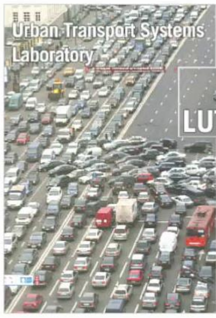
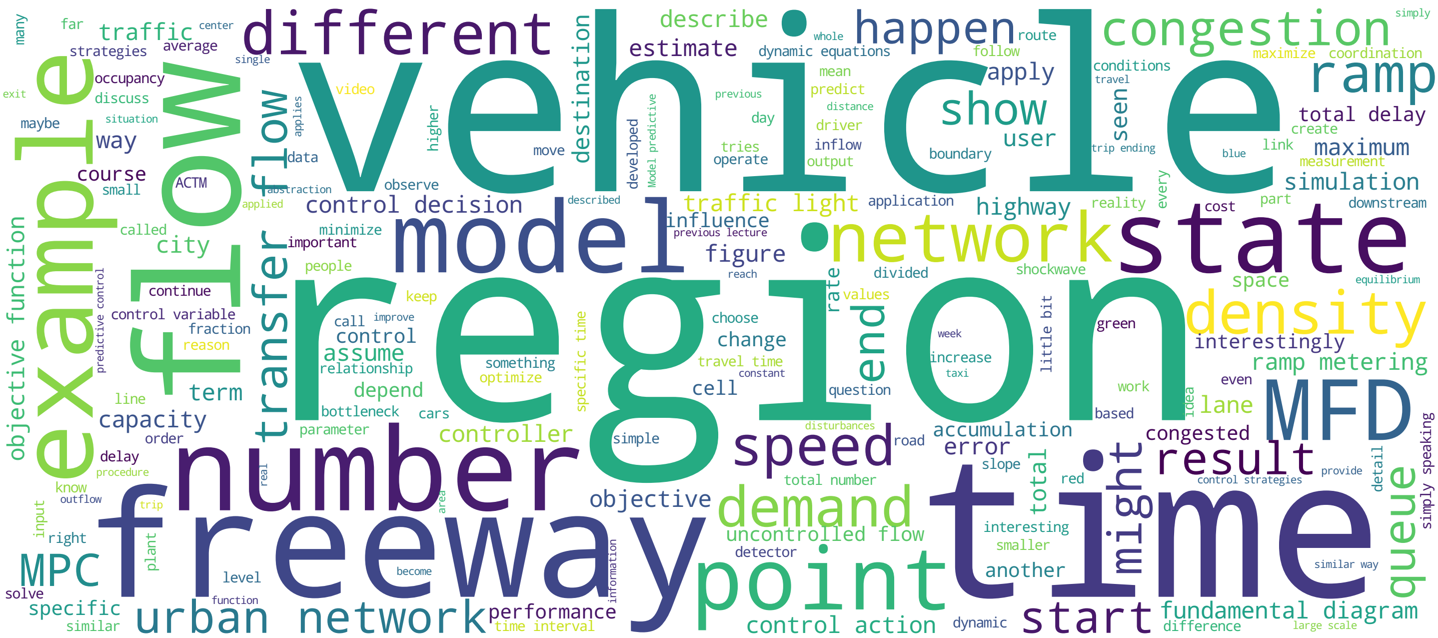
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Model predictive control with multi-region MFDs (part I)

Intro to traffic flow modeling and ITS

Prof. Nikolas Geroliminis





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Video

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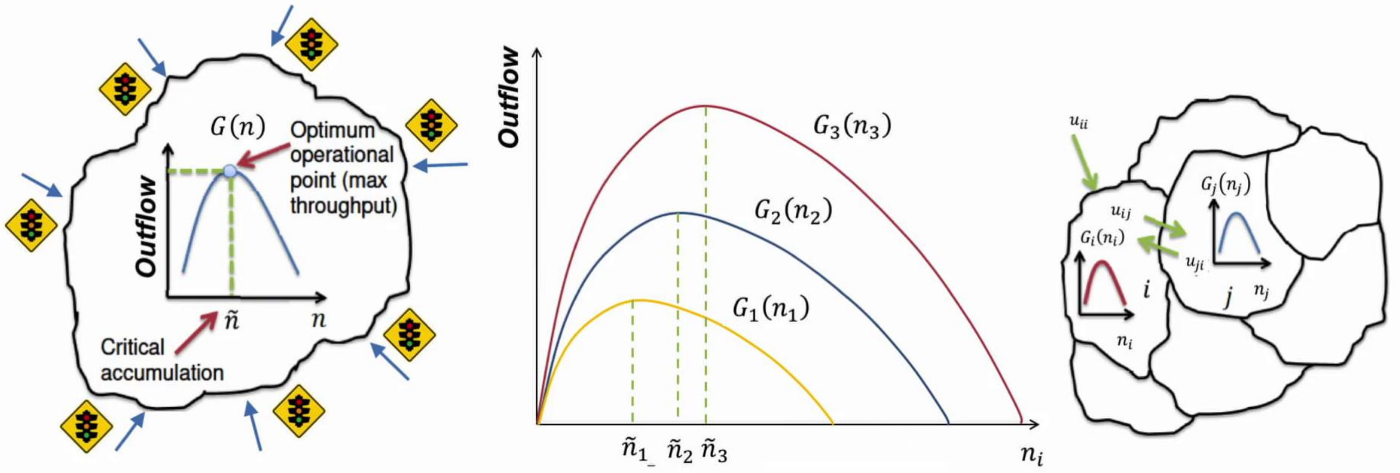
Control logic: from single- to multi-region



WHY MULTI-region?

- heterogeneously congested cities with multiple pockets of congestion (partitioning)
- High internal inflows (controllability)
- Avoid long queues at boundaries (equity)

LUTS publications, 2012-2018

Welcome! So far we have seen that large scale urban networks can be partitioned in an now number of homogeneous regions with well-defined MFDs and traffic management control strategies can be developed where the objective is to keep its region at a specific set point. Nevertheless these strategies might not always be able to improve the mobility in the cities. So during this week we will see a more advanced control tool which is called Model predictive control and if integrated properly it can help us to solve challenging control problems for large scale traffic networks. Please refresh your memory about the control logic we introduced during the previous lectures for seeing and multi region systems. So for single systems the objective is to keep the system at an optimal accumulation. The critical accumulation that maximizes outflow while in the case of multi region systems the idea is that we tried to control not only the inflows to the system but also the transfer flows between different regions with an objective to maximize the output of the system or minimize the total delays.

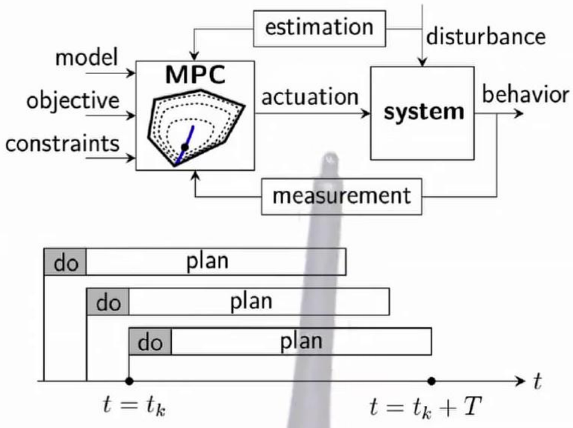
Notes

Summary

0m 05s

Why model predictive control (MPC)?

- A receding horizon procedure
- Can cope with non-linear control problems
- Can handle both state and control constraints
- Proper for real-time implementation
- Robust to prediction error and measurement noise
- Proper for traffic networks with multiple pockets of congestion



Let's first start with the limitations of the previous control techniques we have developed. So we looked at multi variable by regulators. And one of the main uses of these regulators is that they required first of all specific set points but also if they cannot handle constraints concerning the states but also the control decisions. Model predictive control is a receding horizon procedure which can cope with nonlinear control problems. So this figure on the right illustrates how MPC operates. So first of all to deal with MPC we need to have a system which this system is the real world let's say where we have dynamic conditions that change over time. Then what we have we have a model that this model is an abstraction of the real system. We need to define an objective which is what we would like to optimize. So for example we want to minimize the total delays for the system and we also need to define some constraints as for example that the green time cannot be negative or cannot exceed the maximum value. Then if we look in the system we have some control variables that we can use to intervene by constraining let's say the flow that are transferring from one region to the other.

Notes

Summary

1m 29s

Why model predictive control (MPC)?

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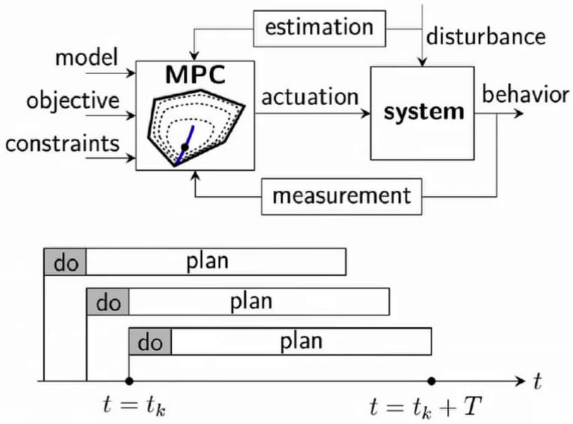
- Proper for real-time implementation
- Robust to prediction error and measurement noise
- Proper for traffic networks with multiple pockets of congestion



This is called actuation but also what we have we have some disturbances that these disturbances are for example uncontrolled flows. So these disturbances can be estimated but this is not mandatory. So this arrow might not be there and MPC by observing what is the state of the system. So by observing for example what is the number of vehicles in each region and by knowing what is over here? It applies an optimization procedure and it identifies what are the control decisions. So these control decisions are applied in the system. The system is updated and then there is a loop and procedure is continued. What they didn't mention yet is that when MPC is optimizing is not optimizing for only one time interval it has a prediction horizon that during these prediction horizon and through the model it tries to predict what will be the state of the system and it tries to identify what are the optimal control actions to minimize this objective over these control horizon and after it may exist control decisions then it applies only a subset of this. So only for a one time interval and then the whole system advances for one time unit and the procedure is repeated until let's say the end of the day or the end of the problem.

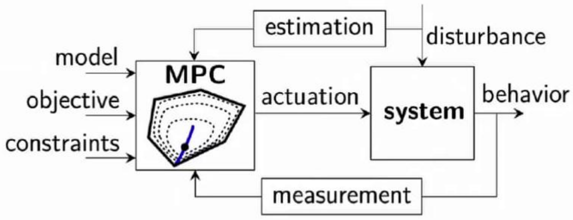
Notes

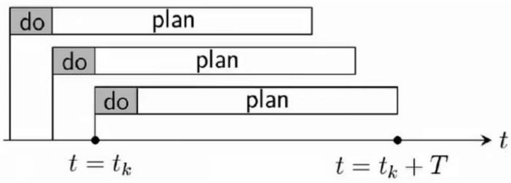
Summary

3m 03s

Why model predictive control (MPC)?

- A receding horizon procedure
- Can cope with non-linear control problems
- Can handle both state and control constraints
- Proper for real-time implementation
- Robust to prediction error and measurement noise
- Proper for traffic networks with multiple pockets of congestion





As we mentioned the MPC can really handle both state and control constraints which is not the case for PI controllers and also is robust to prediction errors and measurement noise as also will see from our applications and is very appropriate for traffic networks with multiple pockets of congestion where the multivariable PI approach with constant set points cannot produce very good results especially when the demand in the city changes over time and space.

Notes

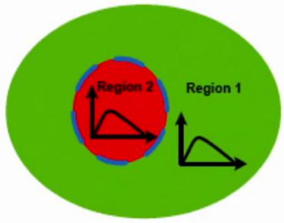
Summary

4m 39s

MPC for MFD systems

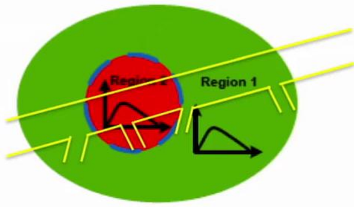
- **Problem I: A two-region MFD system**

Problem I



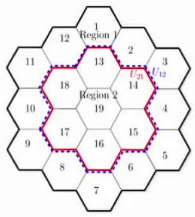
- **Problem II: A mixed freeway-urban system**

Problem II



- **Problem III: A multi-region MFD system with heterogeneity and control hierarchy**

Problem III



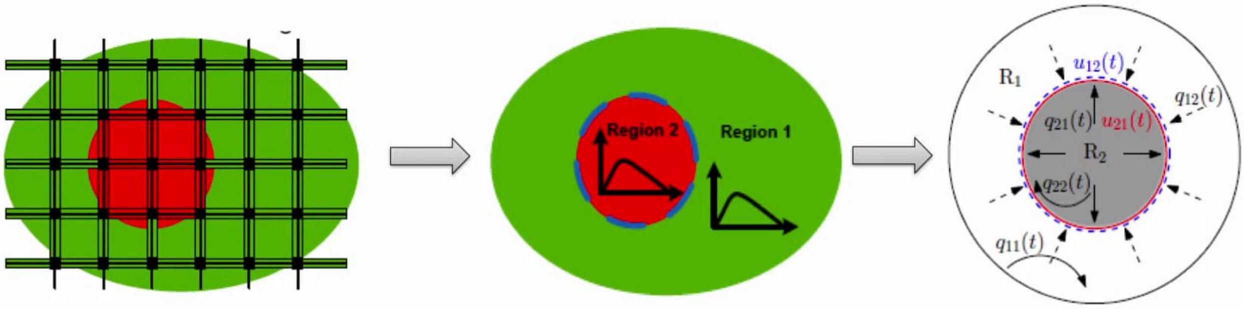
So during these lectures we will look at three different problems that we can handle with MPC and let's say that these problems are of advanced complexity So problem number one is a two region MFD system which we also see in this video. Problem number two is problem number one plus a freeway that crosses the city. While problem number three considers a control hierarchy so an upper and a lower level controller for a system with two regions that also has special heterogeneity.

Notes

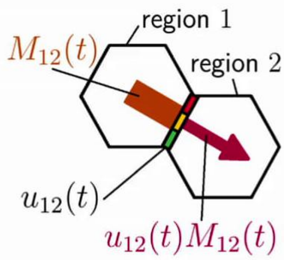
Summary

5m 16s

Problem I statement



- Urban network is partitioned in homogeneous regions
- Two controllers operate on the border
- Objective function is to maximize trip endings



Geroliminis, Haddad, Ramezani (2013) – IEEE Trans. ITS

So let's start with a problem statement for the first problem. So consider that we have an urban network but after some clustering is partition in homogeneous regions. So for simplicity we assume that we have over here two regions but the same problem could be solved without significant effort for more than two regions. The objective function is to maximize the trip ending so to maximize the rate that vehicles reach their destination. And here we see how the network looks like the abstraction would do for the MPC so we have two regions. Each of these regions has a well-defined MFD and then we have two different types of controller. So we have you u12 and u21 that actually can restrict the flow from one to two and from two to one separately. So the idea here is that when you how you are in the boundary between two regions and M12 is the transfer flow from region one 1 to region 2 somebody can intervene by defining and control variable u that this variable for example contains the amount of green time for all the traffic lights in this boundary and in that case by influencing the transfer flow we can change dispersal distribution of congestion in the city and as a result can contains the trip endings and improve the value of the objective function.

Notes

Summary

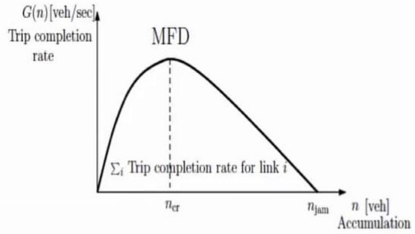
5m 51s

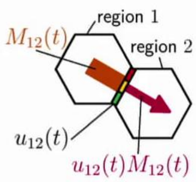
Problem I formulation

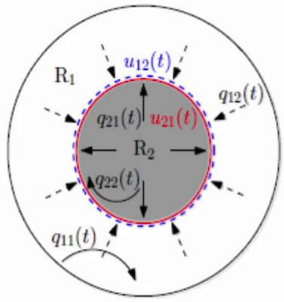
$$J = \max_{u_{12}(t), u_{21}(t)} \int_{t_0}^{t_f} [M_{11}(t) + M_{22}(t)]\mathrm{d}t$$

Subject to:

- $\frac{\mathrm{d}n_{11}(t)}{\mathrm{d}t} = q_{11}(t) + u_{21}(t).M_{21}(t) - M_{11}(t)$
- $\frac{\mathrm{d}n_{12}(t)}{\mathrm{d}t} = q_{12}(t) - u_{12}(t).M_{12}(t)$
- $\frac{\mathrm{d}n_{21}(t)}{\mathrm{d}t} = q_{21}(t) - u_{21}(t).M_{21}(t)$
- $\frac{\mathrm{d}n_{22}(t)}{\mathrm{d}t} = q_{22}(t) + u_{12}(t).M_{12}(t) - M_{22}(t)$
- $M_{ij} = \frac{n_{ij}}{n_i} \times G_i(n_i(t))$
- $u_{\min} \le u_{12}(t) \le u_{\max}$; $u_{\min} \le u_{21}(t) \le u_{\max}$
- $0 \le n_1(t) \le n_{1,\mathrm{jam}}$; $n_1(t) = n_{11}(t) + n_{12}(t)$
- $0 \le n_2(t) \le n_{2,\mathrm{jam}}$; $n_2(t) = n_{21}(t) + n_{22}(t)$







So let's become a little bit more quantitative and look how mathematically we can describe this problem. So what we need for an MPC as we mentioned we need an objective function. We need the dynamic equations of the system and we also need some constraints. So let's call d of n the fundamental MFD of the for each of the two regions. And let's start first of all by looking at the objective function. So the objective function is maximizing the trip endings between a specific time t zero which is the start for optimization and some prediction horizon tf And what is M11 and M22 are the rates that vehicles are finishing their trips in Region 1 and in region 2 respectively. So let's see it we will see in a little more details. How M11 and M22 can be estimated? But let's see now the dynamic equations of the system. So please recall that what we have over here we have some flow conservation equations. So let's look at the first one that shows that the rate the number of vehicles from Region 1 to Region 1 scenes. And this is the unrestricted input in Region 1 which are as you can see in the figure that are generated in Region 1 and they cannot be restricted and then what we have will have the transfer flow from region 2 to region 1.

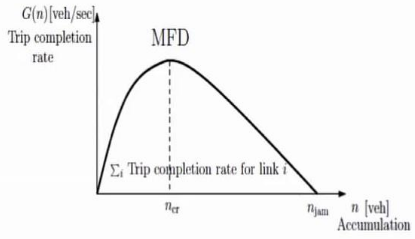
Notes

Summary

7m 22s

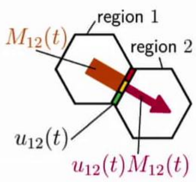
Problem I formulation

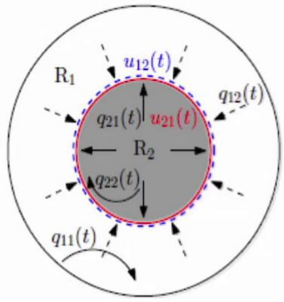
$$J = \max_{u_{12}(t),u_{21}(t)} \int_{t_0}^{t_f} [M_{11}(t) + M_{22}(t)]\mathrm{d}t$$



Subject to:

- $\frac{\mathrm{d}n_{11}(t)}{\mathrm{d}t} = q_{11}(t) + u_{21}(t).M_{21}(t) - M_{11}(t)$
- $\frac{\mathrm{d}n_{12}(t)}{\mathrm{d}t} = q_{12}(t) - u_{12}(t).M_{12}(t)$
- $\frac{\mathrm{d}n_{21}(t)}{\mathrm{d}t} = q_{21}(t) - u_{21}(t).M_{21}(t)$
- $\frac{\mathrm{d}n_{22}(t)}{\mathrm{d}t} = q_{22}(t) + u_{12}(t).M_{12}(t) - M_{22}(t)$
- $M_{ij} = \frac{n_{ij}}{n_i} \times G_i(n_i(t))$
- $u_{\min} \leq u_{12}(t) \leq u_{\max}$; $u_{\min} \leq u_{21}(t) \leq u_{\max}$
- $0 \leq n_1(t) \leq n_{1,\mathrm{jam}}$; $n_1(t) = n_{11}(t) + n_{12}(t)$
- $0 \leq n_2(t) \leq n_{2,\mathrm{jam}}$; $n_2(t) = n_{21}(t) + n_{22}(t)$





But as we mention we can apply some control variable and we can make these smaller or larger depending on what are the conditions in the system and then the outflow of the system is the rate that vehicles reach their destination. And these are the vehicles that are already in the region 1 and they will finish their trips in region. In a similar way of thinking we can describe their dynamic equations for all the different four states of the system and what you see. You see that these depend mainly on the uncontrolled inflow too. It depends on the transfer flows and it also depends on the control variables. What is missing yet to fully define the mathematics of the problem is to identify Mij transfer flows. So, the Mij are really depend on the end of the so what we assume over here we assume that as this system is governed by MFD. The trip completion rate for a region only depends on the number of vehicles in this region i. But as we have different states so then we need to consider the fraction of the vehicles that are associated with this specific transfer flow.

Notes

Summary

9m 11s

Problem I formulation

$$J = \max_{u_{12}(t), u_{21}(t)} \int_{t_0}^{t_f} [M_{11}(t) + M_{22}(t)]\mathrm{d}t$$

Subject to:

- $\frac{\mathrm{d}n_{11}(t)}{\mathrm{d}t} = q_{11}(t) + u_{21}(t).M_{21}(t) - M_{11}(t)$
- $\frac{\mathrm{d}n_{12}(t)}{\mathrm{d}t} = q_{12}(t) - u_{12}(t).M_{12}(t)$
- $\frac{\mathrm{d}n_{21}(t)}{\mathrm{d}t} = q_{21}(t) - u_{21}(t).M_{21}(t)$
- $\frac{\mathrm{d}n_{22}(t)}{\mathrm{d}t} = q_{22}(t) + u_{12}(t).M_{12}(t) - M_{22}(t)$
- $M_{ij} = \frac{n_{ij}}{n_i} \times G_i(n_i(t))$
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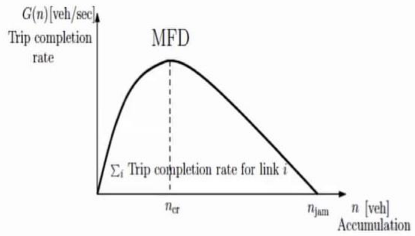

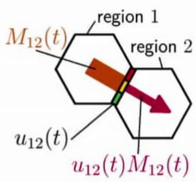

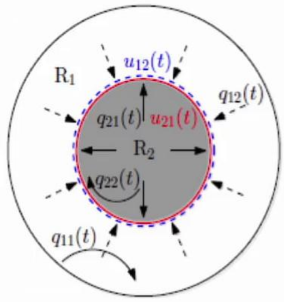

So by assuming that all vehicles in a specific region have approximately equal speeds then what we can estimate we can estimate that the transfer flow from region i to region j is the fraction of nij vehicles over the total numbers of vehicles in region a multiplied by the trip completion rate given by the MFD and also this is applied. This equation is valid for all i and j that take values 1 or 2 based on which region we are. Then we also need to define some constrains which is the minimum and the maximum values that the control decisions state but also the fact that the number of vehicles in each region they cannot exceed the jam number of vehicles. The number of vehicles in one region with different destinations so n11plus n12 is equal to the total number of vehicles in Region 1 and the same applies for region 2.

Notes

Summary

10m 33s

Problem I formulation

$$J = \max_{u_{12}(t),u_{21}(t)} \int_{t_0}^{t_f} [M_{11}(t) + M_{22}(t)]\mathrm{d}t$$

Subject to:

- $\frac{\mathrm{d}n_{11}(t)}{\mathrm{d}t} = q_{11}(t) + u_{21}(t).M_{21}(t) - M_{11}(t)$
- $\frac{\mathrm{d}n_{12}(t)}{\mathrm{d}t} = q_{12}(t) - u_{12}(t).M_{12}(t)$
- $\frac{\mathrm{d}n_{21}(t)}{\mathrm{d}t} = q_{21}(t) - u_{21}(t).M_{21}(t)$
- $\frac{\mathrm{d}n_{22}(t)}{\mathrm{d}t} = q_{22}(t) + u_{12}(t).M_{12}(t) - M_{22}(t)$
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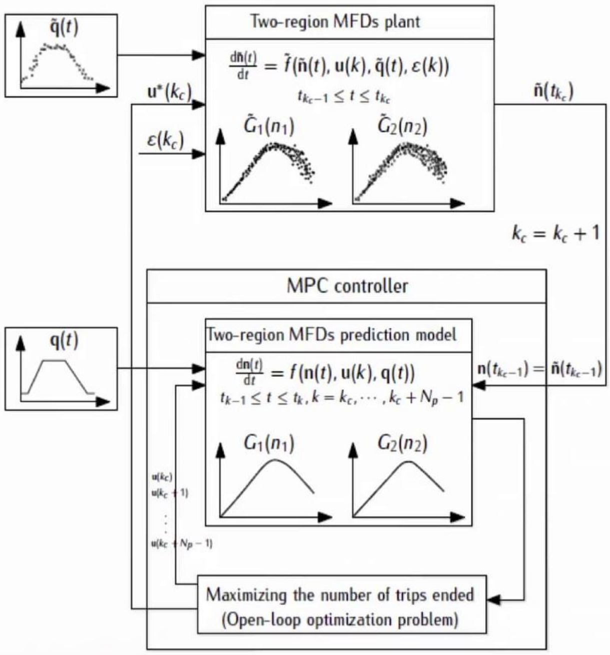


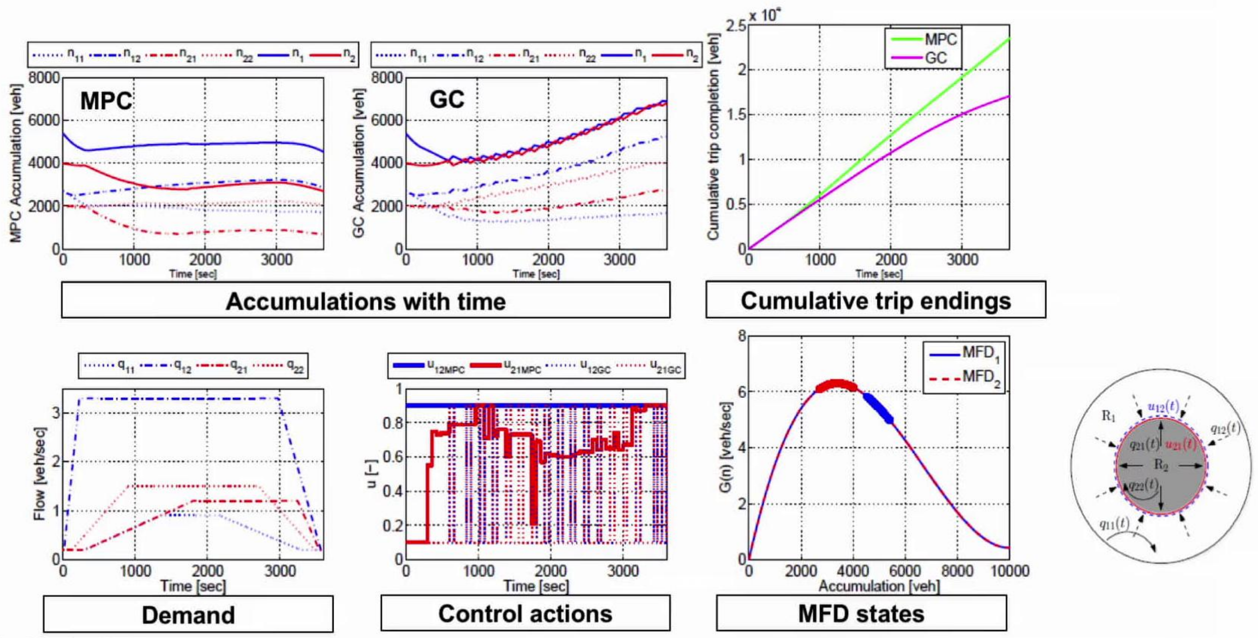
So given this problem formulation the way we solve it, it mainly follows the figure which is described on the right. So this figure actually shows the operation of the MPC. So what we have is called a plant and plant is let's say the real system for this specific word we do some simulations. But the importance is that the plant is different than the model. So for example the MFDs in reality might have some noise. The demand flows too we might not know them with perfect information. So we have the reality and what it happens we assume that we have a model which is an abstraction of this reality that is governed by some state equations. And given that word in a specific time t we use these dynamic equations to predict the state of the system over the horizon between 0 t0 and tf. And after that we try to optimize the system. And after this happens in west image some control yields would take these control decisions and we apply them in the plant. And then the plant evolves it reaches some new states where these states are measured and these states are then input again to our model at the next time interval and the same procedure is repeated until the end of the simulation.

Notes

Summary

11m 32s

Problem I Results (no errors in MFDs)



GC: Greedy Controller

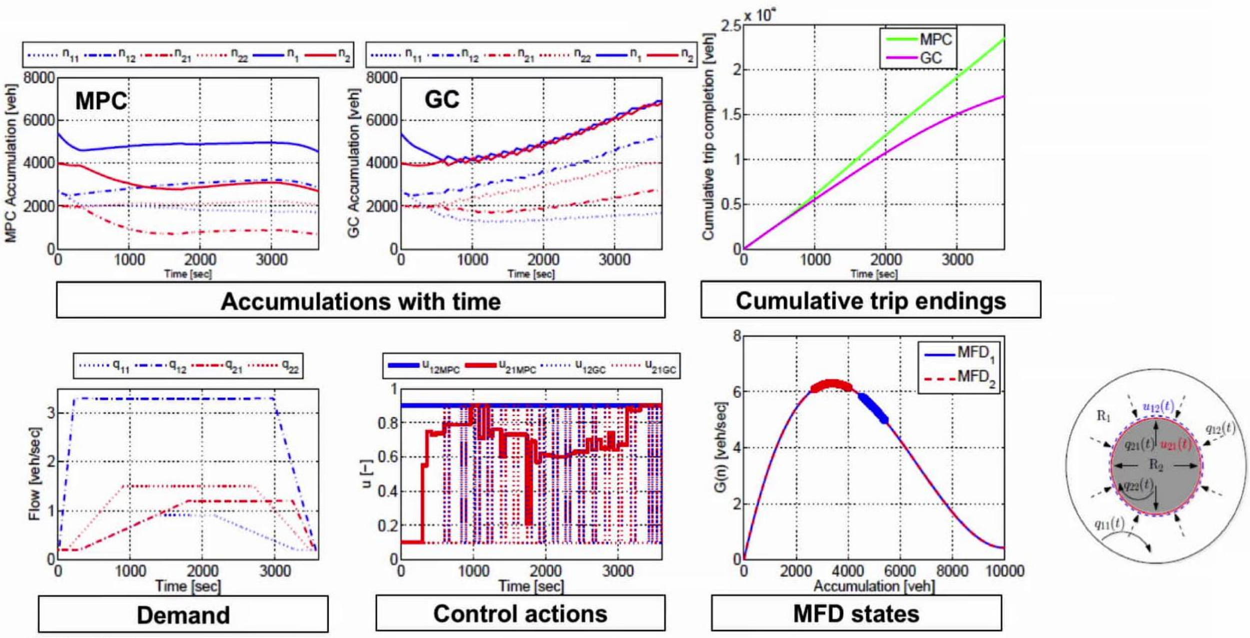
So let's see some results. So over here we compare MPC with a grid controller. A very simple controller where its logic is to try to keep both regions at the critical accumulation so what it does is actually it tries to restrict input to the more congested region. So what do we see in this graph. We see the demand and the demand profile between regions 1 in Region 2 and what we see we see that each of the flow demand profile follow and increasing and maximum and then a decreasing part. But this peak does not happen simultaneously for all zones and let's look also at our initial conditions so this is the accumulation with time. So what we see is that a time zero region blue is which is the region one is congested while Region 1 is almost at capacity. We could see this also here on the MFD. And then what we see over here we see the control actions where in bold are the control actions of the MPC controller where we also see the control actions of the greedy controllers that are very oscillatory and go up and down. So interestingly what we see we see that by the end of the simulation the accumulation of the system is much lower in the case of the MPC compared to the greedy control that has still high accumulations.

Notes

Summary

13m 05s

Problem I Results (no errors in MFDs)



GC: Greedy Controller

And interestingly the total number of vehicles that were able to serve it's much higher for the case on beer MPC compared to the greedy control. And here what we see we see that the MPC it chooses to operate at the maximum for the transfer flow between one and two. While it has different values for the transfer flow from two to one it started from small it goes to large than it retains small and so on and so forth. And what is interesting is that what you could see is that the system is not operating at a single set point as was the case with multiply multi variable PI controller. But there is a range of accumulations where this system operates. So so far these are results by assuming perfect knowledge and know away about MFDs and the demand.

Notes

Summary

14m 43s

Problem I Results (with errors in MFDs)

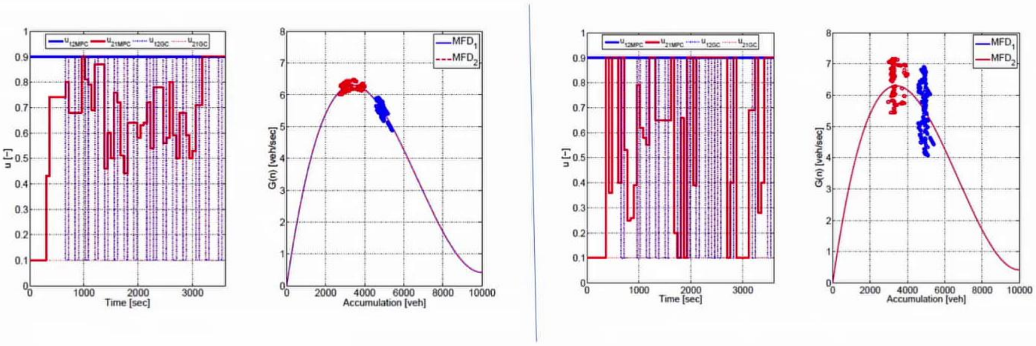


Table 1: The difference between the trip completion corresponding to MPC and greedy control $\cdot 10^3$[veh] without errors, with small and large errors in the plant MFDs.

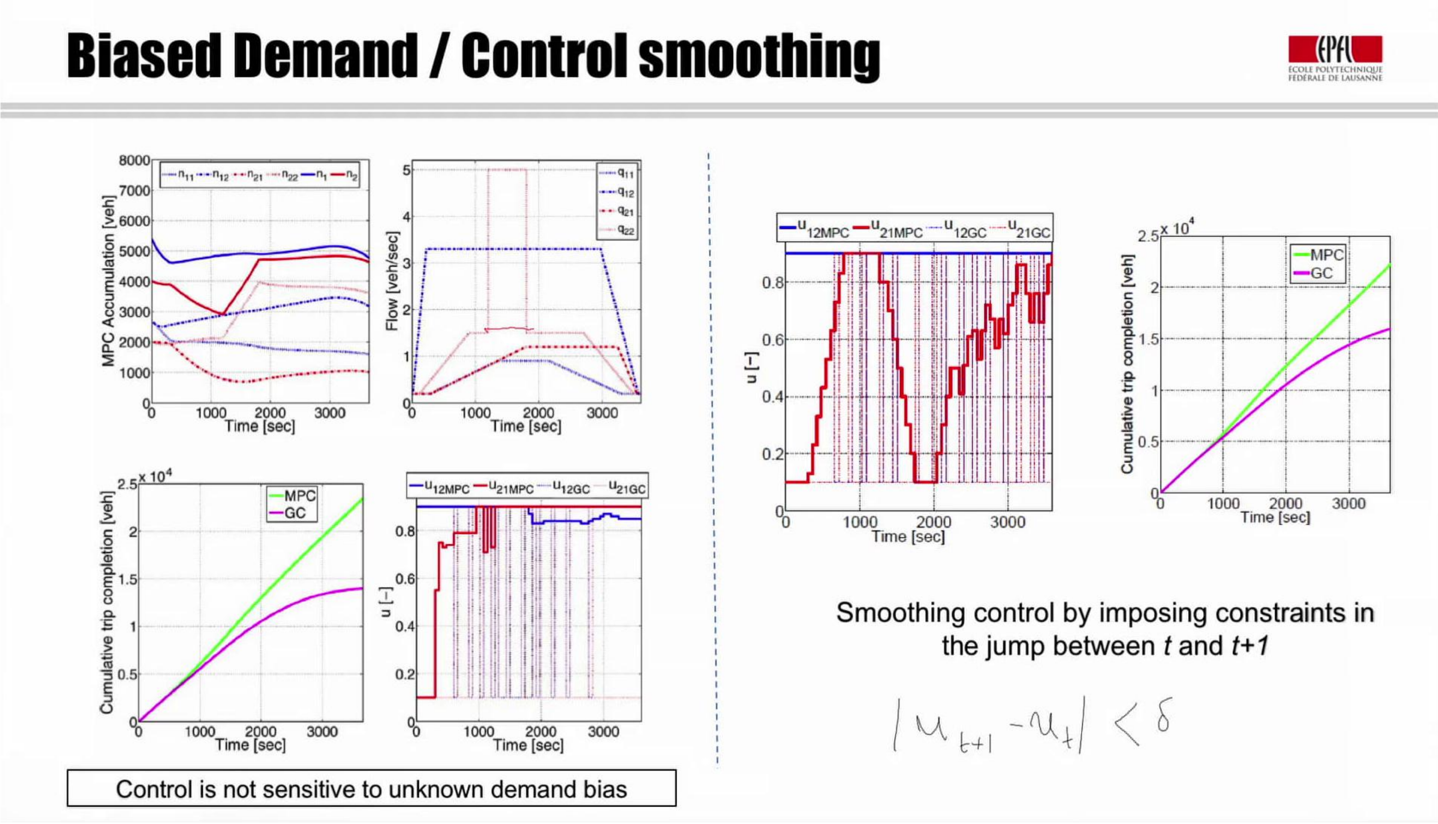
	Example	without errors ($\alpha_1 = \alpha_2 = 0$)	small errors ($\alpha_1 = \alpha_2 = 0.2$)	large errors ($\alpha_1 = \alpha_2 = 1$)
High demand	1	7791.6 (%22.5)	7847.8 (%22.7)	8112.8 (%23.5)
Medium demand	2	6536.8 (%17.3)	6530.7 (%17.3)	6556.1 (%17.4)
Low demand	3	1789.3 (%4.4)	1733.1 (%4.3)	1670.7 (%4.1)

But let's consider actually that maybe the MFDs in the plant and in reality contains some error. So will our controller be able to operate equally well. So we perform different simulations where we have small errors in MFDs and larger errors and if we compare as we've seen the results of this table the improvement of the MPC compared to the greedy controller for low, medium and high demand and for no errors, more error or large error. What we will see we will see that the error in the MFD do not influence the quality of the controller which is a very positive sign what it influences is actually as we see in these two figures that show the control actions is that our controller becomes more oscillatory and this in in reality might be a little bit problematic because it can create a strong stop and go situations and it can also influence the heterogeneity of a region that can also have a bad result on the MFDs shape.

Notes

Summary

15m 34s



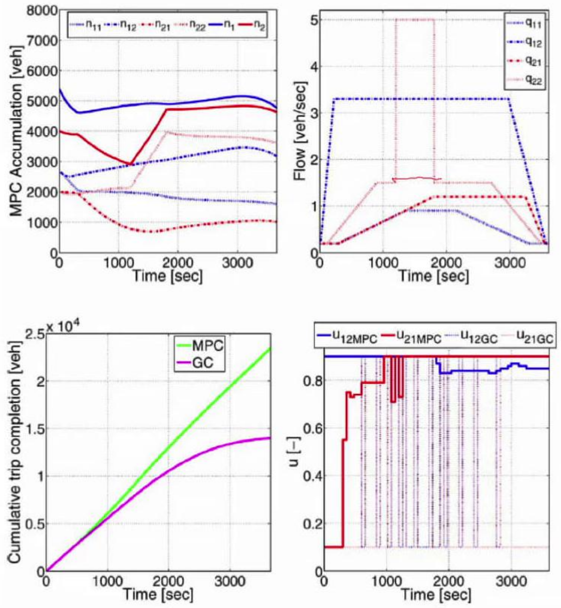
So in that case where we have oscillatory control actions one option is to apply controls smoothong. So what is more controls smoothing we impose some constraints in the control decisions that the control decision between time ut plus one in the control decision between time you should not be larger than a specific value. And in that way so for example here we impose example is not more than 10 percent. And actually what we see how is the control performance the controller is much smoother. But if we look at the performance in terms of trip completion rate we see that MPC is able to succeed equally good performance in the last part I would like to discuss in this work is the effect of bias demand. So it might be a case that there is a certain change in the demand that we're not able to predict. And the question is how this influences our results. So here what we see if we apply a demand profile that for example for these two to two demand between time this time and that time is significantly larger. But the MPC controller has no information about it. So it assumes that the demand remains like that. So in that case we also see that MPC controller performs equally well.

Notes

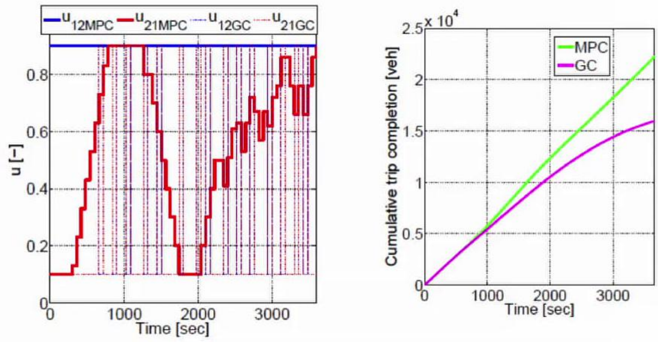
Summary

16m 42s

Biased Demand / Control smoothing



Control is not sensitive to unknown demand bias



Smoothing control by imposing constraints in the jump between *t* and *t+1*

$$|u_{t+1} - u_t| < \delta$$

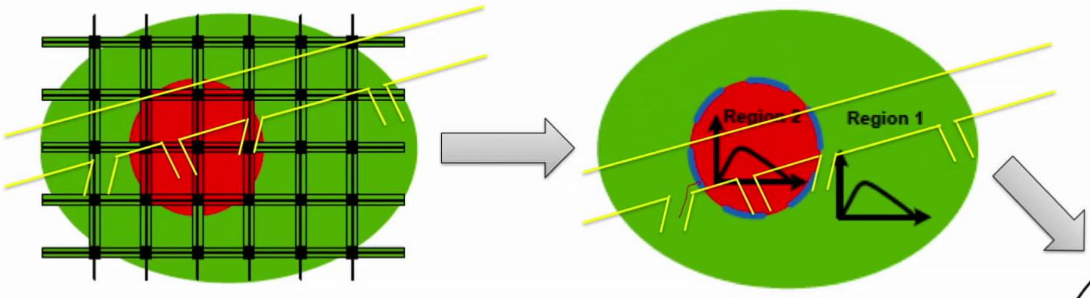
And the main reason is because we're able to have a good estimation of states. So even if we have errors in the demand by using different sensors in reality we can still observe properly the number of vehicles in each region. And that's the reason that we continue to do much better. Interestingly here in this case of errors in the demand what we see we see that the controller here went to does not always remain at the maximum but at some point in time it takes lower value.

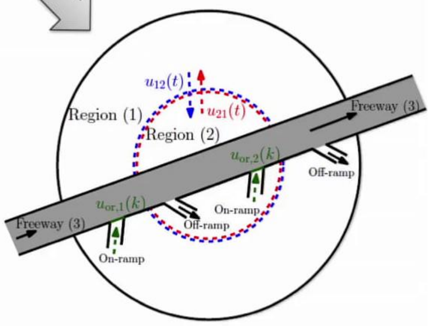
Notes

Summary

18m 16s

Problem II statement





- **Coordinated** control of a mixed freeway-arterial network
- Objective is to minimize the network total delay

- **Controllers are:**
  - 2 Ramp metering
  - 2 Perimeter controllers

Haddad, Ramezani, Geroliminis (2013) – Trans. Res. part B

Let's now look at the second problem which is actually built on the findings of the problem with just look. So again we have read a city and nearby network which is partitioned in two regions with MFDs but what we also have we have a freeway which crosses the center of the region. And this might be the case in many large scale networks. And in that case actually given that we have seen that MFD cannot properly describe freeway networks we have to follow a different procedure. What also makes this problem interesting is that for freeway control we have developed and we have seen in this class adapt metering approach where the idea is that you control the rate that vehicles are entering from the on ramp to the freeway. So to avoid to have high level of concession on the freeway. So now we combine that up metering on the freeways together with perimeter control between the urban regions and our objective now becomes the minimization of the total delay of the system. That includes the delay of Region One Region 2 and also the freeway. So let's see how we can solve this problem.

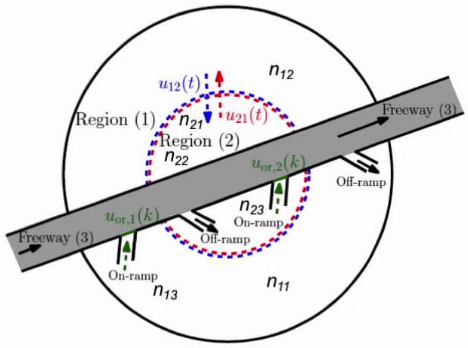
Notes

Summary

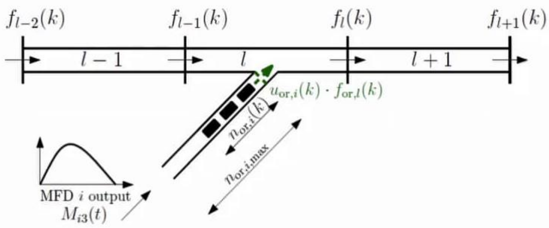
18m 47s

Problem II system characteristics

- The urban regions are modeled with MFDs
- The traffic dynamics of the freeway are modeled with the asymmetric cell transmission model (ACTM)
- Dynamic trip route choice
- Time dependent O-Ds



O\ D	1	2	3
1	$q_{11}: 1 \to 1$ $q_{131}: 1 \to 3 \to 1$	$q_{12}: 1 \to 2$ $q_{132}: 1 \to 3 \to 2$	$q_{13}: 1 \to 3$ $q_{123}: 1 \to 2 \to 3$
2	$q_{21}: 2 \to 1$ $q_{231}: 2 \to 3 \to 1$	$q_{22}: 2 \to 2$	$q_{23}: 2 \to 3$ $q_{213}: 2 \to 1 \to 3$
3	$q_{31}: 3 \to 1$ $q_{321}: 3 \to 2 \to 1$	$q_{32}: 3 \to 2$ $q_{312}: 3 \to 1 \to 2$	$q_{33}: 3 \to 3$



So the first thing we need to answer is how we model a traffic propagation on freeways and ACTM is not a very appropriate tool. What we choose would choose a well-established and numerical scheme which is called Cell transmission model and a description of these have been provided to one of the previous lecture in this book. So the traffic dynamics on the freeway our model actually with a version of ACTM which is called asymmetric CTM and this is asymmetric CTM is an approximation of the classical CTM but it has some linearized standards that make the controller perform faster and I will refer the interested reader to the class material to see in more detail the difference between an asymmetric CTM and an normal CTM. What we also have additionally in this problem is we have dynamic trip through choice. So let's consider that we have three regions. Region 1 is the periphery Region 2 is the center in Region 3 is the freeway and we have a 3 by 3 O/D patterm noted in destination pattern. So let's choose for example origin 1 and Destination 2.

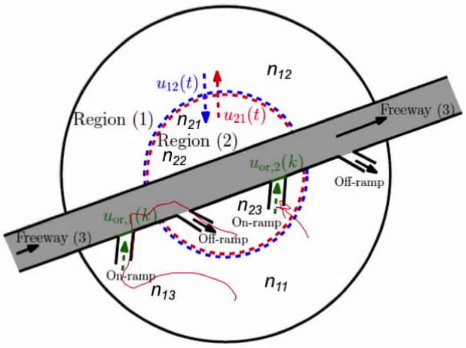
Notes

Summary

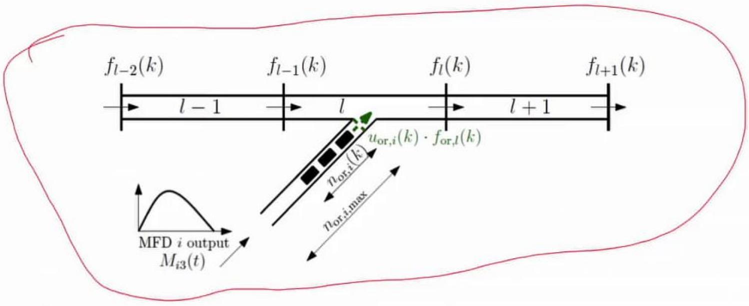
20m 13s

Problem II system characteristics

- The urban regions are modeled with MFDs
- The traffic dynamics of the freeway are modeled with the asymmetric cell transmission model (ACTM)
- Dynamic trip route choice
- Time dependent O-Ds



O\ D	1	2	3
1	$q_{11}: 1 \to 1$ $q_{131}: 1 \to 3 \to 1$	$q_{12}: 1 \to 2$ $q_{132}: 1 \to 3 \to 2$	$q_{13}: 1 \to 3$ $q_{123}: 1 \to 2 \to 3$
2	$q_{21}: 2 \to 1$ $q_{231}: 2 \to 3 \to 1$	$q_{22}: 2 \to 2$	$q_{23}: 2 \to 3$ $q_{213}: 2 \to 1 \to 3$
3	$q_{31}: 3 \to 1$ $q_{321}: 3 \to 2 \to 1$	$q_{32}: 3 \to 2$ $q_{312}: 3 \to 1 \to 2$	$q_{33}: 3 \to 3$

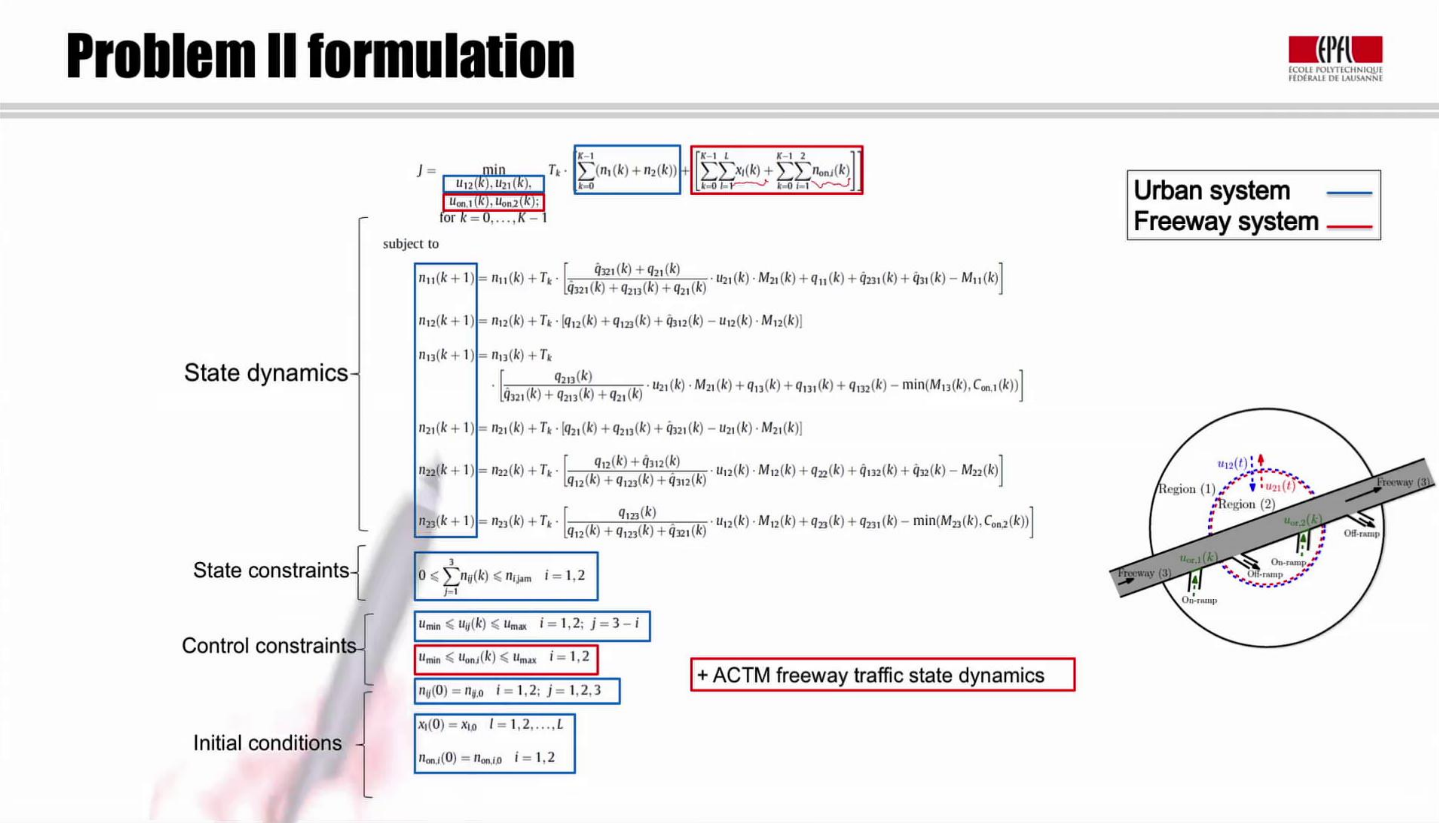


So people can have different options to go from 1 to 2 They can go directly from region to region 2 and if this happens then in that case they need to cross some boundary that this boundary can be can have perimeter control and other alternative is they can go from one to take the freeway traveling the freeway and then exit in the region 2. This is another alternative. So what actually happens in reality is based on the level of conversion on the different parts of the city. People might change their route choice dynamically for example by following online advising system or also based on experience by traveling in different days. So here I also show you how the combined freeway and an MFSs system look like. So here you see the way that we model through the ACTM the freeway system so the freeway system is divided in some cells. We have an on ramp but let's not forget that these on rumP has some maximum standards. So if this on ramp becomes full then it means that this queues will influence the situation inside the urban part of the city. So the only flow that reaches the on ramp is coming from the MFD of some specific region and in a similar way the outflow of the off frames is the input to the regions one in two.

Notes

Summary

21m 30s



So now let's see our problem formulation that might look much more tedious over here but it really follows the same reasoning as the simpler formulation we described in our problem. So here the objective function is to minimize the total delays over a control horizon which is between time zero and time K minus one and the blue represents the urban system while the red represents the freeway system. So they total delay that our travel in the urban system is simply speaking the sum of the number of vehicles a sum of accumulations for each of the regions over the course of time and the same actually is for the freeways where what we have we have the number of vehicles. In its freeway cell believes the number of vehicles in the onramp and also the off ramp which is part of it. So then what we have we have state dynamics for the end of the system and we also have dynamic equations for the ACTM that are not presented over here. And we also have state constraints and control constraints and also initial conditions as in the previous problem. So I will describe only one of the dynamic equations. But the rest of them have some similar trends. So let's see at the first one.

Notes

Summary

23m 15s

Problem II formulation

Urban system
Freeway system

$$J = \min_{\substack{u_{12}(k),\, u_{21}(k), \\ u_{\mathrm{on},1}(k),\, u_{\mathrm{on},2}(k); \\ \text{for } k=0,\dots,K-1}} T_k \cdot \left[\sum_{k=0}^{K-1}(n_1(k)+n_2(k))\right] + \left[\sum_{k=0}^{K-1}\sum_{l=1}^{L} x_l(k) + \sum_{k=0}^{K-1}\sum_{i=1}^{2} n_{\mathrm{on},i}(k)\right]$$

subject to

State dynamics

$$n_{11}(k+1) = n_{11}(k) + T_k \cdot \left[\frac{\hat{q}_{321}(k)+q_{21}(k)}{\hat{q}_{321}(k)+q_{213}(k)+q_{21}(k)} \cdot u_{21}(k)\cdot M_{21}(k) + q_{11}(k) + \hat{q}_{231}(k) + \hat{q}_{31}(k) - M_{11}(k)\right]$$

$$n_{12}(k+1) = n_{12}(k) + T_k \cdot [q_{12}(k) + q_{123}(k) + \hat{q}_{312}(k) - u_{12}(k)\cdot M_{12}(k)]$$

$$n_{13}(k+1) = n_{13}(k) + T_k \cdot \left[\frac{q_{213}(k)}{\hat{q}_{321}(k)+q_{213}(k)+q_{21}(k)} \cdot u_{21}(k)\cdot M_{21}(k) + q_{13}(k) + q_{131}(k) + q_{132}(k) - \min(M_{13}(k), C_{\mathrm{on},1}(k))\right]$$

$$n_{21}(k+1) = n_{21}(k) + T_k \cdot [q_{21}(k) + q_{213}(k) + \hat{q}_{321}(k) - u_{21}(k)\cdot M_{21}(k)]$$

$$n_{22}(k+1) = n_{22}(k) + T_k \cdot \left[\frac{q_{12}(k)+\hat{q}_{312}(k)}{q_{12}(k)+q_{123}(k)+\hat{q}_{312}(k)} \cdot u_{12}(k)\cdot M_{12}(k) + q_{22}(k) + \hat{q}_{132}(k) + \hat{q}_{32}(k) - M_{22}(k)\right]$$

$$n_{23}(k+1) = n_{23}(k) + T_k \cdot \left[\frac{q_{123}(k)}{q_{12}(k)+q_{123}(k)+\hat{q}_{321}(k)} \cdot u_{12}(k)\cdot M_{12}(k) + q_{23}(k) + q_{231}(k) - \min(M_{23}(k), C_{\mathrm{on},2}(k))\right]$$

State constraints

$$0 \leqslant \sum_{j=1}^{3} n_{ij}(k) \leqslant n_{i,\mathrm{jam}} \quad i=1,2$$

Control constraints

$$u_{\min} \leqslant u_{ij}(k) \leqslant u_{\max} \quad i=1,2;\ j=3-i$$

$$u_{\min} \leqslant u_{\mathrm{on},i}(k) \leqslant u_{\max} \quad i=1,2$$

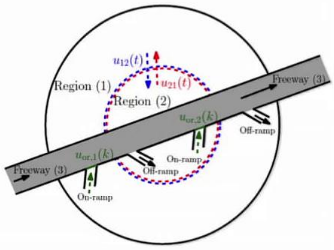
Initial conditions

$$n_{ij}(0) = n_{ij,0} \quad i=1,2;\ j=1,2,3$$

$$x_l(0) = x_{l,0} \quad l=1,2,\dots,L$$

$$n_{\mathrm{on},i}(0) = n_{\mathrm{on},i,0} \quad i=1,2$$

+ ACTM freeway traffic state dynamics



So this shows the categorization of conservation low where what we see we see that the number of vehicles in the region one with destination one at a specific time K plus 1 is equal to the previous time plus the input minus the output for this specific accumulation. So let's look at the different terms. So M11 which is the easiest term is the output which is similar as in our problem 1. And now let's see the inputs. So what I see here. I see q11 which is the uncontrolled flow which is generated in Region 1. Then we have additional uncontrolled flows and these are the uncontrolled flows that are coming from the off ramp to Region 1. So these are the uncontrolled flows q31 but also the uncontrolled flows that started from Region 2 and through the freeway the go to Region 1. So these and that and the only reason that we use a hat in these uncontrolled flows q231 in q31 is simply speaking because the CTM is not able to describe and keep track of the origin of the vehicles when they reach there off ramp. So this is something we estimate by considering what are the initial or details of the system and I would like to look at the course material for additional information and then let's look at this term.

Notes

Summary

24m 46s

Problem II formulation

Urban system
Freeway system

$$J = \min_{\substack{u_{12}(k),\, u_{21}(k),\\ u_{on,1}(k),\, u_{on,2}(k);\\ \text{for } k=0,\dots,K-1}} T_k \cdot \left[\sum_{k=0}^{K-1}(n_1(k)+n_2(k)) + \left[\sum_{k=0}^{K-1}\sum_{l=1}^{L} x_l(k) + \sum_{k=0}^{K-1}\sum_{i=1}^{2} n_{on,i}(k)\right]\right]$$

subject to

State dynamics

$$n_{11}(k+1) = n_{11}(k) + T_k \cdot \left[\frac{\hat{q}_{321}(k)+q_{21}(k)}{\hat{q}_{321}(k)+q_{213}(k)+q_{21}(k)} \cdot u_{21}(k) \cdot M_{21}(k) + q_{11}(k) + \hat{q}_{231}(k) + \hat{q}_{31}(k) - M_{11}(k)\right]$$

$$n_{12}(k+1) = n_{12}(k) + T_k \cdot [q_{12}(k) + q_{123}(k) + \hat{q}_{312}(k) - u_{12}(k) \cdot M_{12}(k)]$$

$$n_{13}(k+1) = n_{13}(k) + T_k \cdot \left[\frac{q_{213}(k)}{\hat{q}_{321}(k)+q_{213}(k)+q_{21}(k)} \cdot u_{21}(k) \cdot M_{21}(k) + q_{13}(k) + q_{131}(k) + q_{132}(k) - \min(M_{13}(k), C_{on,1}(k))\right]$$

$$n_{21}(k+1) = n_{21}(k) + T_k \cdot [q_{21}(k) + q_{213}(k) + \hat{q}_{321}(k) - u_{21}(k) \cdot M_{21}(k)]$$

$$n_{22}(k+1) = n_{22}(k) + T_k \cdot \left[\frac{q_{12}(k)+\hat{q}_{312}(k)}{q_{12}(k)+q_{123}(k)+\hat{q}_{312}(k)} \cdot u_{12}(k) \cdot M_{12}(k) + q_{22}(k) + \hat{q}_{132}(k) + \hat{q}_{32}(k) - M_{22}(k)\right]$$

$$n_{23}(k+1) = n_{23}(k) + T_k \cdot \left[\frac{q_{123}(k)}{q_{12}(k)+q_{123}(k)+\hat{q}_{321}(k)} \cdot u_{12}(k) \cdot M_{12}(k) + q_{23}(k) + q_{231}(k) - \min(M_{23}(k), C_{on,2}(k))\right]$$

State constraints

$$0 \leqslant \sum_{j=1}^{3} n_{ij}(k) \leqslant n_{i,jam} \quad i=1,2$$

Control constraints

$$u_{min} \leqslant u_{ij}(k) \leqslant u_{max} \quad i=1,2;\ j=3-i$$

$$u_{min} \leqslant u_{on,i}(k) \leqslant u_{max} \quad i=1,2$$

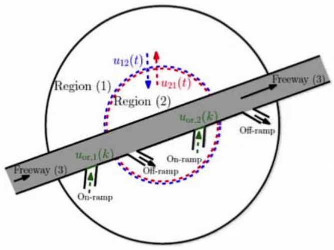
Initial conditions

$$n_{ij}(0) = n_{ij,0} \quad i=1,2;\ j=1,2,3$$

$$x_l(0) = x_{l,0} \quad l=1,2,\dots,L$$

$$n_{on,i}(0) = n_{on,i,0} \quad i=1,2$$

+ ACTM freeway traffic state dynamics



So this is the transfer flow from region 2 to region 1 which actually can be controlled by changing the control variable u21 but not all vehicles that go from two to one have as destination region one. So this is what this term over here shows. So here we take the fraction of the inflows of the system with destination 1 which is the flow from 2 to 1. And also the flow from the freeway to region to the region one divided by the all the flows related to the transfer from two to one. And in a similar way of thing you can also describe the other equations.

Notes

Summary

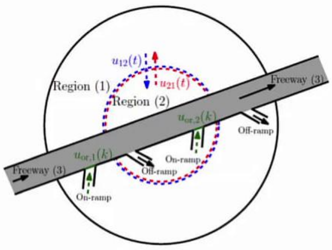
26m 29s

Performance of different controllers

1. **ALINEA Ramp metering:**

$$f_{or,l}(k+1) = f_{or,l}(k) + \kappa\big(x_l(k) - x_{ref}\big)$$

2. **Urban MPC + ALINEA**
3. **Decentralized MPC (Urban MPC + Freeway MPC)**
4. **Centralized MPC (Network MPC)**



	ALINEA	Urban MPC + ALINEA	D-MPC	MPC
Region 1	524	504	537	196
Region 2	121	162	279	143
Freeway	129	129	124	218
On-Ramp 1	13	13	16	8
On-Ramp 2	16	16	16	16
Network	803	823	972	581

So now let's look at the performance of different controllers. So here I would like to draw your attention a on the importance of coordination. So this makes systems of urban and freeways. What is important is that the two entities the freeway and the Urban Network coordinate and collaborate. Otherwise as we will see the result of the control actions can be very problematic and sometimes can be disastrous for the system. So here we look at four different control strategies. So the first one is a very classical ramp metering strategy a local ramp metering strategy ALINA that is widely used both in theory but also in applications and is a very simple controller which actually tries to keep the occupancy of the downstream cell of the on ramp at a reference value. And then what we have we have a controller number two where is a combination of ALINA for freeways and an urban MPC for the Urban Network so the urban MPC is actually what we applied in problem number one. Our objective is to see minimize delays for only the urban network and we cannot intervene in the freeway and then we imply decentralized MPC with the he Urban Network tries to optimize what is best for the Urban Network and the freeway tries to optimize what is best only for its part.

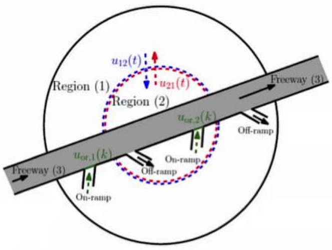
Notes

Summary

27m 18s

Performance of different controllers

1. **ALINEA Ramp metering:**
$$f_{or,l}(k+1) = f_{or,l}(k) + \kappa\big(x_l(k) - x_{ref}\big)$$
2. **Urban MPC + ALINEA**
3. **Decentralized MPC (Urban MPC + Freeway MPC)**
4. **Centralized MPC (Network MPC)**



	ALINEA	Urban MPC + ALINEA	D-MPC	MPC
Region 1	524	504	537	196
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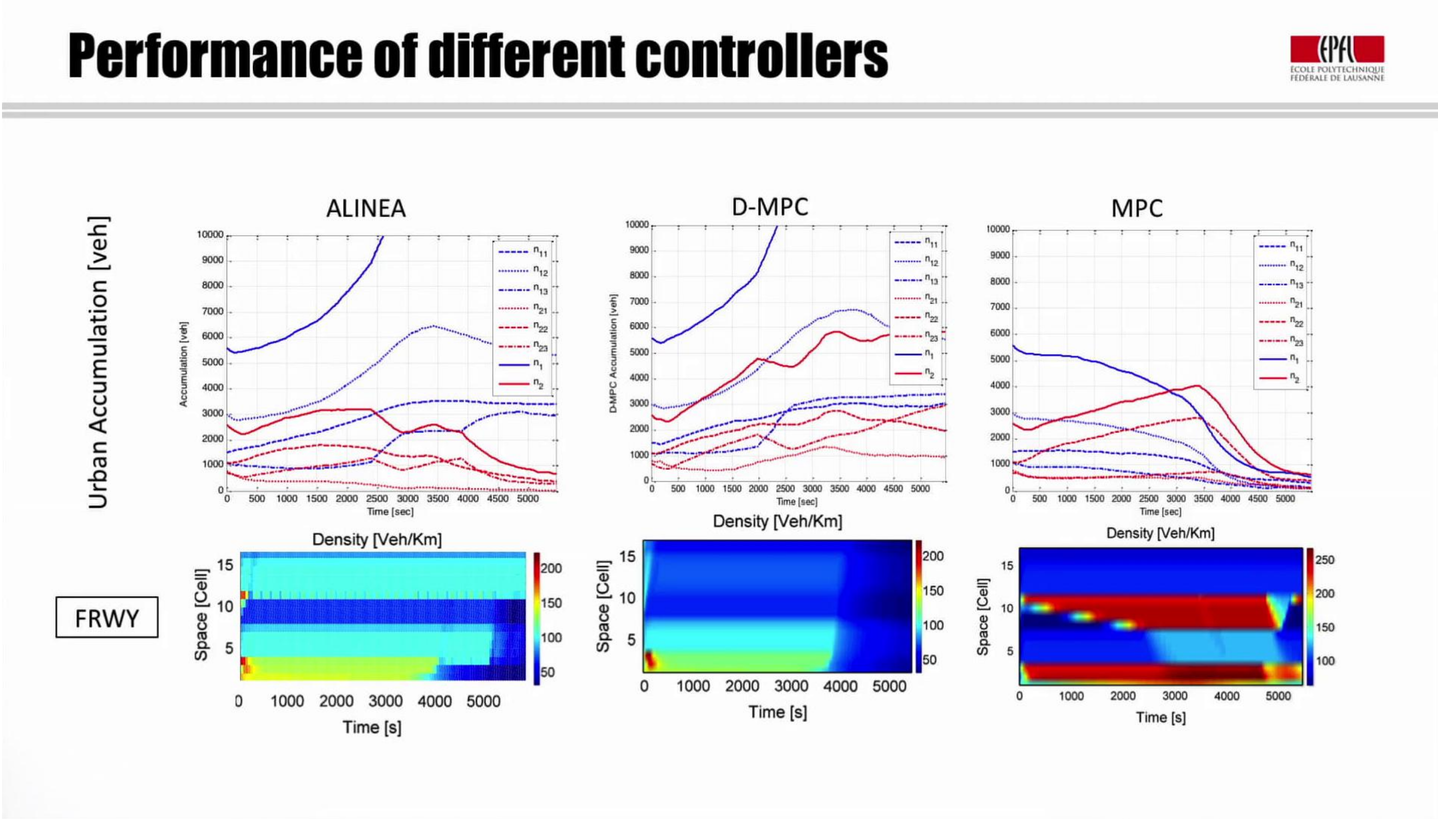
Freeway	129	129	124	218
On-Ramp 1	13	13	16	8
On-Ramp 2	16	16	16	16
Network	803	823	972	581

So he's only the blue and the red terms of the objective function. In the final part what we have we have the centralized MPC which is the full formulation we saw in the previous lecture. So what do we see in this table. We see what are the total vehicles traveled in each of the regions with four different controllers. So for example if we do ramp metering what we see we see that the total delays in the freeway are very small. But there are significant delays in the urban region one of the system. And this as a result have the total network delay to be very high. If we apply decentralized strategies two or three what we see we see that what we have we have again significant issues because actually the two strategies are antagonistic. Why if we do a centralized strategy okay we might penalize a little bit the freeway. So we see the freeway has some vehicle travel a little bit more compared to the other strategies but the overall improvement for the whole system is very significant.

Notes

Summary

28m 58s



Let's look at some additional graphs show over here in the top we see for three strategies. What is the performance of the Urban Network with time series and what is that performance for the freeways. So over here what we see we see that in the case of ALINA our decentralized MPC region number on accumulations significantly increase in world must reach a state of greedy while during that the freeway operates extremely well. So maybe I need to better define this is our control plots where the horizontal axis is time the vertical axis is space so zero is the beginning of the freeway and 15 is the end of the freeway and the color over here shows what is the density of veh. So red means very high congestion while blue means very low congestion so I linear MPC are able to operate the freeway at very good conditions as we see over here with very low density while when we do coordination we might need to keep some vehicles on the freeway but creating some congestion in some queues as we see by these red areas. But as a result what we succeed we succeed that the urban network even if we start from congested conditions by the end all regions become uncongested. So this is the added value of coordination and actually the state of practice is not at this level and definitely this is something we need to consider as traffic engineers.

Notes

Summary

30m 22s

Summary

So what we have seen so far is that by applying model predictive control for dynamic problems of urban networks or mixed urban freeway networks represented by MFDs we can significantly improve the level of congestion nd this can be performed better for all the users of the system. We will continue in the next video with a more complicated case study also related to MPC.

Notes

Summary

31m 59s