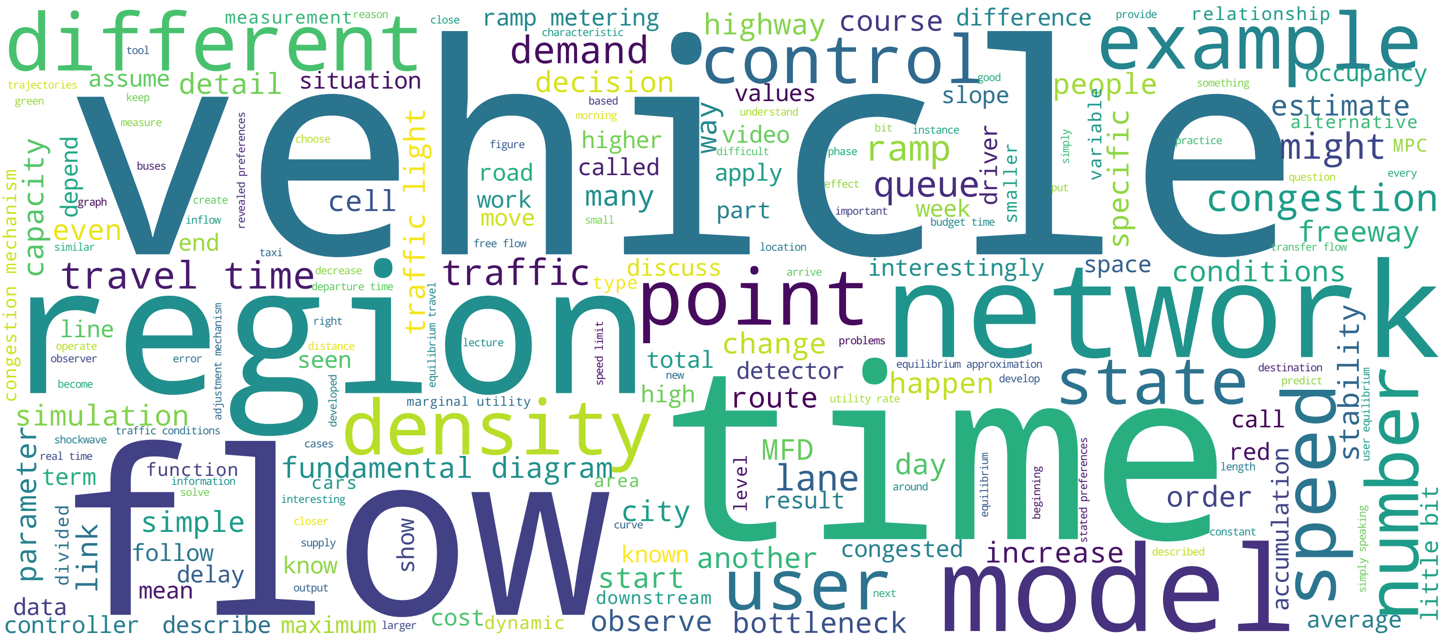
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

How good is the equilibrium approximation?

Intro to traffic flow modeling and ITS

Raphaël Lamotte



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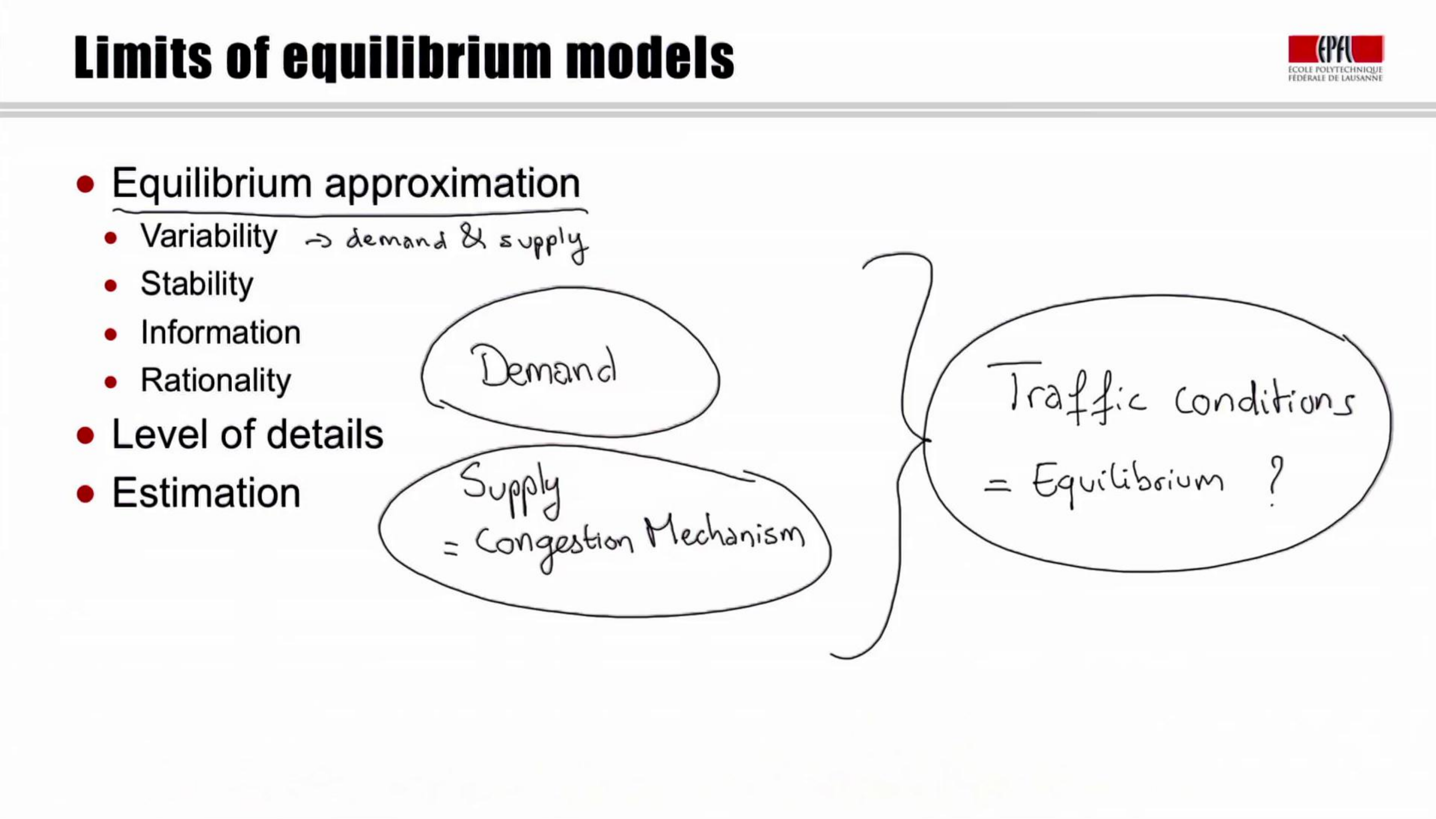
Video

Hello. This last video is about the limits of the equilibrium approximation. The Equilibrium approximation is now largely used, both in theory and in practice and we sometimes forget that many factors influence the quality of this approximation.

Notes

Summary

0m 04s

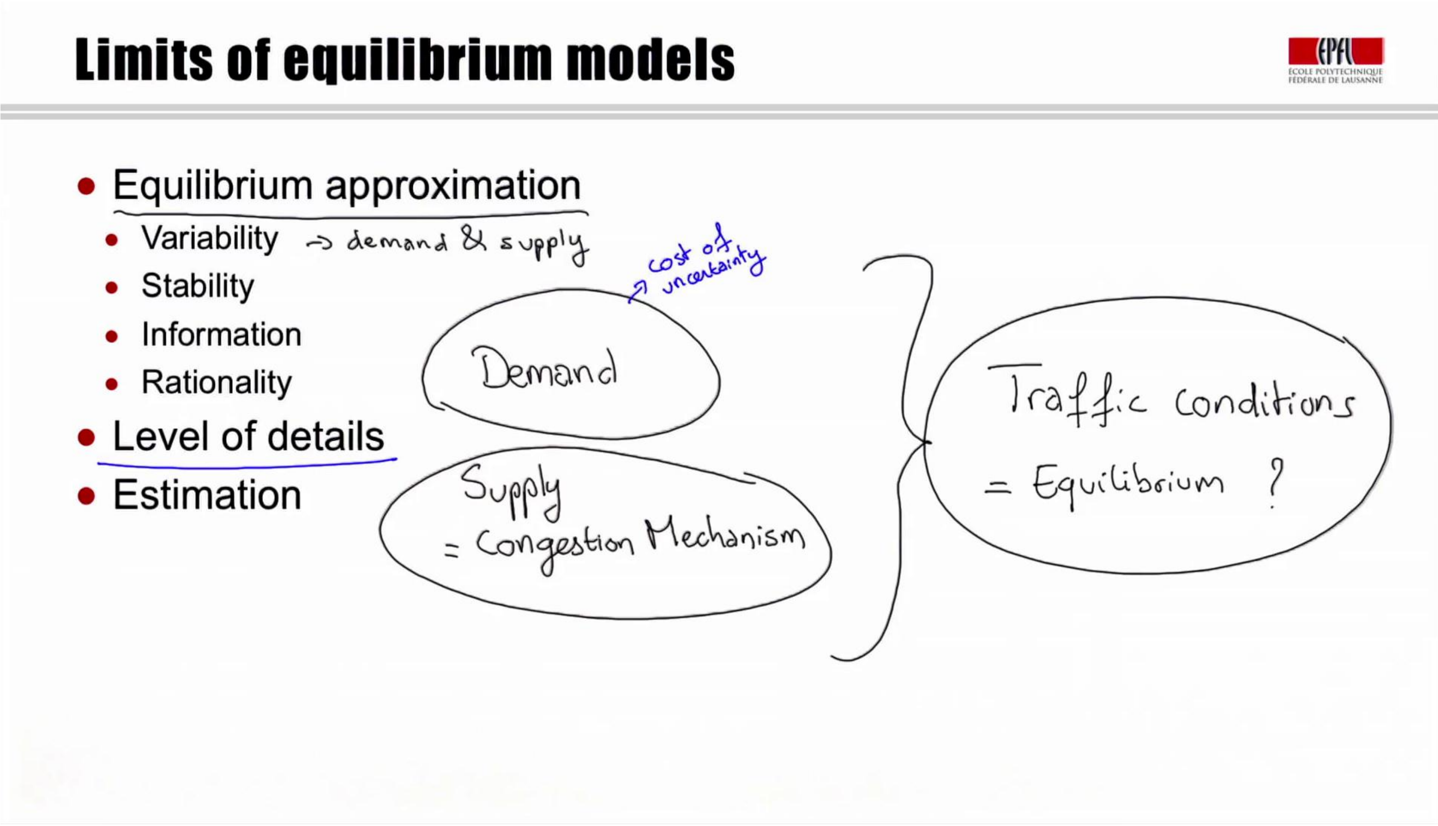


This video does not aim at making a full list of these factors, it only highlights a few that I think especially important and often forgotten Schematically, the system that we consider involves some demand some supply that is a congestion mechanism and together, they lead to some traffic conditions that may or may not be at equilibrium. In order to predict well the traffic conditions, we need to have good models of the demand, of the supply, and we need the equilibrium approximation to work well. These are quite strong requirements. In practice, there are three main issues. First, the approximation may not be reasonable. Second, it's very difficult to find the right level of details for our models. And third, we need to estimate the parameters of our models. These problems are quite general, but I would like to highlight a few specific issues related to the cases that we have explored in the previous videos. So let's start with the equilibrium approximation. Learning about an issue is that the conditions vary from day to day. This applies to the demand, which usually varies of other week and over the seasons, and to the supply, which varies with the weather and with road works for instance.

Notes

Summary

0m 19s

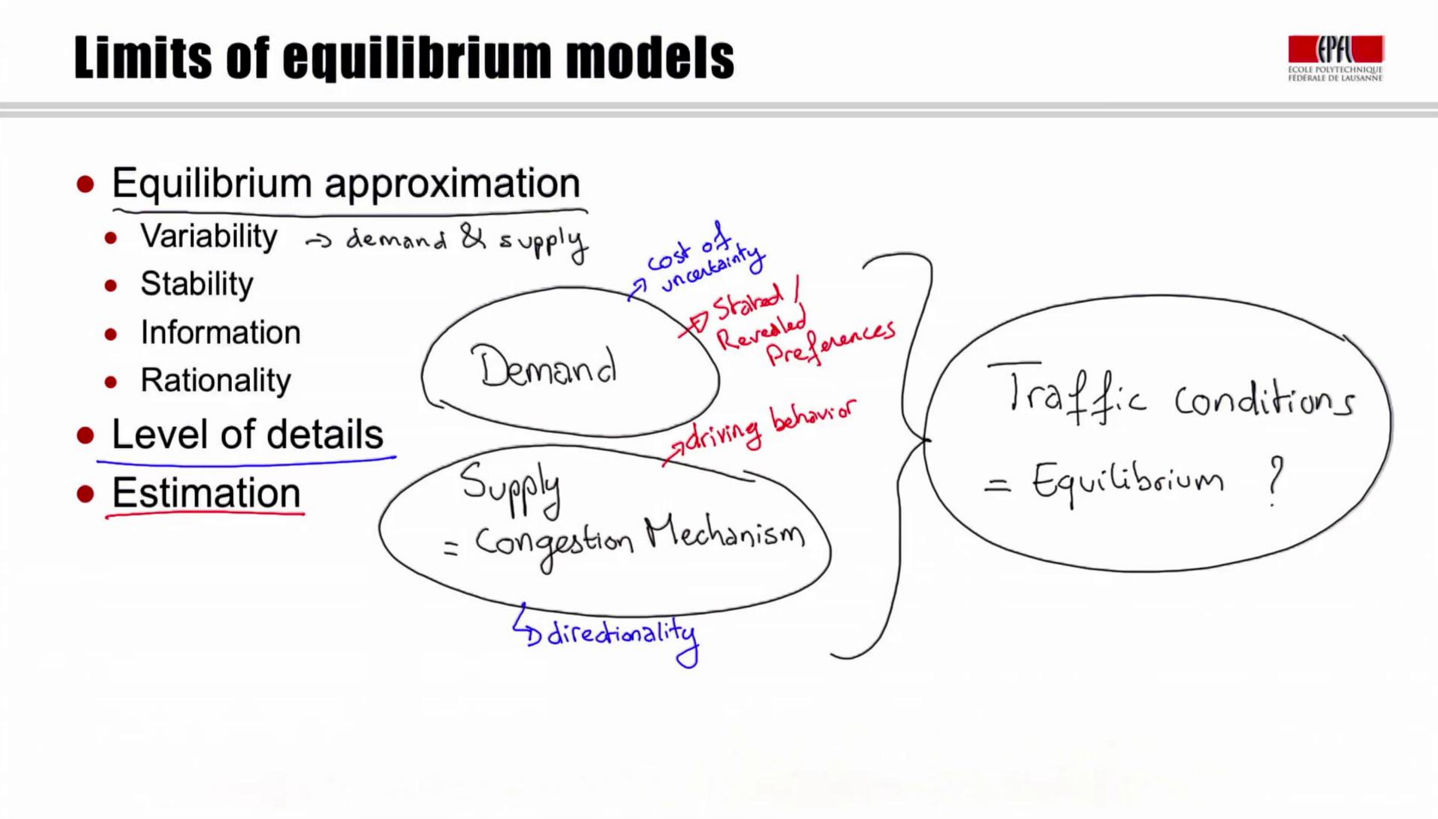


Another important issue is stability. Indeed, we have seen that equilibria may exist, but nothing guarantees that equilibria are actually attracting and repelling but we will cover this in more details in the next slides. Then, users may not be fully informed, or they might not be fully rational. Although this is changing as nowadays, people start using navigation apps now and more often. And with this, they become better informed, and perhaps also more rational. Let's now discuss a bit the level of details. The models we have described in the previous videos were very simple. Of course, some simplifications are always necessary to keep some intuition. But we should not omit important phenomena. One important thing that we have mostly left aside is the cost of uncertainty. If we have viability, we have some uncertainty, for instance, in troubled times, and this is known to be one of the key factors influencing much choice for example. And by focusing on a deterministic equilibrium, we somehow ignore this which is a quite questionable choice. And the supply side, the congestion mechanism that we considered were all very simple.

Notes

Summary

1m 52s

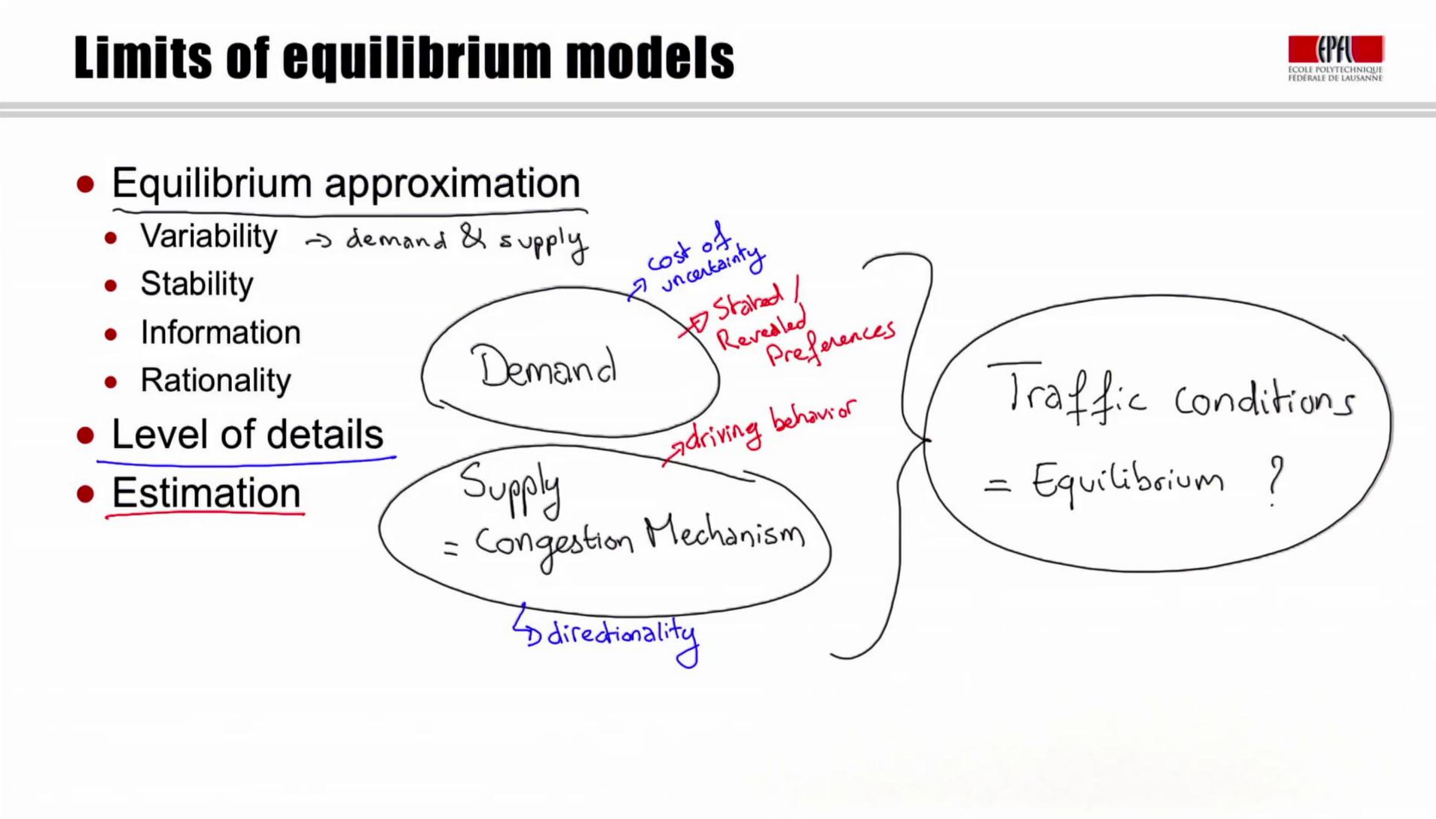


Actually, they were all unidirectional that is that the congestion only occurred between users that traveled on the same link and in the same direction. In practice, different directions often interact, for instance, there are traffic signals or roundabouts. Finally, even if he could find perfect models, we would still have great difficulties to estimate them. Congestion mechanisms are perhaps simpler, because the described almost physical phenomena. But the parameters of this phenomenon still depend on some behavioral parameters like the driving aggressiveness of erection time. To estimate the demand, we use two types of methods which are called stated preferences and revealed preferences. With stated preferences, we basically asked people about what they would do in different situations while with revealed preferences, we try to infer people's preferences from the choice that actually make in real life. Of course, when they're available, revealed preferences are much more reliable. With stated preferences, people might have difficulties to imagine the situation. And even if the situation is relatively simple, and well known from them, users may reply in one way and act in another.

Notes

Summary

3m 07s



The problem with revealed preferences is that we can only observe existing alternatives. And these typically differ in more than one aspect. The only exception is perhaps hotlane, where you can pay a small toll to use a lane that goes a bit faster and the traffic sign at the beginning of the road indicates the expected travel times on the normal road and under hotlane.

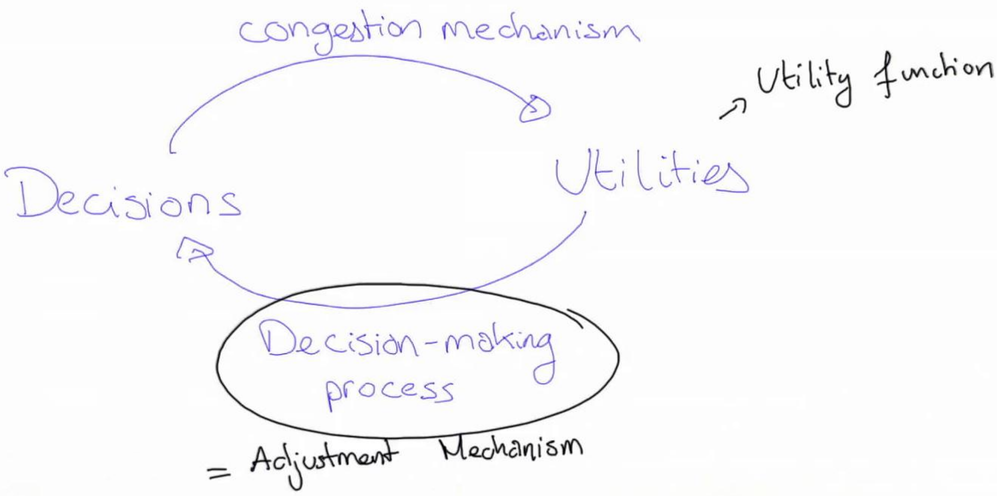
Notes

Summary

4m 34s

Equilibrium stability

- Day-to-day dynamics

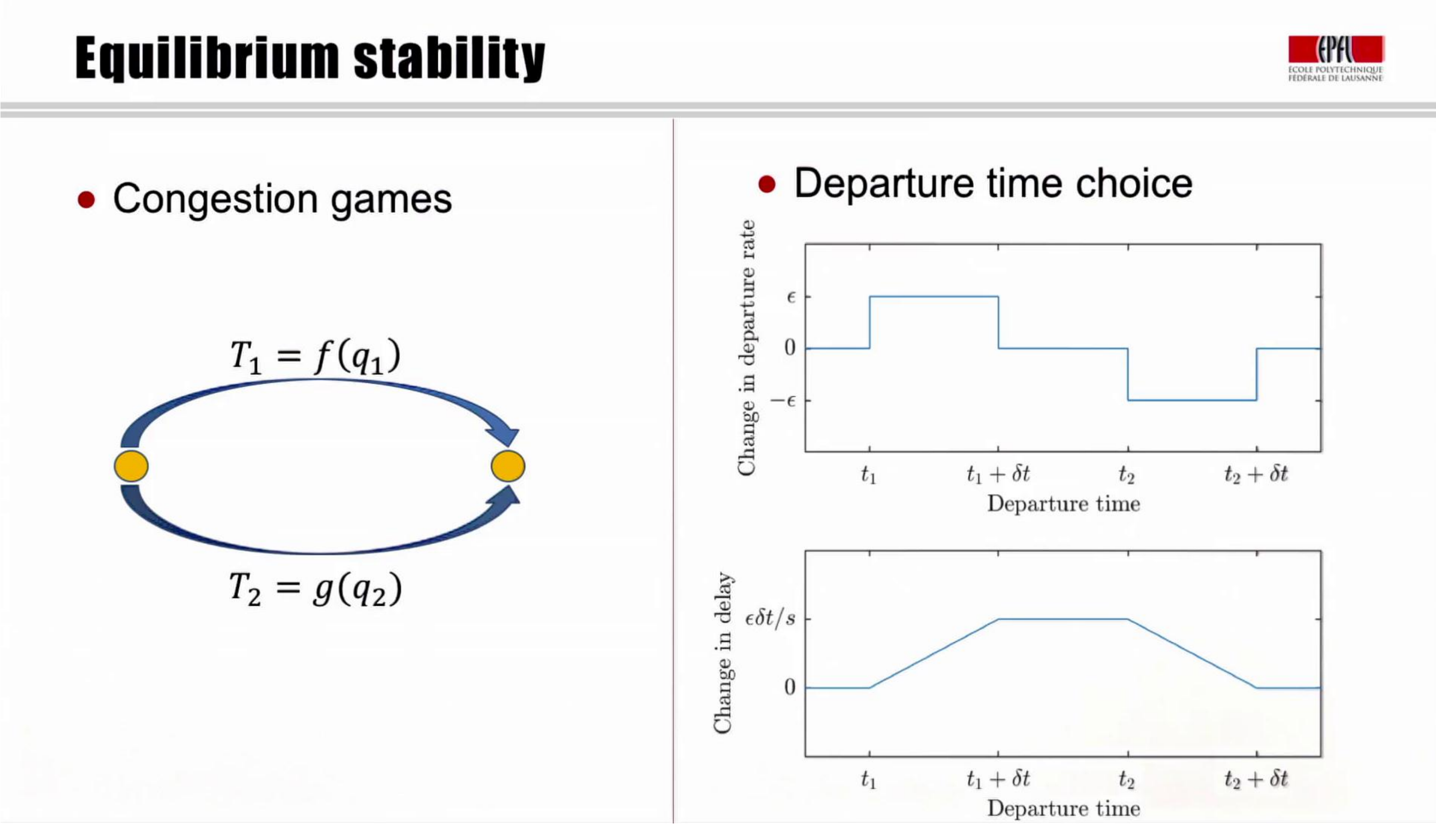


Let's now discuss about stability, it is difficult to obtain general results because the stability of a user equilibrium depends not only on the condition mechanism and under scheduled preferences, but also in how users adjust the decisions. It should remember from the first video of this week, I made this drawing where this is the process that people use to update their decision. This is also called the adjustment mechanism. So the stability of a user equilibrium depends on the congestion mechanism and the adjustment mechanism and on the utility function. If you want to be rigorous, stability can be quite technical. Here, I will just give you the main intuition.

Notes

Summary

4m 58s

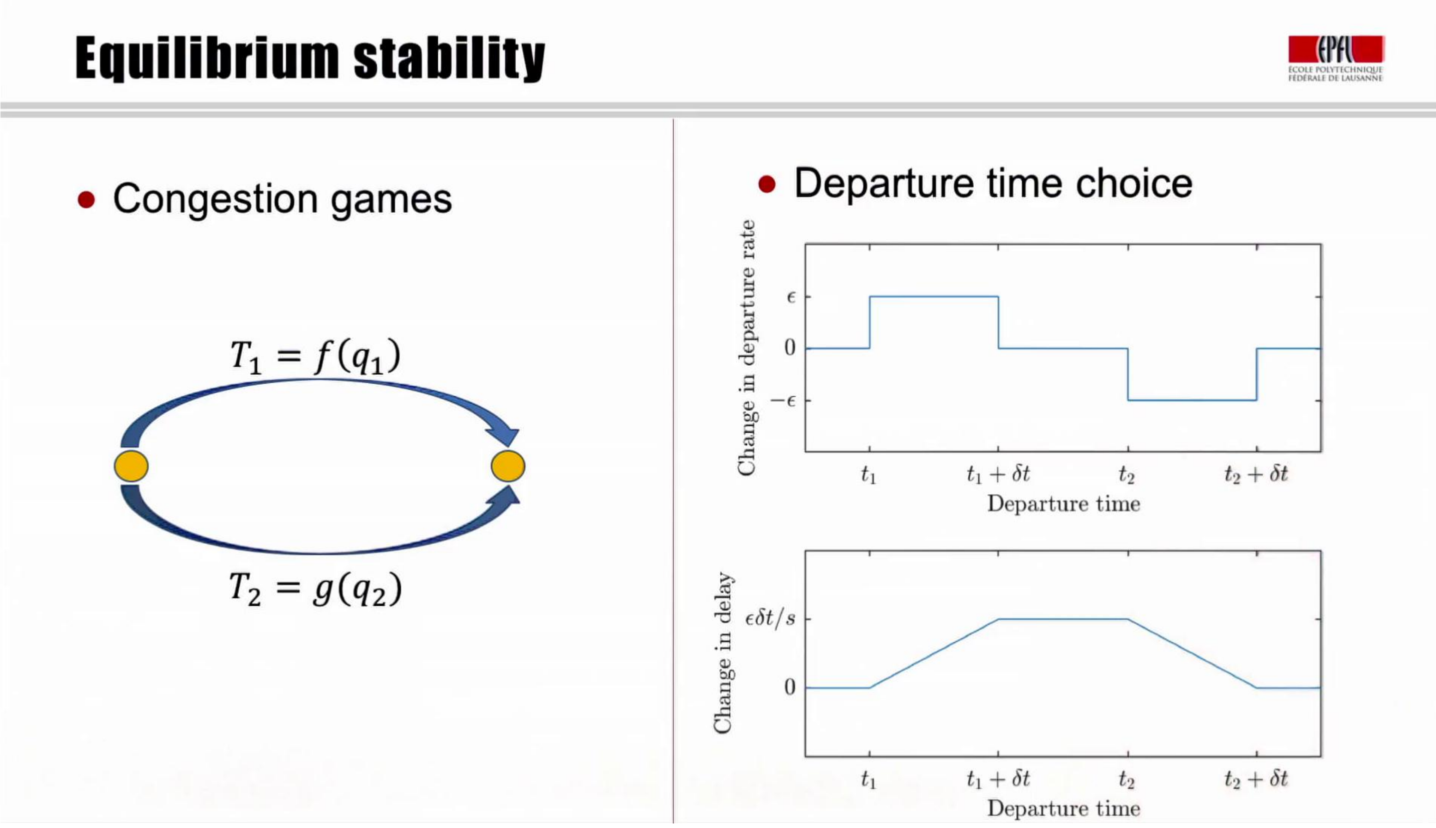


In congestion games, the equilibrium is likely to be stable, because the cost of an alternative increases with the number of people that chooses this alternative. For instance, if we are in a situation like this, and the travel time is a bit larger on Route one, then people are likely to switch from route one to route two which will increase a little bit the travel time on route two, decrease the one on one, and both will get closer to each other. So we will get closer to equilibrium. With the budget time choice however, this is a completely different story. If we consider that this whole time period is congested, and we add when user arrives at the bottleneck at time zero, then the effect on the others is no for the users that arrived at the bottleneck before that user because the queue, they will experience will be the same. But all the users that arrived afterwards will see a queue that is when user longer. So if we consider a system that is near equilibrium, and that we change it a little bit by taking some users from this range of the budget time to that range of the budget time, the consequence in the delay is as follows. For users departing before T1 or after T2 present there is no change.

Notes

Summary

5m 40s



For users departing between T1 and T2, they see that there is this many users just before them, so it increases a delay. And in between it varies linearly. Intuitively, this is problematic as users at the same incentive to switch where users were added and where users were removed.

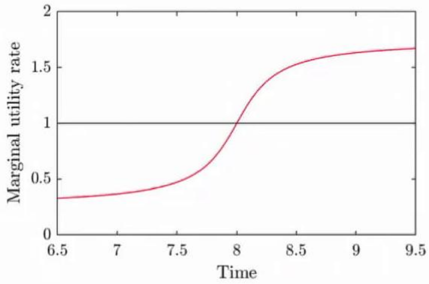
Notes

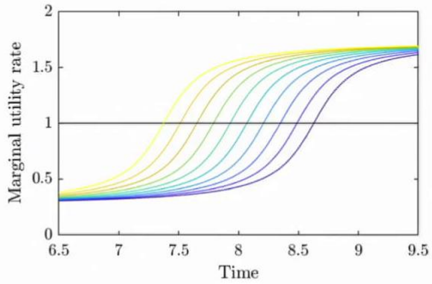
Summary

6m 59s

Equilibrium stability

- A simple simulation experiment:
  - Single OD pair, with unidirectional bottleneck
  - Schedules preferences





  - Smith's adjustment mechanism:

  proportion of population changing from alternative i to j proportional to $\lfloor U_j - U_i \rfloor_+$

Let's make a simple simulation experiment. So we consider a single origin destination pair with the unidirectional bottleneck and schedule preferences that have this shape. So the marginal utility rate at home is constant, and the one at the destination increases like this smoothly. And this is the case with homogeneous users. And with heterogeneous users, we just consider a small offset in the marginal utility rate at the destination. So we translate a little bit this curve in different directions. And in order to go from one day to the next, the adjustment mechanism that we use is known as Smith's adjustment mechanism and it is such that the proportion of the population that changes from alternative A, to alternative J is proportional to the difference, [UJ-UA] if the difference is positive, and it's zero, if it's negative.

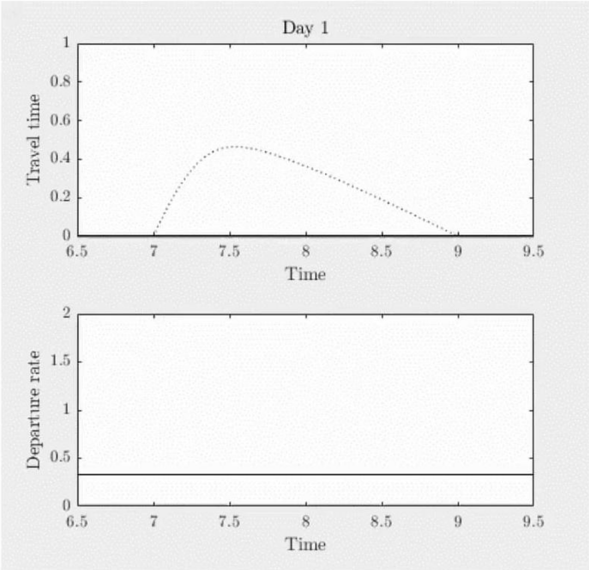
Notes

Summary

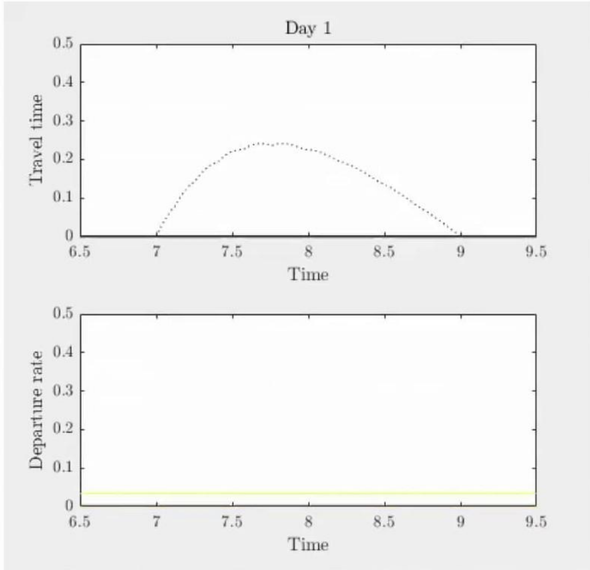
7m 23s

Equilibrium stability

- Homogeneous users



- Heterogeneous users

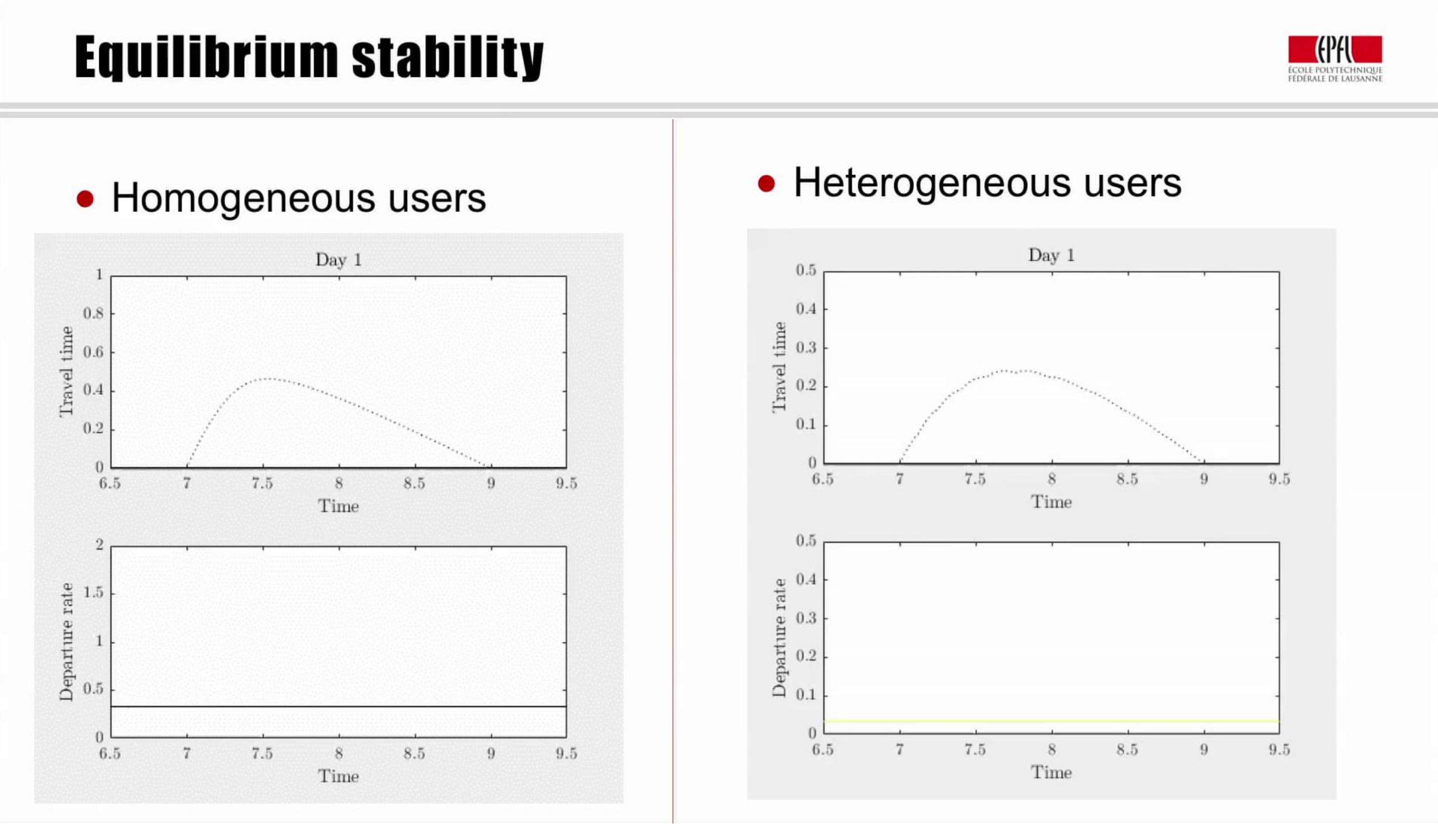


Let's now see some simulations. We start with the departures that uniformly distributed between 6:30 in the morning and 9:30 in the morning, then people will adjust based on the conditions they observe. So if we start a simulation, that's what we observe. So the departure rate adjusts as a function of the observed travel time and the dashed line that you see here is equilibrium travel time and as we see, the travel time in simulation is always quite different and it exhibit some oscillations that are not so close to the equilibrium travel time. Intuitively, this is perfectly consistent with what we described just before for the bottleneck model. Fortunately, with heterogeneous users, we will see that there is a bit more stability. So again, we start with departures that are uniformly distributed between 6:30 and 9:30 but now let's see what it gives once we went through the simulation. So the different curves represent the departure rate for different families of people with different offset in their marginal utility rate and we see that as time goes by this different families segregate themselves, the one that have that it's relatively too late tend to depart quite late and altogether this confers some stability to the system and the oscillation that we observe much closer to the equilibrium travel time.

Notes

Summary

8m 28s



This static code choice problems are expected to be with a stable while for departure time choice, the stability really depends on the amount of heterogeneity it and probably also on other factors.

Notes

Summary

10m 03s

Summary

We are reaching the end of this lecture and it is now time to conclude. We have seen that equilibria are very useful to predict the advancement of travelers to any type of path formation. To ensure tractability and intuition, We have only considered two decisions would and departure time and we consider them separately that nothing prevents us from combining these or even other decisions in simulations. Simulations also allow for greater level of details and largely use in practice. We should however, keep in mind that some limitations remain and in particular, that there is no point in using models that we cannot accurately estimate. Thank you and goodbye.

Notes

Summary

10m 15s