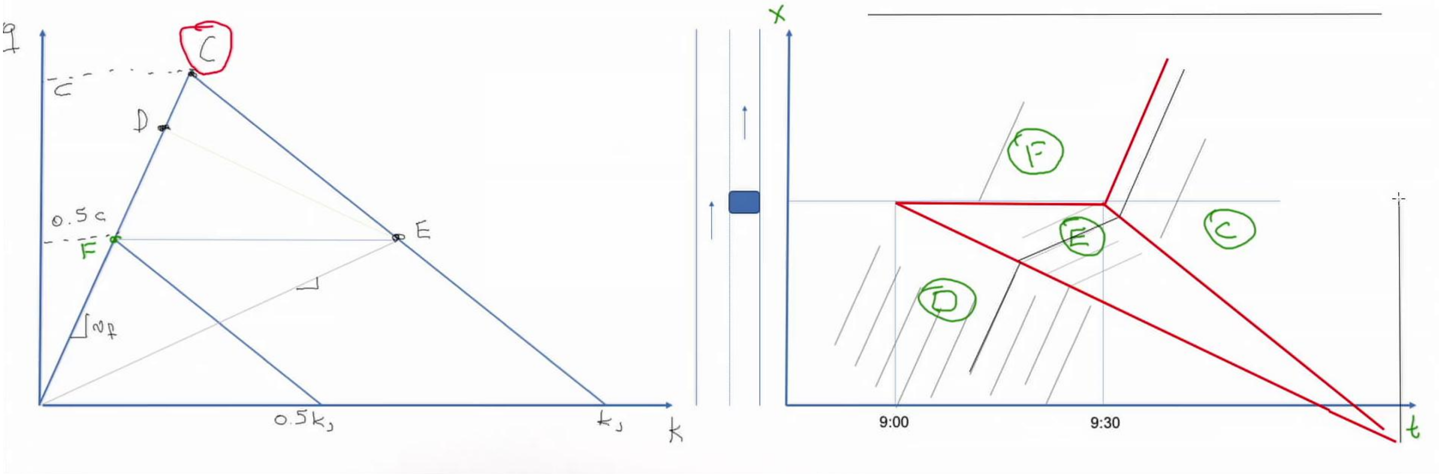
4m 50s

Example I: A freeway accident

Consider a two-lane freeway with a triangular FD with the characteristics below. Traffic is in the uncongested regime with flow q=0.8c. At 9am an accident happens that blocks one lane for 30min.

Estimate the maximum queue length.

What is the clearance time of the accident?



So this answers the two questions for example one and I would like to ask you that you do on your own some simple calculations to estimate the exact values and the solution will also be provided in the class material.

Notes

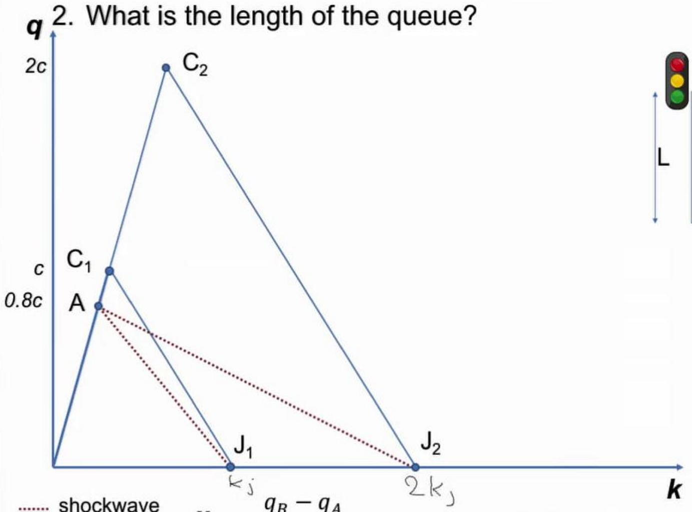
Summary

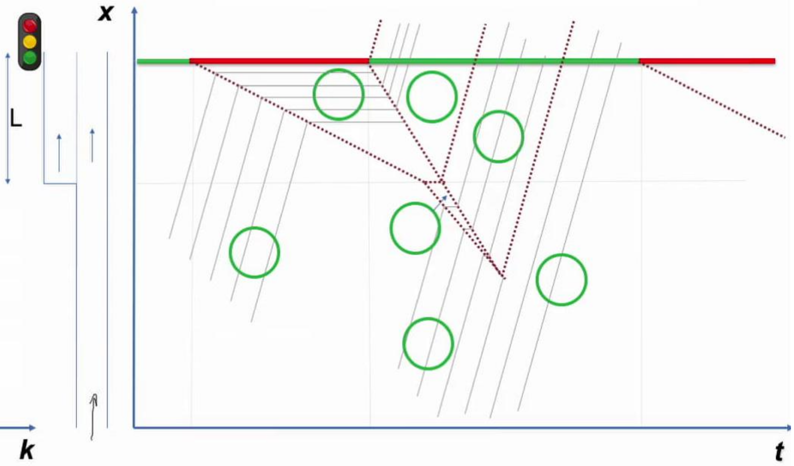
6m 51s

Example II: A complicated traffic light

Consider an arterial road with a traffic light (R=30sec, G=45sec). The road is one-lane but at distance L upstream of the stop-line widens to two lanes. FDs are shown below. Vehicles approach the intersection from upstream at free-flow speed u_f and flow q=0.8c (c is the road capacity of one lane).

1. Draw a time space diagram with all FD states and waves.
2. What is the length of the queue?





...... shockwave
FD state

$$U_{AB} = \frac{q_B - q_A}{k_B - k_A}$$

Let’s look now at the more complicated example. What we have here, we have an arterial road that has a traffic light downstream and let’s assume that the red phase is thirty seconds, the green phase is forty five seconds while the yellow phase is omitted and this road is one lane but in the last L meters before it reaches the traffic light, it enlarges and widens the two lanes. So the question is that to identify what is the effect in the performance and in the queues by widen the road from one to two lanes. So what we see over here, we see the fundamental diagram for the one and for the two lanes and as these two lanes have identical characteristics, what we can say is the capacity of the two lanes is ‘2c’ while the capacity of the one lane is only one ‘c’ and in a same way of thinking the jam density of the two lanes is ‘2k jam’ and that jam density of the one lane is ‘k jam’. Here what we see, we see different states so A is the arrival state of vehicles from upstream that they have a flow equal to 80% of the capacity of the one lane and then we have C1 which is the state of maximum flow for one lane, C2 which is the state for capacity for the two lanes and J1 and J2 which are the jam states for one and for the two lanes.

Notes

Summary

7m 08s

Example II: A complicated traffic light

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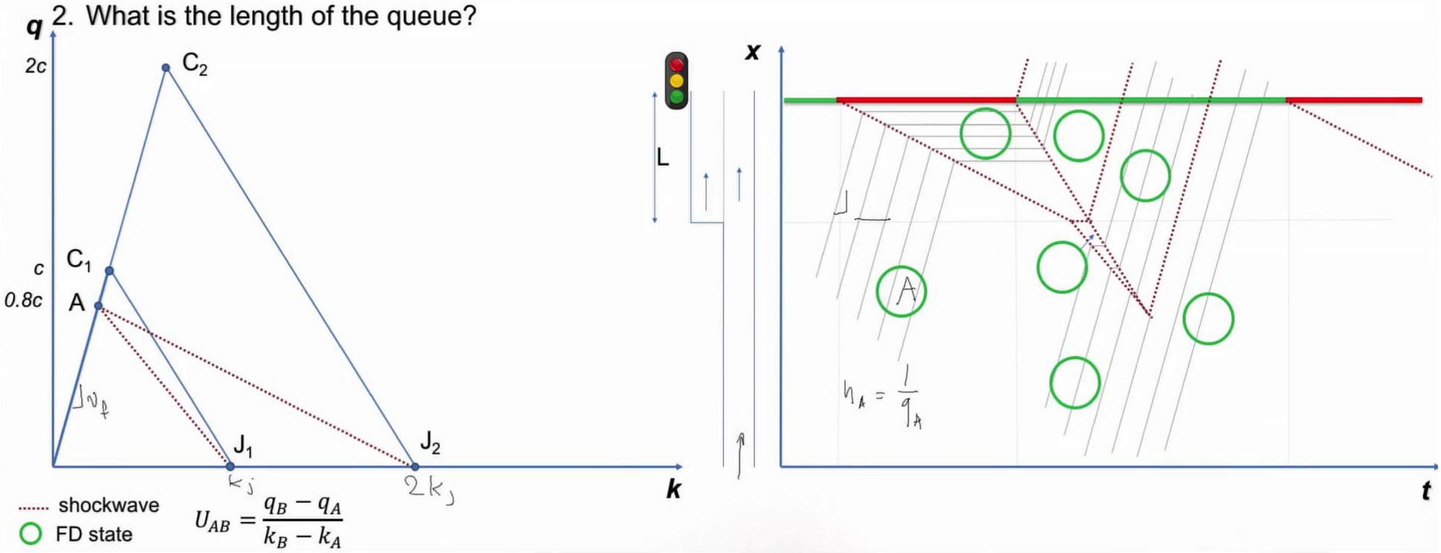
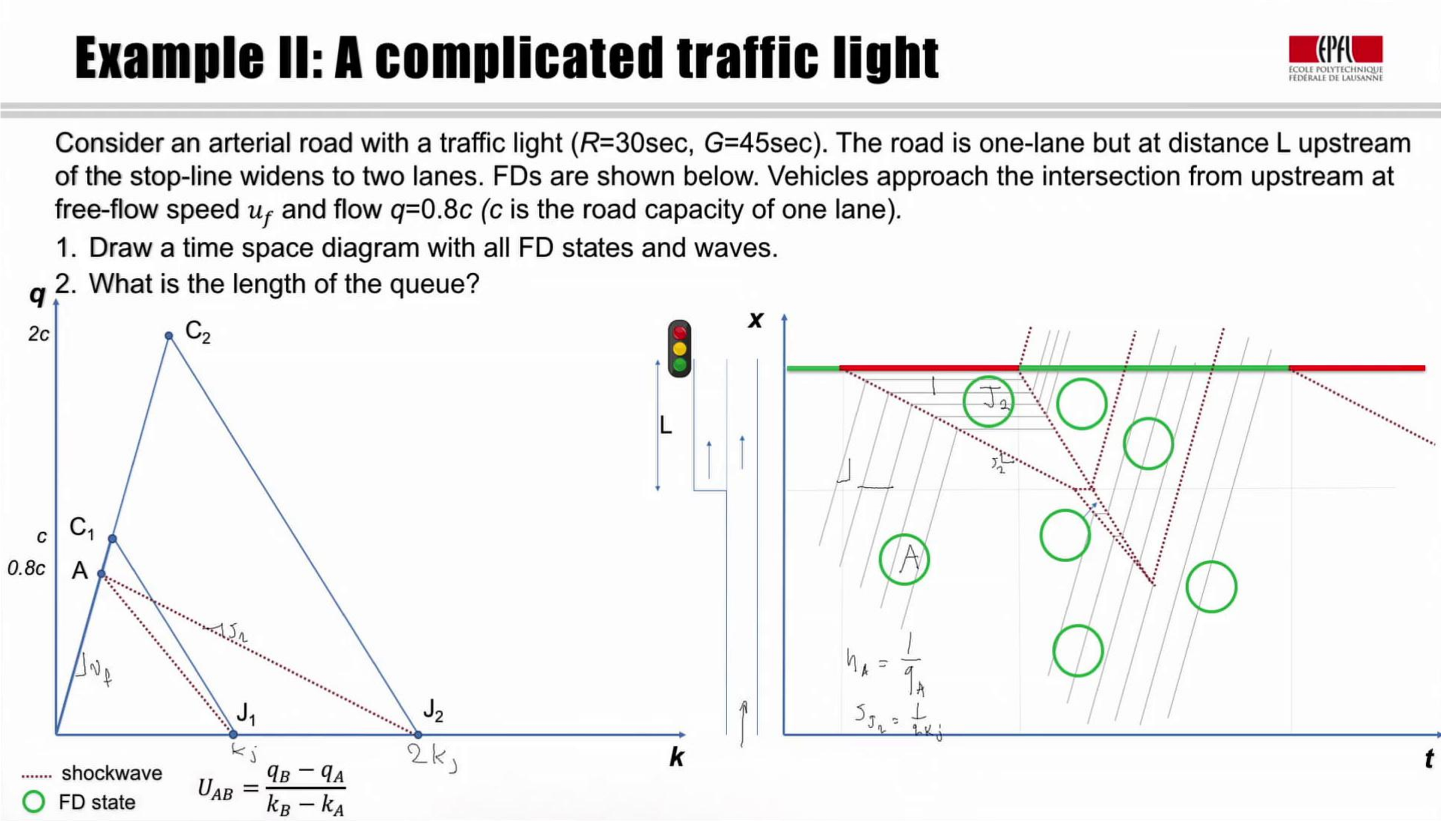


We will utilize shockwaves also states on the fundamental diagram. And let me refresh your memory by saying that when we move from one state of the fundamental diagram to another state and there is a sharp change, this is associated with a shockwave that this shockwave has a slope equal to the change in flow divided by the change in density between the two states. So here, vehicles are approaching the intersection from upstream at state A which has flow 80% of the capacity, so these vehicles over here are at state A and if we ask ourselves how we can identify and draw these trajectories, the answer is simple. So the slope of this trajectory is the free flow speed where the free flow speed is the slope of this line in the fundamental diagram. And what about the distance between these trajectories, so the horizontal distance between these trajectories is the headway and as we have learned the headway for state A is one over the flow for state A. We know the flow from here so we can identify what is the distance between these lines. So as this traffic stream is approaching the traffic light which we see it is red, the vehicles have to stop. So vehicles will move from a moving state of free flow speed to a stopped state.

Notes

Summary

8m 38s



And the stop state is the jam state that over here the geometry has two lanes. So vehicles, they move in the state which is jam for the two lanes which, simply speaking, is state J2, so this is state J2. And what we see over here, we see that the trajectories of these vehicles. Vehicles are stopping and if we want to estimate what is the vertical distance between these vehicles, so that this is the spacing. So the spacing at state J2 is equal to one over ‘2k jam’. So now we would like to identify how these two states A and J2 are separated by each other. So what we learned so far about the principles of LWR theory is that when we have an increase in the density, there is a rapid change and this rapid change is expressed by a shockwave. So actually between state A and state J2 in the time-space diagram, we want to put a shockwave and very interestingly the slope of this shockwave is identical that is given by this expression and we can simply take it from the flow-density diagram. So states between A and J2 have this shockwave, so we take this line the same slope and we move it until we put it exactly here. So simply speaking, let’s call this slope S2 is identical with that slope S2.

Notes

Summary

10m 15s

Example II: A complicated traffic light

Consider an arterial road with a traffic light (R=30sec, G=45sec). The road is one-lane but at distance L upstream of the stop-line widens to two lanes. FDs are shown below. Vehicles approach the intersection from upstream at free-flow speed u_f and flow q=0.8c (c is the road capacity of one lane).

1. Draw a time space diagram with all FD states and waves.
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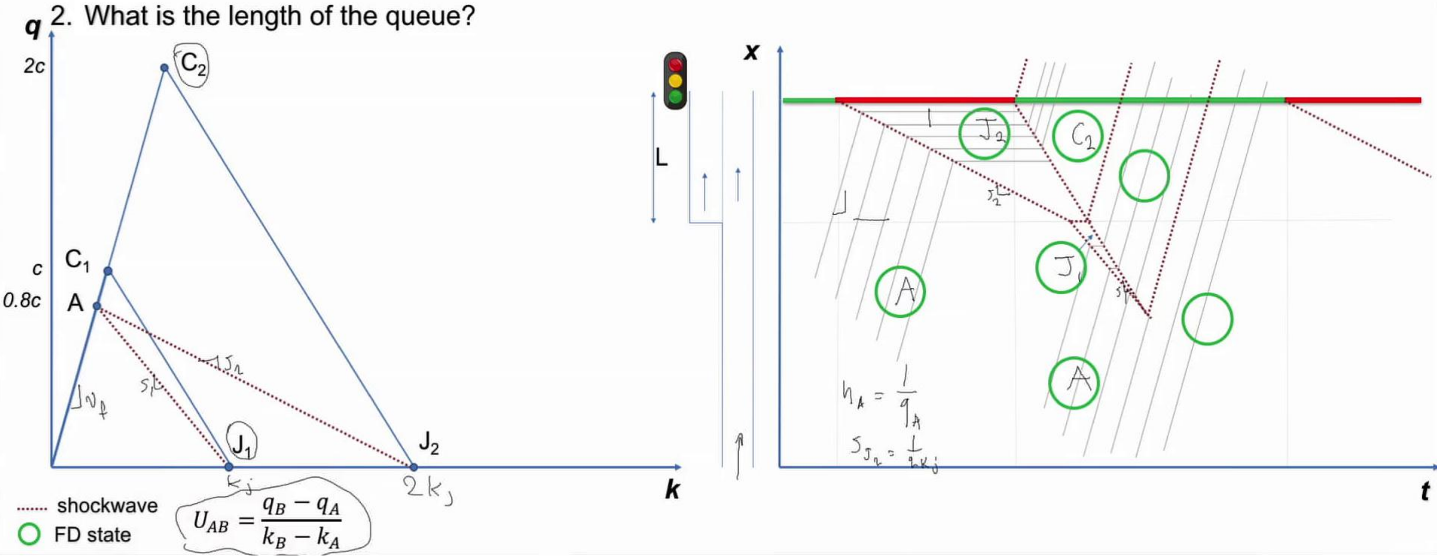

Now what it happens, this queue grows and actually at some point this shockwave will hit the one lane. So there the behavior will traffic will change because also the geometry changes. So now we are in the fundamental diagram of one lane and what we have is that we have that vehicles again have to stop in this small triangle that we see over there. But they have to stop at the jam density of the one lane. So this state over there is state now J1 and that state upstream continues to be state A. So now in a same way of thinking between state A and state J1, we need to put another shockwave where this shockwave is the one that has slope S1, so we come here, we take this shockwave AJ1and we move it parallel without changing the slope and we put it exactly there. And as we mentioned this has the slope of S1. When the traffic signal turns green, what we have learned is that according to the entropy conditions, we have to move in a state where the flow is maximum and actually as we have a queue, the queue should discharge at capacity. So from J2 slope J2 state we will move in the state C2 and we have to put an interface, a kinematic wave that this kinematic wave is coming is [inaudible 13:35:00] again by the same relationship like that.

Notes

Summary

11m 58s

Example II: A complicated traffic light

Consider an arterial road with a traffic light (R=30sec, G=45sec). The road is one-lane but at distance L upstream of the stop-line widens to two lanes. FDs are shown below. Vehicles approach the intersection from upstream at free-flow speed u_f and flow q=0.8c (c is the road capacity of one lane).

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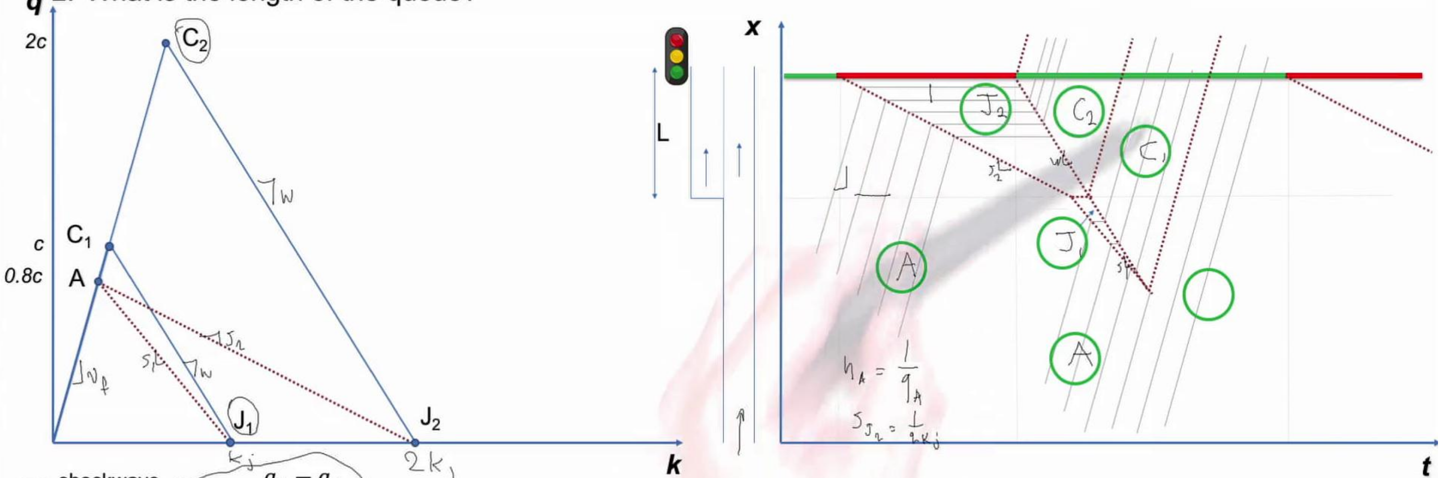

$$U_{AB} = \frac{q_B - q_A}{k_B - k_A}$$

So let's call this slope 'w' which is the maximum congested speed of the shockwave, so we grab this line and we move it parallel until we hit over there. So this is the shockwave that we put over here that this shockwave has slope 'w'. And this shockwave will start in the beginning of green; it will expand all the way back and let's see what happens when we hit the one lane. So when we hit the one lane, we have another shockwave that this shockwave should consider that we move from state J1 to what state, actually the state we move is state C1, is the maximum flow for the one lane. So this is how these vehicles over here they will discharge from their queue. So between J1 and C1 there is another shockwave but interestingly C1J1 is parallel to states C2J2 and it has the same slope 'w'. So we grab C1J1, we move it there but clearly the shockwave is the continuation of the other one, so when if the states change we don't realize this when we draw the shockwaves. And what it happens these vehicles discharge from the queue at state C1 and when these vehicles are reaching the two lanes, they don't change their behavior. They continue to travel at free flow.

Notes

Summary

13m 39s

Example II: A complicated traffic light

Consider an arterial road with a traffic light (R=30sec, G=45sec). The road is one-lane but at distance L upstream of the stop-line widens to two lanes. FDs are shown below. Vehicles approach the intersection from upstream at free-flow speed u_f and flow q=0.8c (c is the road capacity of one lane).

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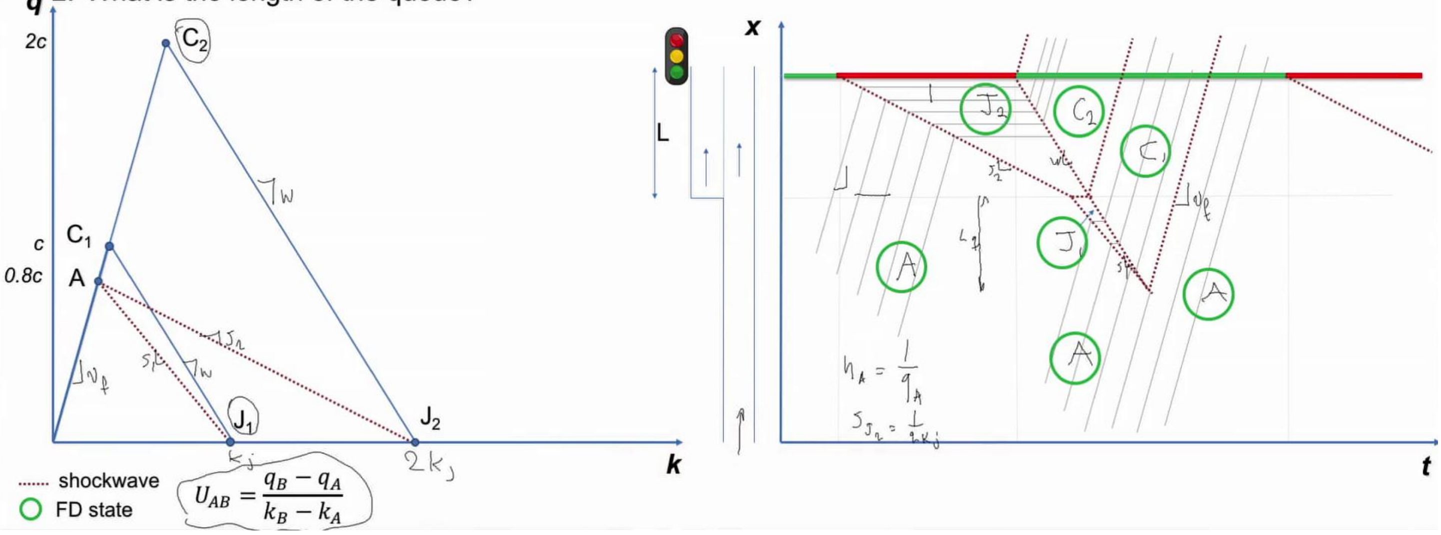
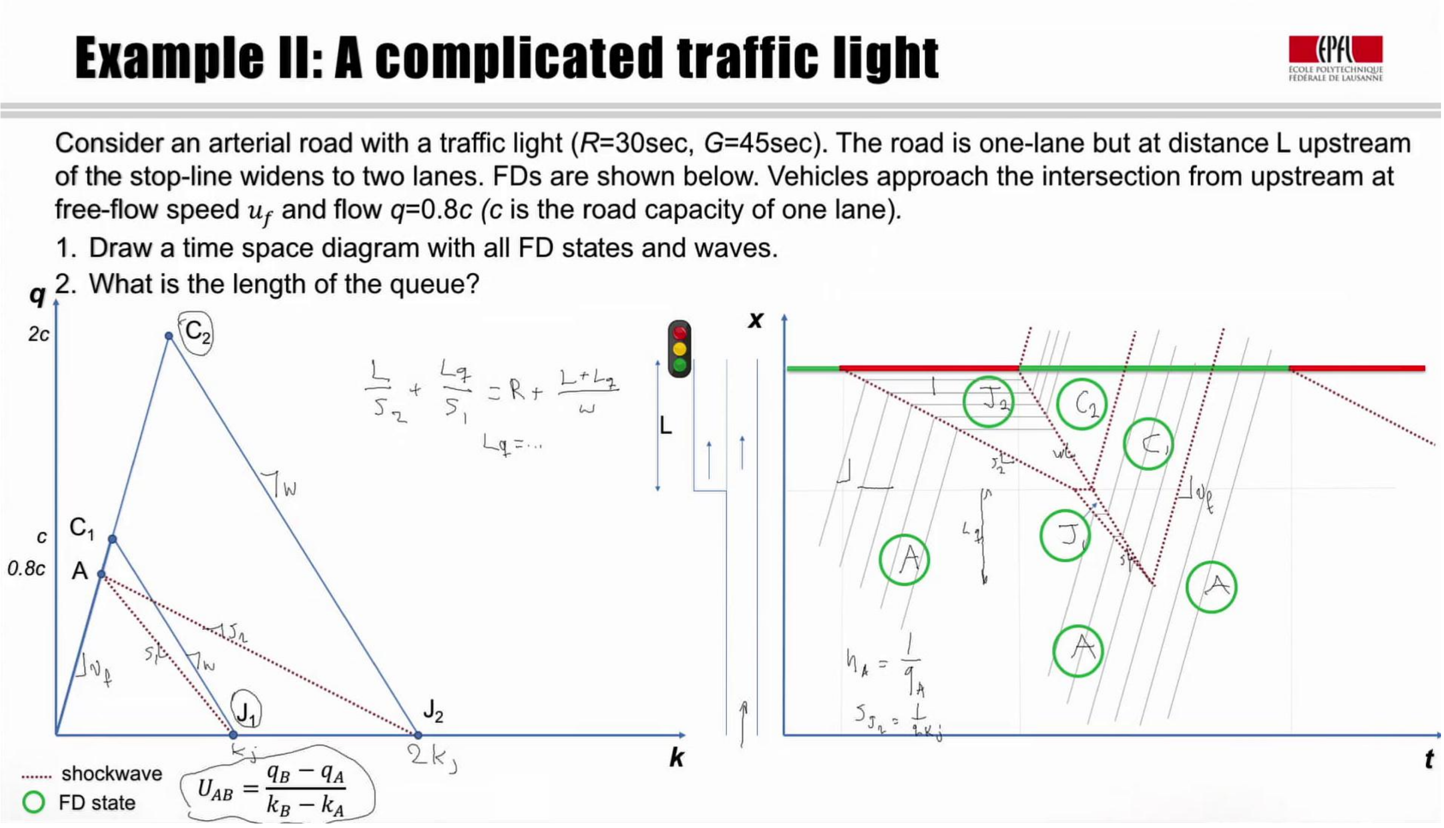


So that's the reason that again the state remains to be C1 even if the vehicles move from one to two lanes. And now between C1 and C2 again we need to put an interface but very interestingly as states C1 and C2 have the same speed for the vehicles, the free flow speed, the interface is parallel to this trajectory so we cannot see it. And then in the same way of thinking this is the point where the queue clears and vehicles arrived after this time, they don't need to join the queue they continue at free flow. So vehicles arrive from upstream at state A and they continue at state A. So again between state A and state C1, we have to put another interface which is parallel to the free flow speed, so all these lines have a slope of free flow speed. So I think here we have concluded our analysis and we draw a time-space diagram that shows all the different states that has occurred in this network. So now the question is to estimate what is the length of the queue? So over here we already mentioned that the queue clears at this point and then the length of the queue is the length of the geometry that has two lanes plus the additional length, let's call it 'Lq'.

Notes

Summary

15m 09s

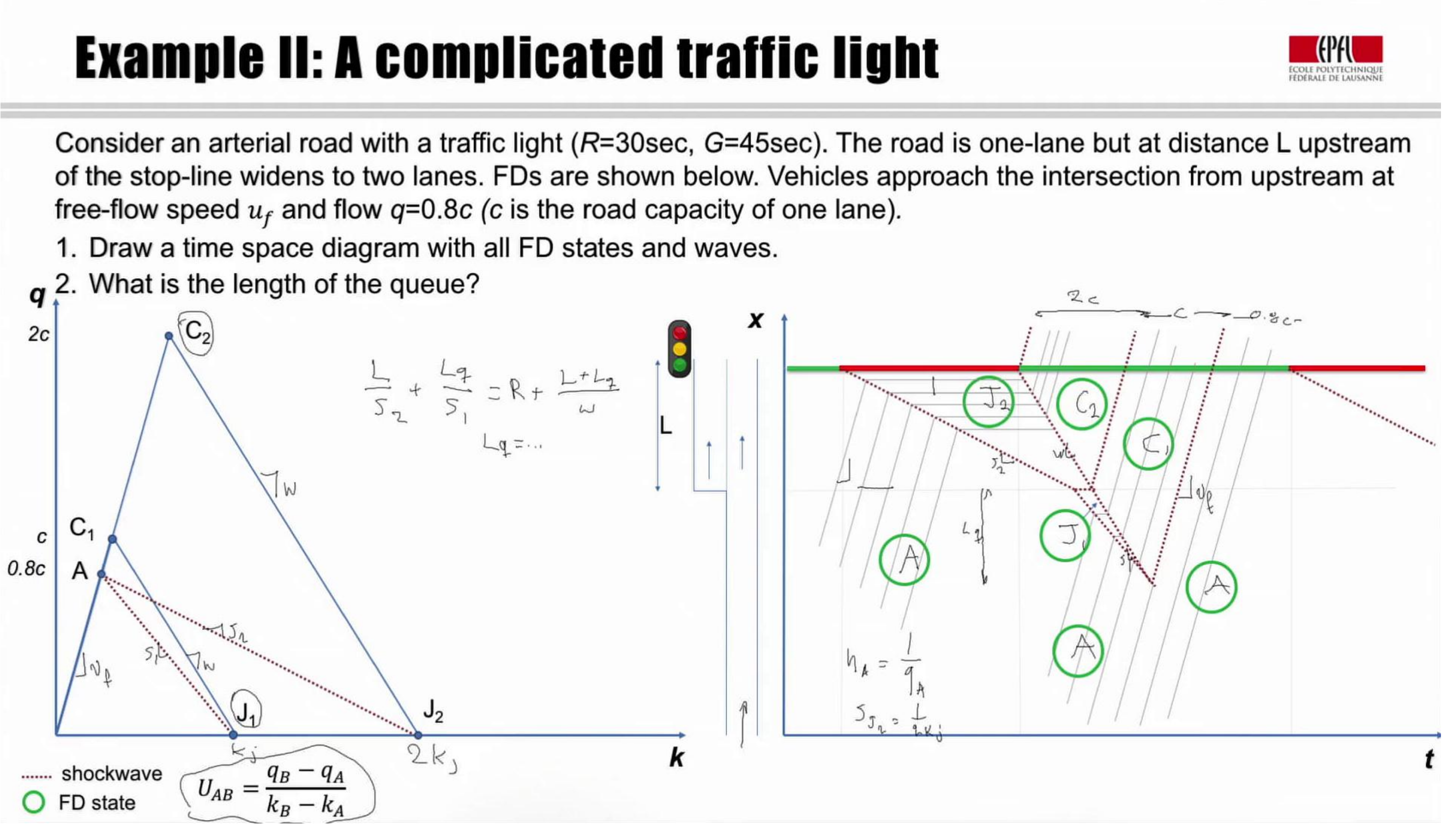


So now I would like to draw your attention in this polygon that I show with my pen. If we take all these points and we project them on the time dimension, we can estimate differences between events that happen over time. So first of all, I grab this point and that point with the upper point is the beginning of red phase where the lower point where I have my right hand, is the point in time where shockwave S2 hits the geometry of one lane, so what I am saying is that the difference between these times. So as the difference between these two times is equal to the difference between red duration plus the difference between these two times. Ok. So what is this duration? This duration, simply speaking, is this distance which is ‘L’ divided by the speed of the shockwave. So what we have, we have ‘L’ divided by ‘S2’ plus ‘Lq’ divided by ‘S1’ is equal to the red phase plus the time between this point and that point. So how much is this time? This slope is already ‘w’. So what we have, we have ‘L’ plus ‘Lq’ divided by ‘w’. So here we know ‘w’, we know ‘R’, we know ‘L’, ‘S1’ and ‘S2’ can be estimated by this formula. So we can solve for ‘Lq’ and we can estimate what is the queue length and I leave the algebra to the students.

Notes

Summary

16m 34s



So here I would like to summarize by saying the following. So what we have noticed is that by adding a part of the road with two roads what we succeed, we succeed that we have an interval of time that during this interval the signal operates at a capacity of ‘2c’ which is the capacity of the two lanes. But this cannot continue forever because this two lane geometry has the limited length. So after that the signal will continue to operate and when the queue clears the vehicles will be served at the capacity point eight c which is, somebody could play with this length ‘L’ by making it shorter or larger and in that case you can update the shockwaves by changing the slopes and I believe after this lecture you know how to do it, and in that way you can possibly estimate what is the ideal length which will minimize the queues and also it can operate this traffic light even if the flow is higher than point eight c. If you want to practice on your own, please solve again this problem but use ‘q’ equal to ‘c’ and try to identify what is the ideal length ‘L’ so that you minimize the queue length and keep in mind that there is a critical point that by further increasing ‘L’, we don’t gain anything.

Notes

Summary

18m 29s

LWR Theory - Summary

So I would like to thank you for your attention. During this week and in the last two lectures, we saw the principles of LWR theory and we solved some examples; one easy and one more difficult, how we can apply LWR theory in real life problems. I would like to ask you to go in the class material and also try to solve some additional exercises for different type of problems that will help you practice and learn the material better. See you soon.

Notes

Summary

20m 04s