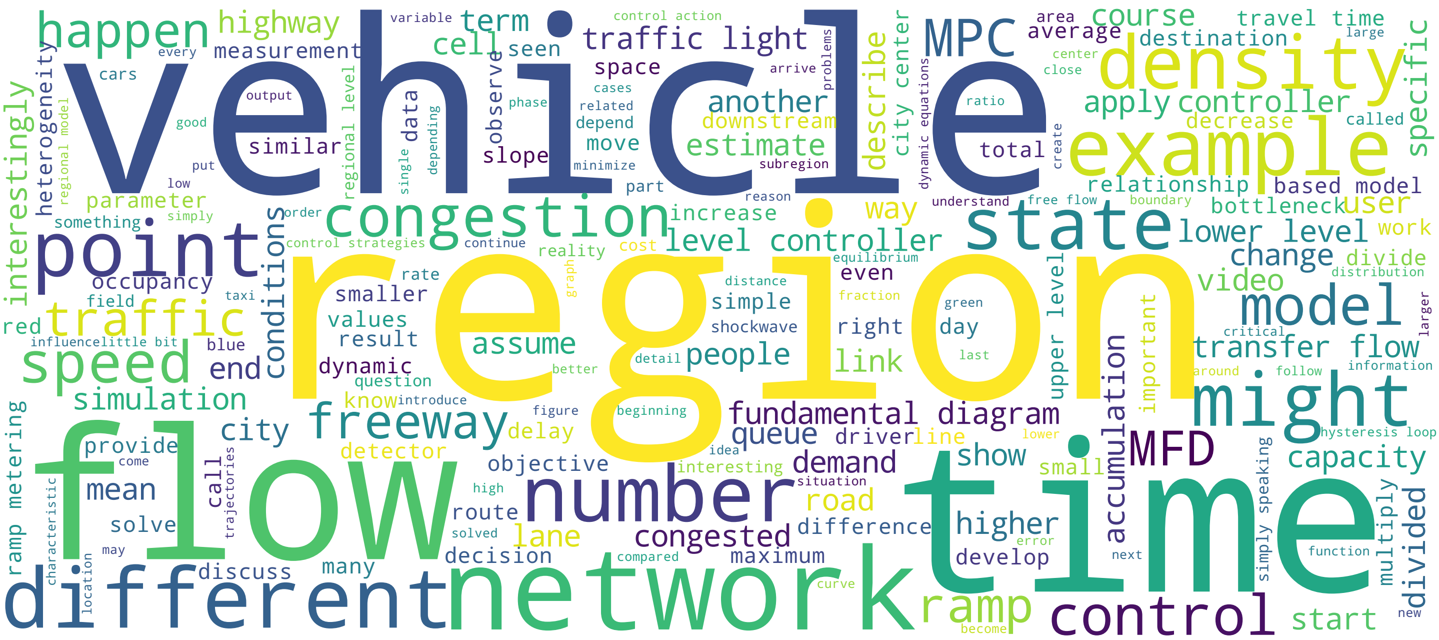
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Model predictive control with multi-region MFDs (part II)

Intro to traffic flow modeling and ITS

Prof. Nikolas Geroliminis



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Video

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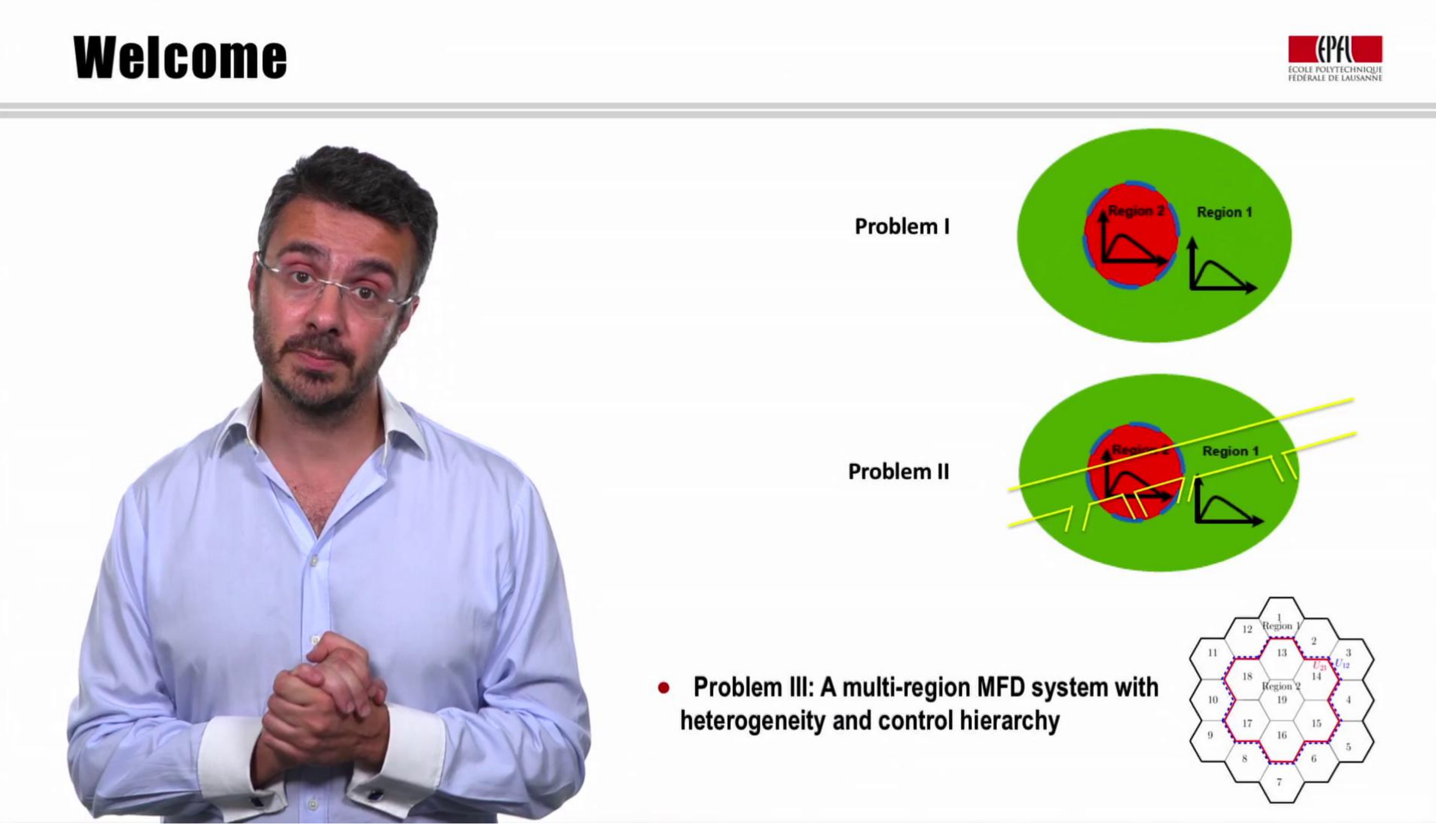
Welcome



Welcome. Let's continue our lectures with respect to applying model predictive control for systems governed by MFD. So, after the previous part dealing with problem 1 and Problem 2 so to urban regions or to urban regions crossed by freeway let's now try to understand better. In this video a little bit more complicated case which it will consider a controller that has two layers an upper and a lower layer. But it will also try to capture the spatial heterogeneity of congestion during optimization.

Notes

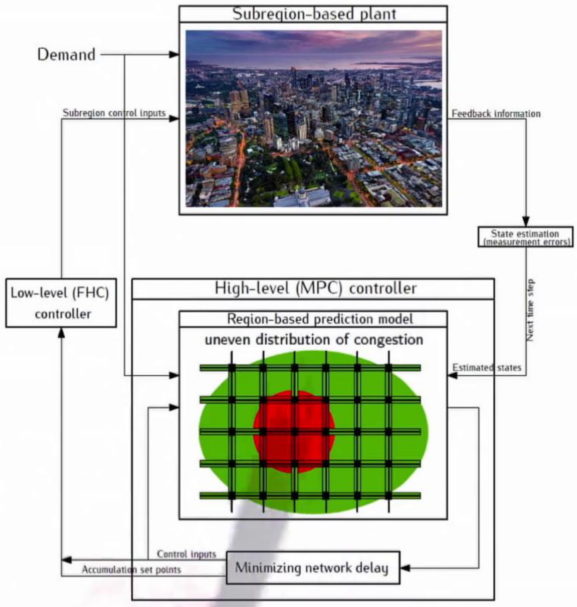
Summary

0m 05s

Problem III statement

Motivations

- Investigating the effect of heterogeneity on the MFD
- Designing perimeter control strategies to control the heterogeneity
- Hierarchical control (upper and lower level)



Ramezani, Haddad, Geroliminis (2015) –Trans. Res. part B

So compared to a classical problem like which solved with two regions where each region has a well-defined MFD we would like to make additional extensions. So, extension number one is that within a region we might have heterogeneity that changes during the time of the day either because of differing demand or even because of control actions. The second extension would like to do is related with the control we apply so so far we only looked at upper level perimeter control. So there is a single u that controls the transfer flow between one region and its neighbor. But actually if we want to apply this on the field in the real world order among this perimeter there might be plenty of roads or they might be plenty of traffic lights. So the question is how we can combine the upper level controller with a lower level controller to succeed in the objectives of minimum delays for our system. So this is what we're gonna see in the problem here. So as before we consider that our city is divided in a number of regions for simplicity let's assume too. But interestingly each of these regions is divided in a number of sub regions and we might not have in reality perfect information about the conditions that happen in the sub regions would because we might not have sufficient censoring techniques.

Notes

Summary

0m 42s

Problem III statement

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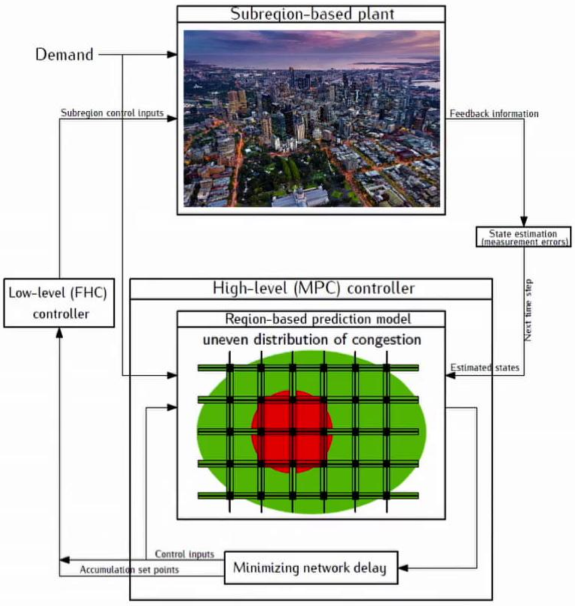
❑ Motivations

- Investigating the effect of heterogeneity on the MFD
- Designing perimeter control strategies to control the heterogeneity
- Hierarchical control (upper and lower level)



Ramezani, Haddad, Geroliminis (2015) –Trans. Res. part B

But the reality will be represented by these sub regional based model which is more detailed. So, here what would develop we develop an MPC framework that has a higher level MPC which is solved at the regional level. And then there is a lower level which is solved between the regional and the sub regional model. In the reality the plant is operating at the sub regional level were actually some estimation of the states of the system performed over here with some measurement errors. In these field again the upper level MPC and there is an open optimization loop to solve this problem.

Notes

Summary

2m 21s

Subregion-based model

$$\frac{dn_{ii}(t)}{dt} = q_{ii}(t) - m_{ii}(t) + \sum_{h\in\mathcal{H}_i} u_{hi}(t).\hat{m}^i_{hi}(t)$$

$$\frac{dn_{ij}(t)}{dt} = q_{ij}(t) - \sum_{h\in\mathcal{H}_i} u_{ih}(t).\hat{m}^h_{ij}(t) + \sum_{h\in\mathcal{H}_i;h\neq j} u_{hi}(t).\hat{m}^i_{hj}(t) \quad \forall j \in \mathcal{H}_i$$

$$\frac{dn_{ir}(t)}{dt} = q_{ir}(t) - \sum_{h\in\mathcal{H}_i} u_{ih}(t).\hat{m}^h_{ir}(t) + \sum_{h\in\mathcal{H}_i} u_{hi}(t).\hat{m}^i_{hr}(t) \quad i \neq r; \forall r \notin \mathcal{H}_i$$

$$0 \leq u_{min} \leq u_{ih}(t) \leq u_{max} \leq 1$$

$$m_{ii}(t) = \frac{n_{ii}(t)}{n_i(t)} \times \frac{p_i(n_i(t))}{l_i(t)}$$

$$m^h_{ij}(t) = \theta^h_{ij}(t) \times \frac{n_{ij}(t)}{n_i(t)} \times \frac{p_i(n_i(t))}{l_i(t)}$$

$$\hat{m}^h_{ij}(t) = \min(m^h_{ij}(t), \frac{m^h_{ij}(t)}{\sum_k m^h_{ik}(t)} r_{ih}(n_h(t)))$$

$q_{ij}(t)$: demand from subregion *i* to *j* [veh/s]

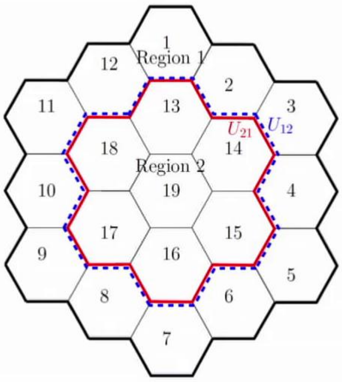
$n_{ij}(t)$: accumulation in subregion *i* with final destination *j* [veh]

$n_i(t)$: accumulation in subregion *i* [veh]

$p_i(n_i(t))$: production in subregion *i* [veh.m/s]

$l_i(t)$: average trip length in subregion *i* [m]

$m^h_{ij}(t)$: transfer flow from subregion *i* to *h* with final destination *j*



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So I would like now to define the sub regional and also the regional model. So here is our case study to a network example. So consider that we have two large regions the perimeter of the city and also the city center which are separated by these boundaries the blue and the red line. And what we have we have perimeter controllers that are operating between these boundary. Region one is the periphery region and region two is the center and then each of these regions is divided in a number of sub regions. So we have seven subregions for the center and twelve sub regions for the perimeter. So, let's now see how first of all this average and based model looks like. So there is no need to describe these from scratch because this modeling is very similar to what we have seen in some of problems one and problem two of MPC we deal with. But let's take as an example this dynamic equation. So what we see we see that the rate that nii changes is equal to the uncontrolled inflow which is generated from region i with destination region minus the outflow within the region plus the transfer flows that come in the region.

Notes

Summary

3m 07s

Subregion-based model

$$\frac{\mathrm{d}n_{ii}(t)}{\mathrm{d}t} = q_{ii}(t) - m_{ii}(t) + \sum_{h \in \mathcal{H}_i} u_{hi}(t).\widehat{m}_{hi}^{i}(t)$$

$$\frac{\mathrm{d}n_{ij}(t)}{\mathrm{d}t} = q_{ij}(t) - \textstyle\sum_{h \in \mathcal{H}_i} u_{ih}(t).\widehat{m}_{ij}^{h}(t) + \sum_{h \in \mathcal{H}_i; h \neq j} u_{hi}(t).\widehat{m}_{hj}^{i}(t) \qquad \forall\, j \in \mathcal{H}_i$$

$$\frac{\mathrm{d}n_{ir}(t)}{\mathrm{d}t} = q_{ir}(t) - \textstyle\sum_{h \in \mathcal{H}_i} u_{ih}(t).\widehat{m}_{ir}^{h}(t) + \sum_{h \in \mathcal{H}_i} u_{hi}(t).\widehat{m}_{hr}^{i}(t) \qquad i \neq r; \forall\, r \notin \mathcal{H}_i$$

$$0 \leq u_{min} \leq u_{ih}(t) \leq u_{max} \leq 1$$

$$m_{ii}(t) = \frac{n_{ii}(t)}{n_i(t)} \times \frac{p_i(n_i(t))}{l_i(t)}$$

$$m_{ij}^{h}(t) = \theta_{ij}^{h}(t) \times \frac{n_{ij}(t)}{n_i(t)} \times \frac{p_i(n_i(t))}{l_i(t)}$$

$$\widehat{m}_{ij}^{h}(t) = \min\left(m_{ij}^{h}(t), \frac{m_{ij}^{h}(t)}{\sum_k m_{ik}^{h}(t)} r_{ih}(n_h(t))\right)$$

$q_{ij}(t)$: demand from subregion *i* to *j* [veh/s]

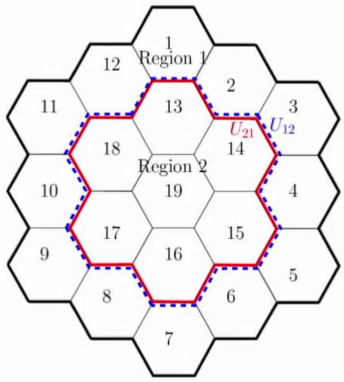
$n_{ij}(t)$: accumulation in subregion *i* with final destination *j* [veh]

$n_i(t)$: accumulation in subregion *i* [veh]

$p_i(n_i(t))$: production in subregion *i* [veh.m/s]

$l_i(t)$: average trip length in subregion *i* [m]

$m_{ij}^{h}(t)$: transfer flow from subregion *i* to *h* with final destination *j*



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So hi are the neighboring regions of i and then the transfer flow that happens and is actually controlled by some variables uhi and u over here is a lowercase. And what is this term over here. This is the transfer flow but we represent it with three terms. So is the transfer flow from region h with final destination region i where i in the top this index is the next destination after h. So here you see that these two i's are identical but if for example we look at another differential equation what we will see we will see over here we have some transfer flows that have terms mhji which are described as the transfer flow as we see over here from some region i to h with final destination with j. And over here what we also see we see that we have some terms with only m and we have also sometimes with m hat. So let's describe this now. So mi is the ending flow within a reservoir within a sub region and actually what we have we have that it subregion is governed by a production nft pi ni of t which shows the relationship between vehicles meters or vehicle kilometer traveled per unit time and accumulation.

Notes

Summary

4m 37s

Subregion-based model

$$\frac{\mathrm{d}n_{ii}(t)}{\mathrm{d}t} = q_{ii}(t) - m_{ii}(t) + \sum_{h\in\mathcal{H}_i} u_{hi}(t).\widehat{m}_{hi}^{i}(t)$$

$$\frac{\mathrm{d}n_{ij}(t)}{\mathrm{d}t} = q_{ij}(t) - \sum\nolimits_{h\in\mathcal{H}_i} u_{ih}(t).\widehat{m}_{ij}^{h}(t) + \sum\nolimits_{h\in\mathcal{H}_i;h\neq j} u_{hi}(t).\widehat{m}_{hj}^{i}(t) \qquad \forall\, j \in \mathcal{H}_i$$

$$\frac{\mathrm{d}n_{ir}(t)}{\mathrm{d}t} = q_{ir}(t) - \sum\nolimits_{h\in\mathcal{H}_i} u_{ih}(t).\widehat{m}_{ir}^{h}(t) + \sum\nolimits_{h\in\mathcal{H}_i} u_{hi}(t).\widehat{m}_{hr}^{i}(t) \qquad i \neq r; \forall\, r \notin \mathcal{H}_i$$

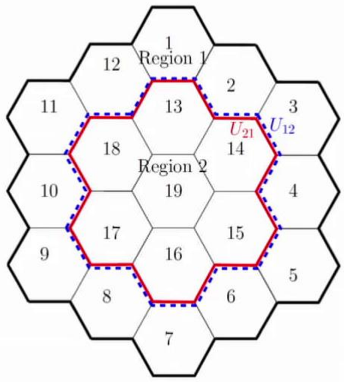
$$0 \leq u_{min} \leq u_{ih}(t) \leq u_{max} \leq 1$$

$$m_{ii}(t) = \frac{n_{ii}(t)}{n_i(t)} \times \frac{p_i(n_i(t))}{l_i(t)}$$

$$m_{ij}^{h}(t) = \theta_{ij}^{h}(t) \times \frac{n_{ij}(t)}{n_i(t)} \times \frac{p_i(n_i(t))}{l_i(t)}$$

$$\widehat{m}_{ij}^{h}(t) = \min\left(m_{ij}^{h}(t), \frac{m_{ij}^{h}(t)}{\sum_k m_{ik}^{h}(t)} r_{ih}(n_h(t))\right)$$

- $q_{ij}(t)$: demand from subregion *i* to *j* [veh/s]
- $n_{ij}(t)$: accumulation in subregion *i* with final destination *j* [veh]
- $n_i(t)$: accumulation in subregion *i* [veh]
- $p_i(n_i(t))$: production in subregion *i* [veh.m/s]
- $l_i(t)$: average trip length in subregion *i* [m]
- $m_{ij}^{h}(t)$: transfer flow from subregion *i* to *h* with final destination *j*



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And we divide these by lit which is the average trip length in the region li so that this ratio shows three endings and then we need to multiply as before by the fraction of vehicles that really are associated with these specific airflow. The difference now if we look at mij is that of course we can see we need to consider that when somebody goes from i to j it passes through a different sequence of region and here the next region after i region is h. So these thetas represent route choices at the regional level of travellers. So the idea here in this sub regional model is that every time vehicles are transferred from one region to another after they transfer they recalculate what is the optimal route they would like to take to reach their destination from their personal point of view and then the rest is very similar. What I did not define yet I didn't define yet the transfer flows with a hat and actually I would like to refresh your memory about what we discussed in the past about boundary capacity. So this means that if the receiving region is very congested then you might not be able to send as many vehicle as we want because there is no available room for these vehicles treats.

Notes

Summary

6m 23s

Subregion-based model

$$\frac{\mathrm{d}n_{ii}(t)}{\mathrm{d}t} = q_{ii}(t) - m_{ii}(t) + \sum_{h \in \mathcal{H}_i} u_{hi}(t).\widehat{m}_{hi}^{i}(t)$$

$$\frac{\mathrm{d}n_{ij}(t)}{\mathrm{d}t} = q_{ij}(t) - \sum_{h \in \mathcal{H}_i} u_{ih}(t).\widehat{m}_{ij}^{h}(t) + \sum_{h \in \mathcal{H}_i; h \neq j} u_{hi}(t).\widehat{m}_{hj}^{i}(t) \quad \forall j \in \mathcal{H}_i$$

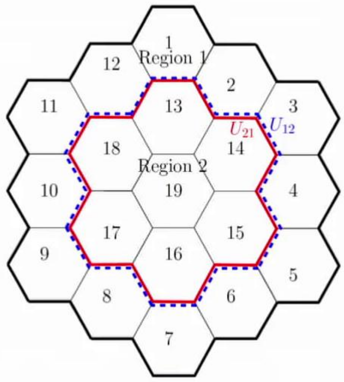
$$\frac{\mathrm{d}n_{ir}(t)}{\mathrm{d}t} = q_{ir}(t) - \sum_{h \in \mathcal{H}_i} u_{ih}(t).\widehat{m}_{ir}^{h}(t) + \sum_{h \in \mathcal{H}_i} u_{hi}(t).\widehat{m}_{hr}^{i}(t) \qquad i \neq r; \forall r \notin \mathcal{H}_i$$

$$0 \leq u_{min} \leq u_{ih}(t) \leq u_{max} \leq 1$$

$$m_{ii}(t) = \frac{n_{ii}(t)}{n_i(t)} \times \frac{p_i(n_i(t))}{l_i(t)}$$

$$m_{ij}^{h}(t) = \theta_{ij}^{h}(t) \times \frac{n_{ij}(t)}{n_i(t)} \times \frac{p_i(n_i(t))}{l_i(t)}$$

$$\widehat{m}_{ij}^{h}(t) = \min\left(m_{ij}^{h}(t), \frac{m_{ij}^{h}(t)}{\sum_k m_{ik}^{h}(t)} r_{ih}(n_h(t))\right)$$



$q_{ij}(t)$: demand from subregion i to j [veh/s]
$n_{ij}(t)$: accumulation in subregion i with final destination j [veh]
$n_i(t)$: accumulation in subregion i [veh]
$p_i(n_i(t))$: production in subregion i [veh.m/s]
$l_i(t)$: average trip length in subregion i [m]
$m_{ij}^{h}(t)$: transfer flow from subregion i to h with final destination j

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So here what we have a minimum of what can be sent and what is this. This is the boundary capacity of the receiving region but we have to multiply by the fraction of vehicles which are associated with a specific transport. So, this is how the sub region based model works and these controlled variables uhi are actually all the control variables between the boundary in the sub region over here and now let's look at the region based model which is an abstraction as we only have two regions but 19 sub regions in this network show over here.

Notes

Summary

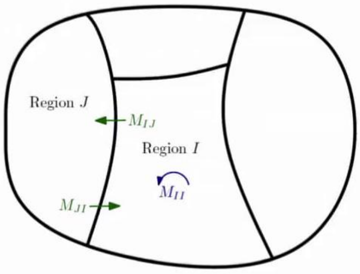
7m 58s

Region-based model

$$\frac{\mathrm{d}N_{II}(t)}{\mathrm{d}t} = Q_{II}(t) - M_{II}(t) + \sum_{J \in \mathcal{H}_I} U_{JI}(t).M_{JI}(t)$$

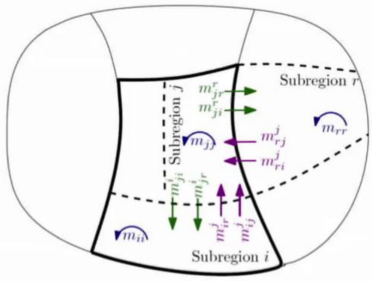
$$\frac{\mathrm{d}N_{IJ}(t)}{\mathrm{d}t} = Q_{IJ}(t) - \sum_{J \in \mathcal{H}_I} U_{IJ}(t).M_{IJ}(t) \qquad \forall J \in \mathcal{H}_I$$

$$0 \le U_{min} \le U_{IJ}(t) \le U_{max} \le 1$$



$$M_{II}(t) = \frac{N_{II}(t)}{N_I(t)} \times \frac{P_I(N_I(t), \sigma(N_I(t)))}{L_{II}(t)}$$

$$M_{IJ}(t) = \frac{N_{IJ}(t)}{N_I(t)} \times \frac{P_I(N_I(t), \sigma(N_I(t)))}{L_{IJ}(t)}$$



$Q_{IJ}(t)$: demand from region I to J; $J \in \mathcal{H}_I$ [veh/s]

$N_{IJ}(t)$: accumulation in region I with direct destination J [veh]

$N_I(t)$: accumulation in region I [veh]

$P_I(N_I(t), \sigma(N_I(t)))$: production in region I [veh.m/s]

$L_{II}(t)$: average trip length for trips in region I [m]

$L_{IJ}(t)$: average trip length for trips from region I to region J [m]

The dynamic equations for our regional based model are much simpler. They're almost identical with what we used for the problem one of the MPC. There is one significant difference the significant difference is on the MFD so as we have seen in previous lectures MFD is influenced by spatial heterogeneity in the distribution of congestion. So this means that if the sub regions that belong in one region are having different level of congestion these will influence the shape of the MFD. So this is expressed by considering that the regional MFD depends not only on the accumulation but the standard deviation of the sub regional accumulations. And of course we again we need to divide by some trip plans and multiply by ratio of vehicle associated with this transfer. So now what we see we see that we only have 2 control decisions which are the capital Uij and Uji.

Notes

Summary

8m 42s

Two-level hierarchical control

MPC objective function (High level control):

- $J = \min_{U_{IJ},U_{JI}} \sum_{I\in\mathcal{R},J\in\mathcal{H}_I} \int_0^{t_f} (N_{II}(t) + N_{IJ}(t))\mathrm{d}t$

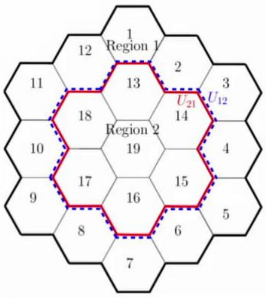
Low-level I controller (feedback homogeneity controller - FHC):

$$u_{ij}(k_c) = u_{ij}(k_c-1) + K_1 \cdot \left(\frac{N_J(k_c+K_p-1)}{|\mathscr{SR}_J|} - n_j(k_c)\right) - K_2 \cdot \left(\frac{N_I(k_c+K_p-1)}{|\mathscr{SR}_I|} - n_i(k_c)\right)$$

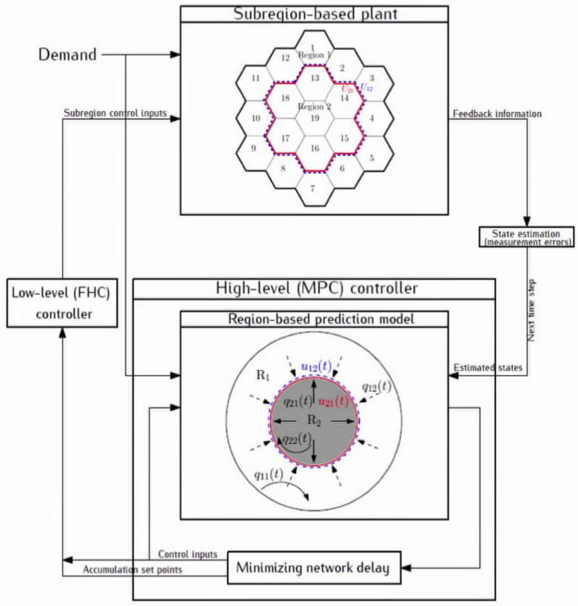
subject to:

$$\left|\sum u_{ij} - U_{IJ}\right| < \delta$$

$$0 \le U_{\min} \le u_{ij}(k) \le U_{\max} \le 1$$



K_p: MPC prediction horizon



So now after describing the dynamics of the system were ready to provide the hierarchical control framework that we will solve through MPC. So, the first level the higher level is the regional level where we only optimize the capital Uij and Uji U 1 and U 2 1 in the system. And our objective is to minimize the delays and this is subject to the dynamic equations we showed in the previous slide and now we introduce a lower level controller so the lower level controller has as an objective to make the sub regions having similar accumulation. Because in that way by decreasing the heterogeneity of the system what we expect. We expect that the flow of them everyday will be higher. So there is a relationship between the lower and the upper level which is given by this equation. So what we see we see that we put some constraint here where the lower level controllers should be consistent with the upper level. So Delta is a small value that actually tries to constrain our flexibility on choosing the lower level controller. We don't put Delta exactly equal to zero because this will make our control problem sometimes infeasible or very challenging. So the lower level controller is more like a tracking problem.

Notes

Summary

9m 47s

Two-level hierarchical control

MPC objective function (High level control):

- $J = \min_{U_{IJ}, U_{JI}} \sum_{I \in \mathcal{R}, J \in \mathcal{H}_I} \int_0^{t_f} (N_{II}(t) + N_{IJ}(t)) \mathrm{d}t$

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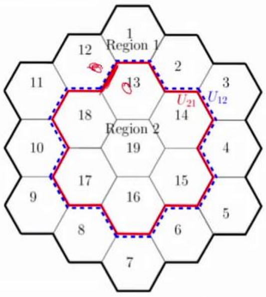
$$u_{ij}(k_c) = u_{ij}(k_c - 1) + K_1 \cdot \left(\frac{N_J(k_c + K_p - 1)}{|\mathcal{SR}_J|} - n_j(k_c)\right) - K_2 \cdot \left(\frac{N_I(k_c + K_p - 1)}{|\mathcal{SR}_I|} - n_i(k_c)\right)$$

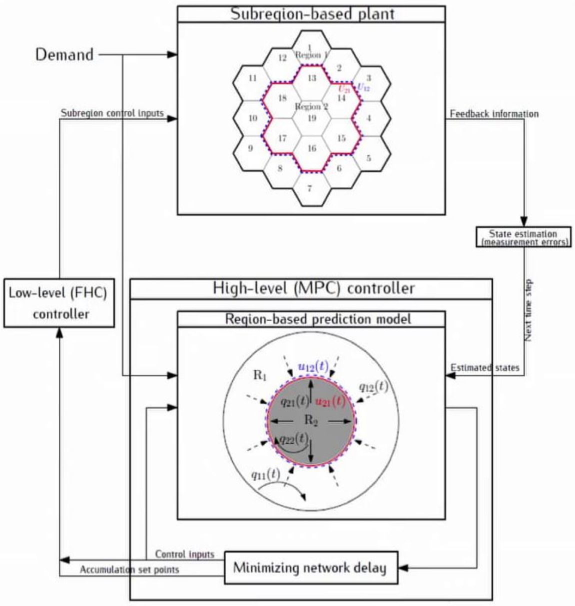
subject to:

$$\left|\sum u_{ij} - U_{IJ}\right| < \delta$$

$$0 \le U_{\min} \le u_{ij}(k) \le U_{\max} \le 1$$



K_p: MPC prediction horizon



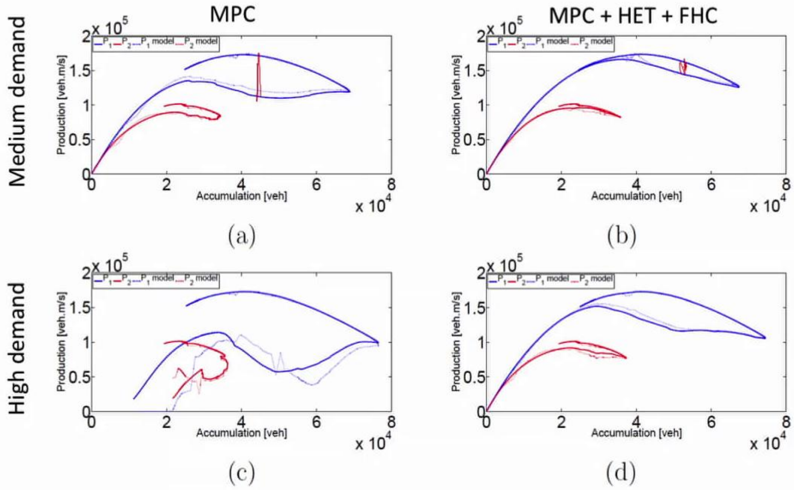
So what we have we have a set point for their individual controller for this set point is the average accumulation of all sub regions. So we take mg which is the total accumulation of the whole region. And we divide the denominator is the number of regions. And this is the targeted accumulation we want is sub region to have. The only difference is that we don't estimate this time tc which is a time we take the decision but we estimate these over the prediction horizon of the MPC so kp minus one steps later. So here what we have we have two gains one gain so if we look at the one specific boundary Uij what we look we look what is the accumulation on the region on the left and on the region on the right. And what we have these two sub regions have some specific values and depending how close they are to the target values this determines what will be the control action at the next step.

Notes

Summary

11m 18s

Performance of different controllers



Total network delay (veh.sec × 10^6)

Demand	No Control	MPC	MPC + HET + FHC
Medium	1069.4	573.8	518.0
High	1204.4	930.5	636.8

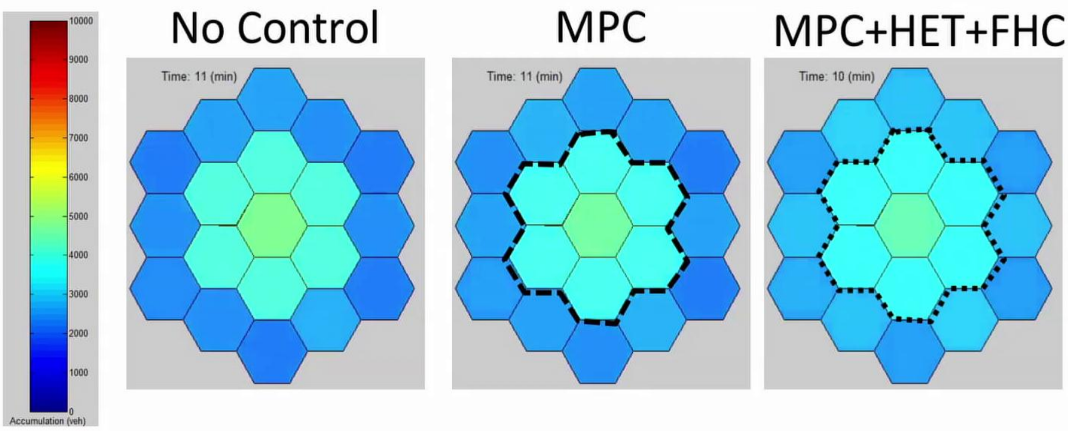
So let's see how these controllers operate in our case study. So we solve two cases with medium and high demand. And over here we test two different controllers. The one controller is only MPC without considering heterogeneity and while without considering the lower level feedback homogenized controller. And the second one is MPC plus HET plus FHC. And what do we see here. We see the MFD of the two regions. So what is very interesting if we ignore the heterogeneity or MPC will do better than no control. But unfortunately we will observe strong hysteresis loop and this hysteresis loop is simply speaking because the level of heterogeneity is much higher in the offset of congestion. While if we apply both upper and lower level controller what we see we see that our network is much more homogeneous and the hysteresis loop is smaller and similar results can be concluded for higher demand. But when conditions are much more congested also we are able to decrease the loop but the hysterics doesn't disappear and over here we see in this table that total network delay in the case of no control MPC and hierarchical control for medium and high level.

Notes

Summary

12m 32s

An illustration of different controllers



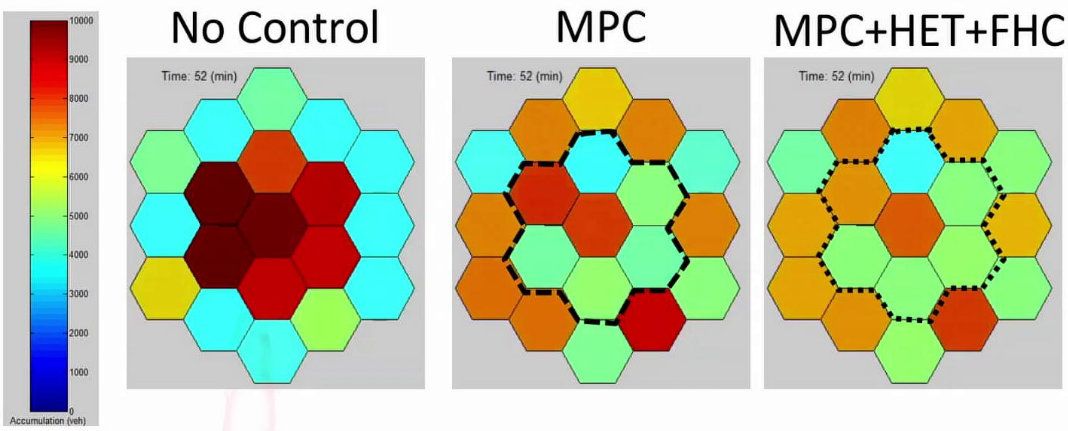
So let's see here an illustration for the three different controllers in this store network by observing a movie. So we have three control strategies no control, MPC and MPC plus heterogeneity and the different color represents the number of vehicles in each of the different sub regions so blue is uncongested. Dark red is congested and as the movie starts we will see how conditions change in the sub regions for the three cases and let me refresh your mind that what we have we have an increasing demand profile so demand is low then it increases and by the end of stimulation it decreases again.

Notes

Summary

13m 57s

An illustration of different controllers



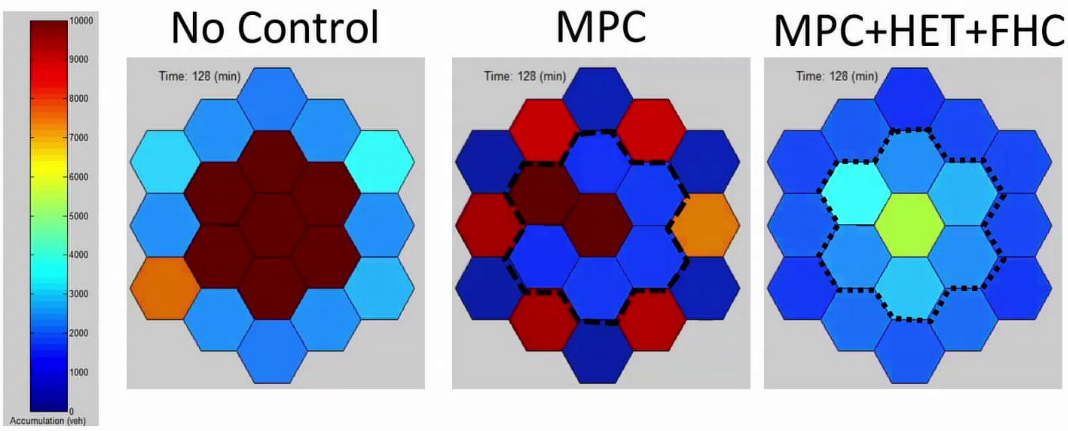
So interestingly let's look at the no control case as a lot of people want to go in the city center, the city center will become highly congested.

Notes

Summary

14m 38s

An illustration of different controllers



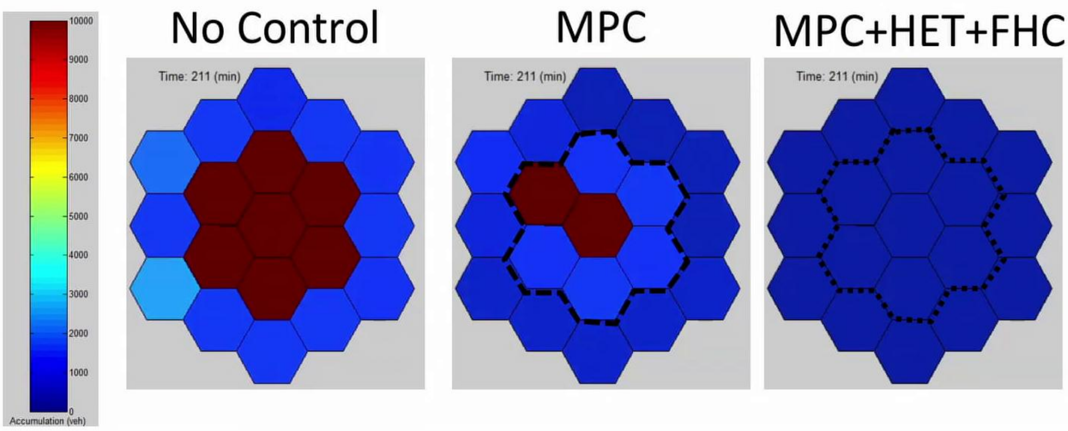
It will reach almost a state of gridlock and it will remain like this until the end of this simulation.

Notes

Summary

14m 45s

An illustration of different controllers



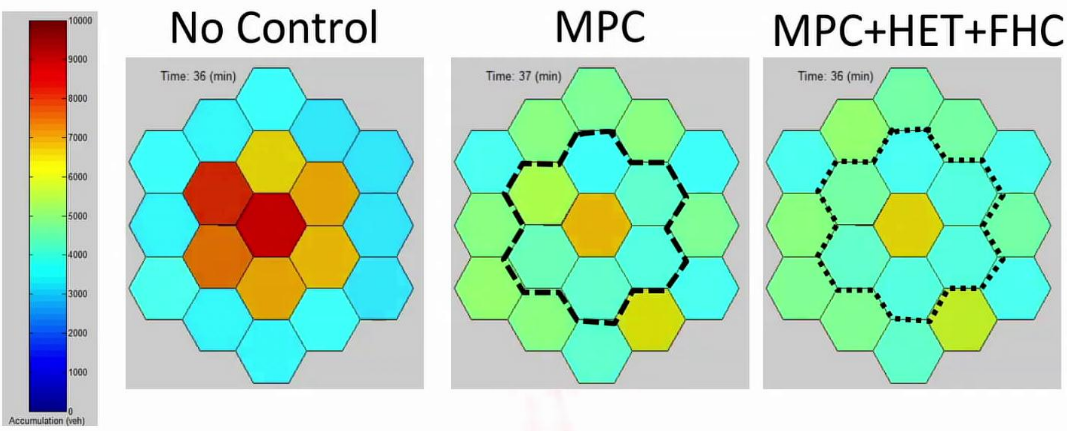
So the periphery is very uncongested but the city center remains very congested.

Notes

Summary

14m 52s

An illustration of different controllers



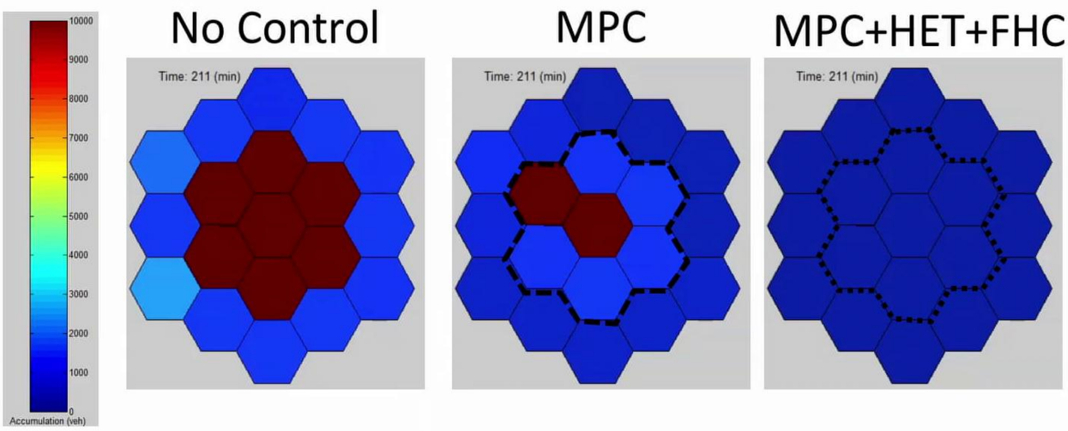
So let's see the video again from the beginning. And now let's look at MPC.

Notes

Summary

14m 58s

An illustration of different controllers



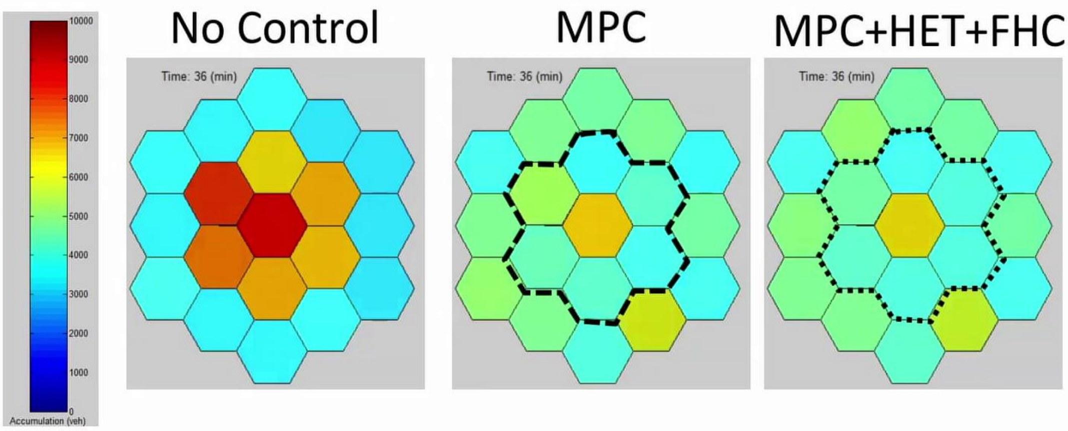
So by applying MPC in this boundary we succeed to avoid very high congestion. But actually there are some uncontrolled flows within the subregions in the center. What we see we see that these two sub regions remain highly congested and we cannot do much to save it.

Notes

Summary

15m 05s

An illustration of different controllers



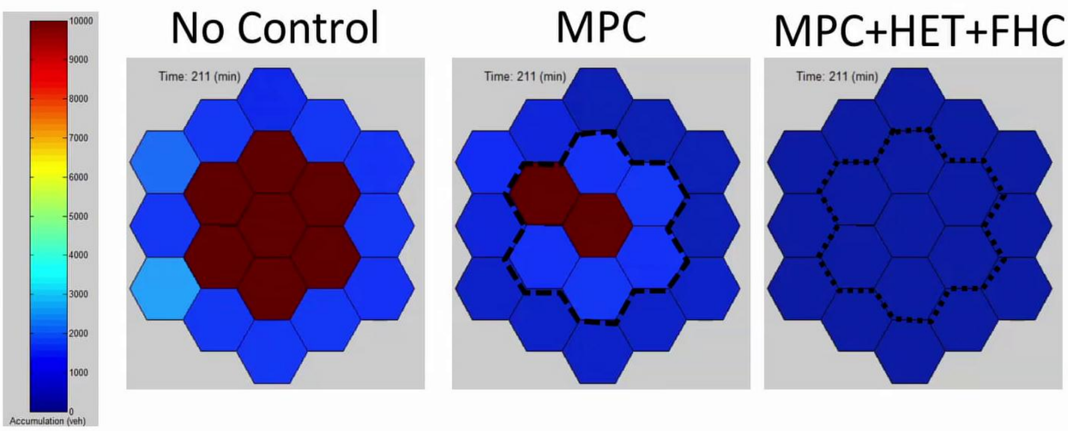
And let's watch the video one more time in the last case.

Notes

Summary

15m 27s

An illustration of different controllers



So here what we see we see that by applying in combination of MPC and the lower level controller we avoid having very high congestion in some say in some subregions because you're able to distribute accumulations in a better way. An answer a large by the end of the simulation all vehicles reach their destination at much faster time.

Notes

Summary

15m 32s

MFD control - Summary

So what we have seen is that if we combine proper dynamics for transportation networks through MFD models with advanced control tools. We are able to develop control strategies that decrease the level of contention in ways that traditional strategies were not able to perform. And more effort is needed to apply these MFD strategies on the field. And what I would say is that this is under development so thank you for your attention and see you soon.

Notes

Summary

15m 58s