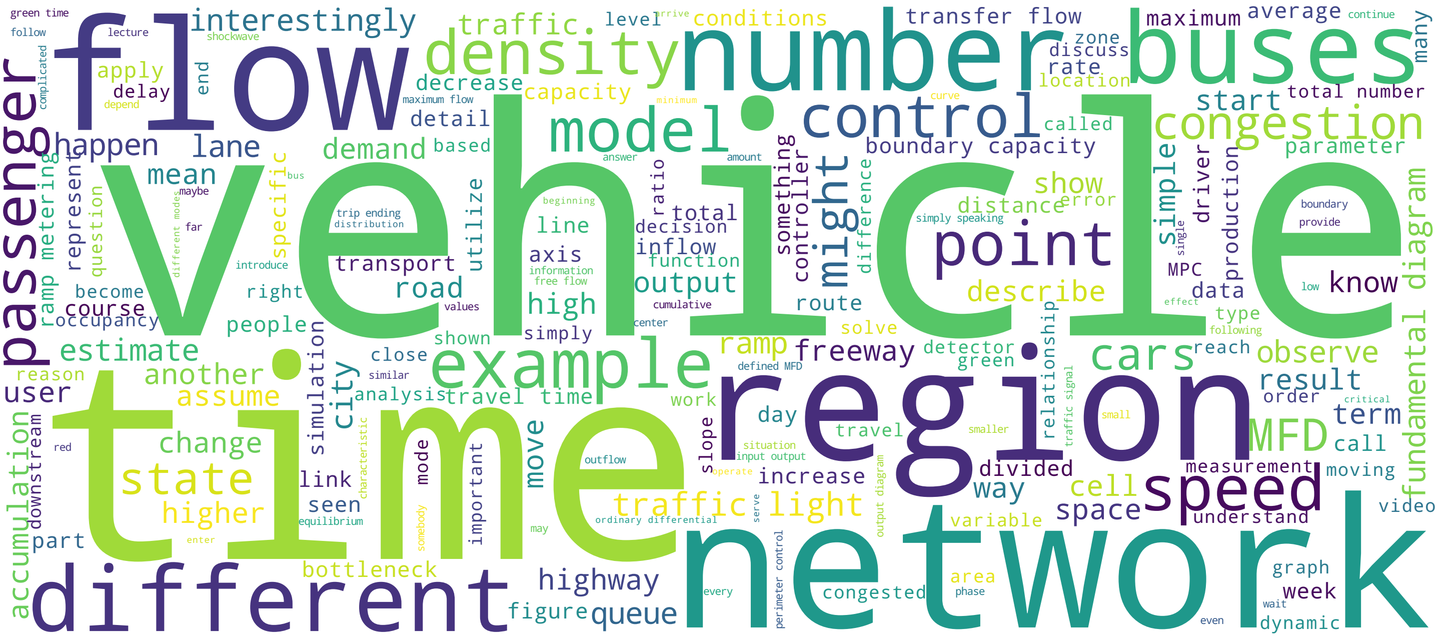
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Macroscopic Fundamental Diagram:

Existence, Physical Properties and Dynamic Modeling

Intro to traffic flow modeling and ITS

Prof. Nikolas Geroliminis

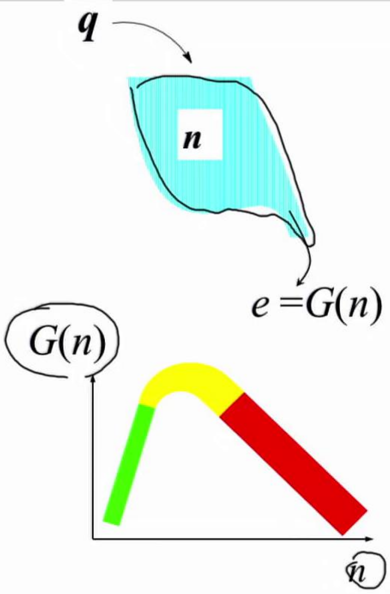


Search MOOC

Video

EPFL

MFD Dynamics of a single-reservoir system



$$\frac{dn}{dt} = q(t) - G(n(t))$$

Daganzo, 2007

So far we have seen that an MFD exists under some conditions like the spatial homogeneity in the distribution of congestion for some type of networks. The question is, can we utilize this MFD tool to develop dynamic models of congestion? So the answer is yes and we will see in this presentation, dynamic modeling for different types of networks. Single region, multiple region, but will also how an MFD world if we have a network with more than one mode of transport like buses and cars for example. So let's start with a single region of a city that we can assume that it has a well-defined MFD with loss cutter. So as we see in that a picture we can represent a city like a single reservoir where simply weights are coming in and after they travel some distance, they exit from the reservoir and if the lot of vehicles are there, then the level will increase. So if we assume that we have an MFD between the number of vehicles and the output of the network G(n) that shows the trip endings, the exit flow, then we can describe the dynamics of this simple system with an ordinary differential equation shown over here.

Notes

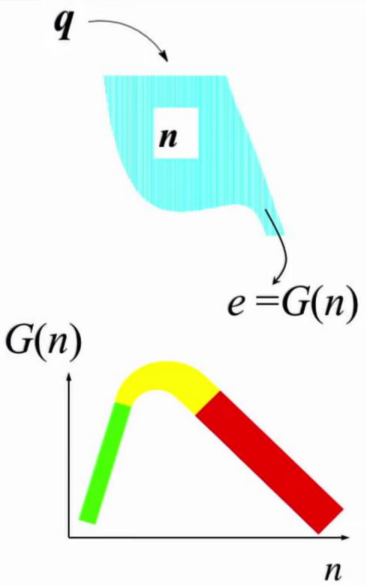
Summary

0m 06s

MFD Dynamics of a single-reservoir system

$$n_1 = n_0 + q_0 - G(n_0)$$
$$n_2 = n_1 + q_1 - G(n_1)$$
$$\vdots$$
$$n_t = n_{t-1} + q_{t-1} - G(n_{t-1})$$

$$n_t = n_0 + \sum_{j=0}^{j=t-1} q_j - \sum_{j=0}^{j=t-1} G(n_j)$$



$$\frac{dn}{dt} = q(t) - G(n(t))$$

Daganzo, 2007

So this is a simple equation that says that the rate d(n) over d(t) that the number of vehicle chains is equal to the input at time t minus the output at time t which is actually is given by an MFD. So this is a simple flow conservation equation. And actually we can utilize this to solve a system and understand how congestion will evolve with time. So let's look at a discrete version of this ordinary differential equation. The way we can write it is now that index represents time. So the number of vehicles in the system at time one is equal to the number of vehicles on the system at time zero plus the inflow times zero minus the outflow of time zero which is a function of the MFD. We can write this discrete version of these or D for different times between 1 and t. So what we can observe is that if we do some simple algebra, the number of vehicles at time t is equal to the number of vehicles at time zero, plus the total number of vehicles that enter the network the inflow of the network from the beginning up to time t minus 1, minus the output of the network so the total trip endings which is the cumulative sum of the trip endings between time 0 and time t-1.

Notes

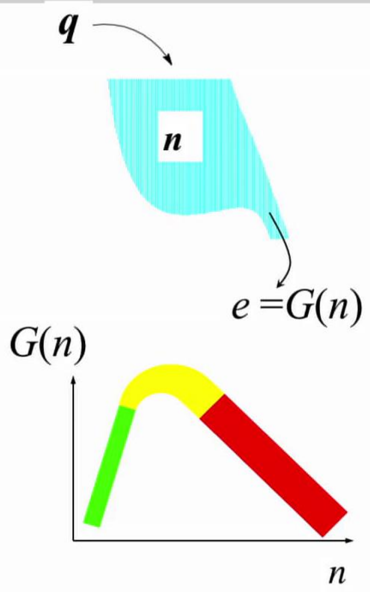
Summary

1m 35s

MFD Dynamics of a single-reservoir system

$$n_1 = n_0 + q_0 - G(n_0)$$
$$n_2 = n_1 + q_1 - G(n_1)$$
$$\vdots$$
$$n_t = n_{t-1} + q_{t-1} - G(n_{t-1})$$

$$n_t = n_0 + \sum_{j=0}^{j=t-1} q_j - \sum_{j=0}^{j=t-1} G(n_j)$$



$$\frac{dn}{dt} = q(t) - G(n(t))$$

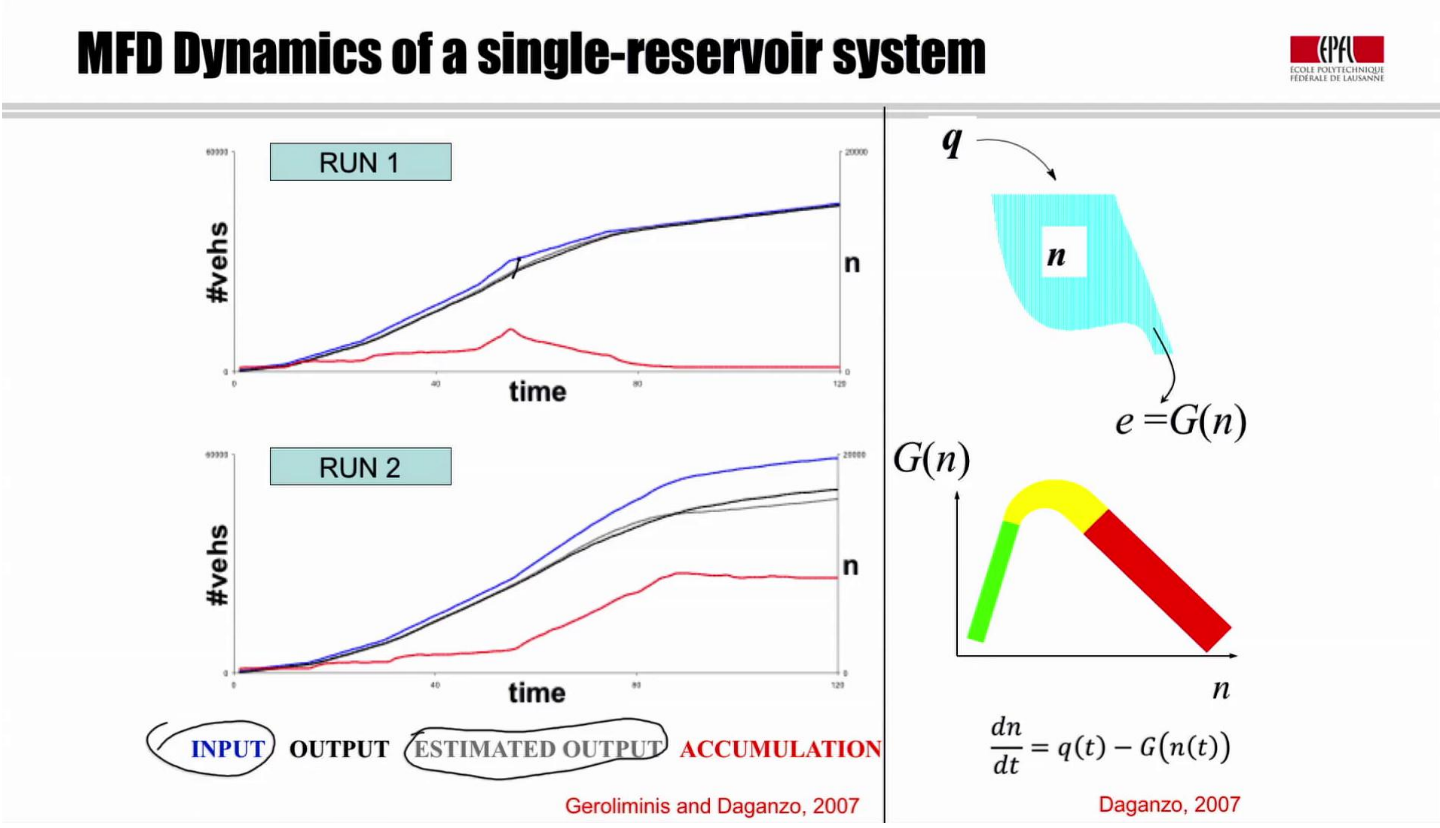
Daganzo, 2007

So if somebody knows initial conditions for this network, so the total number of vehicles at time 0. And now is the inflow of the network and we assume that we have a well-defined MFD, then we can also estimate the number of weeks the network at time t like by using input-output diagrams by following this equation. So let's see some results.

Notes

Summary

3m 06s

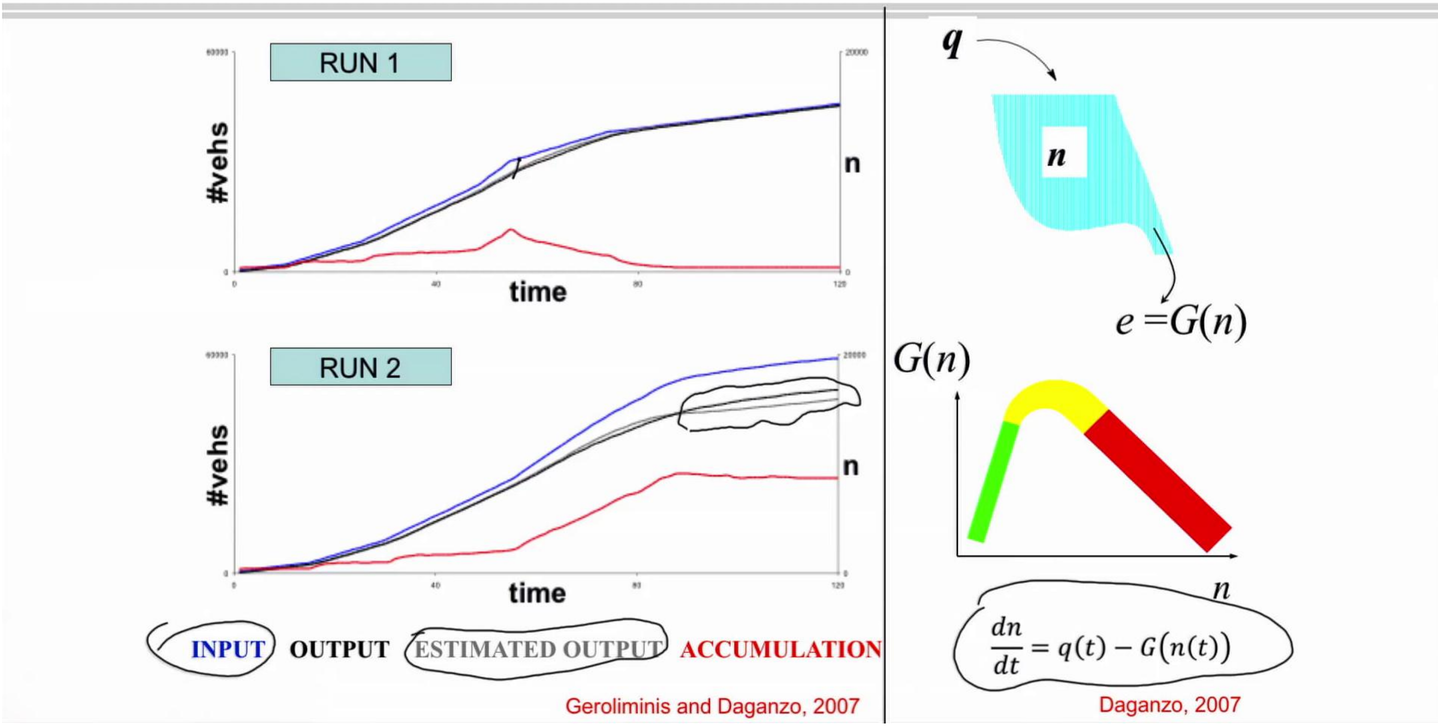


So over here, we see the results of the analysis from this discretized version of this ordinary differential equation for two different scenarios in the micro simulation of San Francisco that we have seen in one of the previous videos. So here, we only know the total number of vehicles in the network at time 0 and we know how many vehicles are entering the network with the time of the day. And this is the blue curve which is cumulative and also if we ran our discretized version of this ODE from the previous slide what we can observe and we can estimate, we can estimate the output. And please recall from input-output diagrams, if we take the vertical difference, we can estimate the accumulation in the network which is shown in red and it's based on the right y axis. So what we observe here is that the gray and the black lines are very close to each other, which means that we can estimate accurately the out of the network without knowing the details for the individual vehicles, the number of vehicles in every link, the only information we have is the total input flow. And we see the same result for a different scenario.

Notes

Summary

3m 35s



Interestingly what we see, we see that when accumulation is very high and the situation is very close to gridlock, our estimation has some level of error and this is due to the fact that under heavily congested conditions that approach gridlock, the MFD might experience some higher scatter as we also observed in some of the previous lectures. But the interesting finding is that with a very simple ODE, we can predict with good accuracy the number of vehicles in the network over the time of the day.

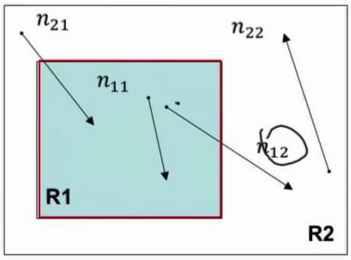
Notes

Summary

5m 03s

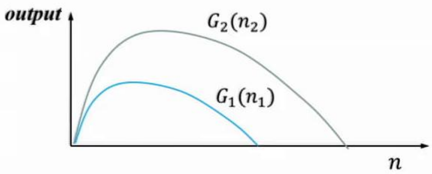
Dynamics of a two-reservoir system

- State Variables
- Dynamic Equations



$$\frac{dn_{11}}{dt} = q_{11} + Q_{1\to 2} - \frac{n_{11}}{n_1} * G_1(n_1)$$

$$\frac{dn_{12}}{dt} = q_{12} - Q_{1\to 2}$$



So after this simple example for a single reservoir city, let's consider a more complicated case. So [07:50 inaudible] congested city by applying some clustering algorithm can be divided in a number of regions. And so let's see the dynamics of a multi region system. And let's look at a simple example with only two regions. So here first of all what we have, we have a central region which is more congested and a periphery zone which is less congested, but both of them have a well-defined MFD with low scatter between output an accumulation G1(n1) and G2(n2). So first of all we need to introduce four state variables to describe the system while in the single reservoir case, we only had one. And these four state variables are shown in the figure. So let's describe for example n12 is the number of vehicles in reservoir 1 that they have as final destination reservoir 2. And now, let's see how the dynamic equations look like for some of these state variables. So let's see first the accumulation and n11 which is a number of vehicles in region one that has final destination the same region. So the rate that n11 changes dn11 over dt is again, a flow conservation equation that has the total inflow minus the total outflow.

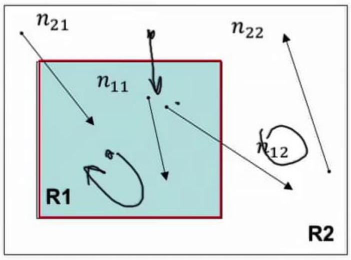
Notes

Summary

5m 41s

Dynamics of a two-reservoir system

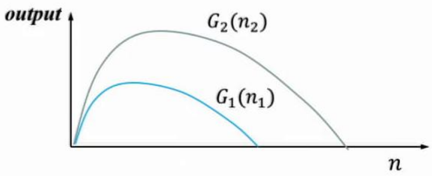
- State Variables
- Dynamic Equations



$Q_{2\to1}$

$$\frac{dn_{11}}{dt} = q_{11} + \cancel{Q_{1\to2}} - \frac{n_{11}}{n_1} * G_1(n_1)$$

$$\frac{dn_{12}}{dt} = q_{12} - Q_{1\to2}$$



And the total inflow has two terms. The first term is 211 which is the new inflow that is generated in region 1 and it has as destination region 1 is something like new trips that are started when people turn on the MZ. But there is also some additional inflow which is the inflow from region 2, region 1. So clearly here we have a typo and this should be Q1 and then what we havek, we have the outflow of the network, the output and this output is the rate that vehicles finish their trips within the zone. And simply speaking if we assume that all the vehicles in zone 1 have the same speed then we can estimate that the part of the output with respect to these vehicles n1 over n1 multiplied by the total output of accumulation from this MFD is the output of the n11 vehicle. And there is one more assumption we make here is that all these vehicles have approximately the same trip length, but this equation can be adjusted even if they have different lengths. And in the same way we can write an equation for accumulation n12, the number of weeks in region 1 that have a size and destination region 2. So here we have some inflow which says the new generated the and from region 1 to region 2 minus the transfer flow from region 1 to region 2 which is Q12. So let's describe now in a few more details how these transfer flows look like.

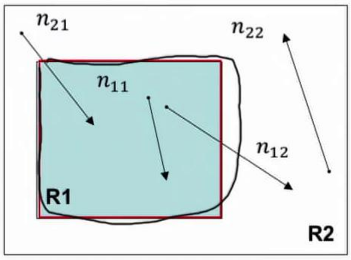
Notes

Summary

7m 14s

Dynamics of a two-reservoir system

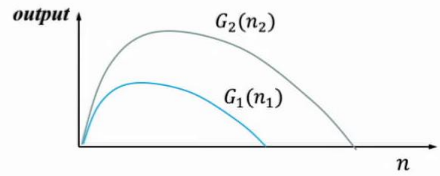
- State Variables
- Dynamic Equations
- Boundary Capacity
- Perimeter Control

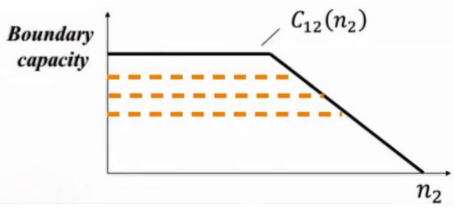


$$\frac{dn_{11}}{dt} = q_{11} + Q_{1\to2} - \frac{n_{11}}{n_1} * G_1(n_1)$$

$$\frac{dn_{12}}{dt} = q_{12} - Q_{1\to2}$$

$$Q_{1\to2} = \min\left(C_{12}(n_2), \frac{n_{12}}{n_1} * G_1(n_1)\right)$$





And let's look at Q12. So Q12 is the minimum of two terms. The one term is the flow which can be sent from region 1 to region 2 and this is given by this term which is simply the total output of this reservoir multiplied by the ratio of this specific type of vehicle so n12 divide it by n1. And the second term c12 of n2 is what we call boundary capacity. And there is some analogy that we also utilized in some of the previous lectures for the cell transmission modelling. So boundary capacity it means that the transfer flow from region 1 to region 2 depends on the number of vehicles in region 2. So if region 2 is very congested, this means that there is not enough space for vehicles to move over there so the boundary capacity will decrease while for smaller values of n2, the boundary capacity will remain that it's high value that mainly depends on the amount of time that the traffic signals in the boundary of the network is green. Here, I would like to draw your attention that this boundary capacity actually can be influenced by changing the traffic lights. So if for example at the same boundary, I go and decrease the amount of green time for movements towards the center of the city, then this boundary capacity will become smaller and if I further decrease even more it will become even smaller.

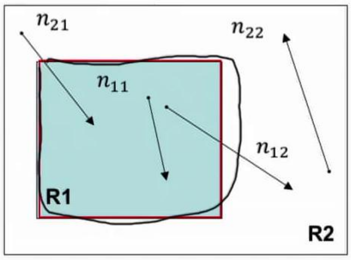
Notes

Summary

9m 16s

Dynamics of a two-reservoir system

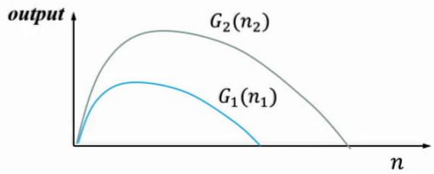
- State Variables
- Dynamic Equations
- Boundary Capacity
- Perimeter Control

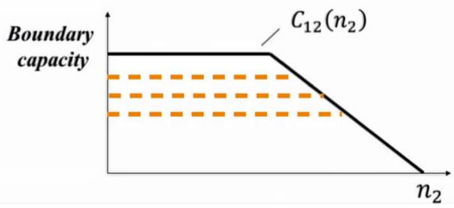


$$\frac{dn_{11}}{dt} = q_{11} + Q_{1\to2} - \frac{n_{11}}{n_1} * G_1(n_1)$$

$$\frac{dn_{12}}{dt} = q_{12} - Q_{1\to2}$$

$$Q_{1\to2} = \min\left(x_{12} C_{12}(n_2), \frac{n_{12}}{n_1} * G_1(n_1)\right)$$





So actually in that case I have to multiply this boundary capacity by a value let's say x12 which is the fraction of the green time that is reduced for all the traffic signals in the boundary. And actually this is a very interesting idea because by doing that, I can control how vehicles are moving from one region to another. I define this as perimeter control and in the next week of this move we will see in details how perimeter control can significantly improve the mobility and decrease the travel times for all vehicles.

Notes

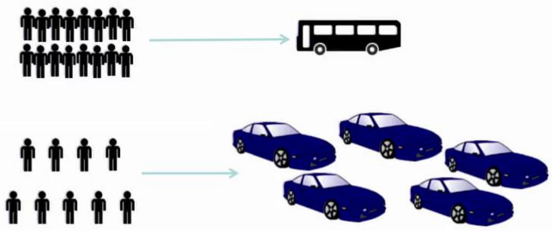
Summary

11m 04s

Multimodal networks

- In urban networks, buses share the same network with the other vehicles.
- Conflicts in multi-modal urban traffic systems:
- Bus stops affect the system like red signals in a single lane (instead of blocking all lanes).

↗ bus frequency
↘ flow of vehicles
flow of passengers



Performance Measures

Vehicle Hours Traveled
Vehicle Kilometers Traveled

Passenger Hours Traveled
Passenger Kilometers Traveled

⇒

Mobility (Accessibility)

Emissions (Environ. Impacts)

Costs (Users, Providers, etc.)

Road Space Used

So far we have seen the dynamics of single and multi region systems where actually only vehicles only cars are moving around, but what we know, we know that in real networks we have different modes of transport like cars versus bicycle strands that are competing for the same space. So an interesting question is, does an MFD exist for multimodal networks? And the emphasis of this analysis will be at the moment for a simple network that consists only of cars and buses. And interestingly what it happens in these networks, there in the multimodal case there are conflicts of the different modes. So for example when a bus makes stop, then you might have a queue of vehicles behind the bus that they are waiting. So it's sounding like you have a small red face in one lane of the road. And this as a result that buses create additional congestion. When the bus frequency increases what will happen because of these effects of the bus stops, the flow of vehicles will go down.

Notes

Summary

11m 46s

Multimodal networks

- In urban networks, buses share the same network with the other vehicles.
- Conflicts in multi-modal urban traffic systems:
- Bus stops affect the system like red signals in a single lane (instead of blocking all lanes).

↗ bus frequency
↘ flow of vehicles
↗ flow of passengers

Performance Measures

Vehicle Hours Traveled
Vehicle Kilometers Traveled

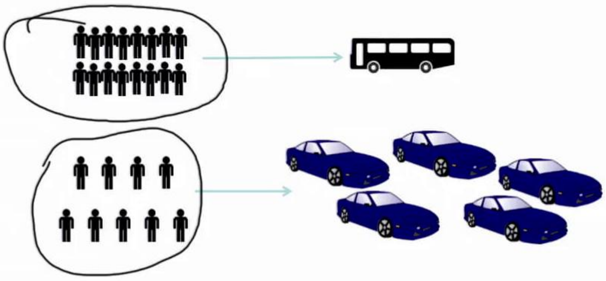
Passenger Hours Traveled
Passenger Kilometers Traveled

Mobility (Accessibility)

Emissions (Environ. Impacts)

Costs (Users, Providers, etc.)

Road Space Used



So somebody can say that buses create additional congestion in the network, but if we consider that buses are carrying many more passengers compared to cars, so maybe a bus can transfer 20 or 30 passengers while if you have something like 5 to 10 passengers, you might need to use four or five cars, then even if the flow of vehicles goes down the flow of passengers in the network can go up which means that buses can be beneficial for the network. So in the case of multimodal networks, we have to switch our focus and the type of performance measures we'll have from vehicles to passengers. So before we define the production of the network as the total vehicle kilometres travel per unit time or we consider the total delay of the network as the total vehicle hours traveled. Now we have to look at passengers for hours and kilometers travel.

Notes

Summary

13m 03s

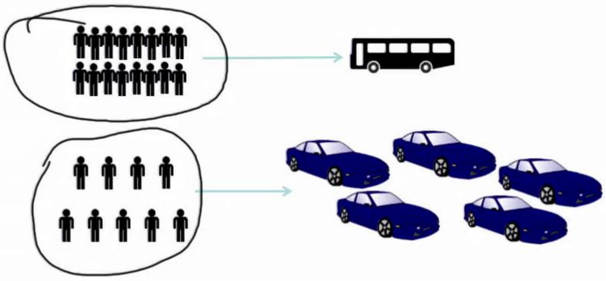
Multimodal networks

- In urban networks, buses <u>share</u> the same network with the other vehicles.
- <u>Conflicts</u> in multi-modal urban traffic systems:
- Bus stops affect the system like red signals in a single lane (instead of blocking all lanes).

↗ bus frequency

↘ flow of vehicles

↗ flow of passengers



Performance Measures

Vehicle Hours Traveled
Vehicle Kilometers Traveled

Passenger Hours Traveled
Passenger Kilometers Traveled

Mobility (Accessibility)

Emissions (Environ. Impacts)

Costs (Users, Providers, etc.)

Road Space Used

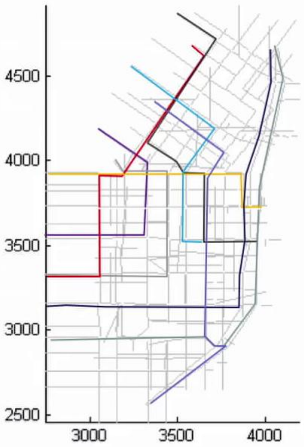
And actually this change is a little bit our direction of research, because if we consider passengers then besides mobility we can also see how accessibility is influenced, because when you utilize a bus you also have some accessibility, so you have some distance to travel to reach the bus stop.\ At the same time because you might have vehicles on average with more passengers, you might have less environmental it but and less emissions, but also in these more complex networks you have to consider the different costs associated with its mode of transport. And also other decision you have to make in multimodal networks is how the road space can be utilized, because somebody can decide to provide individual lanes to buses so to make buses faster.

Notes

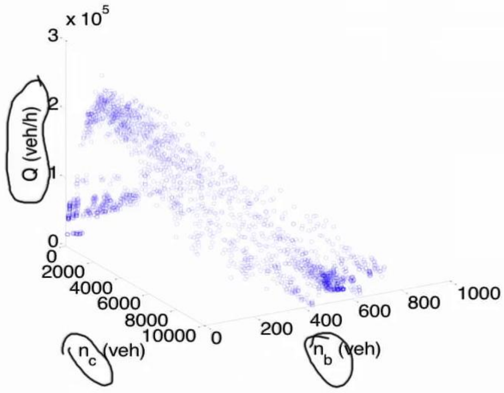
Summary

14m 04s

3-dimensional MFD for bus-car systems



$$Q = Q_c + Q_b = n_c * v_c + n_b * v_b$$



h_c the occupancy of cars (passengers/veh)
h_b the occupancy of buses(passengers/veh)
P_v Production of vehicles (VKT/u.t.)
P_p Production of passengers (VKT/u.t.)

I would like now to introduce a three dimensional macroscopic fundamental diagram for network viewer you have two modes of transport, cars and buses. So one before for the MFD we'll consider only two variables, the number of vehicles in the x-axis versus the flow of vehicles on the y-axis, now we have to consider three. So as we see over here what we have, we have the number of buses in the network versus the number of cars in the network and in the y-axis, we can have the production or the flow of vehicles that are moving in the network per unit time. So here we take the San Francisco network as we analyzed it before in the micro simulation environment, but what we do we add all the real existing bus lines in the network that are shown with these lines of different colors. So now the total production or total circulating flow in the network Q at a specific point in time or a time interval is the sum of the car flow and the bus flow. Please recall that this production is the multiplication of the accumulation with speed for cars and buses.

Notes

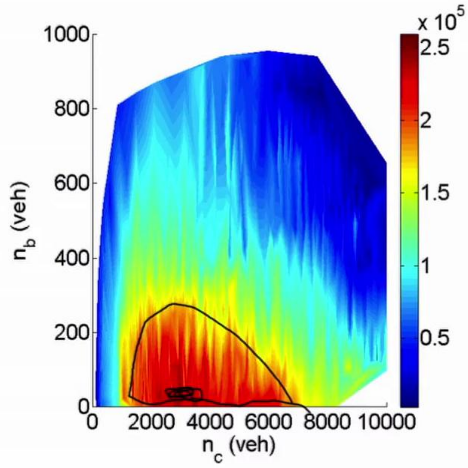
Summary

14m 59s

3-dimensional MFD for bus-car systems

$$Q = Q_c + Q_b = n_c * v_c + n_b * v_b$$

$$P_p = h_c P_c + h_b P_b$$



h_c the occupancy of cars (passengers/veh)
h_b the occupancy of buses(passengers/veh)
P_v Production of vehicles (VKT/u.t.)
P_p Production of passengers (VKT/u.t.)

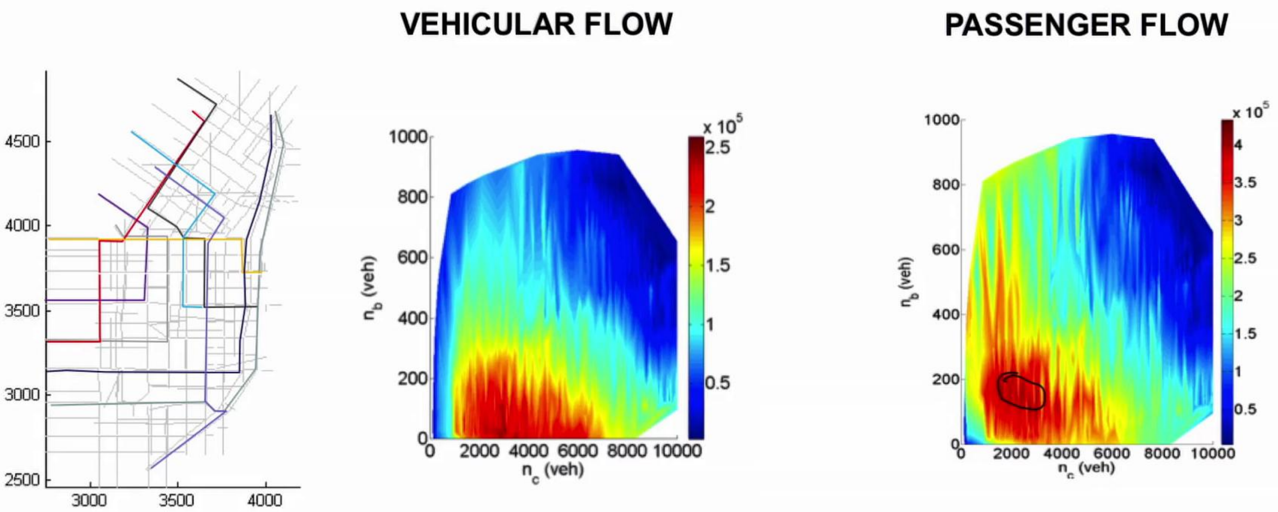
So this is what we see in this three-dimensional graph and interestingly, if we see this more like a counter plot where the x-axis is the number of cars, the other axis is the number of buses, and the corner over here represents the third axis with this scale. So dark red means very high value flows, blue means very low value flows close to zero, this is how this diagram looks like. So what we observe, we observe that this is the area where the flow has the highest values and interestingly what we observe, we observe that the maximum value flow happens when we have almost zero buses in the network. So this is an interesting observation because if somebody translates that, it concludes that the maximum flow in the network happens when I have no buses. So somebody might conclude that buses create problems in the network, but I think this looks quite silly. And the reason is that in this analysis who ignore that buses are carrying much more passengers. So if now we consider that buses have a much higher occupancy compared to carsk, so let's call hb the passengers per vehicle for buses and let's see the passenger of a per vehicle per cars, then we can represent the production of the network as the sum of the production of the two individual modes.

Notes

Summary

16m 21s

3-dimensional MFD for bus-car systems



h_c the occupancy of cars (passengers/veh)
h_b the occupancy of buses(passengers/veh)
P_v Production of vehicles (VKT/u.t.)
P_p Production of passengers (VKT/u.t.)

$$P = P(n_c, n_b) = h_c Q_c + h_b Q_b$$

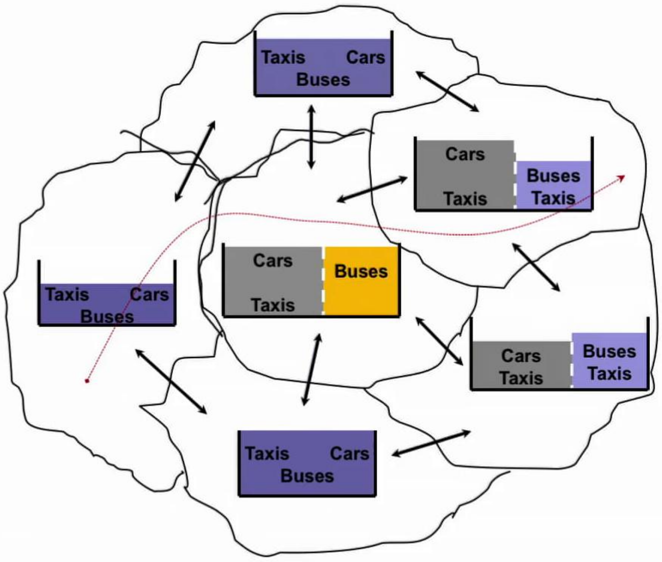
So actually over here Pc and Qc is the same and Pb and Qb is the same but we multiplied by the occupants of the vehicles. So if we do that and if now we plot the same three-dimensional MFD but instead of V vehicular flow, we look at passenger flow let's see how the results ends. So the results are here so for vehicular flow, this is the same graph as before and this is how it looks for passenger flow. So we have again a counter plot and please note that because here we consider the passenger flow, we can have higher values compared to weak flows. So interestingly here what we see, we see that the maximum flow of passengers in the network happens clearly for a nonzero value of buses and you can see here that there is a ratio of buses over cars around 10%. So interestingly what we see, we see if we're able to have networks where at least 10% of the vehicles is buses, then we can have much higher flows and we can serve a larger number of passengers with less congestion.

Notes

Summary

17m 56s

Multimodal multi-reservoir networks



- Spatial allocation of road space
- Special treatment of buses

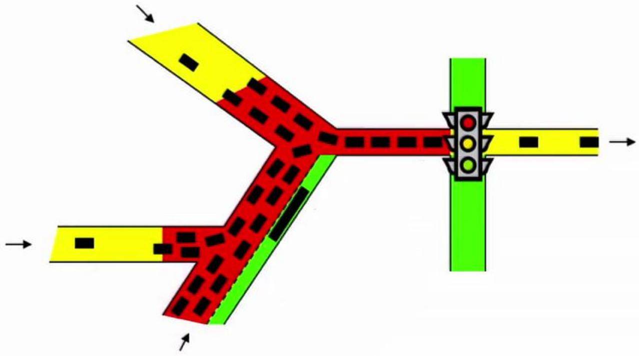
So let's consider now that we have a network with different regions and we'll have multiple modes of transport that are moving in this network. And conceptually we can describe this as in the figure, we see here. While after partition we can have different regions. We might consider that for each region there is some space which is dedicated for specific modes of transport. So for example in the city center that you might have a lot of trips made by bus, you can have dedicated lanes for buses only. While in the periphery of the city you can have all the modes like cars, buses, taxis moving all the lanes of the road. And in some semi-conscious regions we might even allow taxes to use the dedicated bus lanes. So an interesting question that somebody has to answer when we design multimodal multi regional networks is, how to allocate the space between different modes? And in that case we might provide special treatment for buses. And at the same time we can also control the inter transfers between the zones by using traffic lights. So you understand that the multimodal problem becomes more complicated and this analysis might be beyond the scope of this course, but we also provide some references in the course material for the interested reader.

Notes

Summary

19m 13s

An example of bus-car interactions



Queues form at locations with limited capacity, but spill-over to other locations

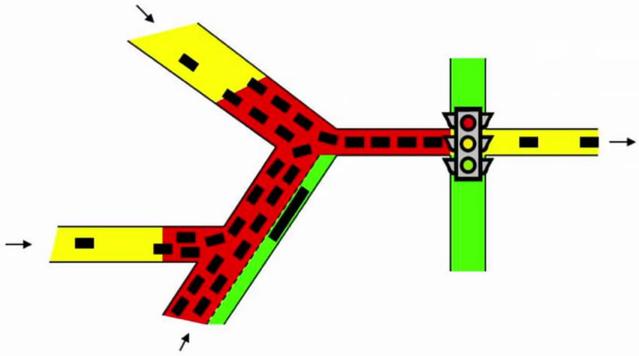
So let's consider now a simple case where buses and cars are interacting and we can see how by simple intervention in simple policies, we can significantly improve the mobility of passengers in this network. So here what we have, we'll have an active Baltic in the center of the city with a traffic light and let's assume that we'll only have one lane of road that moves in this direction and then there is emerged where different roads of multi planes have to pass through this bottleneck. So interestingly what will happen is that as this boat length is active and it has limited capacity queues will propagate in different parts and will spill over to other locations. So if a bus arrives in this bottleneck, this bus has to wait for quite some time until all these vehicles are served before it also passes through the traffic light. And here we don't have space to create a dedicated bus lane because you only have one lane available, but we can have a simple solution. The solution is that for parts of the network where there is available space, we can provide dedicated space for buses like in this line over here. So if this happens actually these buses can overpass these queue of vehicles and can reach this boat link faster.

Notes

Summary

20m 40s

An example of bus-car interactions



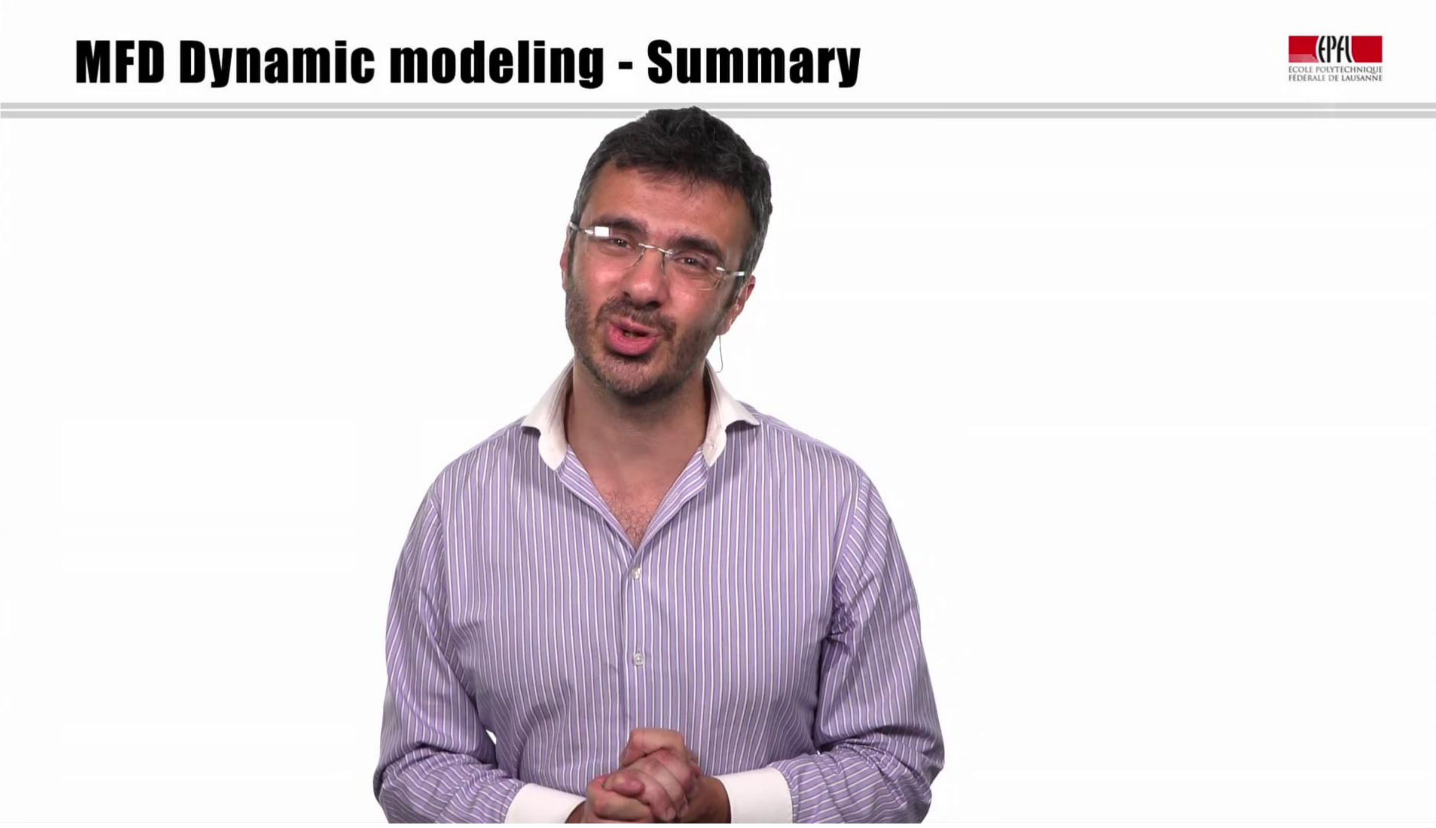
Queues form at locations with limited capacity, but spill-over to other locations

So even if they have to wait to be served, they have to wait by having only a few vehicles in front of them. And what will be there is of this? So the flow of vehicles will not change, it will be approximately the same that our cell from this traffic light but the flow of passengers will be much higher because the frequency of buses that will pass through this traffic light will also be higher. So these are simple policies that can be also applied a network level and by providing special treatment for buses, we can increase the flow of passengers that are moving around the network.

Notes

Summary

22m 11s



So in this lecture we show how we can utilize the MFD tool to model dynamically urban systems with single and multiple regions. And what you have also seen is how the systems will look like and will behave at the aggregated level, if we have multiple modes of transfers competing for the space like buses and cars. Next week we will continue with MFDs but now we will see how we can utilize all the things we learned during this week to develop traffic management schemes. See you soon.

Notes

Summary

22m 48s