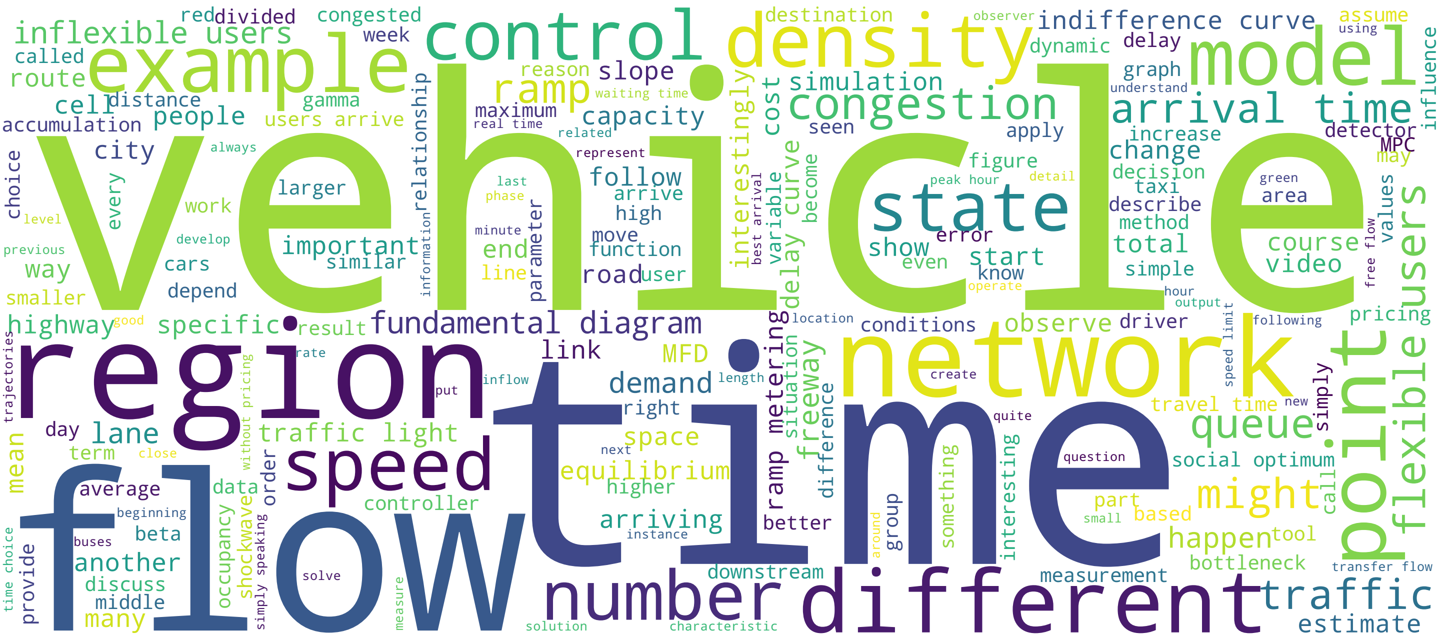

# Departure time choice – part II

## Intro to traffic flow modeling and ITS


Raphaël Lamotte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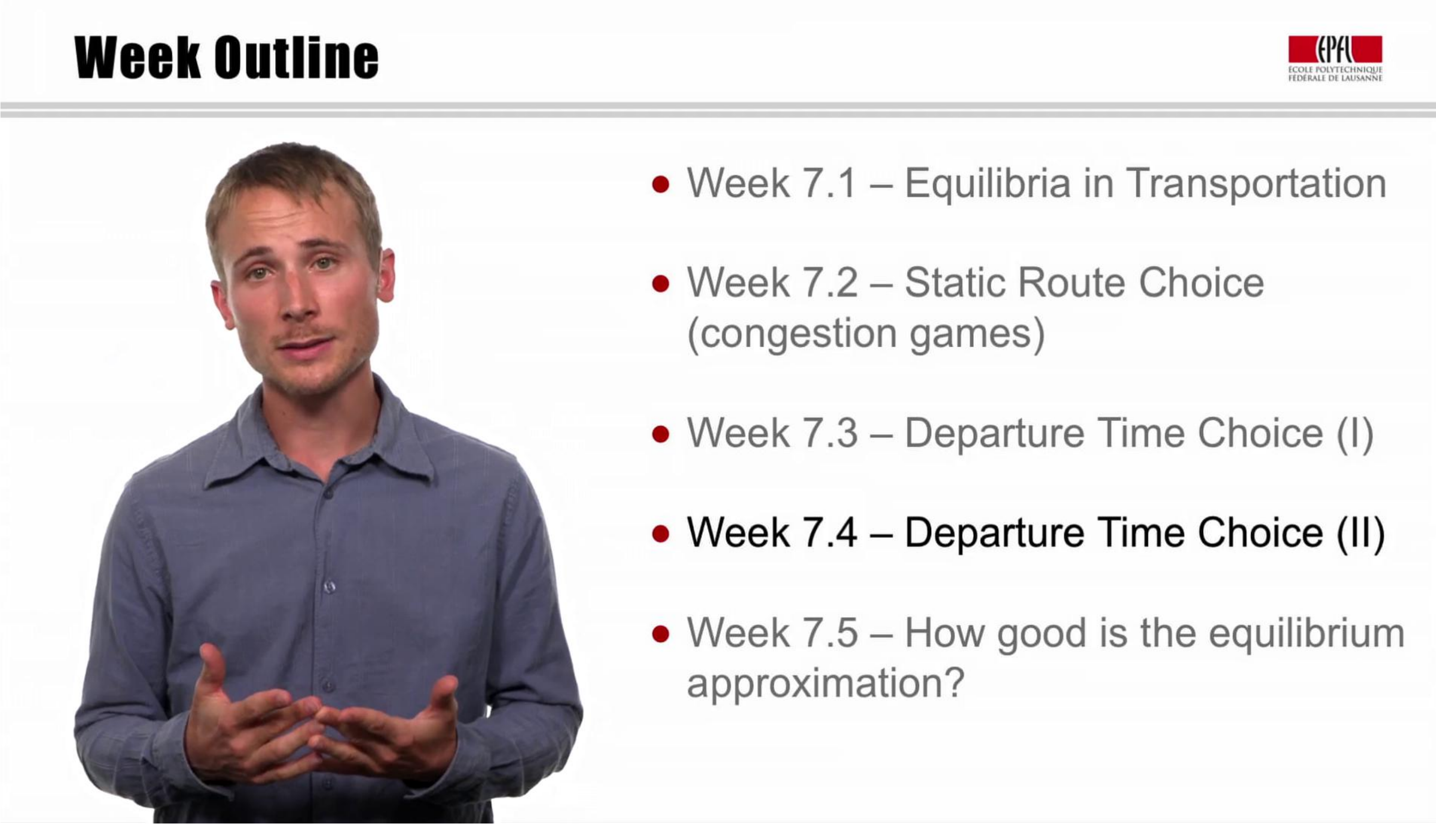


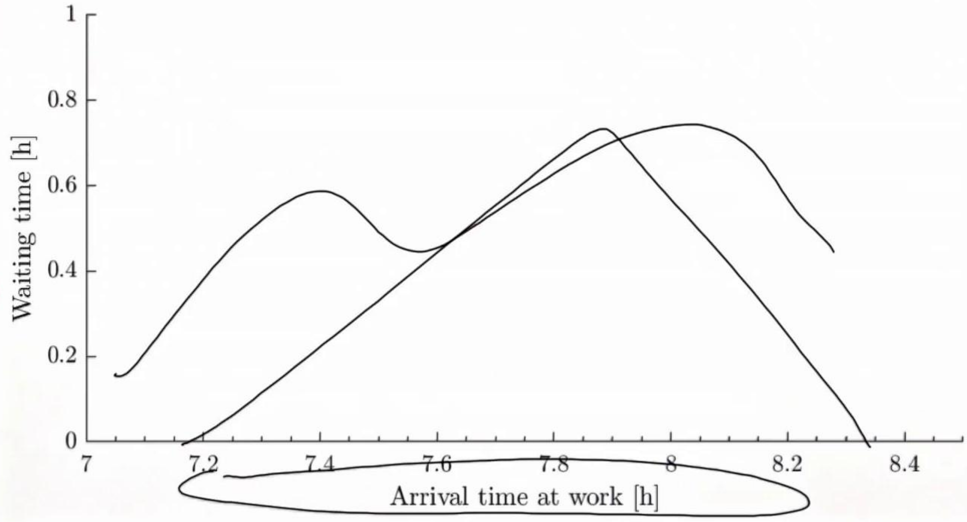

Hello, this is the first video of this week and the second and departure time choice. In the previous one, we introduced a problem and solved it with homogenous users. In this one, we introduced heterogeneity among users. Heterogeneity is very important first, because it influences the equilibrium and second because of Equity. If a government wants to implement a new transport policy, it would figure out whether it is likely to penalize some part of the population and what it can do about it.

Notes

Summary

0m 05s

# Equilibrium characterization



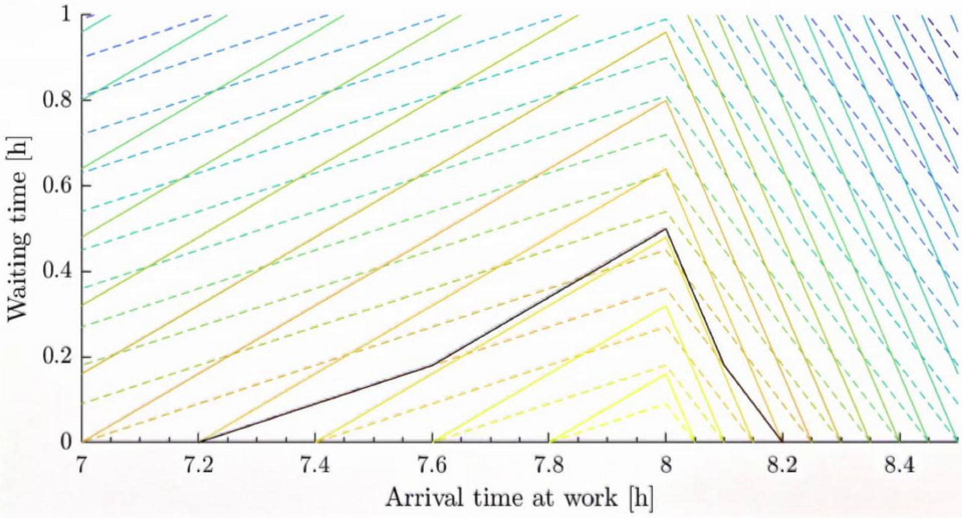

The method we are going to follow is very similar to the one that we use with homogeneous users, we trying to find an indication of arrival times and a curve in the plan of arrival time and waiting time, which together correspond to an equilibrium. With homogenous users, we found equilibrium simply by following when indifference curve that looked, now let's take this here, we have many types of users. So we can have many different sets of indifference curves.

Notes

Summary

0m 36s

# Equilibrium characterization




- Flexible users (--): $\frac{\beta}{\alpha} = 0.45\,; \frac{\gamma}{\beta} = 4$
- Inflexible users (-): $\frac{\beta}{\alpha} = 0.8\,; \frac{\gamma}{\beta} = 4$

Both groups have size $\frac{N}{2}$

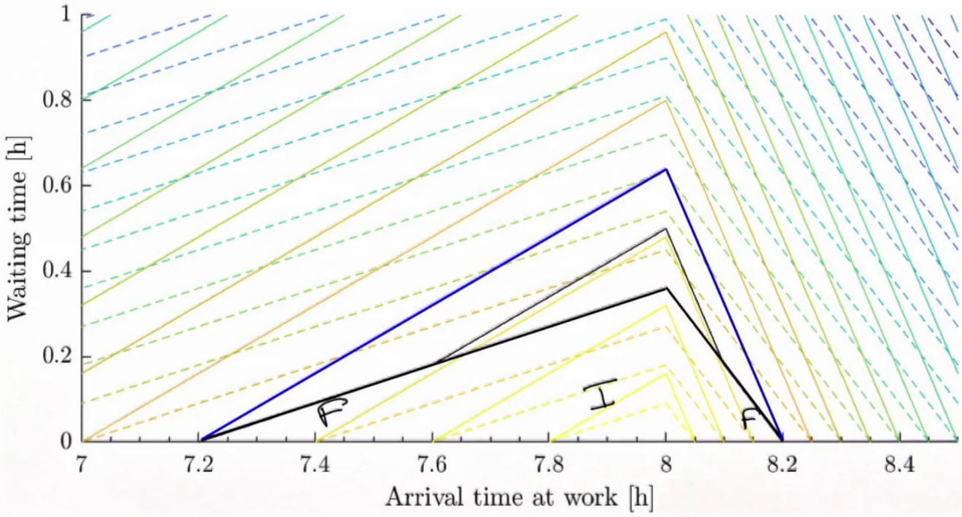
This is with only two families of users, the flexible users, which have the dashed lines, and the inflexible users, which have the solid line, both groups have the same ratio, gamma over beta, which is four, which means that arriving one minute late is as bad as arriving four minutes early. But flexible users have scheduled constraint that relatively weaker compared to the value of time alpha. So here, again, the equilibrium impose it that within each group, all users should have the same cost. It means that at all times were flexible people arrive, the delays should be on the same dashed indifference curve, and at all times were inflexible users arrive, the delays should follow the same solid indifference curve. Besides, given the delay curve, flexible users should prefer their own arrival times to those of indefensible users and vice versa. This constraints the order of arrivals, we see that if during the condition, onset, inflexible users arrive first, and then flexible users arrive second, then inflexible users are not adequately on because the delay curve is below the indifference curve. This the only possible equilibrium is when inflexible users travel at the most desired arrival times like this.

Notes

Summary

1m 09s

# Equilibrium characterization




- Flexible users (--): $\frac{\beta}{\alpha} = 0.45\,; \frac{\gamma}{\beta} = 4$
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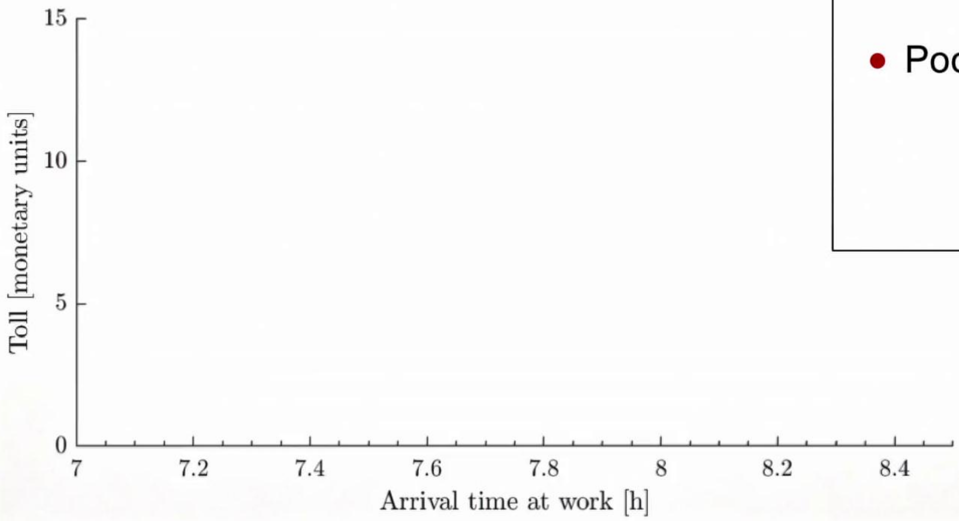
So we have three periods, flexible users arrive in the wings, and inflexible users arrive in the middle of the peak hour. You can see that in this case, the delay curve is always above the indifference curves of the different groups. For instance, here, the indifferent curve, there the delay curve is above, and here the delay curve is there, and the indifference curve would be just below like this. So what is the impact of heterogeneity on the user cost? Actually, it depends on the group. If everybody was flexible, the delay curve would be like this. So flexible users at the same cost as if all the population was like them. If everybody was inflexible, however, the delay curve would be like that. So inflexible users benefit from the fact that some users are not as inflexible as they are.

Notes

Summary

2m 38s

# Social optimum




- Rich & flexible users:

$$\frac{\beta}{\alpha} = 0.45\,;\frac{\gamma}{\beta} = 4;\underline{\alpha = 20}$$

$$\Rightarrow\ \beta = 9\,;\gamma = 36$$

- Poor & inflexible users:

$$\frac{\beta}{\alpha} = 0.8\,;\frac{\gamma}{\beta} = 4\,;\underline{\alpha = 10}$$

$$\Rightarrow\ \beta = 8\,;\gamma = 32$$

Both groups have size $\frac{N}{2}$

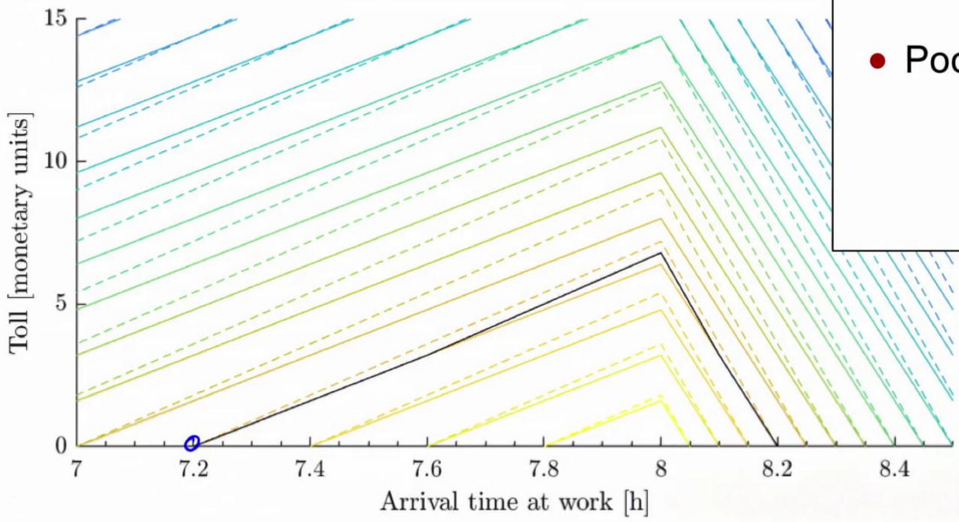
Now let's see what happens with pricing. With queue and without pricing, the best arrival times went to those that were willing to wait the most. Without queues and with pricing, the situation will be essentially the same, except that the best arrival times will go to those that are willing to pay the most. So we will work in this space of arrival time and to. But let's start from the beginning. First, we need to identify the social optimum. Again, there cannot be any queuing at the social optimum, because solutions with queues are always dominated by solutions with the same arrivals that without queues this, we should simply minimize the schedule penalties. And if we consider the same two groups as previously, we should simply give the best arrival times to those with the largest absolute values of beta, and gamma. If we take the same users as before, that we consider that the flexible users have the value of time alpha that is larger than the inflexible users, we can compute the resulting beta and gamma and here we see that counter intuitively perhaps. So flexible users have larger beta and gamma than the inflexible users. That's all because of the different efforts.

Notes

Summary

3m 35s

# Social optimum




- Rich & flexible users:
  $$\frac{\beta}{\alpha} = 0.45\,;\frac{\gamma}{\beta} = 4;\alpha = 20$$
  $$\Rightarrow \beta = 9\,;\gamma = 36$$
- Poor & inflexible users:
  $$\frac{\beta}{\alpha} = 0.8\,;\frac{\gamma}{\beta} = 4\,;\alpha = 10$$
  $$\Rightarrow \beta = 8\,;\gamma = 32$$

Both groups have size $\frac{N}{2}$

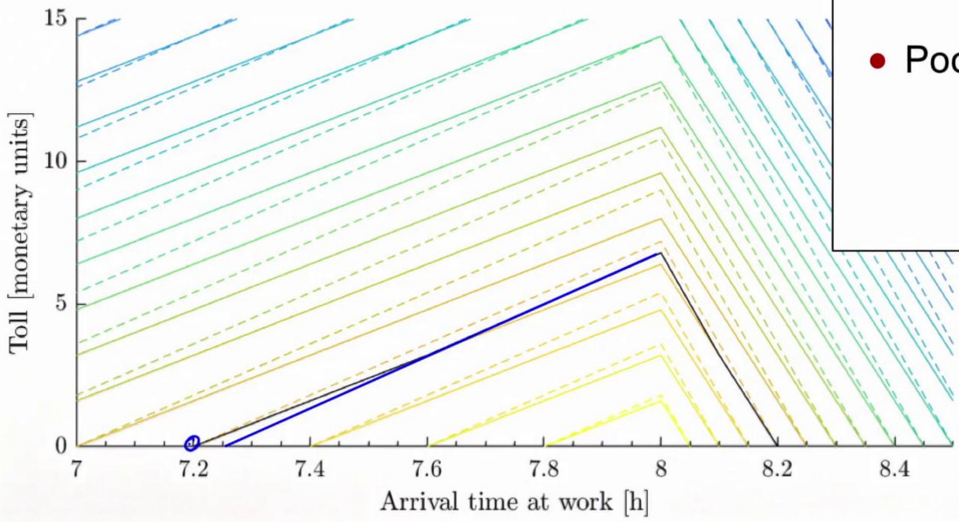
In a case like this, the social optimum is to let the flexible users travel at the most desired arrival times, so it's the opposite order compared to the case without pricing, then we need to find a tool that ensures that this is an equilibrium and to do this, we can follow the same method that we use to convert delays without pricing. We just follow the indifference curves of that space. So here are the indifference curves of the two families, the slopes, plus beta of the early arrivals and minus gamma for the late arrivals. Since flexible users and inflexible users have very similar beta and very similar gamma, the slopes are very similar as well. And to find the equilibrium, we just have to follow the solid curve in the width of the peak hour, and the dash curve in the middle of the peak, so we get that. So this is equilibrium again, we can make competition a source of revenue for the government, rather than impure laws. So it's a very interesting policy. Let's now see what are the impacts of pricing for the different groups before we redistribute the total revenue. By using the point of the first arrival, we can easily draw conclusions.

Notes

Summary

4m 55s

# Social optimum




- Rich & flexible users:
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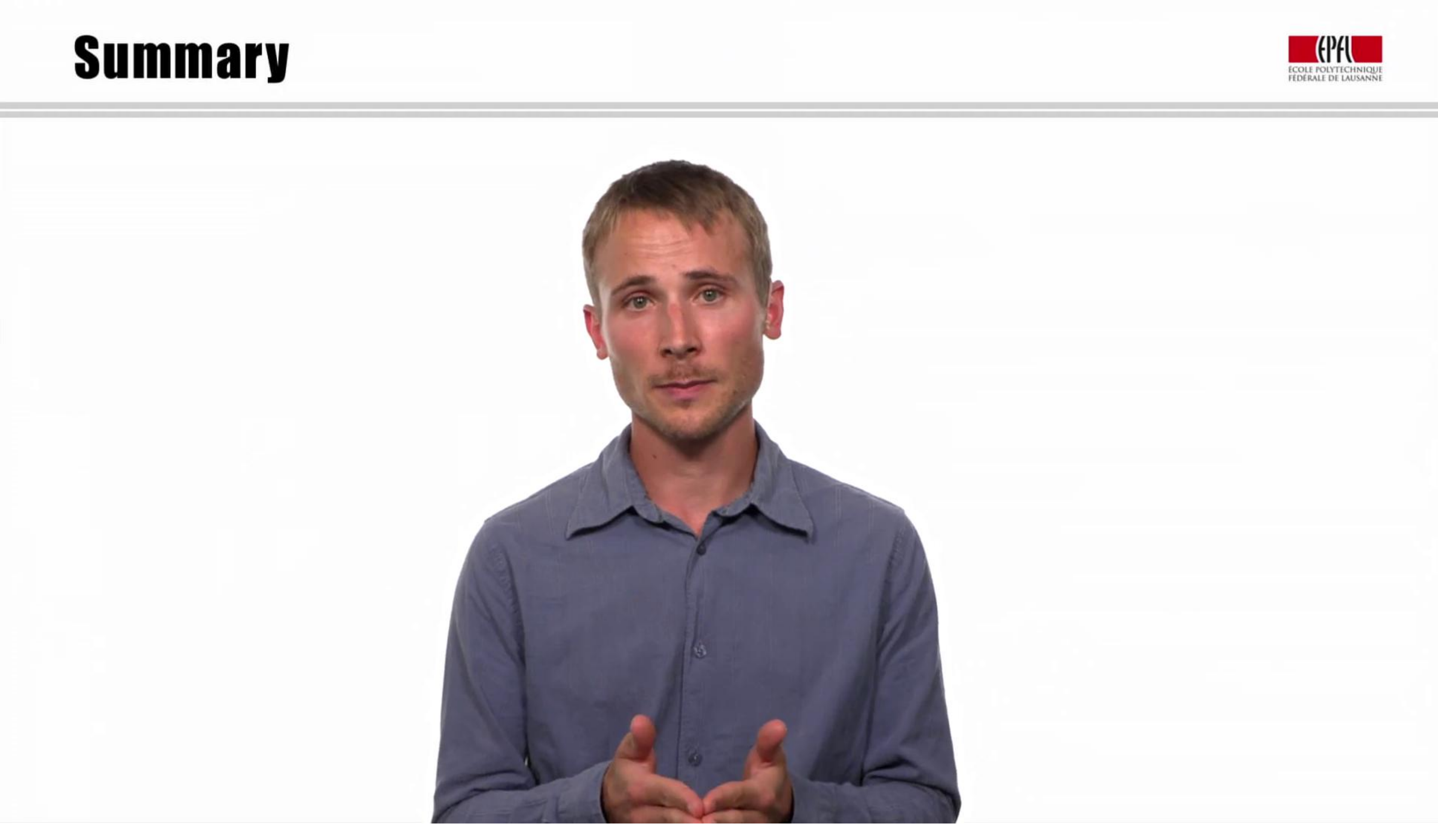
Both groups have size $\frac{N}{2}$

For instance, if we consider the rich and flexible users before they were arriving at time that they were in different between these times, and that time, now they're arriving in the middle and we can see that their cost is smaller than by arriving there because they're indifferent between their current times and this time, which has a smaller schedule penalty than that time. Now, if we look at the pool and inflexible users that were arriving before in the middle, and now arriving in the wings, we have the opposite. Now, they are indifferent between the current arrival time and this time, while before they preferred to arrive in the middle, then there because there it was flexible users that were arriving. So the poor are rushed off and the rich are better have after pricing and before redistributing the toll. This is intuitive because we are basically allowing people to get the best slots by paying instead of waiting, and the price to save some minutes of waiting time is only interesting for people that have a high value of time. Of course, this has been changed by redistributing the money in an appropriate way. We don't explain here how to do this, because the definition of fairness is largely subjective and political. However, this is very important for accessibility.

Notes

Summary

6m 12s




That is the end of this video and I would like to finish with a few highlights. First, we have seen that with a better time choice, a large proportion of the contestant cost is due to schedule penalties. That's one reason why it is so important to understand equilibrium. If we will just measuring the time that people spend queuing on the road, it would largely underestimate the conditioning cost. This is true for the departure time choice, but it's also true for other decisions, like the choice to travel, much choice and residential location. Second, we have seen that the social optimum can be decentralized with an another most time- bearing tool. This is excellent. But I would like to highlight that this is related to our definition of welfare that was based on the Cato-Exploratorium. If we chose a different weighting scheme, the social optimum would probably be more difficult to decentralize. So that's the end of this video. In the next one, we will talk about the limits of the equitable approximation. Thank you.

Notes

Summary

7m 44s