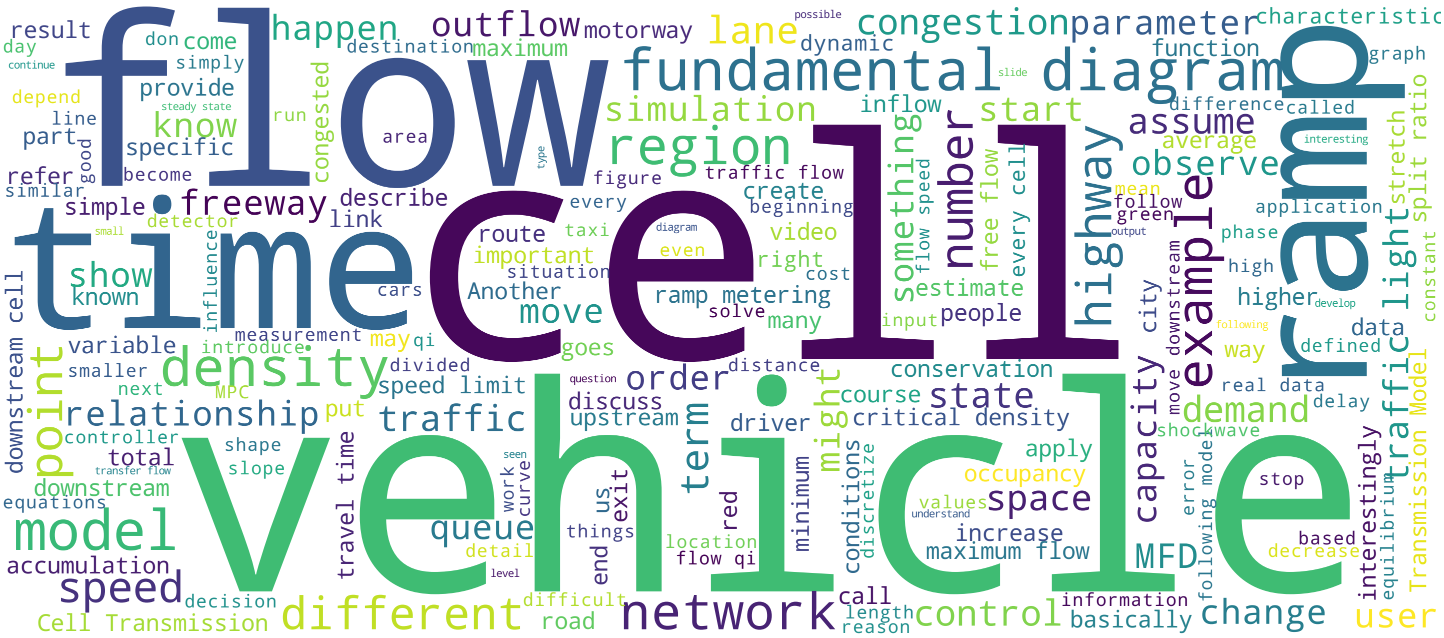
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Cell Transmission Model (CTM)

Intro to traffic flow modeling and ITS

Dr. Anastasios Kouvelas



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Video

EPFL

Cell Transmission Model (CTM)

- Numerical method to solve the kinematic wave equation; consistent with hydrodynamic theory
- Discrete difference equations
- First order discrete Godunov approximation of LWR

Daganzo, C. F. (1994). The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research Part B*, 28(4), 269-287.

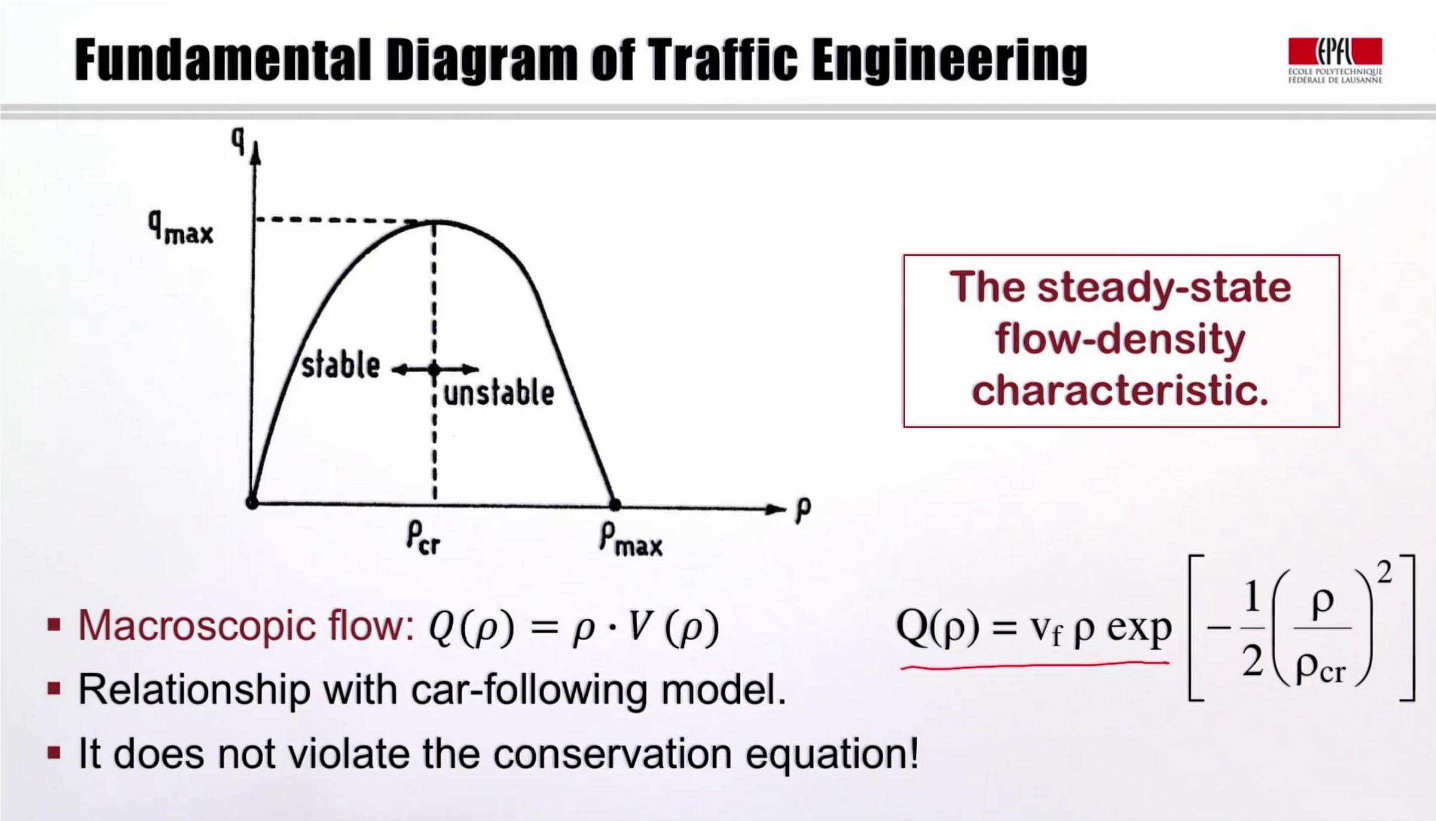
Daganzo, C. F. (1995). The cell transmission model, part II: network traffic. *Transportation Research Part B*, 29(2), 79-93.

Welcome, my name is Anasasios Kouvelas. In this video, we are going to discuss the Cell Transmission Model, which is the most popular method for modeling high way traffic but also arterials. By the end of the video, you should have a good understanding of the dynamical equations of the model and also how to use it in order to run a numerical simulation in your computer. So let us have a look at the Cell Transmission Model. From now on, I would refer to it with abbreviation CTM. CTM is a numerical method to solve the kinematic wave equation and it is also consistent with the hydrodynamic theory. It consist of a set of discrete difference equations. Because we have proved that it is a first order discrete Godunov approximation of the LWR theory. Below, I provide the original references of Carlo Daganzo that introduces this model in 1994 and 1995. If you want to have more information about the model, you can look at these papers.

Notes

Summary

0m 06s

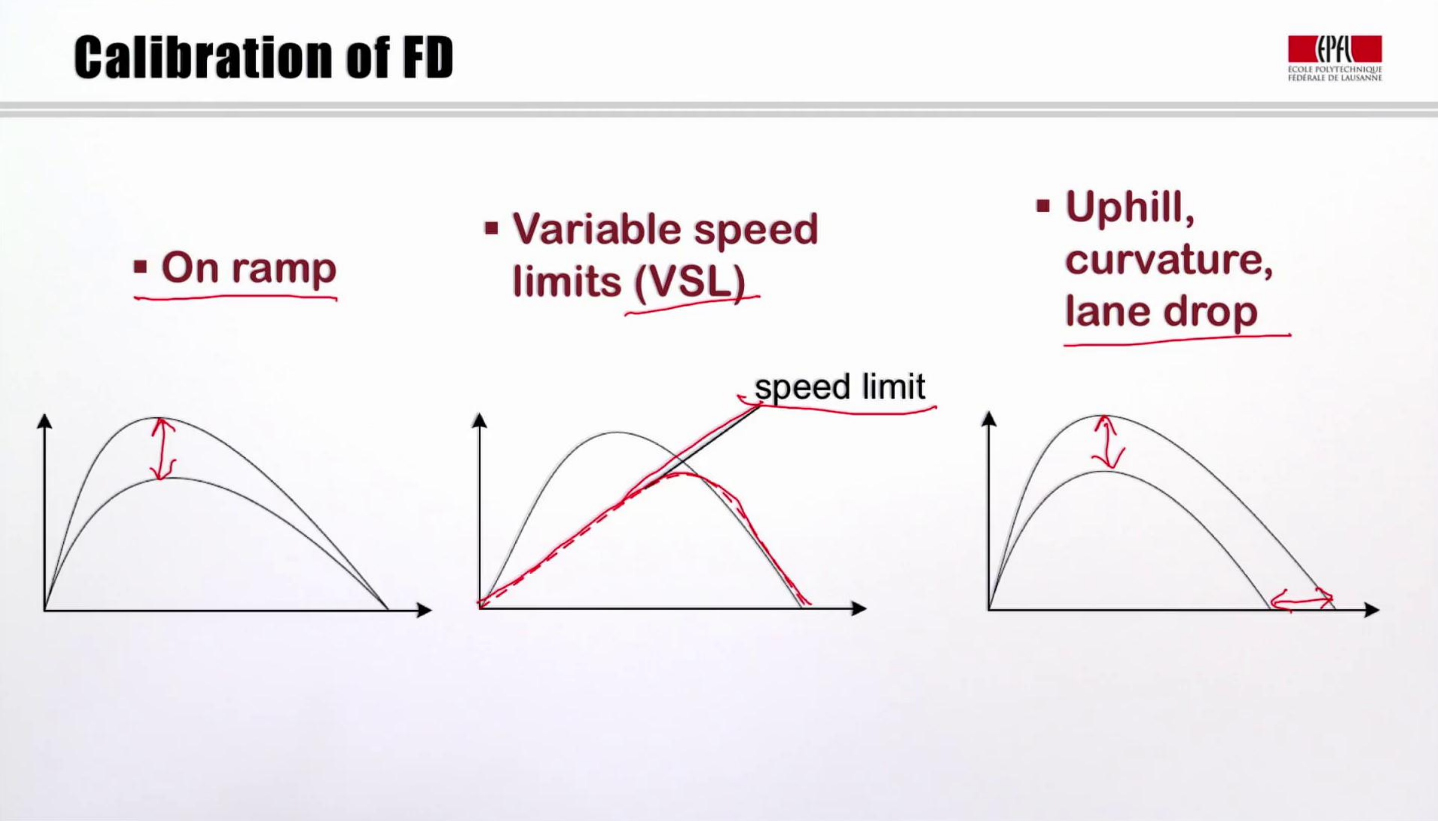


Before going into the details of the model, I would like to introduce again the fundamental diagram of traffic engineering. This is a fundamental relationship between density and the flow and it is an empirical relationship that we observes from real data. So it is this model curve and we have here some parameters like the maximum density or we also call it dome density. The critical density, which is the point where we get this maximum flow. This relationship is the steady state flow density characteristic so someone could derive the characteristic between flow and density like this, which is a function of density and speed. One possible equation to get this is this one, which is an exponent. Vf is the free flow speed, P is the density and we have this exponent of density and critical density. One characteristic if the fundamental diagram is that it has the relationship with the car following model. I am not going to show this here but it has been shown that from the car following model, there is some relationship like the fundamental diagram. Another important attribute of the fundamental diagram is that if we use a relationship like this one, we don't violate the conservation equation, the conservation of this.

Notes

Summary

1m 20s

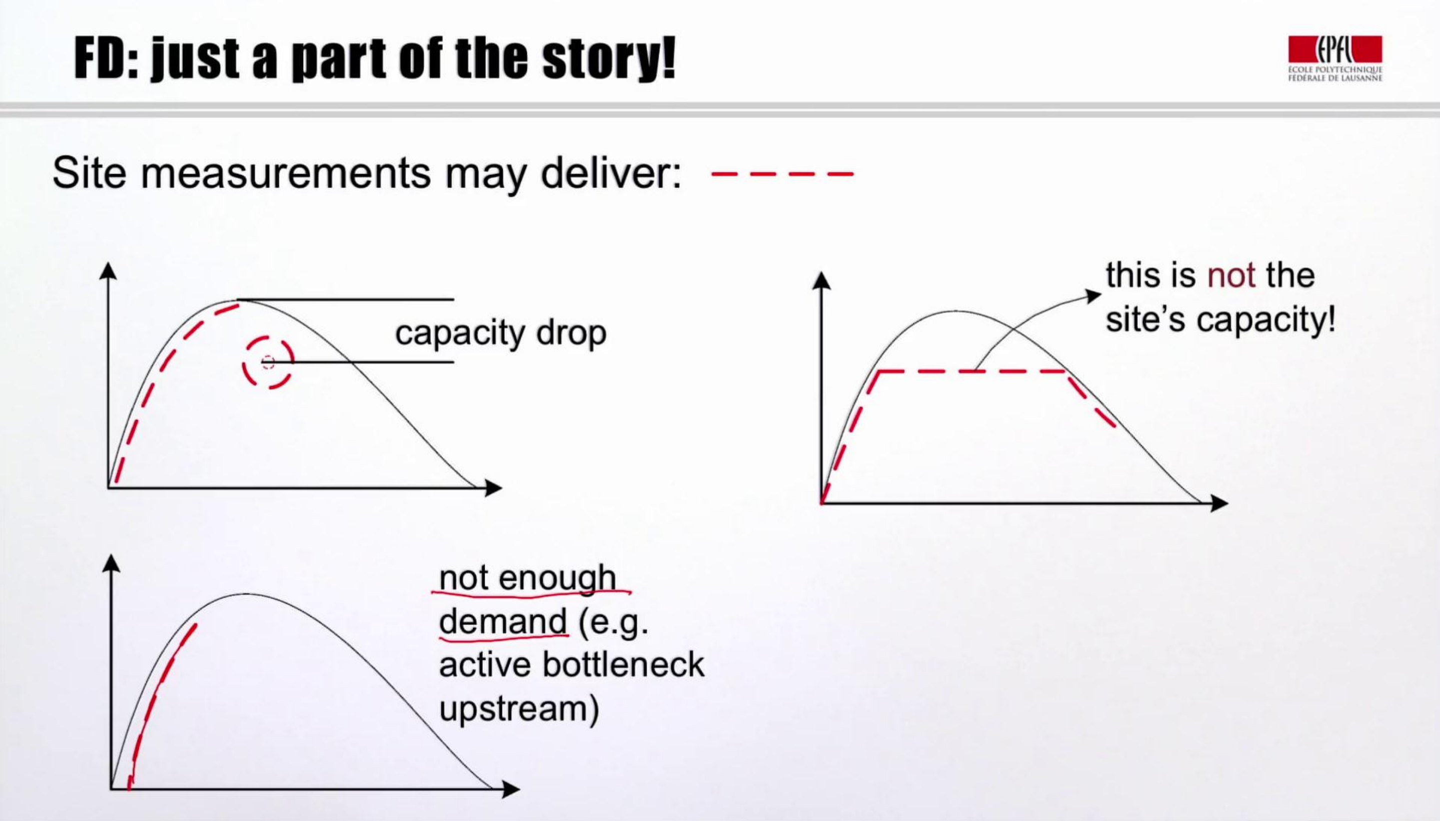


So the fundamental diagram, I also call it FD here is an important tool when we try to model traffic flow. We can calibrate this fundamental diagram in order to replicate different phenomena of traffic. Here I provide three examples, the first one is if we have an on-ramp, we change this FD and increase of decrease the maximum flow according to the signal settings that we have on the on-ramp. The second example is when we have variable speed limits so by changing the shape of the fundamental diagram, I can create something like this. Here, I can have this speed limit. So if I change the speed limit in a section of the highway, I can change the fundamental diagram like this and more than this behavior. Another example is when I have an uphill stretch or a curvature or lane drop. Again, I can model this with the fundamental diagram by changing the maximum flow and also the maximum density of the fundamental diagram. So if I have the data of if I have the network, I can calibrate my fundamental diagrams for every segment of road and model exactly what I want to do.

Notes

Summary

3m 00s

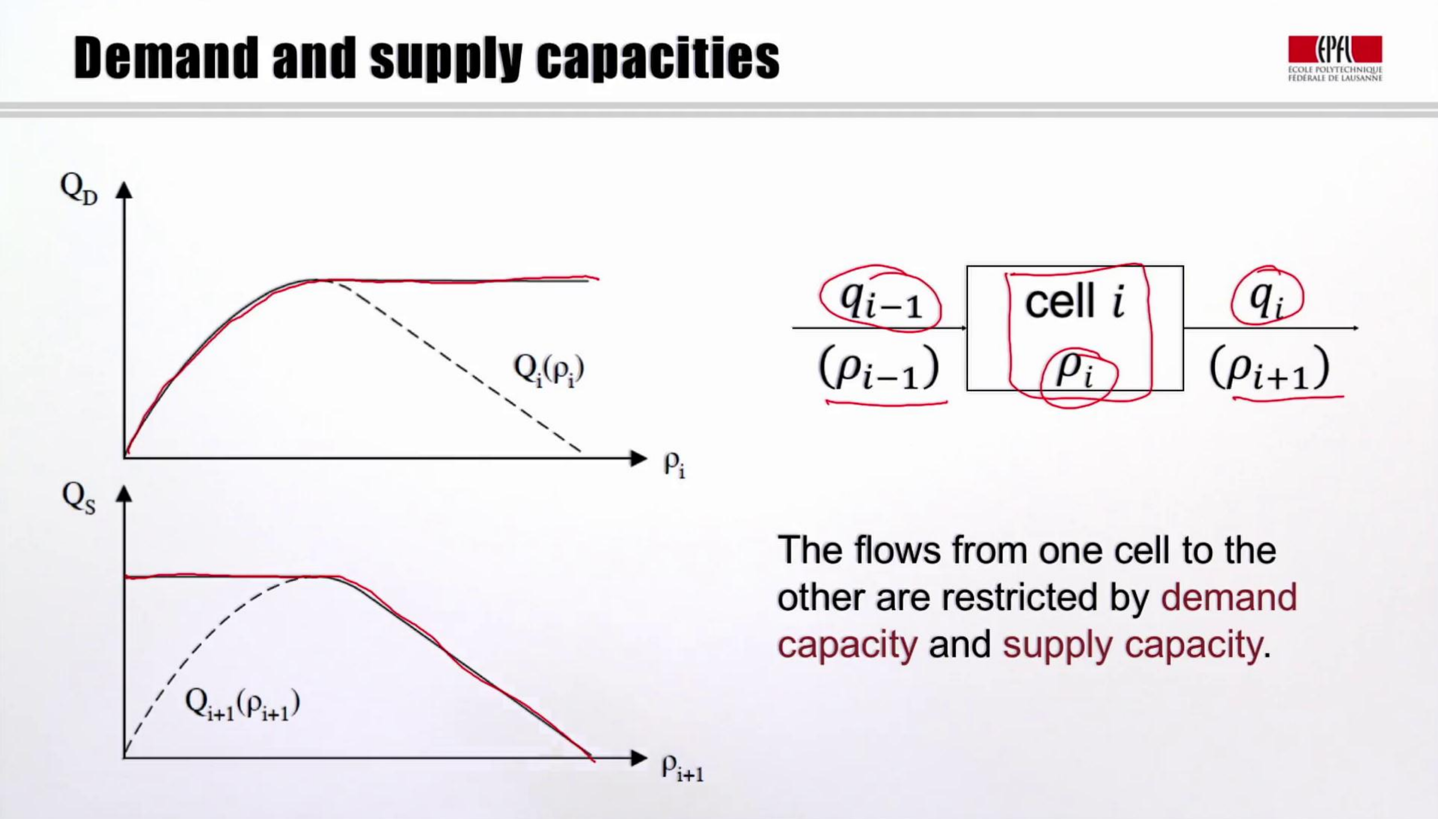


As we discussed, the fundamental diagram is an important empirical tool that we use in traffic flow modelling. However, in real life, we may get data that are different from the fundamental diagram and this is because it refers to steady state conditions. In this slide, I would like to highlight some examples of real data that may differ from what we model with fundamental diagram. Here, with the red dotted line, I show real data that we can observe in the field. So we can get something like this, an increasing curve and then get some points here. This is what we call capacity drop, this is a well known phenomenon in traffic and it can happen in some active bottle neck because we have stop and go waves, which create a synchronous acceleration and deceleration of vehicles. Another example can be something like this, so we can observes a shape for the fundamental diagram like this red one in contradiction with the black curve that we have for our model, which is here. This may happen for example because we have inactive bottleneck downstream. A third example can be something like this red line and we get an incomplete curve for the fundamental diagram. This may happen because we don't have enough demand.

Notes

Summary

4m 23s

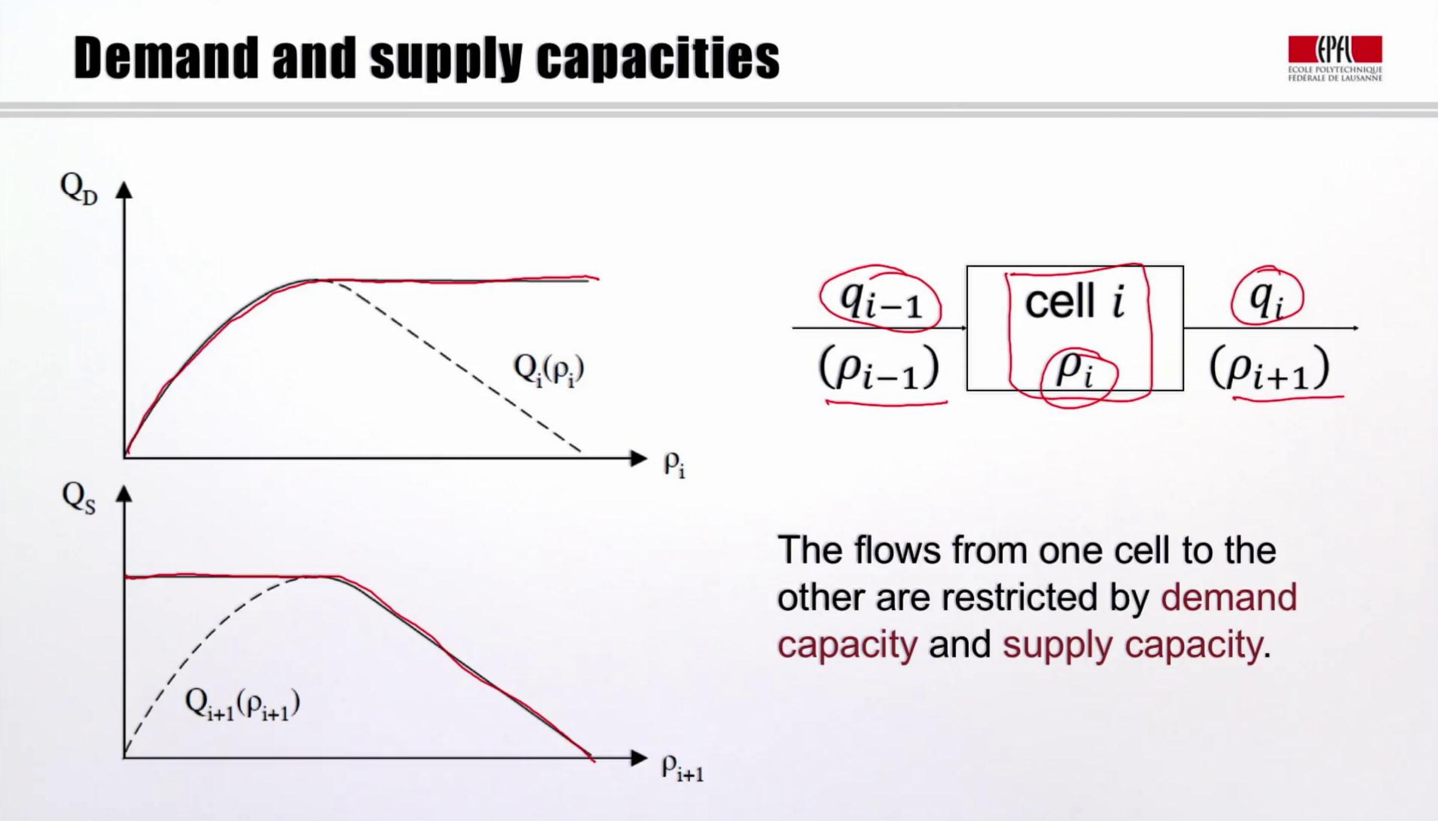


For example, because there is an active bottleneck upstream of the measurement point. Another thing that we can observe from real data is some hysteresis on the fundamental diagram. Again, this can be transient state that are not steady state condition and this may happen because of some congestion discovery. Before describing the situations of CTM, I would like to discuss the basic principle that it is based on. So we assume here that we have a cell, which is part of the highway and this cell has some density and it can have some down flow to the downstream cell and some in flow from the upstream cell. The basic principle that Cell Transmission Model uses is what we call demand and supply capacities. If I assume that very cell has some fundamental diagram, then between two cells, I can define some demand capacity and some supply capacity. So in the figure here on the left, I have a fundamental diagram that goes to the maximum then stays there. So this is what I call demand capacity and this is what this cell wants to send downstream. From downstream, the cell has some supply capacity, how much vehicles I can accept. This is described by another fundamental diagram; the supply fundamental diagram, which has a shape like this.

Notes

Summary

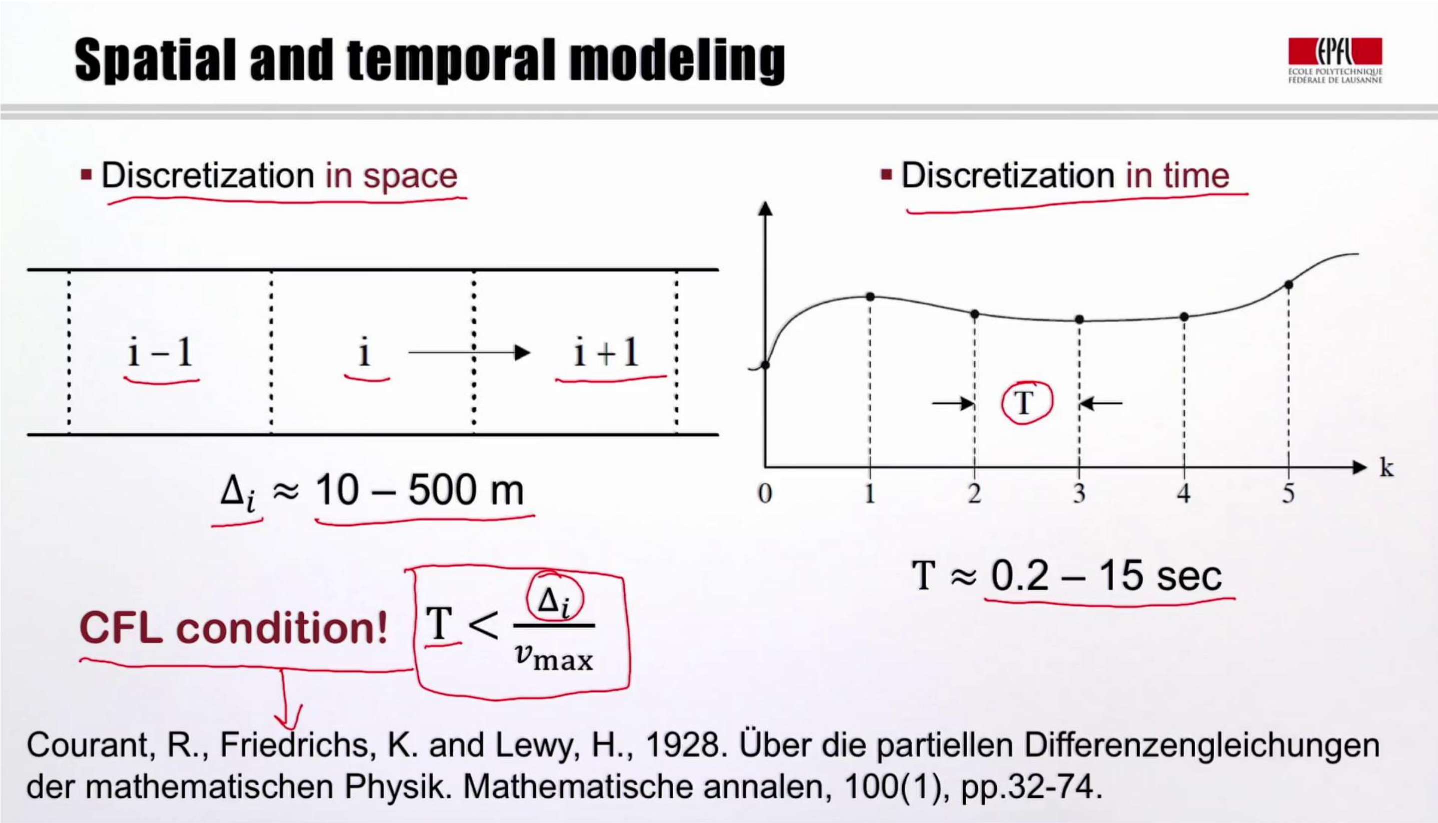
6m 00s



So it is a function that is constant at the beginning and then it is monotonically decreasing. These two fundamental diagrams are a basic modeling tool for the Cell Transmission Model.

Notes

Summary

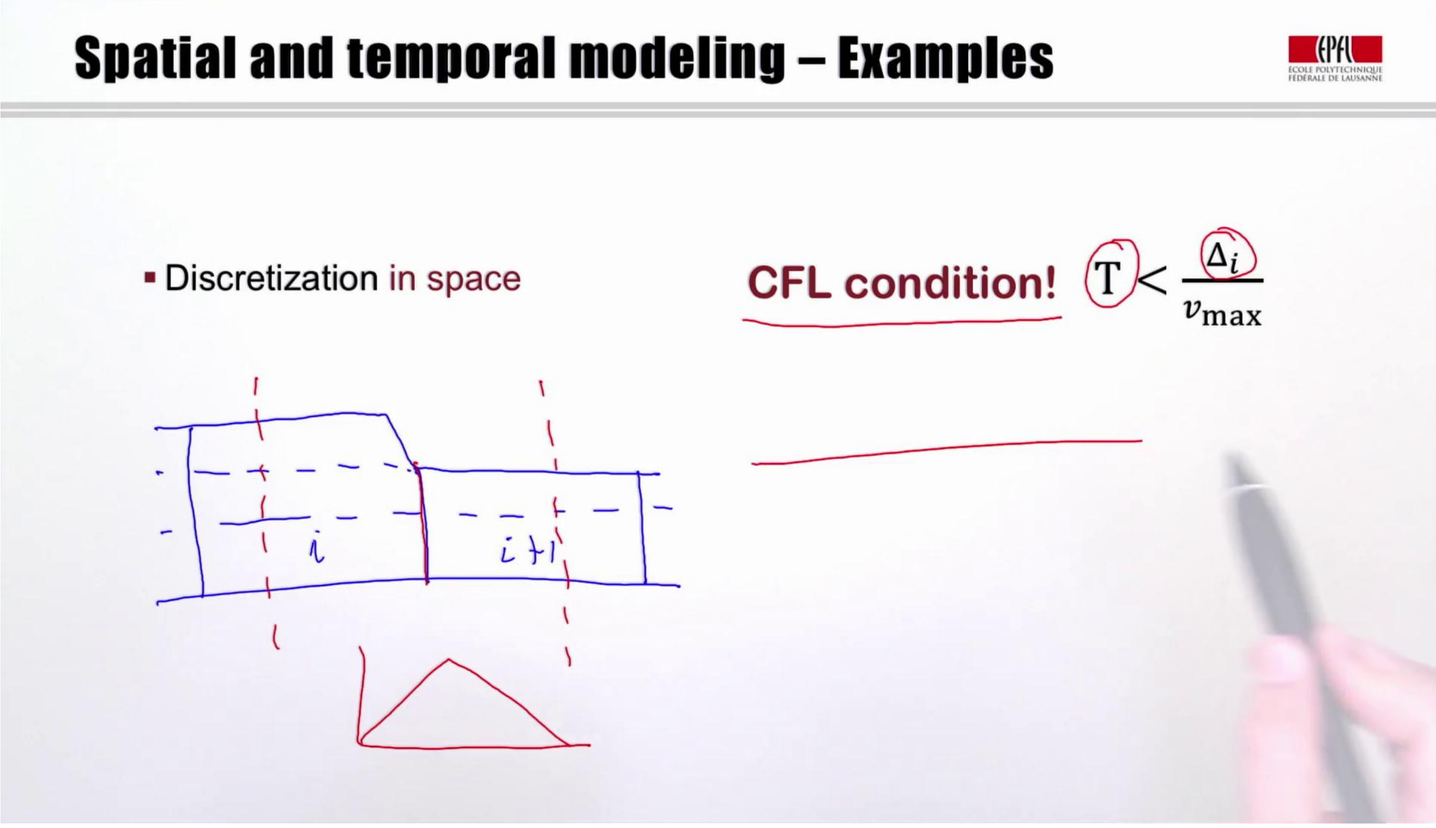


Before writing down dynamical equation of the model, we need to do a spatial and temporal discretization. First of all, we need to discretize the space. If we take a stretch of the highway, we can split in into cells. So I have here, i-1, I, i+1. The length of these cells is typically what u call Δi and it can bet something between 10-500m depending on the applications. Then I need to discretize the time so we can split the time into equal intervals T and this is the sample time of our model. Typically, the parameter T takes values from 0.2-15 seconds but depending on your application, you can modify this value within this range. Another key issue of this type of modelling is what we call CFL condition. This refers to a paper that goes back to 1928 and this is something that we need to take into account when we do numerical simulations. Here, this relationship is shown by this inequality that defines the relationship between T and Δi, this inequality must hold in order not to have any numerical problems.

Notes

Summary

8m 01s

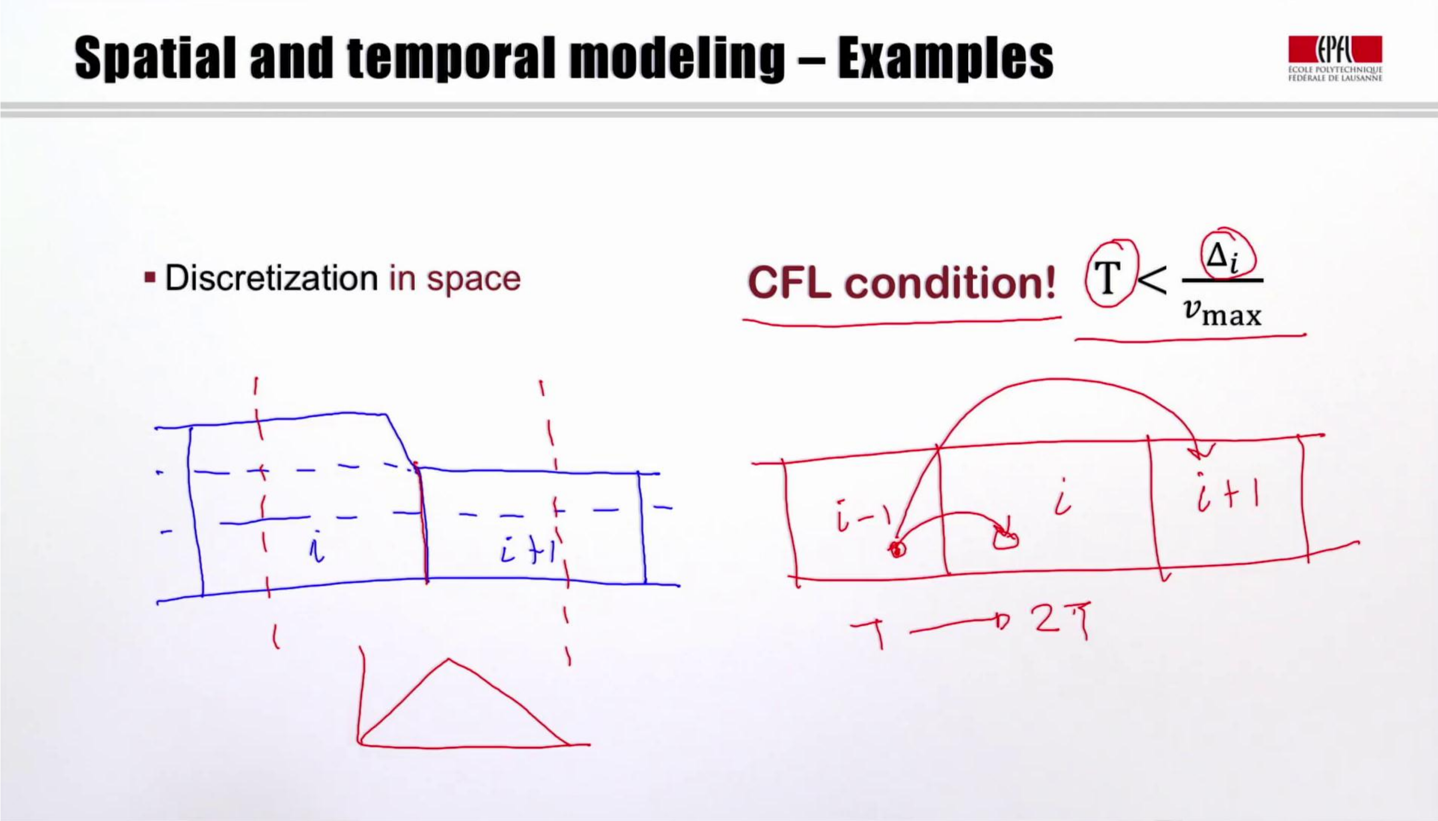


Let me now show you two examples of the discretization in space and the CFL condition. Imagine I have a highway like this that I am drawing here and I have a point that the highway drops from 3 lanes to 2. If I want to create my cells here and discretize the space, I can do something like out one cell here and another cell here. So I have cell i and cell i+1 and actually, this is the correct way to do it. I have one cell here with 3 lanes and the next cell with 2 lanes. So I put the breaking point exactly where I have lane drop. Anyone could put the cell here that starts with 3 lanes and continues with 2 lanes. It is something much more difficult to model because if I want to define the fundamental diagram for this cell, it is very difficult to define the parameters. Like the critical density or the capacity or the dome density. So according to the application, you should be very careful where to place your things and also taking into account what I mentioned before, the CFL condition. So let me explain a little bit what this condition tells us and why it is important for a numerical simulation. Again, my time discretization T and my space discretization Δi must have this inequality where Vmax denotes the maximum speed of the cell.

Notes

Summary

9m 28s

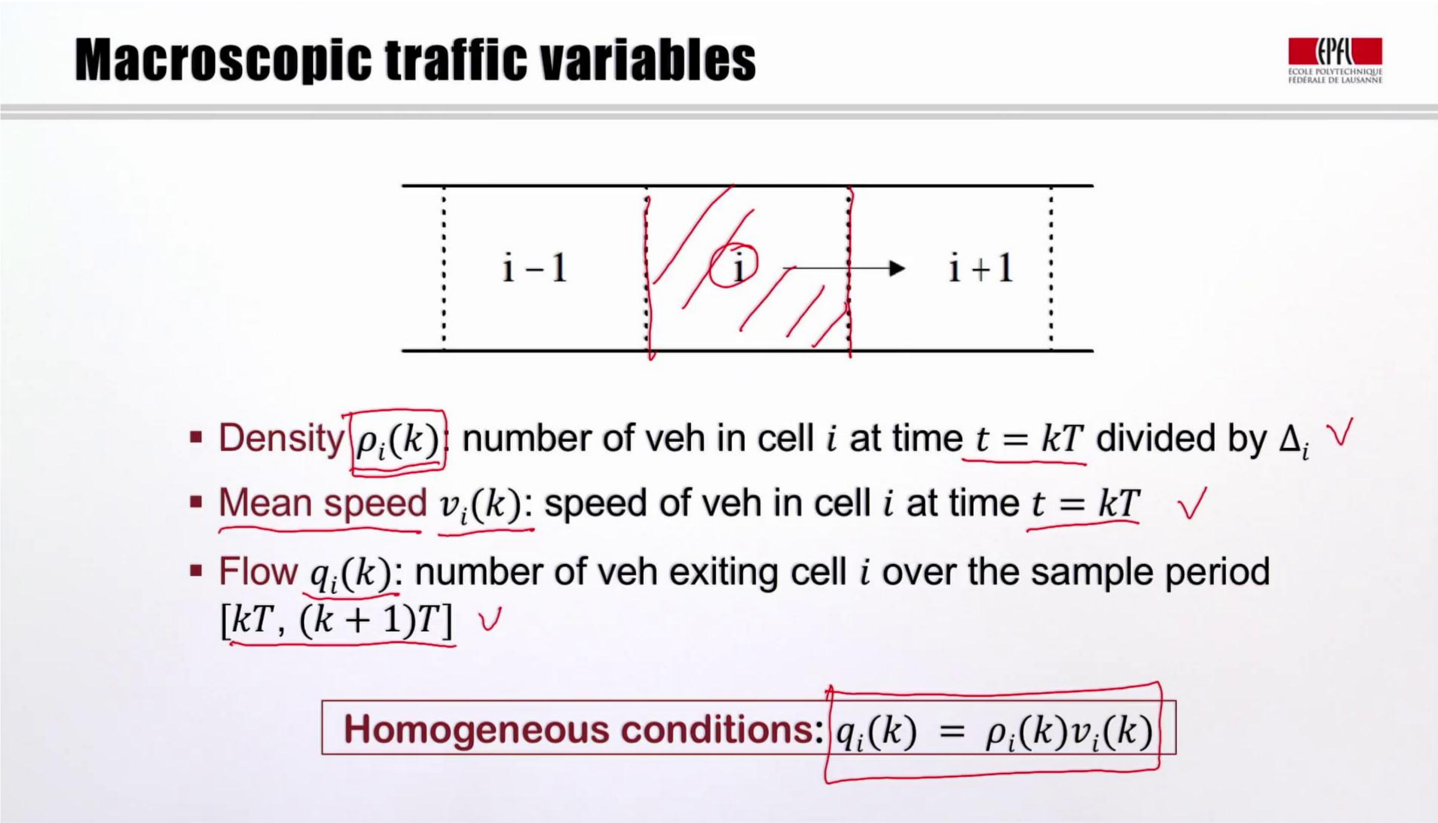


To put it in another way, if I have a stretch to highway and I have 3 cells like this; so I have i-1, cell i and cell i+1. This basically says that at time T, I am at cell i-1 here. Going from T to 2T, the most that I can travel is to go to the next cell so I can either stay in the cell that I am or go to the next one. In one sample time period, I cannot jump this cell and go to cell i+1. This is what basically this inequality says and if it doesn't hold, it can create numerical problems to my simulation.

Notes

Summary

11m 17s

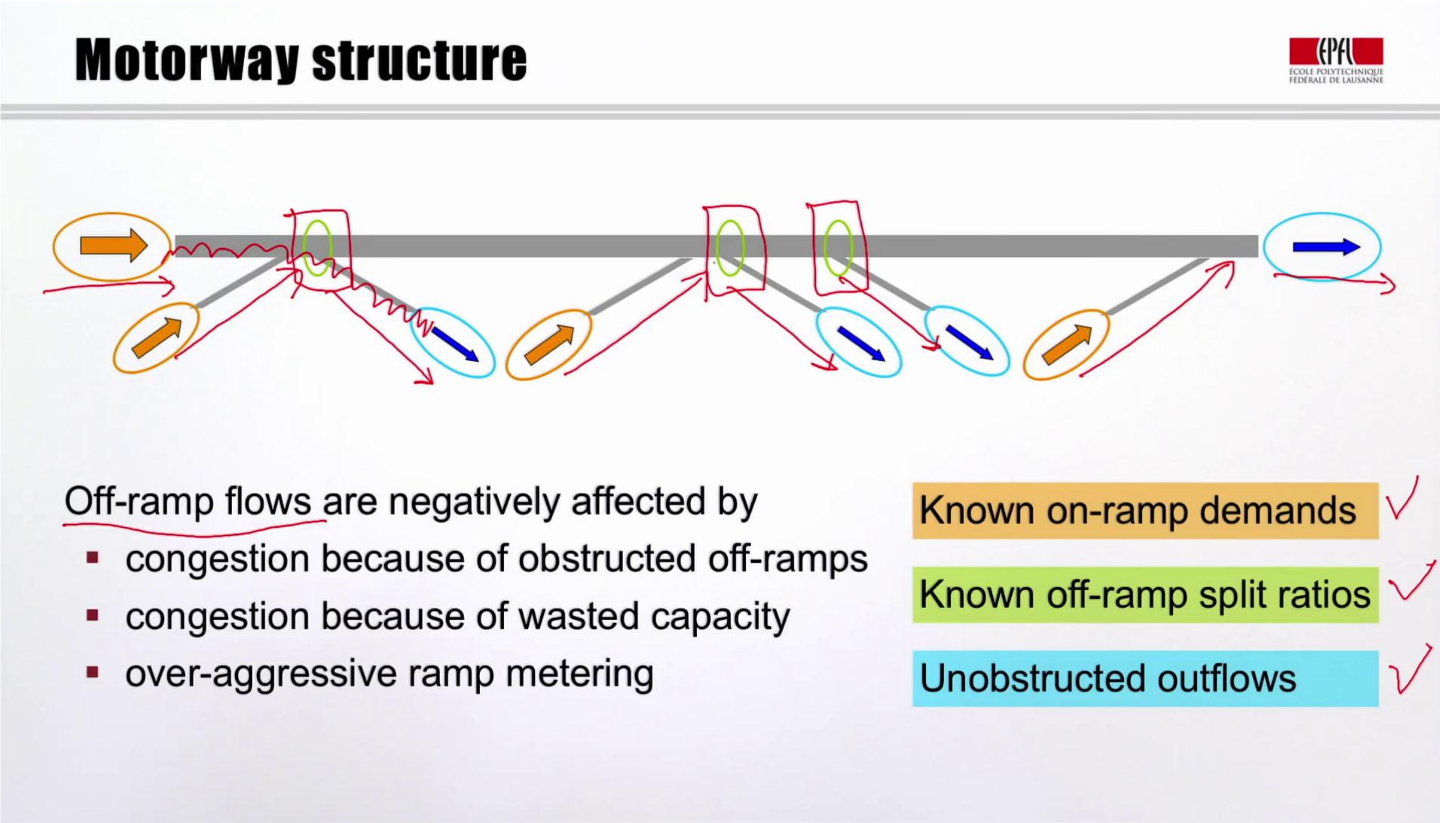


Let us now define the basic macroscopic variables we are going to use. So this are the density, P, the speed V and the flow q. for every cell i, I define these 3 macroscopic variables. Note here that the density is for some time and the speed is also for some time, it is like taking a snapshot of this highway. Whereas, the flow refers to a period, so this flow qi (k) refers to this period [kT, (k+1) T]. so the first 2 variables are variables that are defined for a specific time and the third variable is defined for a time period. A basic assumption of this model is that in the cell i, I have homogenous conditions so this density is homogenously spread in the cell. I have this density everywhere and this mean speed is the same for the cell. Under this homogenous condition, I have this fundamental relationship between the three variables; density, flow and speed.

Notes

Summary

12m 12s

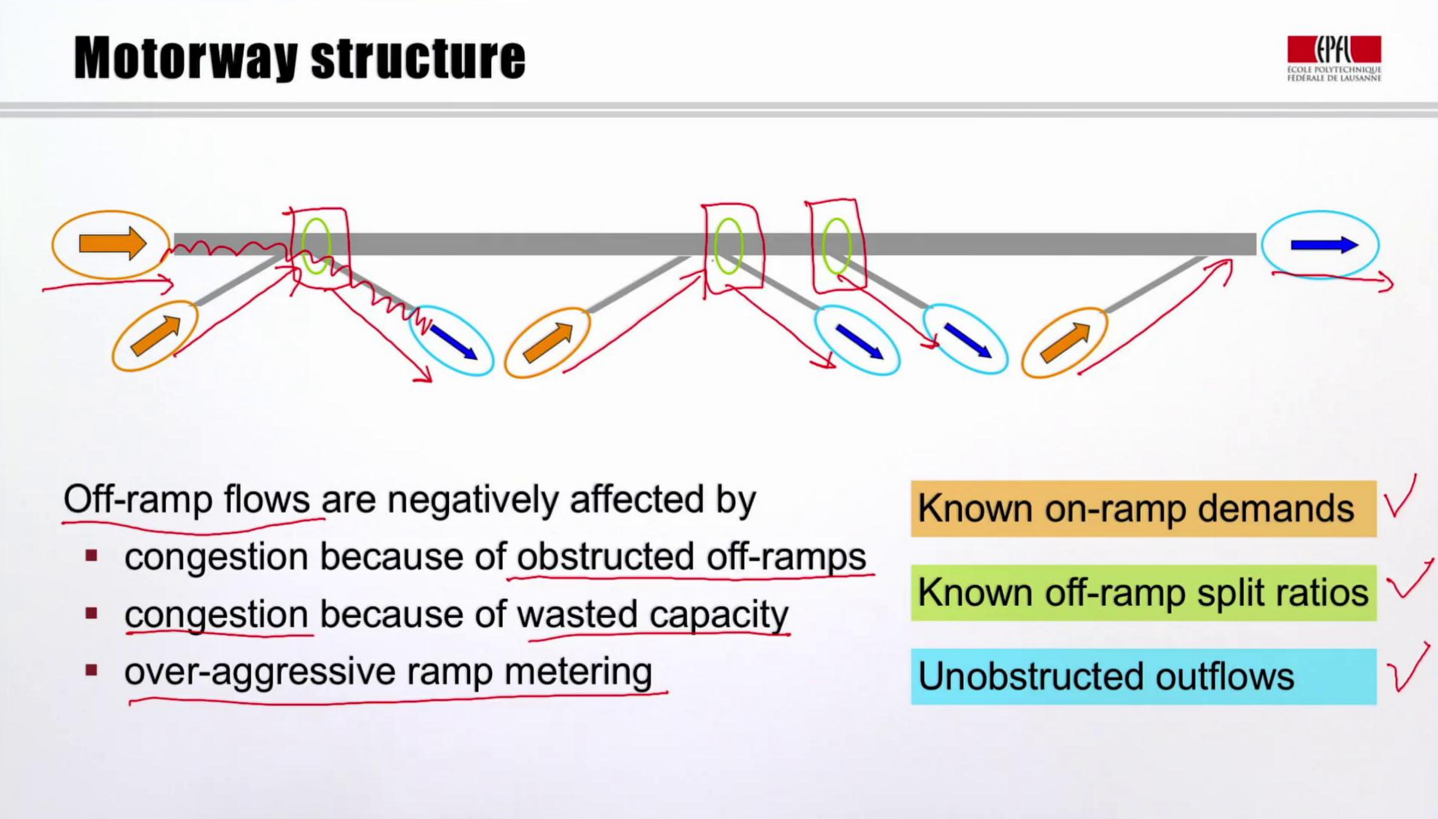


Let us now see the structure of a motorway. In this picture, we have a stretch of a motorway. With orange color, we have the on-ramps where vehicles can get in the motorway and with blue colors, we have all the off-ramps, which are exits. There is also one exit here at the end of the motorway and of course, some in flow at the beginning. In order to apply CTM, we assume these three things here. The first one is that we assume that we know all the demands on the on-ramps, all the flows that come in, we assume they are known over time. With the green color here, we have the divergent points where the main line is connected to the off-ramps and we can use these as exits. So in these locations, we assume that we know the split ratios. 1% of the vehicles would leave the freeway and 1% would move downstream. Another assumption that we do in our modelling is that all the off-ramps are not obstructed from downstream so they are not blocked from downstream. If I have a blocking here, this can create a spin back in the highway and this is something very difficult to model. In general, the off-ramp flows, as I said, we assume they are not blocked and they can leave the freeway freely.

Notes

Summary

13m 40s



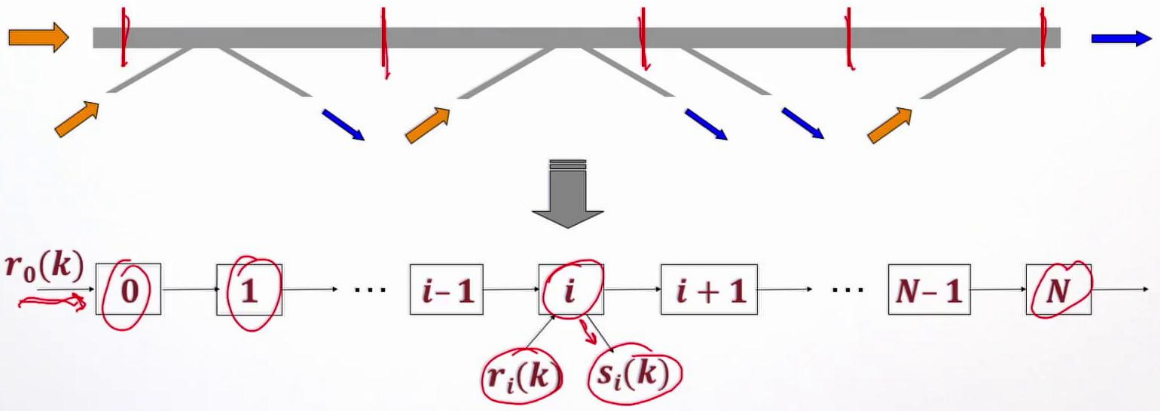
But in general, they can be affected by several reasons. One of them is that I may have some obstructions in the off-ramps or I can have some congestion because of wasted capacity. Another effect is if I have over-aggressive ramp metering, this again can influence the outflow of the highway.

Notes

Summary

15m 15s

Partitioning into cells



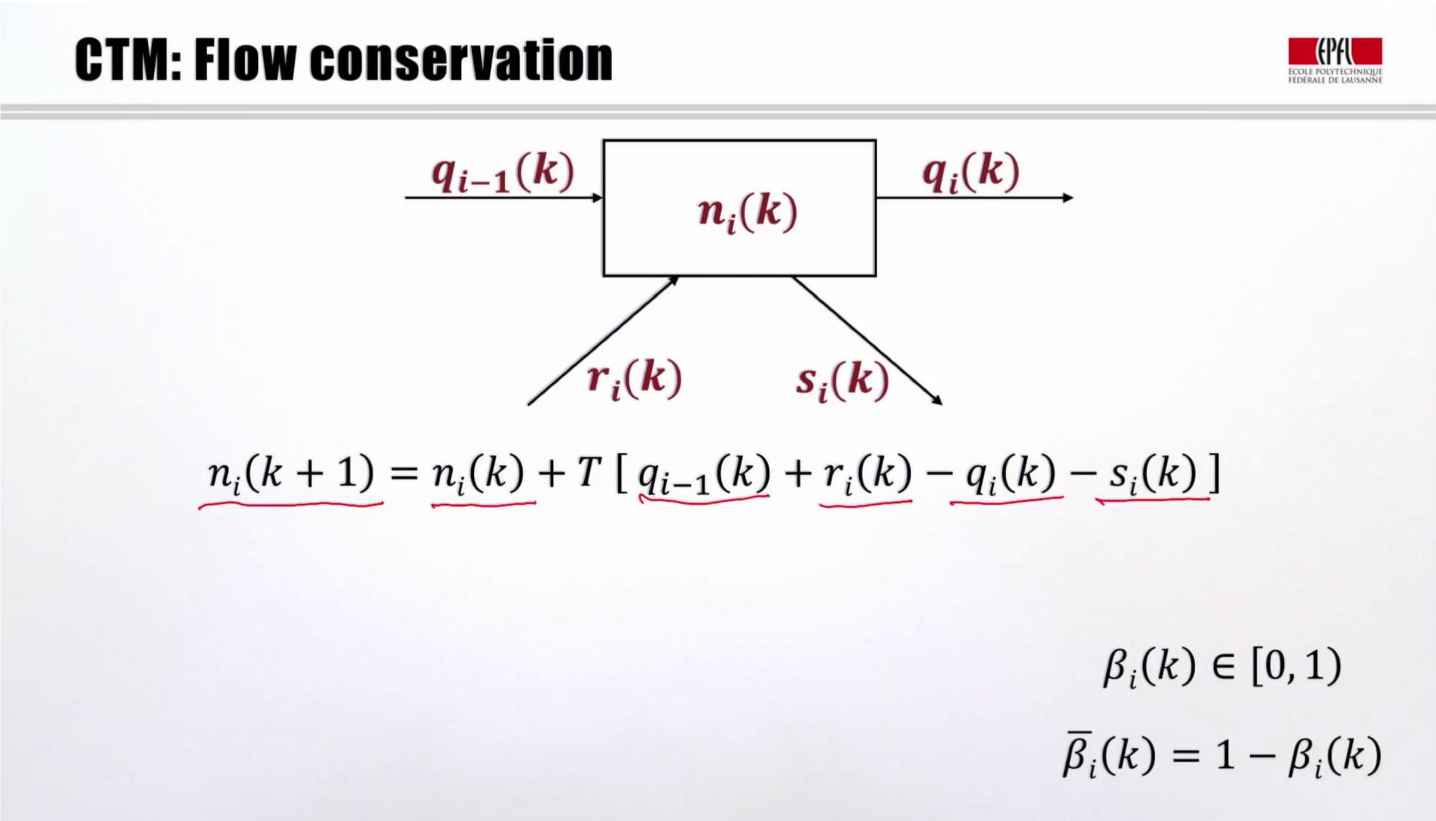
- motorway has $N+1$ cells, $0, 1, \ldots, N$
- each cell has one on-ramp and/or off-ramp
- very upstream cell has specified inflow $r_0(k)$
- all on-ramps have specified inflows $r_i(k)$ and all off-ramps have specified split ratios

Let us see now how we partition is stretch of a highway in the cells. So if I have highway like this one in the cell, I can put my cells here. So I created an abstract model that is here in the graph. So I have some cells; o, 1, i, up to N. I have some demand at the beginning at 0 and I can have on-ramps or off-ramps anywhere in this highway, in this cells. So this motorway has N+1 cells from 0 to N and it is good that each cell has at most one on-ramp, one off-ramp or both of them. So it is good to have more than one off-ramp or on- ramp in one cell if it is possible. Then in the upstream of the motorway, we specify this in flow r0, which is known. On- ramps, we have known inflows ri and for off- ramps, we have known split ratios. So for any off-ramp, I should know what percentage of flow goes out of the highway.

Notes

Summary

15m 40s



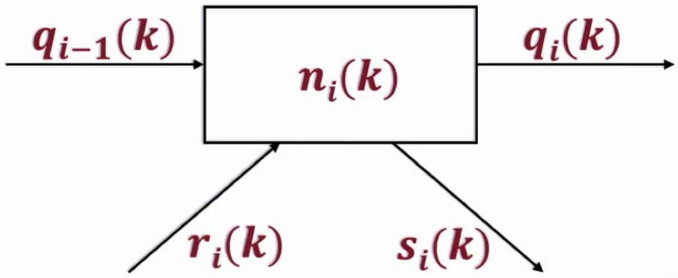
Let us now see that happens in the cell I, I have this and the accumulation of vehicles is defined by ni (K) of time K. for this cell, I have a flow qi that goes to the downstream cell. A flow qi-1 that comes from the upstream cell and if I have an on-ramp, I have some inflow ri that comes in the cell. If I have an off- ramp, I have this outflow here that I define with si (k). Here it is important to define this parameter, Bi and Bi (bar). So these are the split ratios of the cell, the total out flow that goes out of the cell, I can split it with Bi (k) that would exit the highway and with Bi bar (k) that would move downstream. Of course, this value should be between 0-1 and the summation of the 2 should be equal to one. In this model, I assume that this split ratios are known. So I can now right the conservation equation for this cell, which is basically that the number of vehicles in this cell k+1 is equal to the number of vehicles in the previous time stamp. All the inflows minus all the outflows. So this is an inflow from the upstream, this is an inflow from the ramp and this is an outflow for downstream and this is an outflow from the off-ramp.

Notes

Summary

17m 03s

CTM: Flow conservation



$$n_i(k+1) = n_i(k) + T\,[\,q_{i-1}(k) + r_i(k) - q_i(k) - s_i(k)\,]$$

Off-ramp flows are given by split ratios:

$$s_i(k) = \beta_i(k)[s_i(k) + q_i(k)]$$

$$\boxed{s_i(k) = \frac{\beta_i(k)}{\bar{\beta}_i(k)} q_i(k)}$$

$$\beta_i(k) \in [0, 1)$$

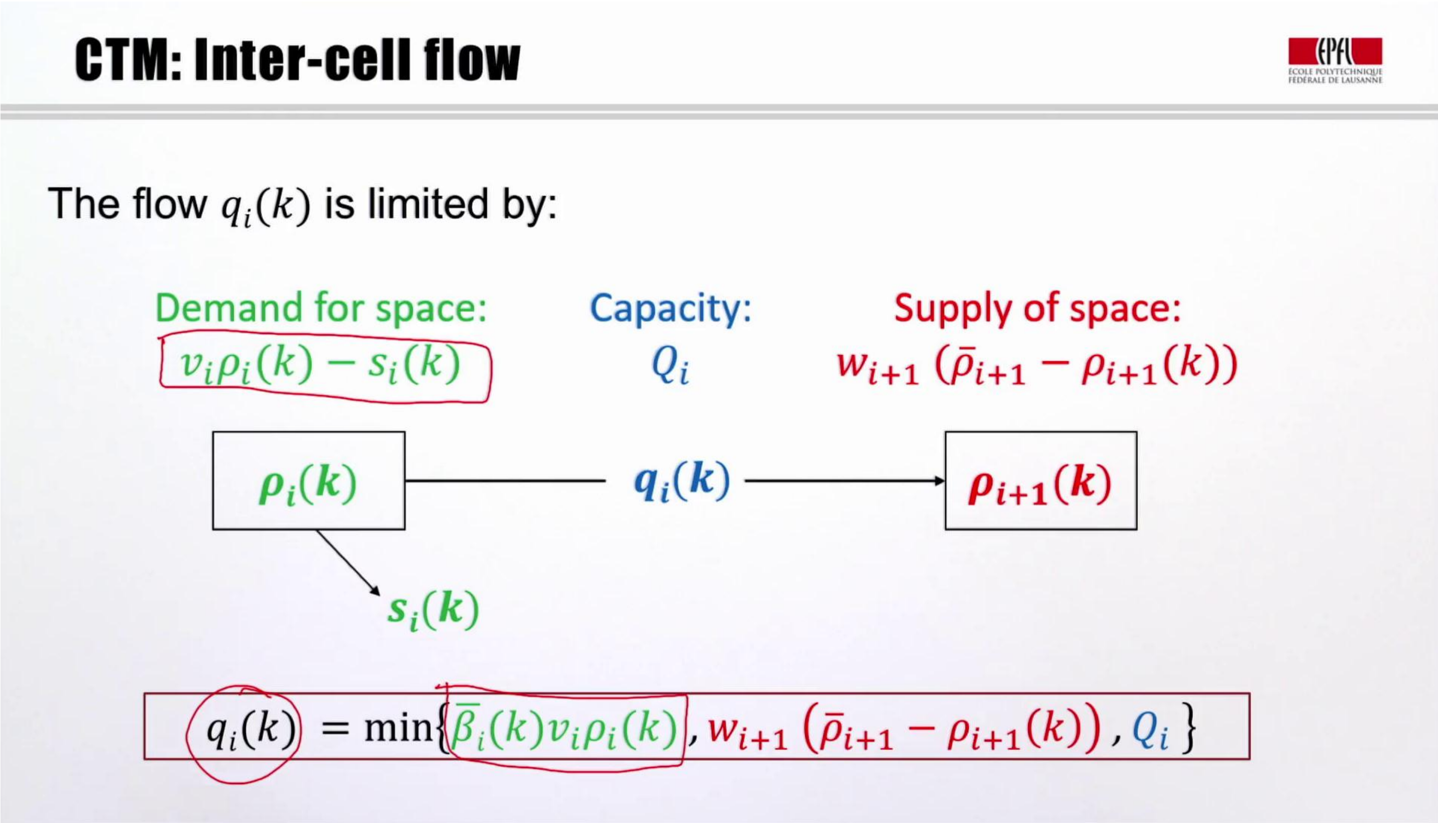
$$\bar{\beta}_i(k) = 1 - \beta_i(k)$$

Note here, that this variable here denotes the accumulation of the cell, which is the number of vehicles within the cell. If I want to make this density, I simply divide by Δi, which is the length of the cell all the time. Now, I have the conservation equation written for densities instead of numbers of vehicles. If I want now to define the number of flows that exit the highway on the off-ramps, I can write these 2 equations. So the outflow si is equal to the split ratio times the summation of the 2 flows; si and qi. If I do the computation here because of Bi and Bi bar, I can also write these relationship that relates si which is the outflow with qi, which is the flow that moves to the downstream cell.

Notes

Summary

18m 50s

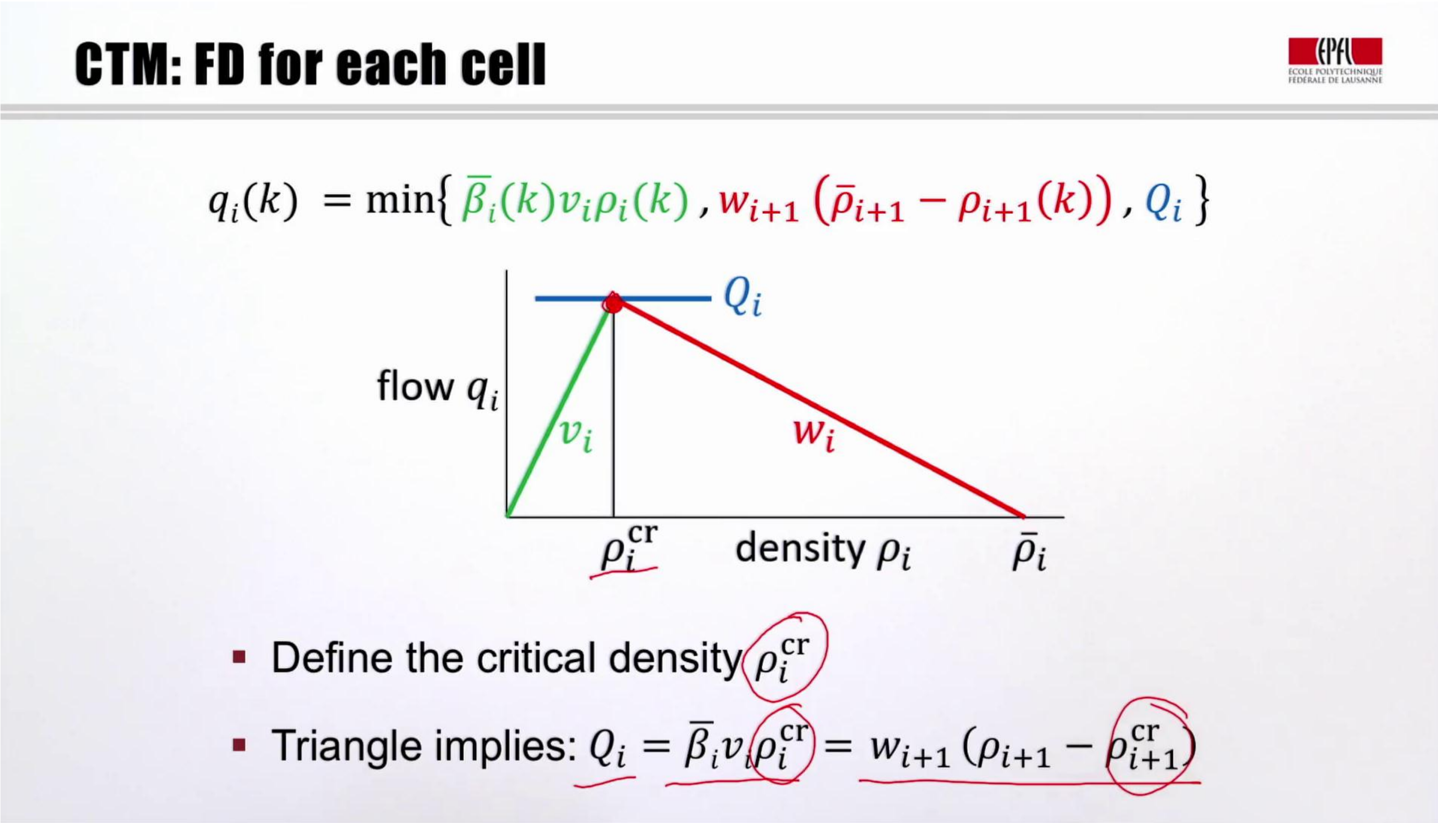


In the previous slide, we saw that happens within a cell with the conservation equation. Let us now look at what happens between cells. So if I have cell i and cell i+1, I want to define the flow that would move from cell i to cell i+1 at time interval K and I denote this with qi (k). Here, I can also have an exit flow from an off-ramp for example. This flow is basically the minimum of 3 terms and these 3 terms are the following. The first one is the demand for space with the green color so this is basically how many vehicles are in cell i and they want to move downstream. The third term with the red color is the supply of space and this is basically how much space is available in cell i+1 to receive vehicles. This is this product, which is the speed W and the difference of the density. The ramp density and the current density of i+1 (k). So by this product, I can find the supply of space and the third term is the capacity of this point. So between 2 cells, I have a point depending on the number of lanes or other factors. This has a certain capacity and the flow cannot go higher than this. Summing up all these 3 terms, the flow qi (k) that would move to cell I or cell i+1 at time k, it is the minimum of these 3 terms. If I compute these 3 terms, I can find the flow. Note here that this term is equivalent to this term because of the definition of the split ratio B.

Notes

Summary

19m 54s



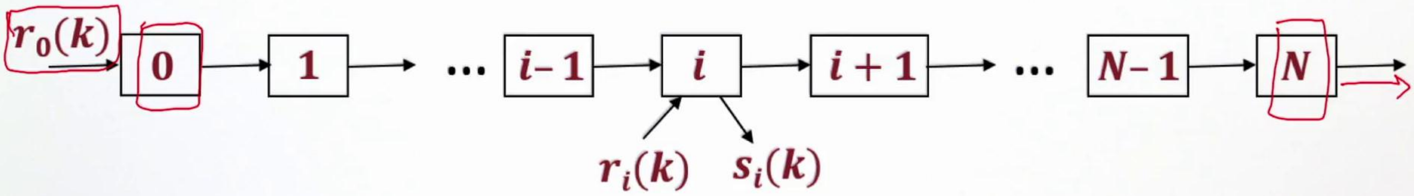
I have said before in order to find the flow that moves from one cell to the other, we need to take the minimum of these 3 cells. In the Cell Transmission model, every cell has a fundamental diagram so for every cell, I need to define this diagram, which have some parameters like the gram density, the critical density, the free flow speed, the shock wave speed of congestion and the maximum flow in the maximum capacity. When defining this fundamental diagram for the cell, some things are important. For example, it is very important to accurately define the critical density. The point of this diagram maximizes the flow, this is a very critical parameter. Another thing that I can see here from this triangle is this relationship shown at the point here at the top. These 3 equations become equal. This relates that critical density of cell i and the critical density of cell i+1.

Notes

Summary

21m 59s

CTM: Overview



$$q_i(k) = \min\{\, \bar{\beta}_i(k) v_i \rho_i(k) \,, w_{i+1}\left(\rho_{i+1} - \rho_{i+1}(k)\right) , Q_i \,\}, \quad 0 \le i \le N-1$$

$$n_i(k+1) = n_i(k) - T\left[\frac{q_i(k)}{\bar{\beta}_i(k)} - q_{i-1}(k) - r_i(k)\right], \quad 1 \le i \le N$$

$$n_0(k+1) = n_0(k) - T\left[\boxed{\frac{q_0(k)}{\bar{\beta}_0(k)}} - \boxed{r_0(k)}\right]$$

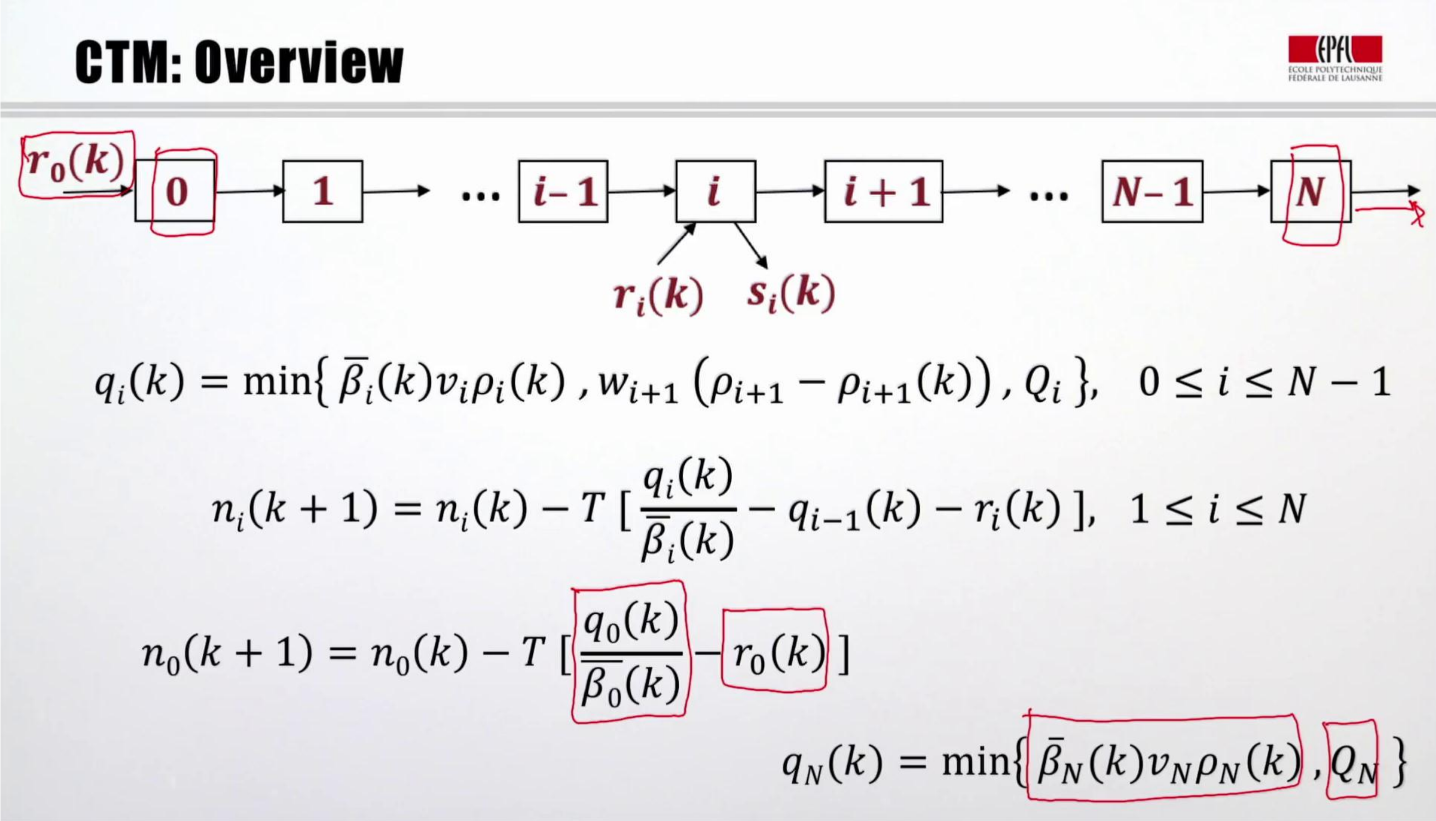
$$q_N(k) = \min\{\, \bar{\beta}_N(k) v_N \rho_N(k) \,, Q_N \,\}$$

So I would like to summarize in this slide all the dynamics of the Cell Transmission Model. Again, we have the stretch of the highway with N+1 cells and first of all, for every cell i from 0 to N-1, we can define the outflow that moves to the downstream cell qi (k). As we explained before, this is the minimum of these 3 terms. The next equation is the conservation equation. For every cell i from N-1, the number of vehicles must conserve, so the number of vehicles supplying k+1 is equal to the number of vehicles supplying k plus all the inflows minus all the outflows. Again, if I divide this by Δi, then I get the densities. So here, I can add the flow that comes from upstream, the flow that comes from the ramp minus this term, which is the flow that moves from cell i to cell i+1 downstream. Note here, that the equations for the first and the last cells are a bit different. So if I take cell 0 and I write the conservation equation, I have this term r0 (k), which is an input. It is the demand that joins the main line of the highway so I don't have to compute any incoming flow. This term is the flow that leads cell 0 to the downstream cell 1. For the last cell N, when I want to calculate the flow that would move downstream.

Notes

Summary

23m 15s



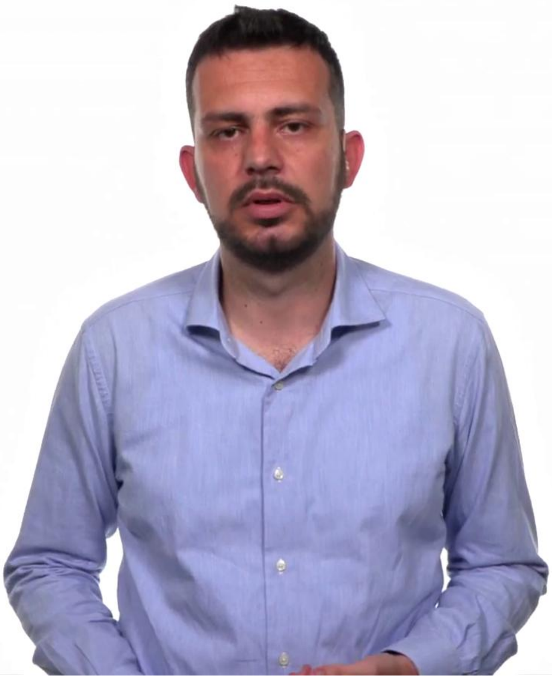
I have 2 terms instead of 3. So I have the capacity and I have the term that shows me the demand for space. But I don't have any term for capacity of space as I assumed that the highway downstream here is empty and nothing can block this outflow.

Notes

Summary

25m 11s

Cell Transmission Model – Summary



This video have provided the basic explanation of the dynamical equations of Cell Transmission Model. Now, you may be ready to run your simulation. If you want to have hands on experience with this model, I would encourage you to take a small stretch of a highway with one on-ramp and one off-ramp for example. Then you can write down the equation and if you define all the required input. You can run the simulation in your computer and look at the results. The result will show you how traffic congestion evolves over time and space for this specific network. See you soon.

Notes

Summary

25m 34s