ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Departure time choice – part I

Intro to traffic flow modeling and ITS

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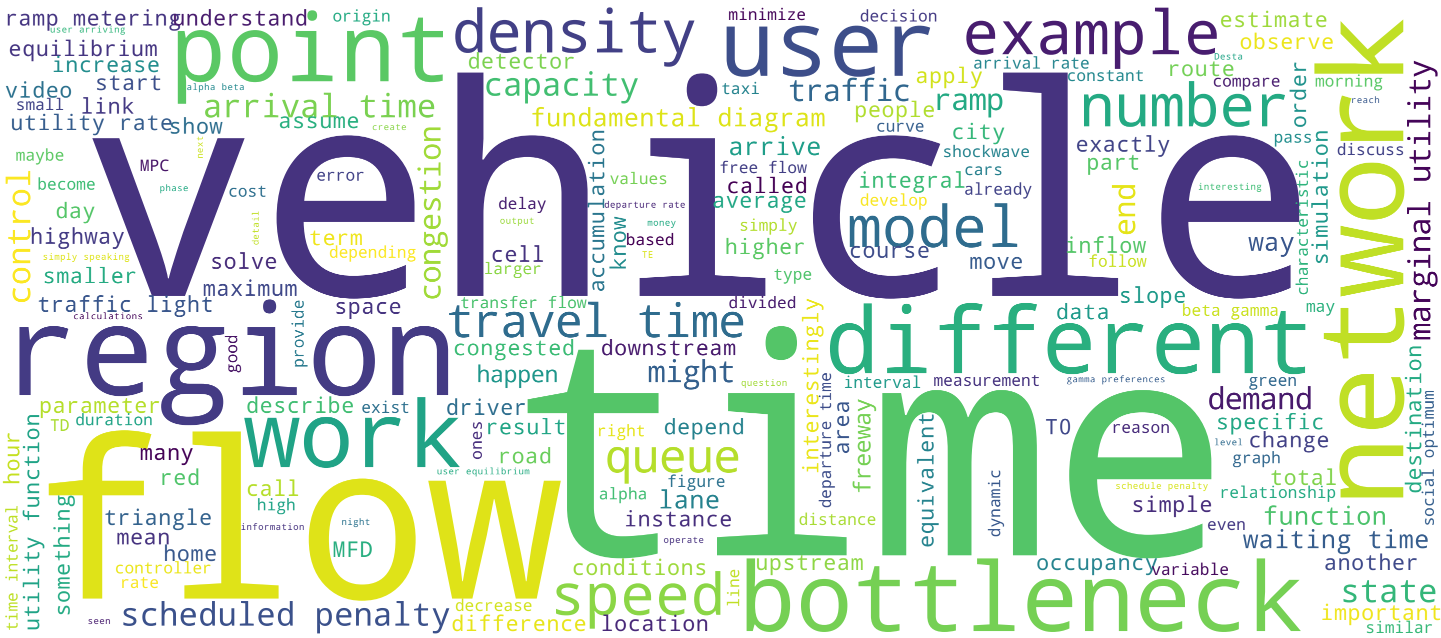

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Video

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Week Outline

- Week 7.1 – Equilibria in Transportation
- Week 7.2 – Static Route Choice (congestion games)
- Week 7.3 – Departure Time Choice (I)
- Week 7.4 – Departure Time Choice (II)
- Week 7.5 – How good is the equilibrium approximation?

Hello, we saw in the last video that cold choice problems are easy to solve when they belong to the category of contestant games catches when the utility function of a traveler only depend on the traffic volume and the arcs that this person uses. This assumption is often reasonable when the arc-flows do not depend on time. But when they do, the problem becomes much more challenging.

Notes

Summary

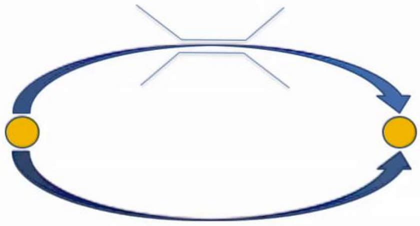
0m 05s

Example of queuing game

- Without departure time choice:

Index	Time	Demand (veh/h)
$j-1$	7:30 – 8:00	1100
j	8:00 – 8:30	1400
$j+1$	8:30 – 9:00	900

$$T_1(j+1) = f(n_1(j), q_1(j+1))$$



$$T_2 = \cdots$$

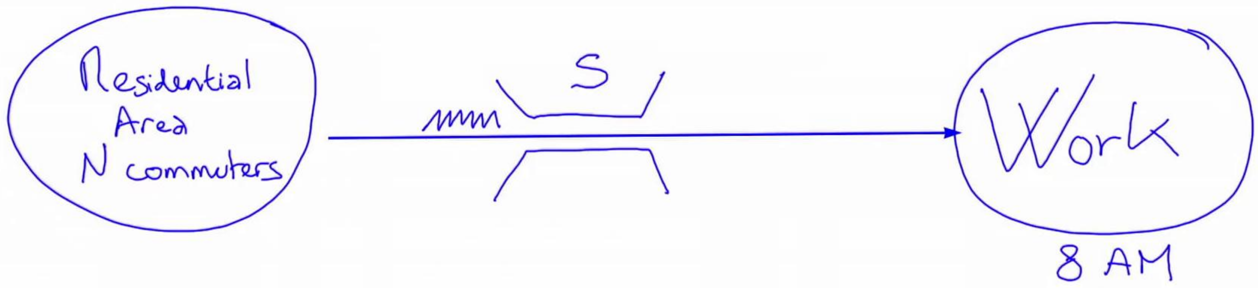
Let's consider again a simple example with two routes. But this time the demand is specified as a function of time. From 7:30 to 8, the demand is 1100 vehicles per hour, from 8 to 8:30, it's 1400 and from 8:30 to 9, it's 900. Let's not consider that the road one as a bottleneck. If all the demand goes through that bottleneck, then that might be queues for instance, in the capacity of this bottleneck is 1200 vehicles an hour, that would be queue developing between 8 and 8:30 and when the next demand will arrive, there will still be some queue, because all vehicles will not be able to pass between 8 and 8:30. So the travel time for vehicles of time interval day + 1 will depend on the number of vehicles on link 1 the end of interval J + on the demand on link 1 during interval [J+1]. This event without departure time choice, it's already more complicated than in the static case.

Notes

Summary

0m 27s

The morning commute problem



This type of example is only one bottleneck per route is relatively simple, so we can actually solve it. But as soon as we start having multiple bootlenecks per route, or multiple routes using the same bottleneck, it becomes quite complex, we can have multiple equilibria and we don't always have a general method to find them. Practitioners use simulations or heuristics that didn't always work. And when they do, we don't know whether the equilibrium are deformed or the only ones. In order to get some intuition, we need to consider very simple cases. So imagine that we have N commuters living in a residential area and that they need to go from home to work in the morning. To go from home to work, they need to pass by bottleneck and this bottlneck as some capacity as an ideally they would like to arrive at work at 8am. They cannot arrive at 8am due to the bottleneck capacity. So what we will have is that queues will develop just upstream of the bottleneck. And some users will arrive early, while others will arrive late.

Notes

Summary

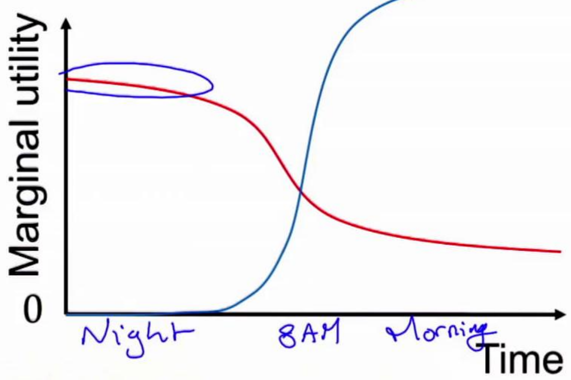
1m 34s

Schedule preferences

- "Marginal utility" point of view:

$$U = \int_0^{t_d} h(t)\mathrm{d}t + \int_{t_a}^{12} w(t)\mathrm{d}t$$

- $h(t)$: marginal utility **at home** at time t
- $w(t)$: marginal utility **at work** at time t,
- Marginal utility of time spent **driving** assumed to be 0 (reference).



To evaluate the cost of these early and late arrivals and to explain why condition appears, we need to specify the schedule preferences. Now different ways of doing this, and one way is based on marginal utilities. So we assume that people value differently the time depending on the location and depending on the time of day. So for instance, this is the marginal utility of being at home at time, T and W, is the marginal utility of being at work at time, T. Usually, every minute that people spend at home during the night is worth a lot for them. So the marginal utility rate is quite high during the night, that in the morning, it's not as important for them to be at home. So this marginal utility rate for being at home decreases. Conversely, people usually don't like to be at work during the night because it's useless. There is nobody else to work with. But it's very important for them to be at work during the day. So around maybe 8am we have a sharp increase in the marginal utility of being at work. And what we consider here a 0 is actually the marginal utility of time spent driving. This is just our reference.

Notes

Summary

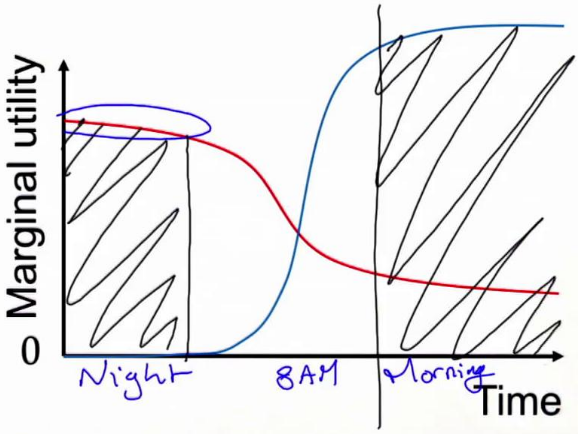
2m 54s

Schedule preferences

- "Marginal utility" point of view:

$$U = \int_0^{t_d} h(t)\mathrm{d}t + \int_{t_a}^{12} w(t)\mathrm{d}t$$

  - $h(t)$: marginal utility **at home** at time t
  - $w(t)$: marginal utility **at work** at time t,
  - Marginal utility of time spent **driving** assumed to be 0 (reference).



And we assume that the marginal utility rates are always positive, that it means that it's always more useful for people to be either at home or at work than in the vehicle. If it was not the case, they could simply stay in the vehicle and drive around their workplace around their home, then the total utility of a user is the integral of the marginal utility rate at the place, this person is of a time. So for instance, imagine that some user lives home at that time and arrives at work at that time, the utility of that user would be this integral which is here plus integral there.

Notes

Summary

4m 17s

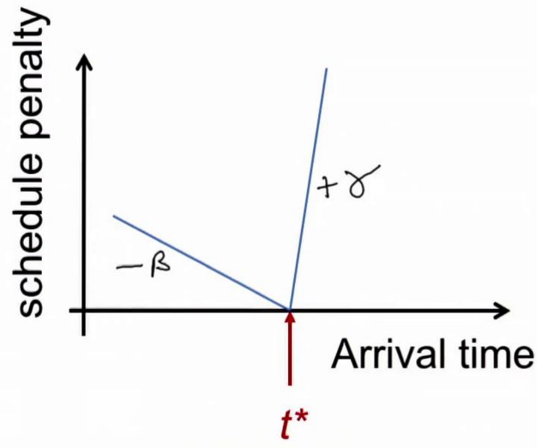
Schedule preferences

- "Schedule penalty" point of view:

$$U = \underbrace{-\alpha(t_a - t_d)} - \underbrace{\mathrm{SP}(t_a)}$$

α : "value of travel time"
$\mathrm{SP}(t_a)$: "schedule penalty" for arriving at t_a

Example: «$\alpha - \beta - \gamma$» preferences



An alternative point of view considers that the utility is the same of EACH other time component and other scheduled energy, the cost of total time is generally assumed to be linear and this coefficient alpha is called the value of time. The scheduled penalty here captures the digitality associated to writing at work either early or late. And it depends only on the arrival time at work. The most common example is perhaps the so called alpha-beta-gamma preferences, it assumes bitwise linear schedule penalty function with slopy minus beta, and here plus gamma. And this time that minimizes the schedule penalty is usually referred to as Desta is scheduled penalty approach is actually the one that is the most common in the literature.

Notes

Summary

5m 04s

Schedule preferences

- "Marginal utility" point of view

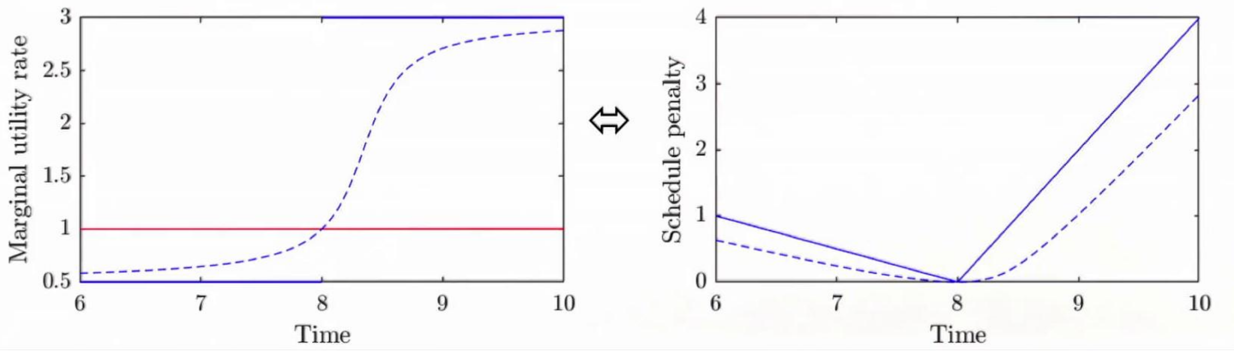
$$U = \int_{\cancel{\emptyset}\; t^*}^{t_d} h(t)\mathrm{d}t + \int_{t_a}^{\cancel{12}\; t^*} w(t)\mathrm{d}t$$

$$= -\int_{t_d}^{t_a} h(t)dt - \int_{t^*}^{t_a} w(t)-h(t)\,dt$$

- "Schedule penalty" point of view

$$U = -\alpha(t_a - t_d) - \mathrm{SP}(t_a)$$

Remark: if $h(t) = \alpha$, both points of view are equivalent



But I also wanted to show you the marginal utility point of view that you understand the limitations of the approach based on scheduled penalties. If we compare the two expressions of the utility, we can see that one is just a specific case of the other. Indeed, I can replace these limits by Desta because the utility functions are only defined up to a constant. So these constant limits do not matter, then I can rewrite this as follows, -6 integral from the time of departure to the time of arrival of H of Tdt, which is equivalent to that turn if H is constant - the integral from desta to TA of W of t - h of t, which is equivalent to datum. So here, I just give an illustration of this equivalence, so that this way is constant marginal utility rate is just the equivalent of the scheduled penalty of the alpha-beta-gamma preferences that is this wise linear. And this is equivalent to that one, as long as we have H of T that is equal to a constant alpha.

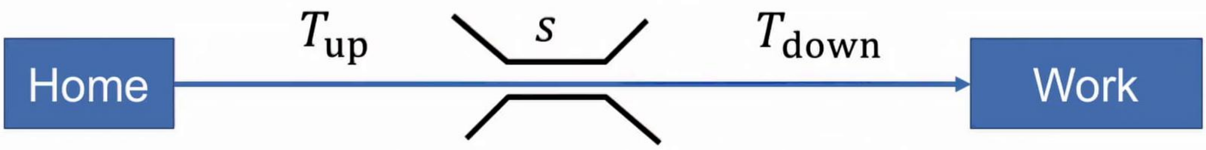
Notes

Summary

5m 56s

The morning commute problem: supply

- “Fluid approximation”: users can depart continuously over time.
- Bottleneck with “vertical” queues:



- Upstream and downstream legs “included” in the schedule preferences:

$$\tilde{U}(t_d', t_a') = U\left(t_d' - T_{\text{up}}, t_a' + T_{\text{down}}\right)$$

- Queue evolution:

$$\dot{q}(t) = \begin{cases} r(t) - s, & \text{if } q(t) > 0 \text{ or } r(t) > s \\ 0, & \text{otherwise.} \end{cases}$$

In short, schedule penalties are nice and intuitive, but they cannot account for the variation in the marginal utility rates at the origin. Let's now describe a bit more precisely the supply side. So we make the fluid approximation, which means that users are considered like a fluid they can depart continuously over time, and not necessarily with small steps of one user. It really simplifies the analysis and if we have a very large number of users, it's actually quite reasonable. Then the setup. So this is a show that is quite similar to the one I drew before, maybe a bit cleaner. And here we have two segments, upstream and downstream of the bottleneck that we will consider that users travel at freefall speed. So they are this constant travel time, T-up and T-down to reach the bottleneck and to go from the bottleneck to work. At the consequence, the flow that leaves from home is the same as the flow that arrives at the bottleneck, but it's translated by a travel time, T-up and similarly, the flow that exists at bottleneck is equal to the flow that arrives at work, but again, translated by the constant term that is T-down.

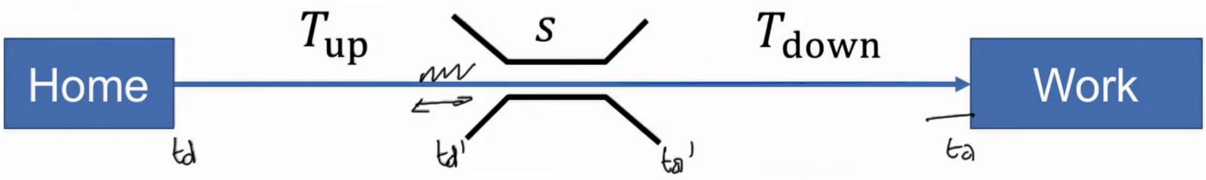
Notes

Summary

7m 24s

The morning commute problem: supply

- "Fluid approximation": users can depart continuously over time.
- Bottleneck with "vertical" queues:



- Upstream and downstream legs "included" in the schedule preferences:

$$\tilde{U}(t_d', t_a') = U(\underbrace{t_d' - T_{up}}_{t_d}, t_a' + T_{down})$$

- Queue evolution:

$$\dot{q}(t) = \begin{cases} r(t) - s, & \text{if } q(t) > 0 \text{ or } r(t) > s \\ 0, & \text{otherwise.} \end{cases}$$

Regarding the cues here, we consider that they are vertical, which means that we keep track of the number of vehicles in the queue, that we do not matter this space that they take, so it does not reduce the free flow travel time before the bottleneck. This simplifies the calculations and in the end, the arrivals at work are exactly the same as if we had normal always untold queues, and the bottleneck. Besides, to further simplify the calculations, we usually include this upstream and downstream legs inside the scheduled preferences. So let's denote TD and TA the departure time and arrival time, and TD' and TA', the time the user arrived at the bottleneck and leaves the bottleneck. Instead of considering this and that, as the decision variables WE will consider these ones and it will be much simpler, because in this way, we can ignore T-upstream and T-downstream. If we do this, okay, it's not exactly correct, because users don't really depart at that time TD'. But to correct for this, we just have to translate the utility function. So if the initial Utility function is a function of TD and TA, we just have to translate it like this.

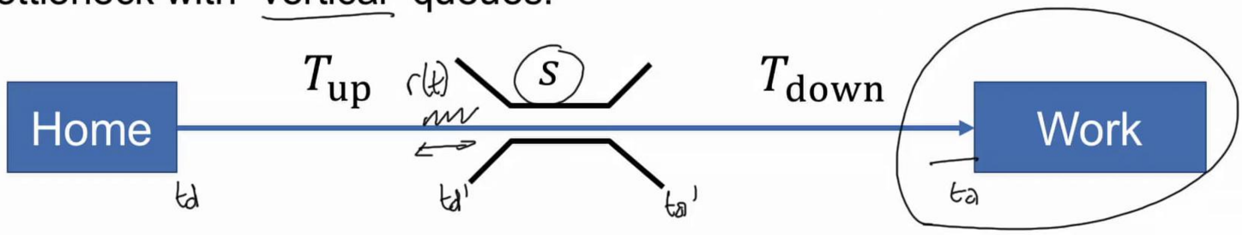
Notes

Summary

8m 48s

The morning commute problem: supply

- "Fluid approximation": users can depart continuously over time.
- Bottleneck with "vertical" queues:



- Upstream and downstream legs "included" in the schedule preferences:

$$\tilde{U}(t_d', t_a') = U\big(\underbrace{t_d' - T_{\text{up}}}_{td}, \underbrace{t_a' + T_{\text{down}}}_{ta}\big)$$

- Queue evolution:

$$\dot{q}(t) = \begin{cases} r(t) - s, & \text{if } q(t) > 0 \text{ or } r(t) > s \\ 0, & \text{otherwise.} \end{cases}$$

And we can define a new function that is a function of TD' and TA' that is exactly identical to the initial one. We can now introduce the main equation, which describes the evolution of the number of vehicle in the queue denoted Q of T as a function of the inflow and time. So, if there is a queue at time T, or if the inflow here of the bottleneck of the is louder than the capacity, then the queue size increases with a rate that is equal to the difference between the inflow rate and what exists the capacity. Otherwise, the queue remains equal to 0. This very simple model provides a description of the arrival rate at work, which is quite accurate, provided that the departure rate is larger than the capacity, S, during a long time interval. If that's not the case, if the departure rate is always smaller than the capacity, S, for instance, this model predicts that vehicle travel at freefall speed, while in the real world, they might still slightly slow down each other.

Notes

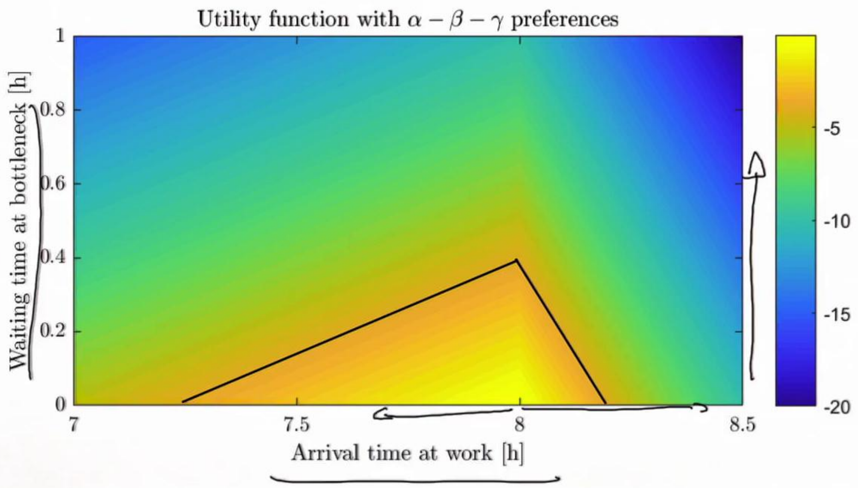
Summary

10m 16s

Equilibrium with homogeneous users

- Equilibrium definition (reminder):
  "no user can be better off by unilaterally changing decision"

$\alpha = 10$ \$/h
$\beta = 5$ \$/h
$\gamma = 20$ \$/h



Let 's now try to solve the departure time choice equilibrium, in a very simple case with homogenous users will have alpha-beta-gamma preferences. With the scheduled penalty point of view, the utility depends on two things, the arrival time at work and the waiting time at the bottleneck. So in that space, the utility function looks like this, it decreases when people arrive further from the preferred over time, which is 8 here. And it also decreases when the waiting time increases. So now what should the equilibrium look like in this space? If we draw a random curve like this, then all the users will want to arrive at the same time, which has the largest utility probably here. But since there is a waiting time at all the other times, it means that there is a queue and therefore some users have to arrive at this time and these users would not be at equilibrium. Therefore, this curve cannot be an equilibrium. So we should find travel time profile such that first, all your arrival times had the same utility, so they should be a triangle like this. Second, all annual arrival times should have utilities that are smaller then the used ones.

Notes

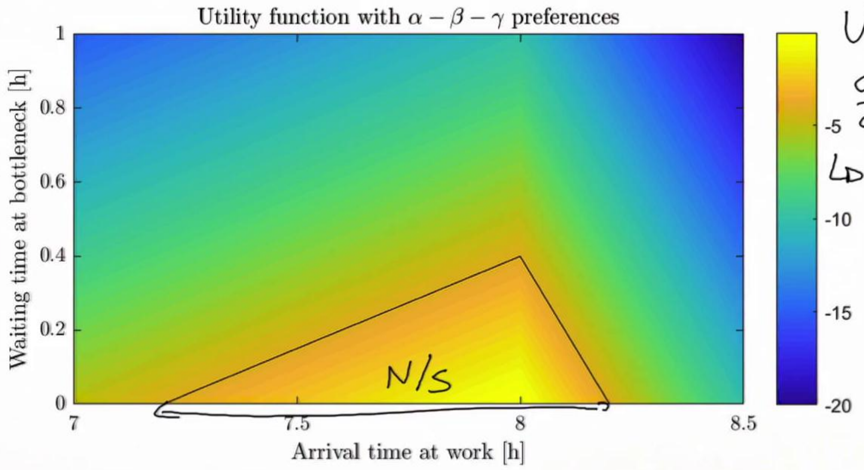
Summary

11m 29s

Equilibrium with homogeneous users

- Equilibrium definition (reminder):
  "no user can be better off by unilaterally changing decision"

$\alpha = 10$ \$/h
$\beta = 5$ \$/h
$\gamma = 20$ \$/h



$U = -\alpha T(t_a) - SP(t_a)$

$\frac{dU}{dt_a} = 0$

$\Rightarrow \frac{dT}{dt_a} = \frac{\beta}{\alpha}$ if $t_a < t^*$

$-\frac{\gamma}{\alpha}$ if $t_a > t^*$

For instance, if the used arrival times of these ones, the basis of this triangle, it's fine because the other travel time here and there, other smaller utility. But what we cannot have is a triangle with us arrival time maybe like this. And maybe like this, because then this unused travel times have a very high utility here, because they have no waiting time. And third, all users should have an arrival time given that their arrival rate at destination is at most S vehicles an hour and users that experience delays had to queue and the arrival rate should be equal to the capacity at arrival. So overall, the solution is this. Where this duration here is exactly equal to N)S, where N is a total demand and S, is the bottleneck capacity. To find what this looks are, we should simply differentiate the utility function. As a reminder, the utility function is given by this function where the travel time is denoted capital, G and the scheduled penalty by SP. If we differentiate this function with respect to the arrival time, and we impose that this is equal to 0, we get that the derivative of travel time with respect to the arrival time should be equal to beta over alpha, if users are early, so if TA is smaller than T*, and it should be equal to minus gamma over alpha, if TA is louder than T* that if users are late.

Notes

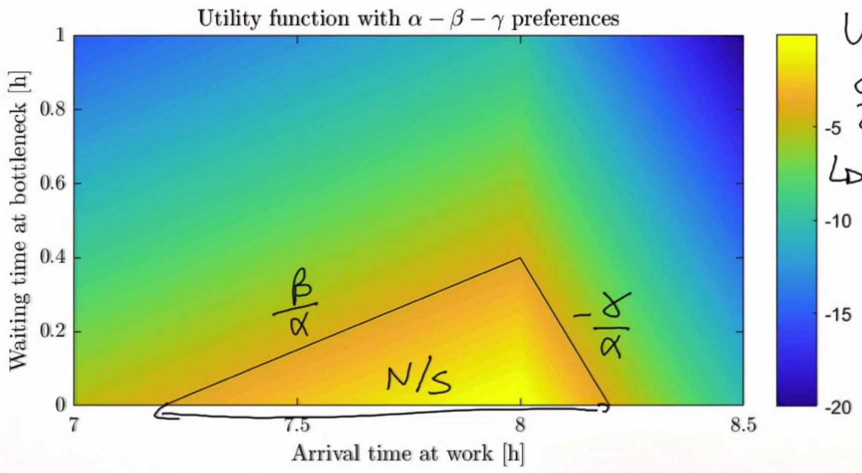
Summary

13m 01s

Equilibrium with homogeneous users

- Equilibrium definition (reminder):
  "no user can be better off by unilaterally changing decision"

$\alpha = 10$ \$/h
$\beta = 5$ \$/h
$\gamma = 20$ \$/h



$U = -\alpha T(t_a) - SP(t_a)$

$\frac{dU}{dt_a} = 0$

$\Rightarrow \frac{dT}{dt_a} = \begin{cases} \frac{\beta}{\alpha} & \text{if } t_a < t^* \\ -\frac{\gamma}{\alpha} & \text{if } t_a > t^* \end{cases}$

So, this slope is beta over alpha, and this slope is minus gamma over alpha. From this, we can derive the departure rate from the origin, that this is something that we do in the exercises.

Notes

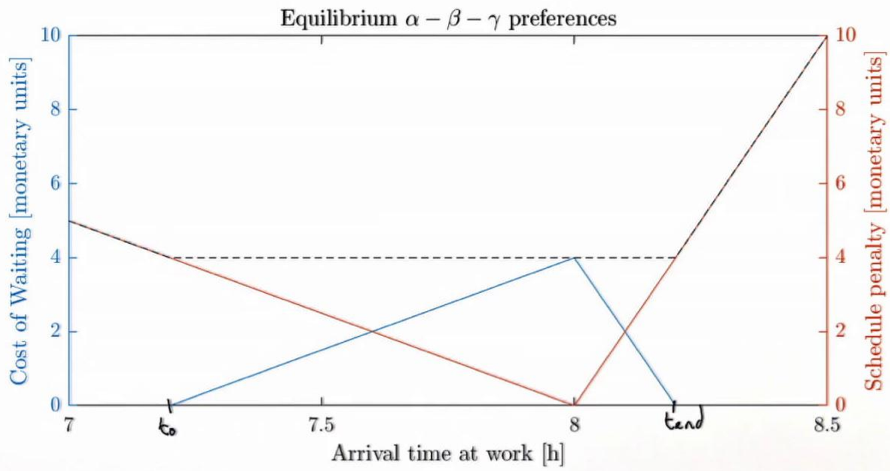
Summary

14m 59s

Equilibrium with homogeneous users

- Equilibrium definition (reminder):
  "no user can be better off by unilaterally changing decision"

$\alpha = 10$ \$/h
$\beta = 5$ \$/h
$\gamma = 20$ \$/h



$$TSP = \int_{t_0}^{t_{end}} SP(t) \cdot s \, dt = s \cdot A$$

$$TT = \int_{t_0}^{t_{end}} w(t) \cdot s \, dt = s \cdot A'$$

Let's now look at the cost decomposition as a function of arrival time. We have the waiting time that we just derived and we have the scheduled penalty as a function of arrival time. And this is from the definition. In order to compare them, we need to convert this waiting time into a waiting cost for this, we multiply it by alpha, which is the value of time and we get this. The dashed line represents the total cost, which is the sum of the scheduled penalty and the waiting cost. We can already see that there is some symmetry between the two. But let's compare the total scheduled penalty and the total travel time. So that but all scheduled penalty is simply equal to the integral of all users of the schedule penalty. And we can convert this with an integral and time from T0 to T- end where this is T0 and this is T-end of the scheduled penalty that is experienced times how many users experiences that is S. So this is equal to S, time the integral of SP, which is the area, A and the red curve from T0 to T-end. Similarly, the total travel time is equal to this integral from T0 to T-end of the waiting time of users arriving at T, times how many users arrive at TSdT, which is equal to S times an area A', which is area below the triangle.

Notes

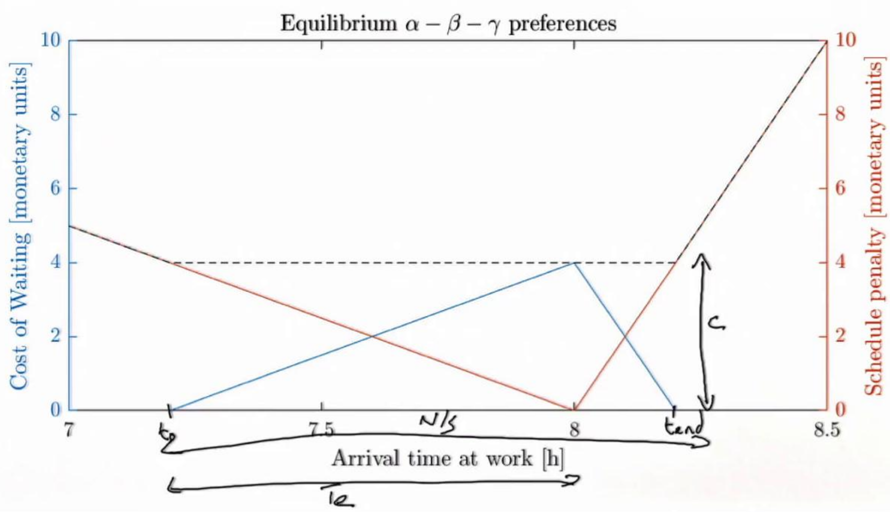
Summary

15m 15s

Equilibrium with homogeneous users

- Equilibrium definition (reminder):
  "no user can be better off by unilaterally changing decision"

$\alpha = 10$ \$/h
$\beta = 5$ \$/h
$\gamma = 20$ \$/h



$$TSP = \int_{t_0}^{t_{end}} SP(t)\cdot s\, dt = s \cdot A$$

$$TT = \int_{t_0}^{t_{end}} w(t)\cdot s\, dt = s \cdot A'$$

$$A' = A$$

$$c = SP(t_0) = SP(t_{end})$$

$$\beta T_e = \gamma\left(\frac{N}{S} - T_e\right)$$

$$T_e = \frac{\gamma}{\beta+\gamma}\frac{N}{S}$$

$$C = \frac{\beta\gamma}{\beta+\gamma}\frac{N}{S}$$

We can already see that both are equal. So A' is equal to A. Let's now determine the average cost that every user experiences that is C. So C is equal for instance, to the scheduled penalty experienced by the user arriving at time 0. And it's also equal to the scheduled penalty for users arriving at T-end. So the scheduled penalty of user arriving at T0 is simply beta TE, where TE is this time, the maximum earliness and the scalar quantity of a user arriving at T-end is simply gamma N over S-TE, because as a reminder, this is N over S. So, what we get is that TE is equal to gamma divided by beta plus gamma times N over S. So, if we use this expression inside the expression of C, we get that see is simply beta gamma, divided by beta plus gamma, N over S. What is important to remember is that in the simple case, the average congestion cost increases linearly with the demand. And that it is made of 50% of delays total over time and a 50% of scheduled penalties.

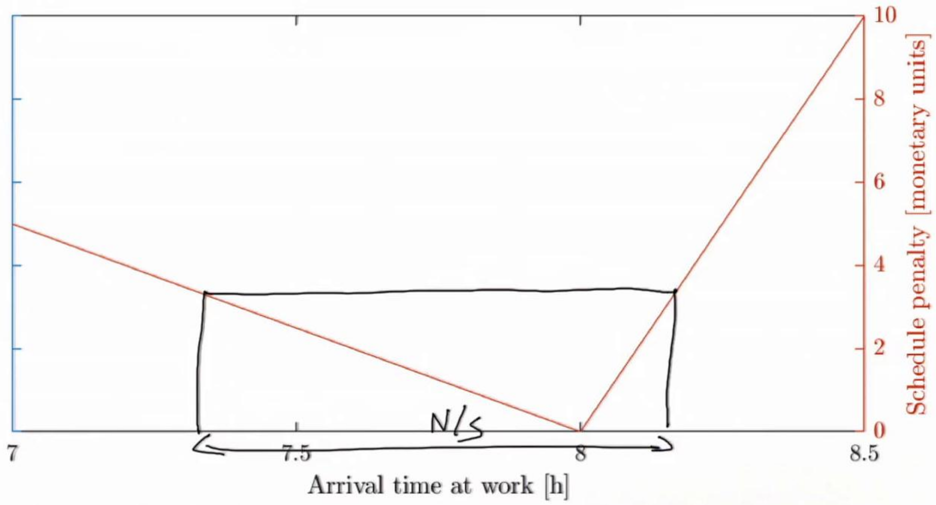
Notes

Summary

17m 07s

Optimal pricing with homogeneous users

$\alpha = 10$ \$/h
$\beta = 5$ \$/h
$\gamma = 20$ \$/h



Let's not see what is a social optimum? First, you should have no queuing, it other locations of arrival times that our physical rescuing are also feasible without killing, people just have to arrive at the bottleneck at the good time. And since killing is a pure loss, it should be avoided. This, the social optimum should just minimize the total scheduled penalty. Since users have to pass the bottleneck at some point. The best here is to make all users but exactly like at the user equilibrium. So we have to find an interval that is like this, where the scheduled penalty and the two extremes are the same and that has a duration, N over S. Then the next question is how we can make users behave like this. One answer is to implement a toll, which plays exactly the same role as waiting time in the user equilibrium without pricing.

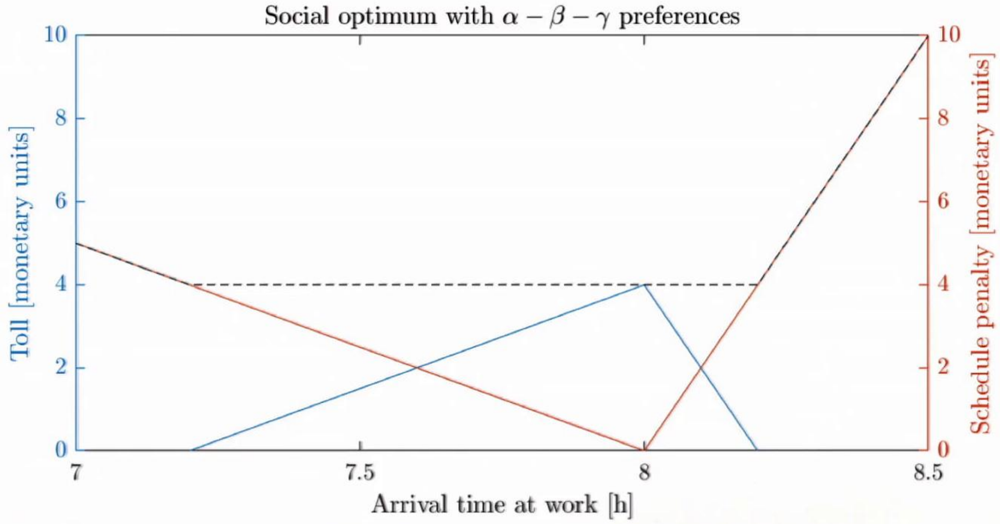
Notes

Summary

18m 55s

Optimal pricing with homogeneous users

$\alpha = 10$ $/h
$\beta = 5$ $/h
$\gamma = 20$ $/h

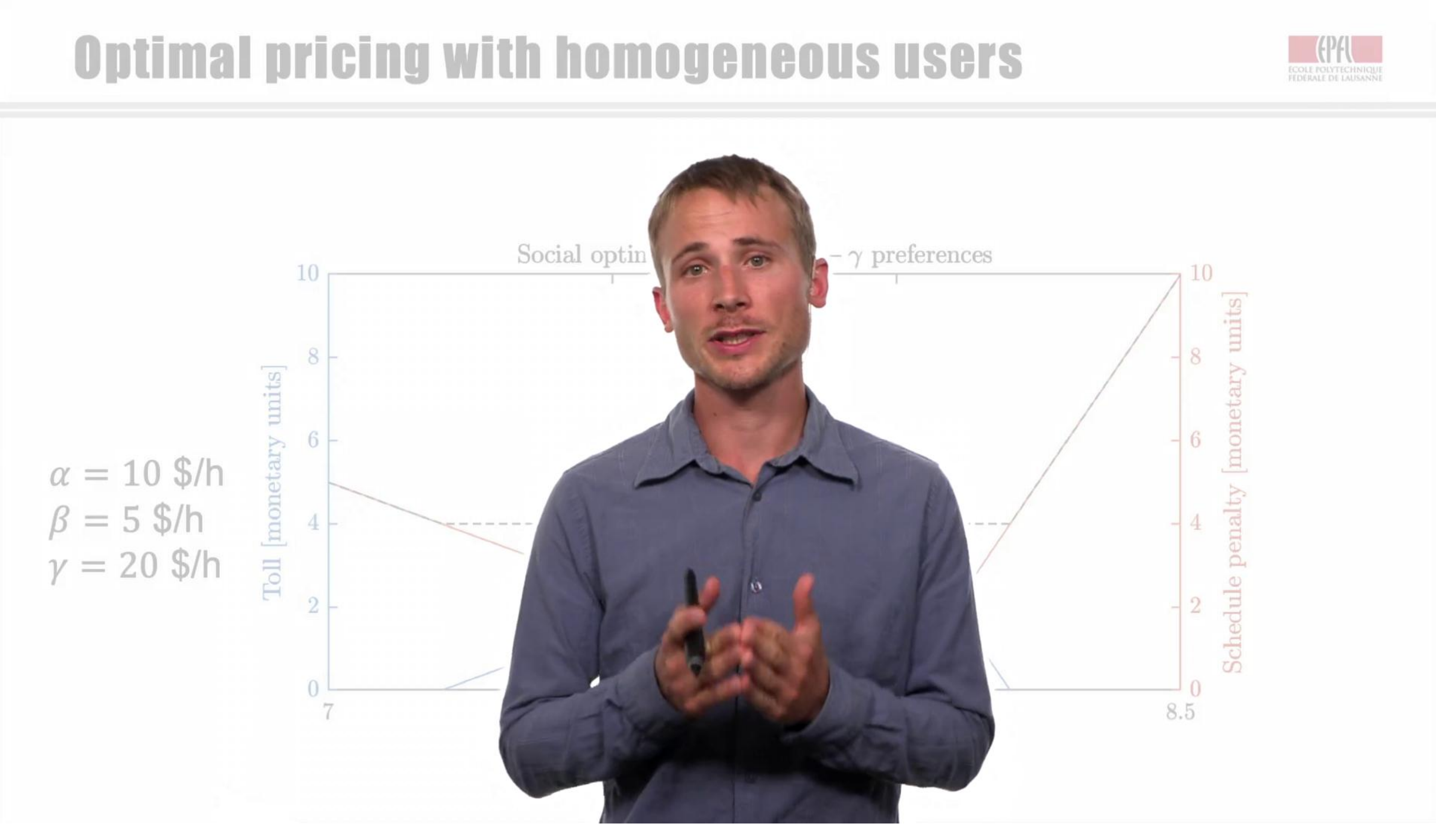


So you can notice that the toll that we have drawn as exactly the same shape as waiting time, the total cost is the same, at least default with distributing the money. So the advantage compared to the case with a waiting time is that this money is collected, it's not lost and it can be redistributed. This is called decentralizing the social optimum, because a central operator using this pricing strategy can then let agents decide by themselves and the equilibrium is the optimum.

Notes

Summary

19m 56s



It usually takes some time to really understand the money problem. So I suggest that you first try to solve the exercises following this video. And once you have a good understanding, you can watch the next one which deals with heterogenous users.

Notes

Summary

20m 26s