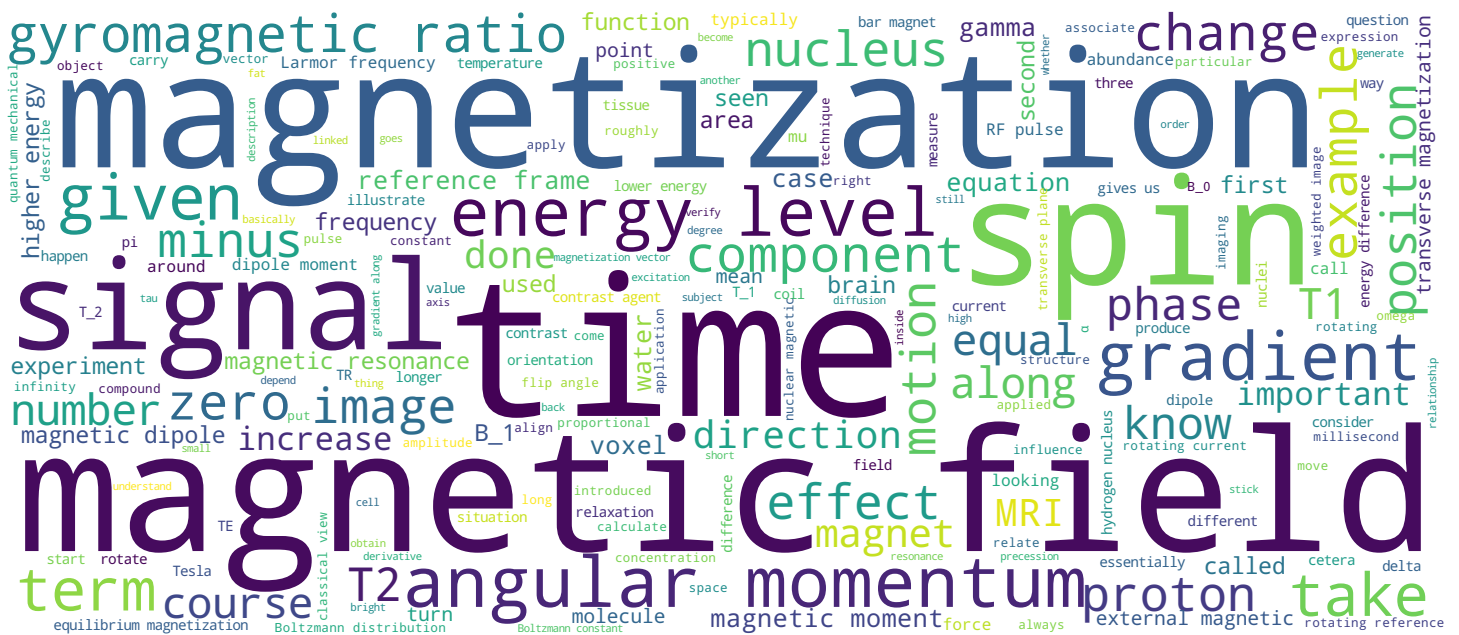


8.2 Basis of nuclear magnetization

Fundamentals of Biomedical Imaging

Prof. Rolf Gruetter



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Video

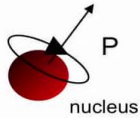


8-2. What is the basis of Nuclear Magnetism ?

Classical and quantum-mechanical view

Nucleus \rightarrow angular momentum L (here called P)

\Rightarrow Rotation of electrical charge (nucleus)



8-8

So the question that now arises is how is nuclear magnetism generated? What is its basis? And we will here talk about the classical view, more on a classical view than the strict quantum-mechanical view but I will make some references to the underlying quantum-mechanical theory. Now it is known from nuclear physics that a nucleus typically has an angular momentum, which we know from our physics classes is denoted L but here it is called P . So we can envisage we have the nucleus here and the nucleus has an angular momentum. Now we have with an angular momentum, classically-- now we'll take here the classical view-- classically we associate with an angular momentum a rotating motion. So we have a rotating motion and we'll stick with the classical view because this gives us an intuitive feeling where the magnetism comes from. So if we feel like in the description that the nucleus is rotating, since the nucleus is positively charged, there is a rotation of the electrical charge. As you can imagine, it's like the earth-- it's positively charged, it's rotating so at the equator the charge is moving and moving charge implies a current.

Notes

Summary



0m 04s

8-2. What is the basis of Nuclear Magnetism ?

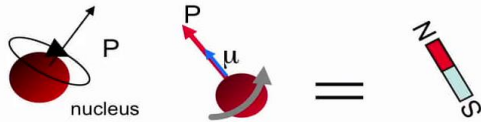
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Nucleus → angular momentum L (here called P)

⇒ Rotation of electrical charge (nucleus)

⇒ Rotating current

⇒ Dipole moment



Magnetic moment μ of individual spin in induction field B_0 . $\vec{\mu} = \gamma \vec{P}$

8-8

So here is the angular momentum and since we have a rotating current-- here's the rotating current-- so we'll associate this with a rotating current and such a rotating current now is associated with a dipole moment. It's just like a current. This is again the classical picture. It's not perfect but it helps us understand where nuclear magnetism comes from. So we'll associate it just like with a ring current with a dipole moment μ that we'll call μ here. Now in terms of magnetism we can picture the nucleus as if it was a bar magnet. The magnetism of the nucleus acts as if it is a bar magnet. It's not a bar magnet per se but it acts electromagnetically like a bar magnet. It has a dipole magnetic field associated with it. So that's the intuitive reason why with some nuclei we have an angular momentum. Once we have an angular momentum we have with it associated a dipole moment. Now if we look at a magnetic moment of an individual spin in a induction field so this dipole moment, this magnetic moment is given by μ , is equal to γ times the angular momentum. γ here is called the *gyromagnetic ratio*, it's an empirical constant.

Notes

Summary



1m 20s

8-2. What is the basis of Nuclear Magnetism ?

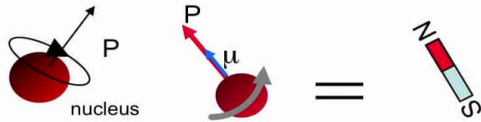
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Magnetic moment μ of individual spin in induction field B_0 . $\vec{\mu} = \gamma \vec{P}$

γ : **gyromagnetic ratio** (empirical constant)

The angular momentum P of a nucleus is quantized:

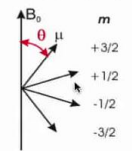
P_z has $2I + 1$ values (m):

$$P_z = \frac{h}{2\pi} \cdot m_I$$

$$|\vec{P}| = \frac{h}{2\pi} \cdot \sqrt{I \cdot (I+1)}$$

Spin $1/2$: $P = h\sqrt{3}/4\pi$

$-I, -I+1, \dots, I-1, I$



8-8

There's a more profound theory behind it on what dictates the gyromagnetic ratio for a given nucleus, but for our intents and purposes it suffices to say that this is an empirical constant. Now what we know from quantum mechanics is that the angular momentum of a nucleus is quantized. So it has, for the z component of the angular momentum, it has $2I + 1$ values and they take on the values $m \cdot i$. So here is, for example, where i can go from minus the angular momentum-- the z component, the quantum number m here-- can go from minus i to plus i and increments in values of 1. So here, in this particular example, we go from minus $3/2$ to minus $1/2$ plus $1/2$ plus $3/2$, that's the z component of the angular momentum. We can calculate the total angular momentum and this is given by this equation here. So it's the quantum number, the angular momentum number here i times $i + 1$ times h over 2π . So if we take this for a spin $1/2$, so the spin $1/2$ can take the values $1/2$ and minus $1/2$ the angular momentum. In this case, the total angular momentum is times the Boltzmann constant times square root of $3/4\pi$ that we can verify by putting here into i : $1/2$.

Notes

Summary



8-2. What is the basis of Nuclear Magnetism ?

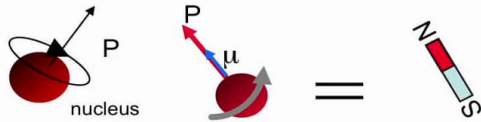
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⇒ Dipole moment



NMR-active isotopes and their **gyromagnetic ratio** γ

$I = 0$ (^{12}C , ^{16}O , etc.)

Even mass # & Even atomic #
No Nuclear spin

$I = 1/2$ (^1H , ^{13}C , ^{15}N etc.)

Spherical charge distribution in nucleus

$I > 1/2$ (^2H , ^{11}B , ^{23}Na etc.)

Odd mass # & Odd atomic # ($I = 1/2$ integer)
Even mass # & Odd atomic # ($I = \text{whole integer}$)
Ellipsoidal charge distribution in nucleus
gives *quadrupolar electric field*

Magnetic moment μ of individual spin in induction field B_0 . $\vec{\mu} = \gamma \vec{P}$

γ : **gyromagnetic ratio** (constant)

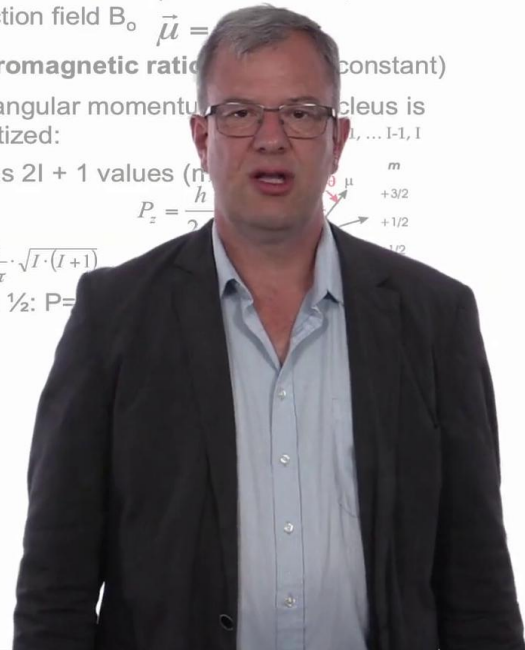
The angular momentum of nucleus is quantized:

P_z has $2I + 1$ values ($m = -I, \dots, -1/2, 0, +1/2, \dots, I$)

$$P_z = \frac{h}{2\pi} \cdot \frac{m}{I} \quad \begin{matrix} +3/2 \\ +1/2 \\ 0 \\ -1/2 \\ -3/2 \end{matrix}$$

$$|\vec{P}| = \frac{h}{2\pi} \cdot \sqrt{I \cdot (I+1)}$$

Spin $1/2$: $P = \frac{h}{2\pi} \cdot \frac{1}{2}$



Notes

So that's for a spin $1/2$. So let's look at some isotopes and the gyromagnetic ratio *gamma*. And first we want to introduce some isotopes that do not carry an angular momentum. Those are carbon-12, oxygen-16, et cetera. They have typically even mass and even atomic number. So they don't carry a nuclear spin, they don't carry a nuclear angular momentum. Then we have nuclei with spin $1/2$. These are the proton, the hydrogen nucleus H-1 , C-15 , and nitrogen-15. Another one is phosphorus-31. They are characterized by a spherical discharged distribution in the nucleus. And then we have the spins that are larger than spin $1/2$. Deuterium is an example, boron-11, sodium-23, lithium-6 is another one, lithium-7, et cetera. Depending on whether they have odd or even mass and an odd atomic number they carry either half integers-- so they're $3/2$, $5/2$, et cetera-- or a whole integer, their spin 1 , 2 , 3 , et cetera. What's particular about these nuclei is that they have an ellipsoidal charge distribution. They generate within the nucleus in a electric field and it's a so-called *quadrupolar electrical field*. They have distinct different properties typically than the spin $1/2$.

Summary



8-2. What is the basis of Nuclear Magnetism ?

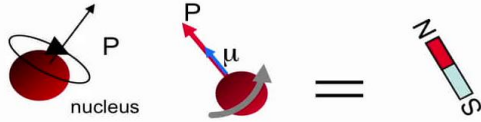
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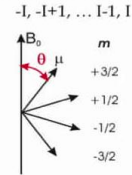
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Ellipsoidal charge distribution in nucleus
gives *quadrupolar electric field*

Spin $1/2$: $P = h\sqrt{3}/4\pi$

Isotope	Net Spin (I)	gyromagnetic ratio $\gamma/2\pi$ [MHz T ⁻¹]	Abundance / %
^1H	1/2	42.58	99.98
^2H	1	6.54	0.015
^{31}P	1/2	17.25	100.0
^{23}Na	3/2	11.27	100.0
^{15}N	1/2	4.31	0.37
^{13}C	1/2	10.71	1.108
^{19}F	1/2	40.08	100.0

8-8

Now let's look at some examples here. So we have the isotope listed. We have the spin, whether it's spin 1/2 et cetera. The gyromagnetic ratio. And here we list the gyromagnetic ratio not as the gyromagnetic ratio but we divide it by 2π so this gives us a number that relates the Larmor frequency to the-- that relates the frequency to the teslas. And finally we have the abundance. So what do we see here? We have hydrogen spin 1/2, gyromagnetic ratio is 42 MHz/T and it's basically every hydrogen atom is a H1. Then we have deuterium, this has a spin 1 so it's a quadrupolar nucleus, very low gyromagnetic ratio and the abundance is about 0.01%. Phosphorus-31 is a spin 1/2, gyromagnetic ratio is about 40% of that of the hydrogen--of the proton-- and the abundance is 100%. Sodium: 100% natural abundance here. It's a spin 3/2 and the gyromagnetic ratio is roughly a fourth of what the same holds for carbon-13, but the abundance for carbon-13 is 1.1%. And fluorine-19 is almost the same gyromagnetic ratio as protons and the abundance is also very high, we just don't have a lot of fluorine in our bodies.

Notes

Summary



Unequal population of Energy levels

Energy of a magnetic dipole in magnetic field B_0 (classical)

$$E = -\vec{\mu} \cdot \vec{B}_0 = -\mu \cdot \cos\theta \cdot B_0 = -\mu_z \cdot B_0$$



8-9

Now for reasons of the abundance and the presence we will be mostly concentrating this course on considering the hydrogen nucleus--that is the proton-- as the source of the MR signal. And for all intents and purposes suffice it to know that there are the nuclei around, how to understand them but most of what we are going to deal with is focusing on the hydrogen nucleus, that is the proton. It has spin 1/2. It has a high gyromagnetic ratio, and we'll see shortly why this is important. And what's also important, it is present in our bodies in raw abundance. So far I've explained that some nuclei carry an angular momentum, they therefore carry a magnetic moment but now how does this generate a macroscopic nuclear magnetization? And that's the question we're going to address here. So let's look at the energy of a magnetic dipole in an external magnetic field and we'll stick here to the classical description. The energy is given by the scalar product of the dipole moment with the external magnetic field. And it is given by μ times *cosine*-- that's because of the scalar product-- or if we say $\mu \cos\theta$ that is the z component of the dipole moment times B_0 . That is the energy of a magnetic dipole.

Notes

Summary



7m 37s

Unequal population of Energy levels

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Energy is minimal, when $\mu \parallel B_0$

(Where is that used ?) $\vec{\tau} = \vec{\mu} \times \vec{B}_0$



8-9

Now this energy is minimal when the magnetic dipole is aligned with the magnetic field. That is actually used. Where is that used? Or where was it used? Well think about Christopher Columbus, he needed to know where the North Pole is so he had to use a way to identify the magnetic North Pole and this was done with magnetized needles orienting in a liquid parallel to the earth's magnetic field. So that was used in navigation. Now what's behind this is that a dipole will align itself to find the minimal energy and there's a torque exerted on the dipole that is given by the magnetic dipole, is vector product of magnetic dipole with the external magnetic field. So let's just demonstrate this in the following experiment where we will demonstrate the force of the external magnetic field on the dipole with a magnetic dipole.

Notes

Summary



9m 08s



As mentioned in the course, the energy of a magnetic moment in atomic-- a nuclear magnetic moment in an external magnetic field is not dissimilar to what happens with a macroscopic magnet in external magnetic field. And I want to illustrate this with this experiment here. We have a magnet here which can rotate, so it can freely rotate. Here is a--it's attached here to show the torque. And here we have two electromagnets. These two electromagnets will develop the magnetic field when I turn them on. So right now there is no force on this. The magnet is like this. And when I now turn on the magnetic field, you can see that the magnet tries to align itself. If we didn't have here this force meter, it would align itself perfectly with the magnetic field. If I reverse the polarity of the magnetic field, the motion is in the other direction in this sense. I can see if I can illustrate this if I loosen this, then now here you can see the perfect alignment if I don't have the force attached to it which is meant here to illustrate the development of the force. So like a macroscopic magnet in a magnetic field, nuclear magnetic moments have a tendency to align themselves in one direction or the other and their energy that they have in the magnetic field depends on their orientation but the minimum is found when the moment is as much aligned as possible with the magnetic field.

Notes

Summary

10m 19s



Unequal population of Energy levels

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Quantum mechanical description:

$$E_I = -\gamma \cdot \frac{h}{2\pi} \cdot m_I \cdot B_0 \quad m_I = -I, \dots, I$$

$$E_1 = +\gamma \cdot \frac{h}{4\pi} \cdot B_0 \quad m = -1/2 \text{ (N}_1 \text{ spins)}$$

$$E_2 = -\gamma \cdot \frac{h}{4\pi} \cdot B_0 \quad m = 1/2 \text{ (N}_2 \text{ spins)}$$

8-9

Okay, now we have stated the energy as a function of the z component of the magnetic moment. So it's this term here-- μ_z times B_0 . So we will now look at the quantum mechanical description and we know that the z component of the angular momentum therefore of the magnetic dipole is quantized, has discrete values and it's given by this expression. This is the gyromagnetic ratio and these are the terms that we have introduced for the angular momentum just in the previous slide. So we have energies that depend on the level of the z component of the magnetic moment that is the angular momentum. Again, m_i can go from minus i to plus i . For a spin 1/2 it's minus 1/2 to plus 1/2. So if we said for spin 1/2 we'll take the energy level below so that is minus γ , h over 4π times B_0 here's the energy level. This is for the m of 1/2 and we'll say that our m_2 spins in that energy level. Then we have the higher energy level, that's where the dipole is opposite-- in opposite orientation-- here the energy is now positive. It's spin 1/2 so it's plus minus 1/2. The calculation's given so m equals minus 1/2.

Notes

Summary



Unequal population of Energy levels

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$$\Delta E = \gamma \cdot \frac{h}{2\pi} \cdot B_0$$

Boltzmann statistics/distribution:
Unequal population of energy levels

$$\frac{N_1}{N_2} = e^{-\frac{\Delta E}{kT}}$$

k : Boltzmann's constant (1.4×10^{-23} J/Kelvin)

NB. At 310K : ~ 1 in 10^6 excess protons in low energy state (1Tesla)

8-9

You can verify this with this equation here. And we'll say we have m_1 spins in that energy level. We now have an energy difference. Whether the nucleus is spinning upwards or downwards, it has an energy difference depending on its orientation that is given by the gyromagnetic ratio times Boltzmann constant divided by 2π times the magnetic field B_0 . Now we know from Boltzmann statistics or a Boltzmann distribution, we know that we are going to get an unequal population of energy levels. This is dictated by Boltzmann distribution. And the relationship between the number of spins in the higher energy level--that is N_1 --to the number of nuclei in the lower energy level, this relationship is given by e to the minus the energy difference divided by the thermal energy, which is kT . K is the Boltzmann constant, here is its value given. It's not really important.

Notes

Summary



13m 16s

Unequal population of Energy levels

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8-9

But what is important if you look at this ratio, and you do the math, you plug this in, you calculate the ΔE divide it by kT , at 310 kelvin-- which is roughly our body temperature-- you'll find that at a field of 1 Tesla, if we look at the proton spins so we take the gyromagnetic ratio of the hydrogen-1 nucleus--the proton-- then you find that for every million protons in the higher energy level there's a million protons, a million and one proton in the lower energy level. That makes a difference, an excess in difference between the two energy levels of one out of a million. So basically, as a rough approximation, we can say N_1 is the number of spins in the upper energy level is almost equal to the number of spins in the lower energy level where N --and we'll call this $N/2$ -- where N is the total number of spins or hydrogen nuclei present in our voxel. So this is the Boltzmann distribution. Now if you look at the situation here, if we are now looking at the distribution of the spins we have them all in the lower energy level and none in the higher energy level. Is such a state even possible? Well, in theory it is.

Notes

Summary



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$\rightarrow N_1 \sim N_2 \sim N/2$ (N = no of spins)

Transitions between E_1 and E_2 induced by photons

$$h\nu = \Delta E$$

8-9

This basically means that N_1/N_2 goes to zero and this is if we let the temperature go to zero. Then $e^{-(1/0)}$ becomes $e^{-(\text{infinity})}$ that becomes zero. So if we cool our sample down to micro- or nanokelvin, you'll be essentially getting something at approximative state. This is the ground state of the system. For us in biomedical imaging, this is not very interesting. We cannot really cool down our subjects for imaging. So if we raise the temperature now gradually, what we have is that the thermal energy will bump some of the spins into the higher energy level. And as we increase the temperature we have more and more in the higher energy level. We'll still have a certain difference in the population unless we have the temperature go to infinity, at very high temperatures, then they are truly equalized. That we can see again here if the temperature goes to infinity, this goes to zero, e^{-0} is equal to 1. So we have this population difference that is generated between the nuclei in the higher energy level and the nuclei in the lower energy level. And this difference is important. Before we go on though, I want to make the point here that we have also these transitions between the energy levels is induced by photons.

Notes

Summary



What is the basis for nuclear magnetization ?

Unequal population of Energy levels

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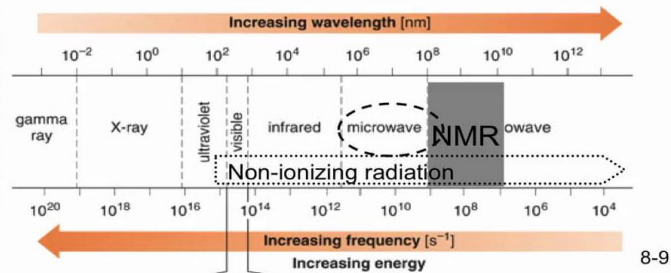
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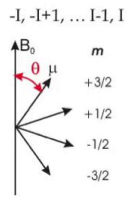
8-9

Photon is the interaction boson for electrodynamics. So we can associate this energy difference here to ΔE with a photon in the $[h\nu]$ and then we can look this up what this value is, to relate this to the x-ray techniques that we have been discussing in the first half of this course. And then we can see actually that for NMR--nuclear magnetic resonance or magnetic resonance--we have the energies are in this range. So they are in this wavelength range here. We are well beyond microwave wavelengths, so it's a very long wavelength. And what is important here-- we are working with non-ionizing radiation in magnetic resonance.

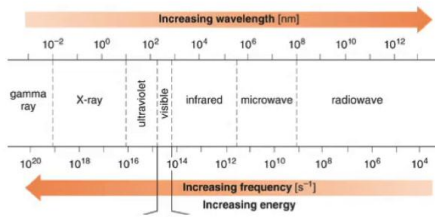
Notes

Summary





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Notes

Summary

