



- Week 3.1
 - Car following models
 - Stability
 - Lane Changing models
- Week 3.2
 - Cell Transmission model
- Week 3.3
 - Ramp metering

Welcome. Drivers when moving in a network, they make different type of decisions at the strategic, the tactical and the operational level. So that strategic level, they decide about their destination. At the tactical, they decide which route to take. While at the operational level, they might make detailed decisions on how to follow other vehicles in front of them, how to make lane changes, how to accelerate or decelerate. This course will be about car following and lane changing models and we will try to explain and quantify how drivers make these decisions. So during this lecture we will analyze car following and lane changing models and we will also see the effect of different parameters on the stability of this model. Later this week, we will focus on the cell transmission model, which is a numerical approximation of LWR theory, which is used to model a congestion mainly in freeways. And in the end of this week, our focus will be on ramp metering, which is the most established traffic management schemes for freeways.

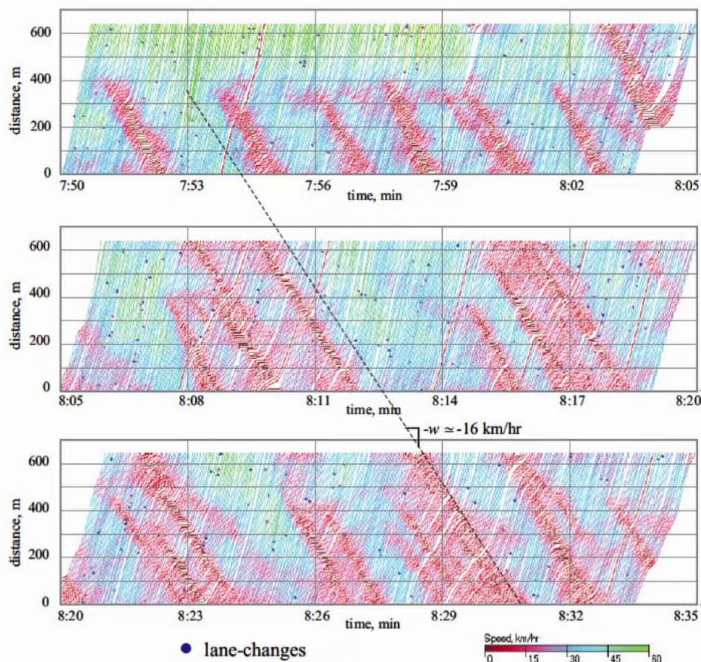
Notes

Summary



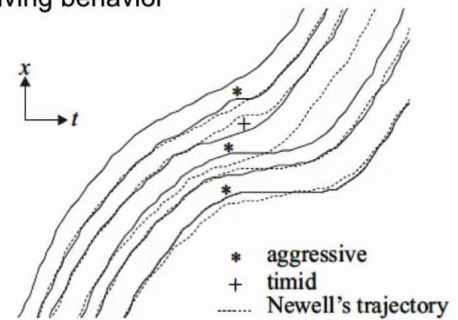
0m 06s

Empirical trajectory freeway data



Observations

- Stop-and-go phenomena
- Propagation of congestion and oscillations
- Different magnitude of oscillations
- Shockwaves
- Driving behavior



Laval and Leclercq, 2010

So let's start by looking at some empirical data from trajectories in a freeway in California during the morning peak. Here we see time, space diagrams for a part of a freeway that has a length of 700 meters for the period between 7:50 and 8:35 about 45 minutes. And the color in this graph represented the speed, green being high speed while red being lower speed. So what do you observe in this graph is that around that point an event happens that makes a vehicle to slow down. And interestingly, there are strong phenomena of stop and go traffic that propagate all the way upstream. And interestingly, the stop and go phenomena are happening quite frequently, every two or three minutes in the same location. What we also see, we see a shock wave that also propagates backward with SPD, constant approximately equal to 16 kilometers per hour. Also we can observe that there are different magnitude of these oscillations. So for example, here oscillations are much smaller while there, they're much wider. And what we can also see, we can also see that there is different in driving behavior. So not all vehicles have the same speed during a stripe of stop and go ways.

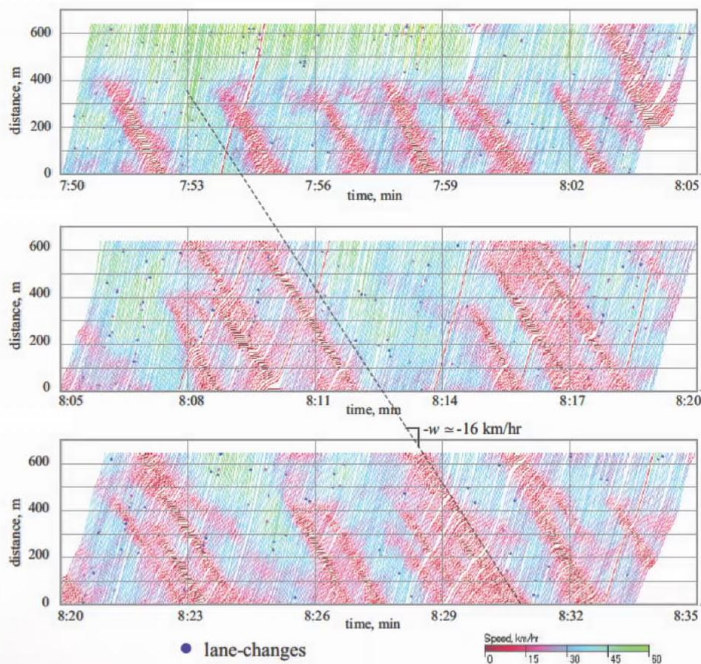
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Summary



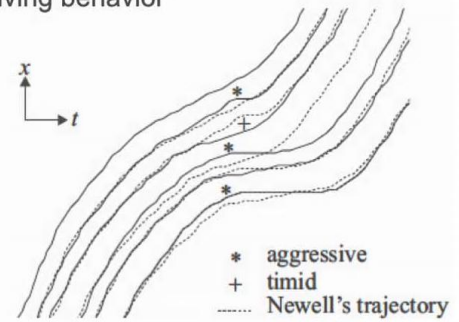
1m 25s

Empirical trajectory freeway data



Observations

- Stop-and-go phenomena
- Propagation of congestion and oscillations
- Different magnitude of oscillations
- Shockwaves
- Driving behavior



Laval and Leclercq, 2010

So let's have a careful look for a few drivers by zooming in in some of the time space for a couple of seconds period. So here what we see, we see that drivers have different driving behavior. So for example, there are aggressive drivers like this one who's on with the star that this driver will follow the car in front of him, but it will not decelerate until he gets very close. So we see the two trajectories that are getting very close and then we see that this driver makes a very set of deceleration. On the other hand, we might have tiny little conservative drivers where they will make [03:35 inaudible] reactions before getting very close to others. And we see that they keep longer and a safer distance, safe distances. And what we also see, we see what we name new will trajectory that are the trajectories of drivers that can be predicted by a car following model. And this is one of the car following models, we will investigate during this lecture.

Notes

Summary



2m 52s

Why a micro modeling approach?

Advantages

- Towards automated and connected vehicles
- More “behavioral”
- Fundamental diagram contains scatter
- Important for safety
- Develop microsimulation models

Challenges

- Calibration
- Stability
- System integration



And initial question is, why we need to develop car following models, which are very detailed models that describe micro movements of drivers? And to answer this question, it will be nice to look at the history of car following models. So some of the first car following models that have been developed in the 50s, where the big car manufacturers in U.S. have some tendency to look towards autonomous and connected vehicles. So what this manufacturers did? They hired top scientists and they gave them the opportunity to do deep research on how vehicles are interacting with each other. They gave them the opportunity to do experiments and actually this is how the research on car following models started. So car following models can be very behavioral, because it can really describe the interactions between individual vehicles. Another advantage is that the more aggregated models like the continue models of traffic flow, they cannot model these driving behavior. So for example, the fundamental diagram of traffic between flow and density contain some scatter that this scatter cannot be explained if somebody doesn't look at individual trajectories.

Notes

Summary



3m 58s

Why a micro modeling approach?

Advantages

- Towards automated and connected vehicles
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- Important for safety
- Develop microsimulation models

Challenges

- Calibration
- Stability
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Also car following models are important from a safety point of view as one vehicles travel, they try to avoid accidents and crosses. And one of the more reason is that our objective is also to develop micro simulation models that can be utilized to replicate realistic conditions and then they can be utilized also to test different type of strategies. But there are significant challenges related with the car falling models. The first one is that these models contain a very large number of parameters, which is difficult to calibrate. So it's difficult to identify proper values of these parameters that will make this model as close to reality as possible. And other issue is that these models can sometimes become unstable and maybe they can create conditions that are not so realistic as drivers might overreact or underreact to different conditions. And then the last challenge is that these kind of falling models should be associated with other detailed models, which might be lane changing models, route choice models where vehicles really drive and interact and make decisions in a network. And this system integration is really challenging.

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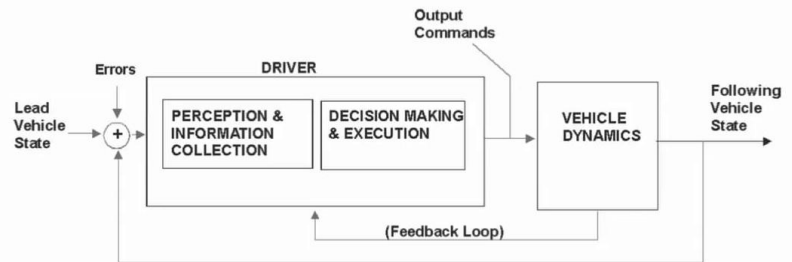
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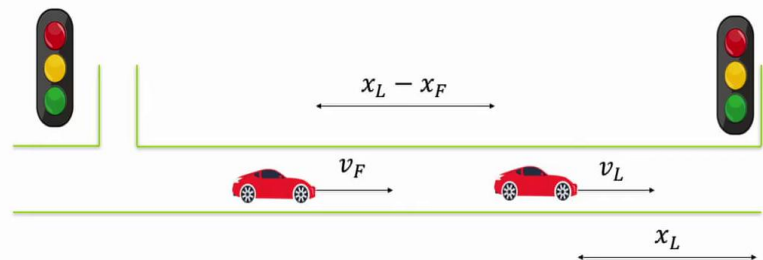
5m 17s

General characteristics of car-following models

- Perception
- Decision Making
- Control/Reaction
- Safety
- Efficiency
- Vehicle Dynamics



Block Diagram of CF models (Rothery, 2001))



So let's describe what are the general characteristics of car following models? The first one is related to perception. So a driver, when he drives, he understands the environment and he notices that some changes have been made. So [06:57 inaudible] these changes. He realizes that he has to also adapt by himself the speed of the car or his position. And then he has to make decisions related to acceleration or deceleration of a vehicle. So after these decisions are made, the driver has to make some control actions, which is mainly, press the brake or accelerate or move the steering wheel to make a lane changing or any other related actions. And this have to pass through the hardware and the engine of the vehicle to become real actions of the vehicles. So here on the right, we see a block diagram of general car following models, which has been developed by Rothery in 2001, where perception, decision making and also control are shown in the graph. What we also need to add over there is that the state of these vehicle and the perception of the driver also contains some level of errors because we don't really understand the exact position or how dense are some conditions can it be.

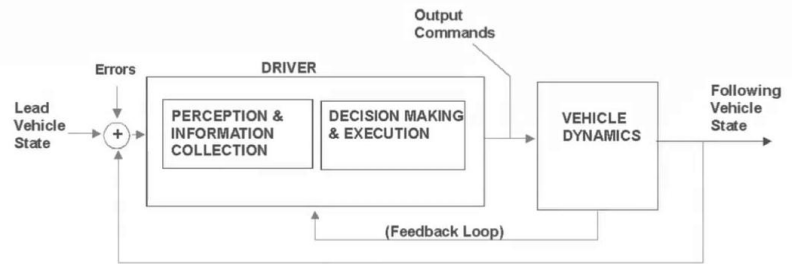
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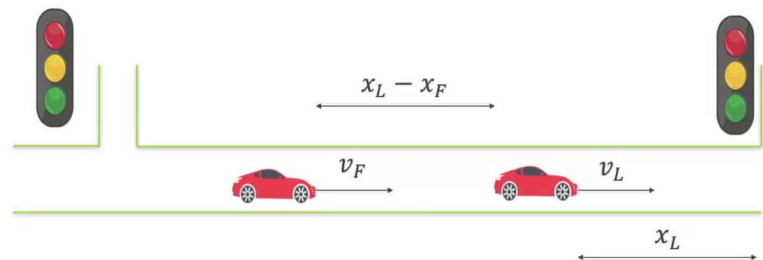


General characteristics of car-following models

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Block Diagram of CF models (Rothery, 2001))



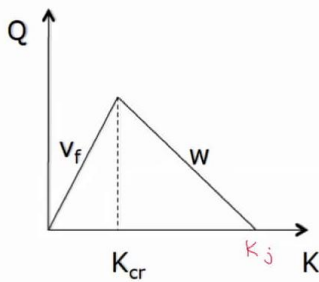
And what it happens, there is a feedback loop where the driver, he updates his decisions and his perception after he makes some reactions and after the dynamics between his vehicle and the other vehicle changes. So here we see an abstraction of the car following model where two vehicles are approaching a traffic light. So L is the leader, F is the follower and these vehicles have different speeds. And actually what we will see, we will see that the speed of the follower is adjusted and adapted based on the speed of the leader and also their relative distance and possibly some other parameters.

Notes

Summary



Consistency of CF and FD

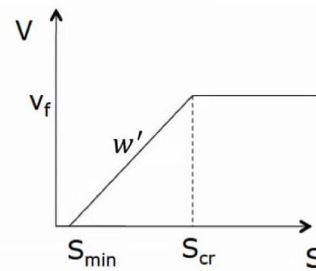


Fundamental diagram

$$w' = \frac{u_f}{\frac{1}{k_{cr}} - \frac{1}{k_{jam}}}$$

$$Q = k \cdot V$$

$$S = \frac{1}{k}$$



Speed – Spacing relation

So let's start building a simple car following models by first looking at the consistency between car following models and more aggregated models of traffic flow. And please refresh your memory that we have seen the fundamental diagram that connects flow and density. So I would like here to provide a simple explanation about the consistency of these models that have different granularity. So here on the left we see a triangular fundamental diagram between flow and density with parameters, free flow, speed, critical density that maximizes the flow and also the maximum shockwaves w . Actually by taking this diagram, we can transform it in another diagram that has characteristics related to car following models. So please refresh your memory that the density is the inverse of the spacing, which is the distance between two cars. So if we try to plot a diagram between speed and spacing by also considering that there is a relationship between the flow, density and speed which is kv and as I mentioned also spacing is 1 over density. Then we can have a diagram that looks like that where there is a minimum value of spacing that actually happens when density is largest for [10:19 inaudible]. There is a critical spacing, which is one over critical density.

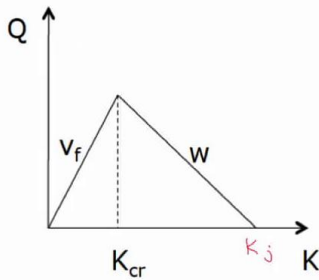
Notes

Summary



8m 50s

Consistency of CF and FD

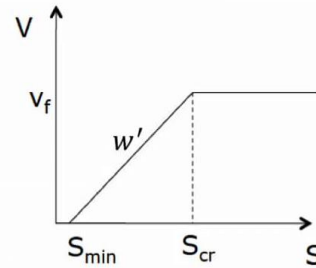


Fundamental diagram

$$w' = \frac{u_f}{\frac{1}{k_{cr}} - \frac{1}{k_{jam}}}$$

$$Q = k \cdot v$$

$$s = \frac{1}{k}$$



Speed – Spacing relation

And what we also see, we see that the speed is constant above the critical spacing. So these straight line is equivalent and has a slope of free flow speed over there. And actually what we can estimate, we can estimate what is the slope of this line over here. And if we do some simple calculations, we can say that this has given by the equation shown. Feel free in your spare time to try to do some additional calculations in this equation and see what will be the final outcome.

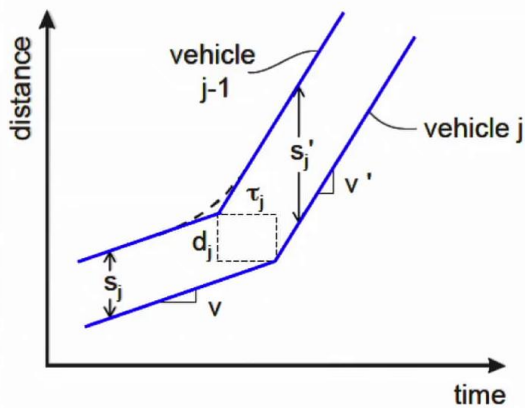
Notes

Summary



Newell's CF model

$$x_F(t + \tau_n) = x_L(t) - d_n$$



- **One simple equation**
- x_F, x_L represent the positions of the follower and the leader
- Same trajectory except for a translation in time and space
- τ_n, d_n May vary across drivers n

Properties of τ_n, d_n

- independent of vehicle's velocity
- drawn independently from a joint probability distribution

Let's now introduce one of the most established and most elegant car following models, which is known as Newell's Car Following model. So this car following model under congested conditions is described by only one simple equation. So what we see here, we see that the position x of the follower is simply the position of the leader. But what we have, we only have a translation in time and space. So this model has only two parameters T_n that has units of time and D_n that has units of distance. And n simply means that the values of these parameters might vary across different drivers. So here in this diagram, the time space diagram, we see the trajectories of two vehicles and simply speaking, we see that if we have one vehicle that goes at a slow speed and then it goes at a higher speed, the following vehicle will follow exactly the same pattern, but this point where speed changes is translated over time and distance according to the parameters, D_j and T_j . Some additional properties of these two parameters is that they're independent of vehicles velocity. So it will be the same even if these vehicles go fast or slow. And also they can have some stochastic characteristics. So they can be drawn independently from joint probability distribution.

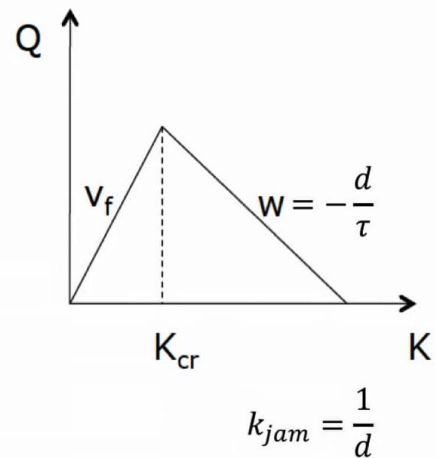
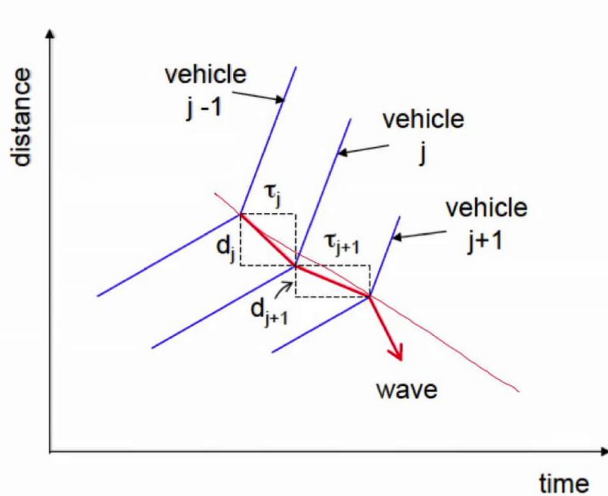
Notes

Summary



10m 56s

Newell's CF multiple vehicles



Each wave propagates as a random walk

So let's see the Newell's model in case of multiple vehicles. So over here, we see a series of three vehicles and actually what we also notice is that each vehicle has its own D_j and T_j . And what we have here, we have a wave which is developed and these wave propagates back actually now like a random walk. If these vehicles have all the same value for the parameters, then what we will see, we will see that this wave will be like a straight line. So in this way, we can identify the relationship between these parameters D and T with respect to triangular fundamental diagram parameters. So what we know, we know that D is one over the value of density where simply speaking, the ratio between D and T is equal to the maximum shockwave speed. The minus was there is because w has a negative slope.

Notes

Summary



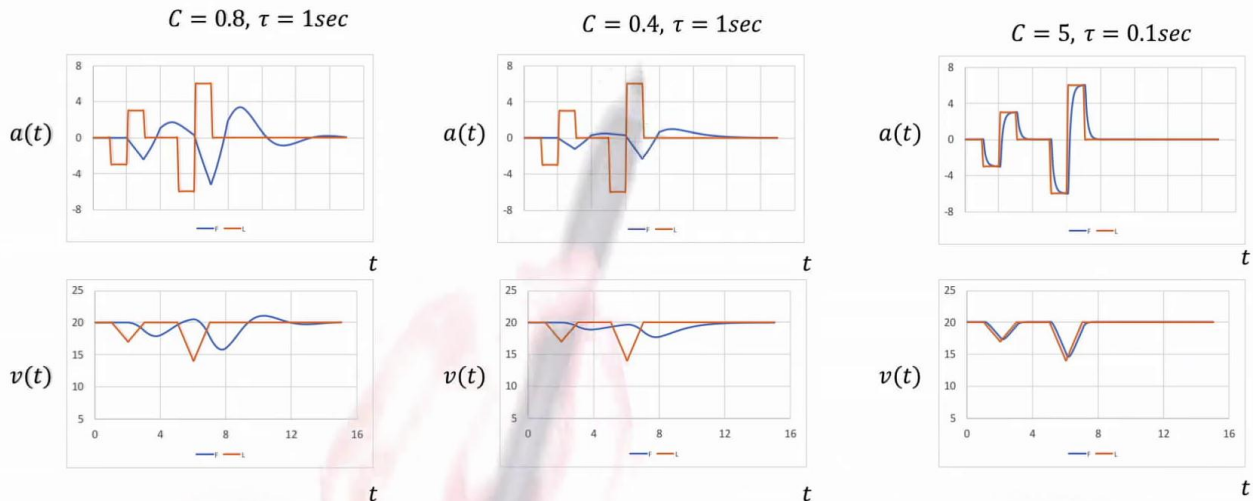
12m 32s

A linear car following model

Response = sensitivity * stimulus

$$\ddot{x}_F(t + \tau) = \frac{C}{\tau} (\dot{x}_L(t) - \dot{x}_F(t))$$

τ : reaction time
 $a(t)$: acceleration
 $v(t)$: speed



So Newell's car following model for constant values of parameters of D and T for its driver is actually identical and consistent with LWR theory for a triangular fundamental diagram. And this is a very nice finding that shows the consistency of the model that have different [14:01 inaudible]. Let's now look at another simple car following model that has leaner characteristics and is represented by an equation again with a small number of parameters. Let's look now at another type of car following model which is known as a linear car following model and is represented by a same linear equation again with a few number of parameters. Qualitatively, the response of the driver depends on each sensitivity and on the stimulus. So let's look this in some more details. So the response of the driver, which is represented as either acceleration or deceleration for the following vehicle. Actually, this decision will have a time lag of T seconds where T is the reaction time of a driver. So what we have, we have a sensitivity parameter over here that shows you the driver overreacts or under reacts and how aggressive he is. And then the stimulus simply is the relative speed between the leader and the follower.

Notes

Summary



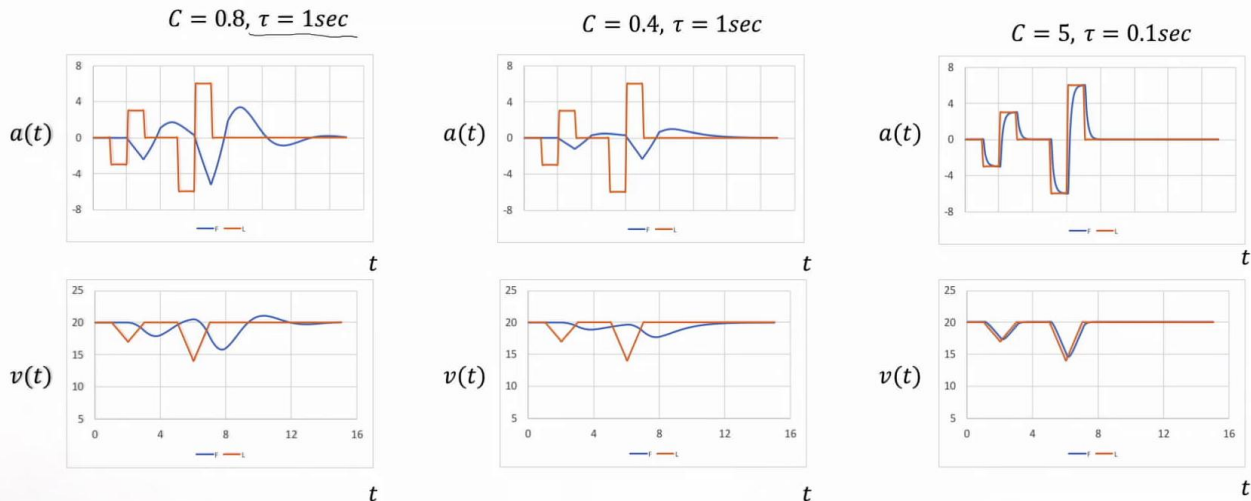
13m 38s

A linear car following model

Response = sensitivity * stimulus

$$\ddot{x}_F(t + \tau) = \frac{C}{\tau} (\dot{x}_L(t) - \dot{x}_F(t))$$

τ : reaction time
 $a(t)$: acceleration
 $v(t)$: speed



So $\dot{x}$ represented the speed. So as this model is simply a linear equation, we can easily take it and translate it in a computer program. And here in these graphs, I show you some examples of applying this in a computer. And this is something you can also do on your own. So here what we have, we look at two vehicles, that these vehicles initially have a distance of 30 meters. And while these vehicles follow each other at a constant speed of 20 meters per second, what it happens is at some point the lead vehicle decides to accelerate to decelerate. So the the lead vehicle decelerates for one second and then it accelerates with the same acceleration, which is three meters per second square for another second. And after some time it decelerates and then accelerates at the higher value, which in these cases six meters per second square. So let's see how the following driver will behave. So what will happen is when these first vehicle decelerates, there is no action for a period of one second, which is the reaction time. So what we see, we see that after one second, the other vehicle will start to decelerate. And as it follows the linear equation and the deceleration is constant, this deceleration will be a straight line.

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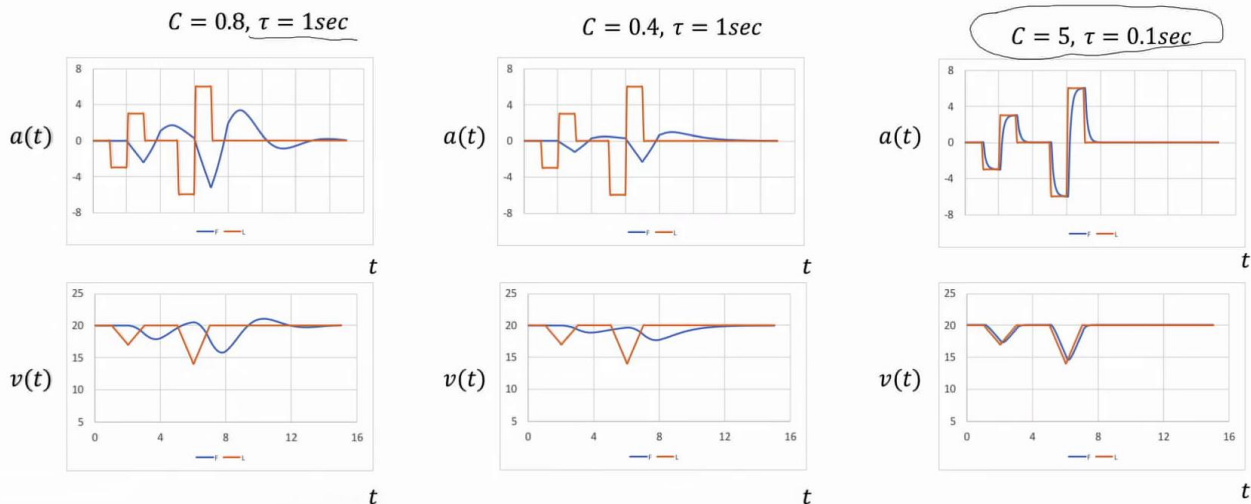


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 $v(t)$: speed



And then after one second, they leading vehicle accelerates again. And then we see a change in the acceleration. And then what will happen is that the other vehicle will go to zero. So this actually will create a, a graph like that. And when the vehicle, the lead vehicle decelerates at the higher rate, the following vehicle will follow, but now you see that, the change and the acceleration profile is [17:13 inaudible]. And the figure below, we see the behavior in terms of the speed of over time. If we utilize a smaller value of sensitivity, so instead of 0.8, we use 0.4, what we see, we see that the profile is similar, but now what we see, we see that this driver chooses to have the smaller accelerations and decelerations, but in the end of the system of the two vehicles is able to stabilize, they follow each other again in the way they started. Interestingly, if we use a very high value of c, but a very small value of the reaction time, we see actually that the vehicles follow each other almost in an identical way. And you could imagine that actually these characteristics might represent autonomous vehicles where the reaction time is close to zero and vehicles can react very half, very fast, what happens in the vehicle in the front.

Notes

Summary



16m 43s

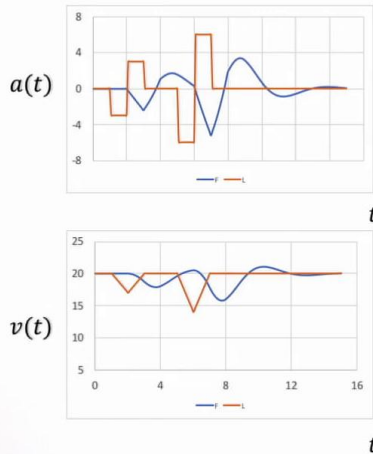
A linear car following model

Response = sensitivity * stimulus

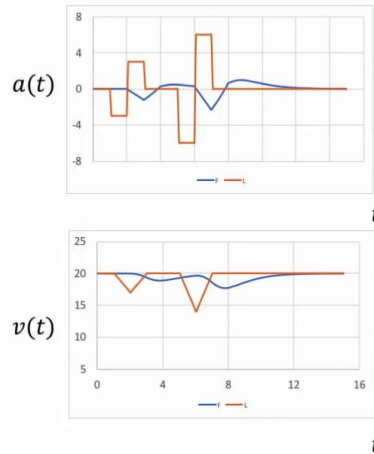
$$\ddot{x}_F(t + \tau) = \frac{C}{\tau} (\dot{x}_L(t) - \dot{x}_F(t))$$

τ : reaction time
 $a(t)$: acceleration
 $v(t)$: speed

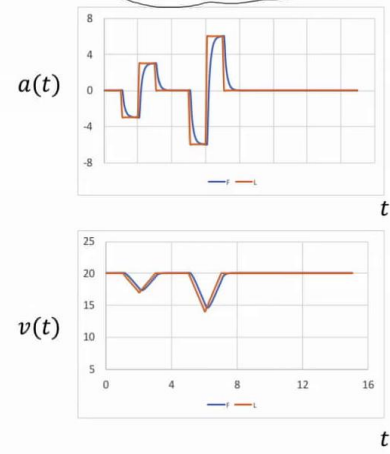
$C = 0.8, \tau = 1\text{sec}$



$C = 0.4, \tau = 1\text{sec}$



$C = 5, \tau = 0.1\text{sec}$



But interestingly, if we keep the reaction time at one second and we will increase c at the higher values, what we will notice, we will notice that this system becomes very unstable.

Notes

Summary

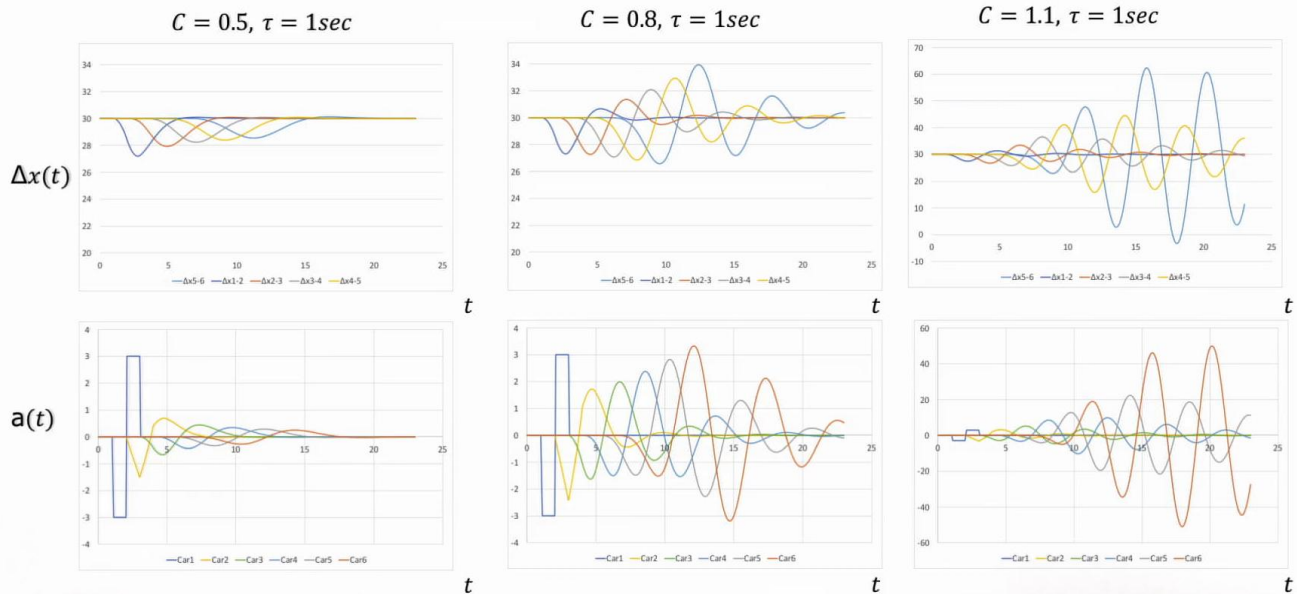


18m 17s

Stability analysis

Local Stability: the response of a following vehicle to a fluctuation in the motion of the leader vehicle directly in front of it.

Asymptotic Stability: how fluctuation in the motion of lead vehicle of a platoon, is propagated through a series of vehicles.



So let's look at more details about stability and now let's consider now a series of vehicles. So we have six vehicles that are following each other with the car following model we described in the previous slide. And over here, we see the values of parameters that are identical for all vehicles. So in the first case that c is 0.5 and T is one second. What we see, we see that when the leading vehicle decelerates and then accelerates again, the relative spacing, so the relative based on distance between the vehicles will be higher in the beginning. So this shows a distance between vehicle one and two, but this profile will smooth out as we moved over towards the back of the queue. If now we use a higher value of sensitivity c 0.8 what will happen? We see that the vehicles that follow have higher accelerations with the time lag. So what you see here, you see waves that go backwards and they increase in size. So you see, even the car number two reacts with a deceleration or acceleration, which is smaller than the next cars, the acceleration or deceleration will get higher values, but then after some time, again, it will smooth out. And then if we use a very high value of c , so we have a system good becomes totally unstable.

Notes

Summary

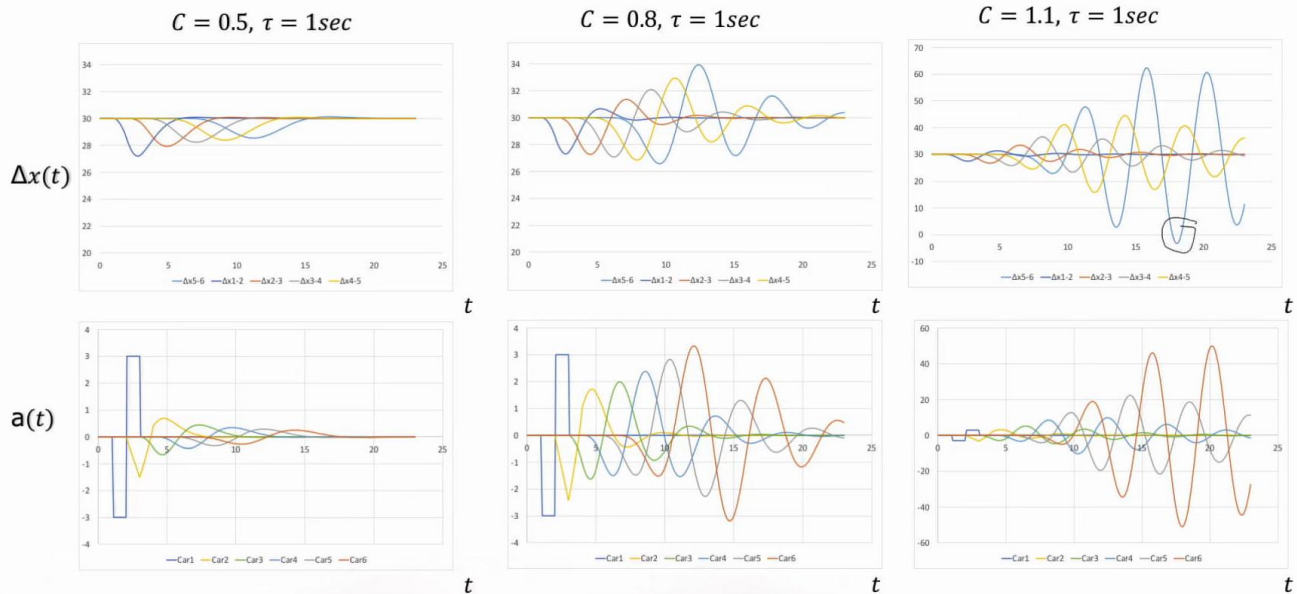


18m 29s

Stability analysis

Local Stability: the response of a following vehicle to a fluctuation in the motion of the leader vehicle directly in front of it.

Asymptotic Stability: how fluctuation in the motion of lead vehicle of a platoon, is propagated through a series of vehicles.



So what we see, we see that as we move backwards in the back of the queue, the vehicle will have a huge value for acceleration or deceleration that actually is very unrealistic. And as a result, what you see, you see that we have unstable situations and we have actually situation where the relative distance becomes negative. Which simply meaning that in this part, we have the crash. So clearly this is an unrealistic model. And with this simple example, I wanted to show you that by properly choosing the values of parameters of car following models, we can have realistic situations or very unrealistic ones. So mathematically speaking, this situation is totally unstable and actually there are two types of stability that can be analyzed. Local stability, that looks at the response of their following vehicle with respect to the motion of the leader and I see total stability that that's when he looks how the fluctuation, the motion of the lead vehicle propagates through the vehicles in the back of the queue. Stability is a strong mathematical term and if you are more interested, I will ask you to look for more information in the class material.

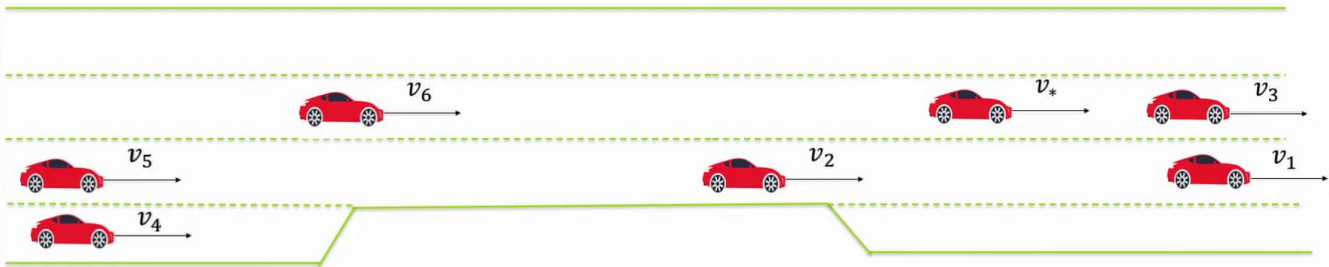
Notes

Summary



Types of lane changing actions

- Mandatory: A vehicle is obliged to change a lane (e.g. on- or off-ramps)
- Anticipatory: A vehicle changes a lane to anticipate congestion downstream
- Discretionary: A vehicle decides to change a lane to be better off
- Cooperative: A vehicle changes a lane to help others



So far for the car following models, we assume that all vehicles follow each other in a single lane and there are no changes of lanes. As we know in reality, drivers can do lane changes so it will be nice to introduce some concept on how lane changing is performed. So if we look at this figure, we see vehicles at different lanes and let's assume for simplicity that v_3 is the speed is much smaller compared to the start. So this vehicle slow down significantly and then that vehicle has to evaluate if he's willing to make a lane change and actually move on that lane. So to do that, this is a more complicated process compared to a car following, because he has to see what happens on the other lane both in front and back of him. So here what we have, we have two gaps. The lead gap with the front wheel and the lack gap and actually based on the values of these gaps, the vehicle will make a decision if the existing gaps are acceptable. And in that case he will decide to move in a different lane. There are different type of lane changing actions that can happen in a real environment. And let's try to describe by looking here at the movements of multiple vehicles in a freeway that has an on ramp and also an off ramp.

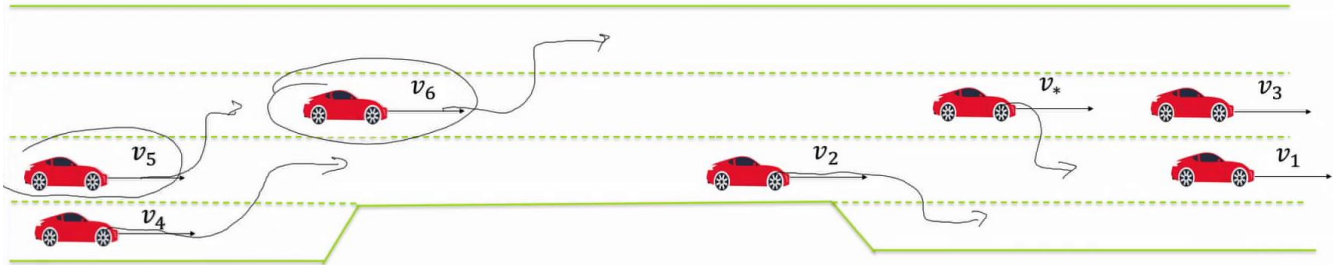
Notes

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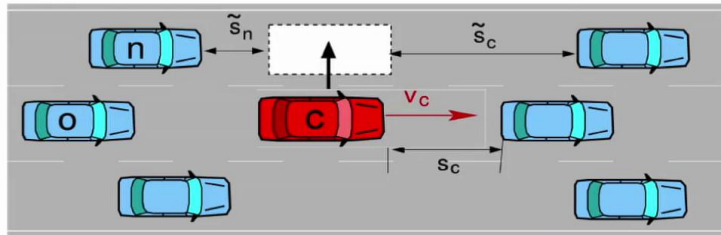
So first of all, there are mandatory lane changes where a vehicle is obliged to change the lane. So for example, this vehicle before, it has to change the lane to be able to enter the freeway. And also for example, these vehicle v_2 , if it wants the exit to or from it also have to make a mandatory lane change. Then what we might also have, we might have what we call anticipatory lane changes. So in that case, a vehicle like this one sees downstream there is a lot of congestion so it might be proactive and it might decide to change lane so that it avoids meeting congestion further downstream. Other type of lane changes are discretionary lane changes where a vehicle decides to change lane to be better off. And this is the lane change we described in the previous case where v^* is larger than v_3 and this vehicle decides to move to the right lane. And then we can also consider competitively change where a vehicles might decide to change lanes to help others. So here might be an example where these vehicle five might decide to change lane so that it gives more space for the vehicle four to enter the freeway.

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Summary



An example of a lane-changing model



Builds on a car following model with an acceleration function

$$\frac{dv_\alpha}{dt} = a^{\text{mic}}(s_\alpha, v_\alpha, \Delta v_\alpha)$$

Minimizing

Overall

Braking decelerations

Induced by

Lane changes

Safety criterion

$$\tilde{a}_n \geq -b_{\text{safe}}$$

Incentive criterion

$$\underbrace{\tilde{a}_c - a_c}_{\text{driver}} + p(\underbrace{\tilde{a}_n - a_n}_{\text{new follower}} + \underbrace{\tilde{a}_o - a_o}_{\text{old follower}}) > \Delta a_{\text{thr}}$$

Kesting, Treiber, Helbing, 1999

Let's look now at a quantitative example of a lane changing model which is widely used both in terms of research and also in practice in the transportation world. So this lane changing model has is called [24:23 inaudible], which the initials mean mean is migrating over all breaking decelerations in use by lane changes. So the concept of this model is similar to what we described before. So what we have, we have a target vehicle that has some specific speed and this vehicle had some spacing in his current lane while at the same time it has a lead and a gap spacing at the target lane where he might consider to do end lane things. So this model has three important characteristics. So first of all, it requires that a car following model exists in this kind of falling model has an acceleration function. So this means that vehicles decide for their acceleration based on the spacing for example, the speed and the relative speed with a vehicle in front of them. Then what we have, we have a safety criterion which is related that when a vehicle changes lane, it has to guarantee that this acceleration will not create issues to other vehicles.

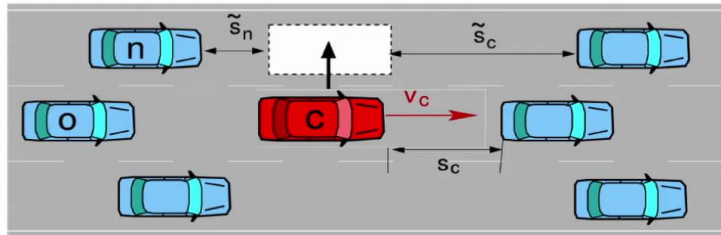
Notes

Summary



24m 08s

An example of a lane-changing model



Builds on a car following model with an acceleration function

$$\frac{dv_\alpha}{dt} = a^{\text{mic}}(s_\alpha, v_\alpha, \Delta v_\alpha)$$

M inimizing

O verall

B raking decelerations

I nduced by

L ane changes

Safety criterion

$$\tilde{a}_n \geq -b_{\text{safe}}$$

Incentive criterion

$$\underbrace{\tilde{a}_c - a_c}_{\text{driver}} + p(\underbrace{\tilde{a}_n - a_n}_{\text{new follower}} + \underbrace{\tilde{a}_o - a_o}_{\text{old follower}}) > \Delta a_{\text{thr}}$$

Kesting, Treiber, Helbing, 1999

So for example, if a vehicle behind is traveling very fast and it approaches that vehicle, then no lane change will happen. And then there is an incentive criterion where simply speaking, the acceleration of the driver in the car and lane and in the new lane, new lane is represented by Tilda. So the driver will evaluate what is his benefit. And if he can go faster than most probably he will make a lane change, but what he will also evaluate, he will evaluate what is the effect in the rest of the travelers. So by moving this driver, we'll have a new follower. So if by moving, it influences significantly the acceleration of the new car, he might decide not to make this lane things. And what we have, we have a parameter P that this is something like a politeness factor that can get values between zero and one. And also we have a threshold. That this threshold mainly has to highlight that a driver needs to have at least a minimum lane changing benefit before he makes an action. Otherwise, if you don't have a threshold, we might have various laterally behaviors where drivers make a very large number of lanes changing, which might be unrealistic.

Notes

Summary



25m 31s

Summary



In summary, in this lecture we saw different type of car following and lane changing models. And interesting aspect I would like to highlight here is that these models contained in large number of parameters. So to use them in an efficient and effective way in real transportation problems, we have to be sure that they represent reality. So very proper calibration of these parameters is an important task, which is beyond the scope of this course, but it has to be done very carefully. Thank you and see you next week.

Notes

Summary



26m 54s