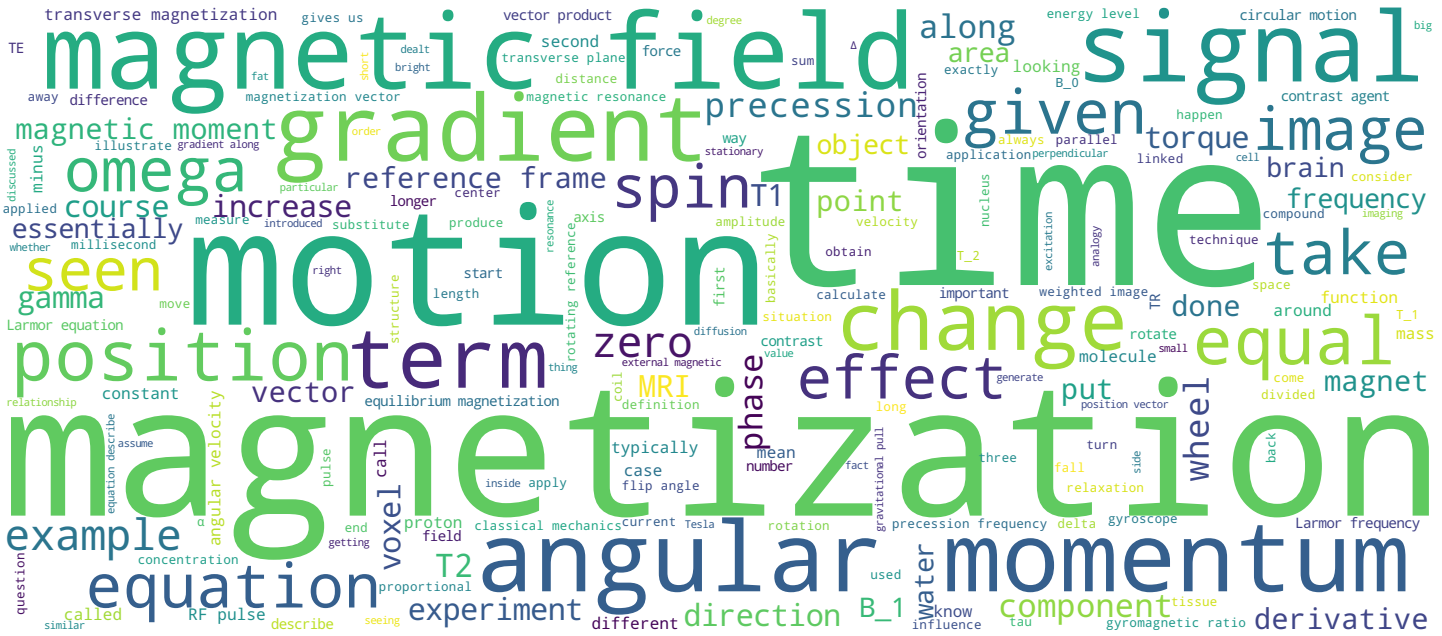
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

8.3 Motion of magnetization

Fundamentals of Biomedical Imaging

Prof. Rolf Gruetter



Search MOOC

Video

EPFL

3-3. How to classically describe the motion of magnetization ?

View each spin as a magnetic dipole μ
(a tiny bar magnet).
Classically: torque τ of a dipole μ in B

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

2nd law of rotations (P: angular momentum)

$$\vec{\tau} = \frac{d\vec{P}}{dt}$$

So now we have dealt with the difference in population, we have dealt with the introduction of a magnetic moment. Now we are going to have to address the question: "How does this generate a macroscopic magnetic field?" And this is what we call the magnetization. So how do we see this motion? Now we'll do--by the following analogy again, to classical physics-- we'll view each spin, each nucleus as a magnetic dipole *mu*-- that is, we can visualize it without it having any resemblance to the actual situation. It's just the electromagnetic properties but we can think of it as a virtual tiny bar magnet. Now classically, as I mentioned before, the magnetic field exerts a torque on that dipole, a torque *tau* and this torque is given by the vector product of the magnetic moment of that spin with the external magnetic field. Now we'll take the second law of rotations and here *P* denotes the angular momentum. So we know from the second law of rotations that the derivative of angular momentum with time is equal to the net torque. Here we have one torque. And we can now use this and substitute the angular momentum with the magnetic moment.

Notes

Summary

0m 04s

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$$\vec{\tau} = \frac{d\vec{P}}{dt} \qquad \vec{\mu} = \gamma \vec{P}$$

$$\frac{d\vec{\mu}}{dt} = \vec{\mu} \times \gamma \vec{B}$$

Sum over all $\mu_k \rightarrow$ Magnetization $\vec{M} \equiv \sum \vec{\mu}_k$

Larmor equation

$$\frac{d\vec{M}}{dt} = -\gamma \vec{B} \times \vec{M}$$

What motion does the Larmor equation describe ?

A brief tour back to rotational kinematics

8-10

We'll do this on both sides and then we are getting that the derivative, classically, of a spin. The derivative of its magnetic moment is equal to the magnetic moment times *gamma B* where *B* is the external field. So this is the motion for an individual spin equation of motion, classically. Now if we sum over all the magnetic moments that are in our voxel, all the hydrogen nuclei, all the protons that we have in there, we sum all the magnetic moments then we get the macroscopically measured magnetization. So this is--we define this magnetization as the sum of all magnetic moments present in our voxel. Then we can do the summation over this differential equation that we have here and we obtain the so-called *Larmor equation*, which is given here by the derivative of the magnetization with time is equal to minus *gamma B* cross *M*, that's the vector product. Now what does this Larmor equation describe? Have you seen a similar equation already? Well, let's look at this. So what does this Larmor equation describe? And to make the analogy, I'll go back to your first year physics course and we'll look at rotational kinematics. We'll do a brief digression here.

Notes

Summary

1m 36s

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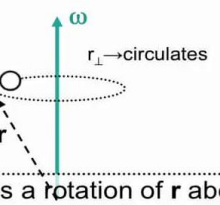
$v = \omega r = |\Delta r_\perp / \Delta t|$

$\vec{v} = \vec{\omega} \times \vec{r}$

$$\frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$$



Describes a rotation of **r** about ω with frequency f=ω/2π

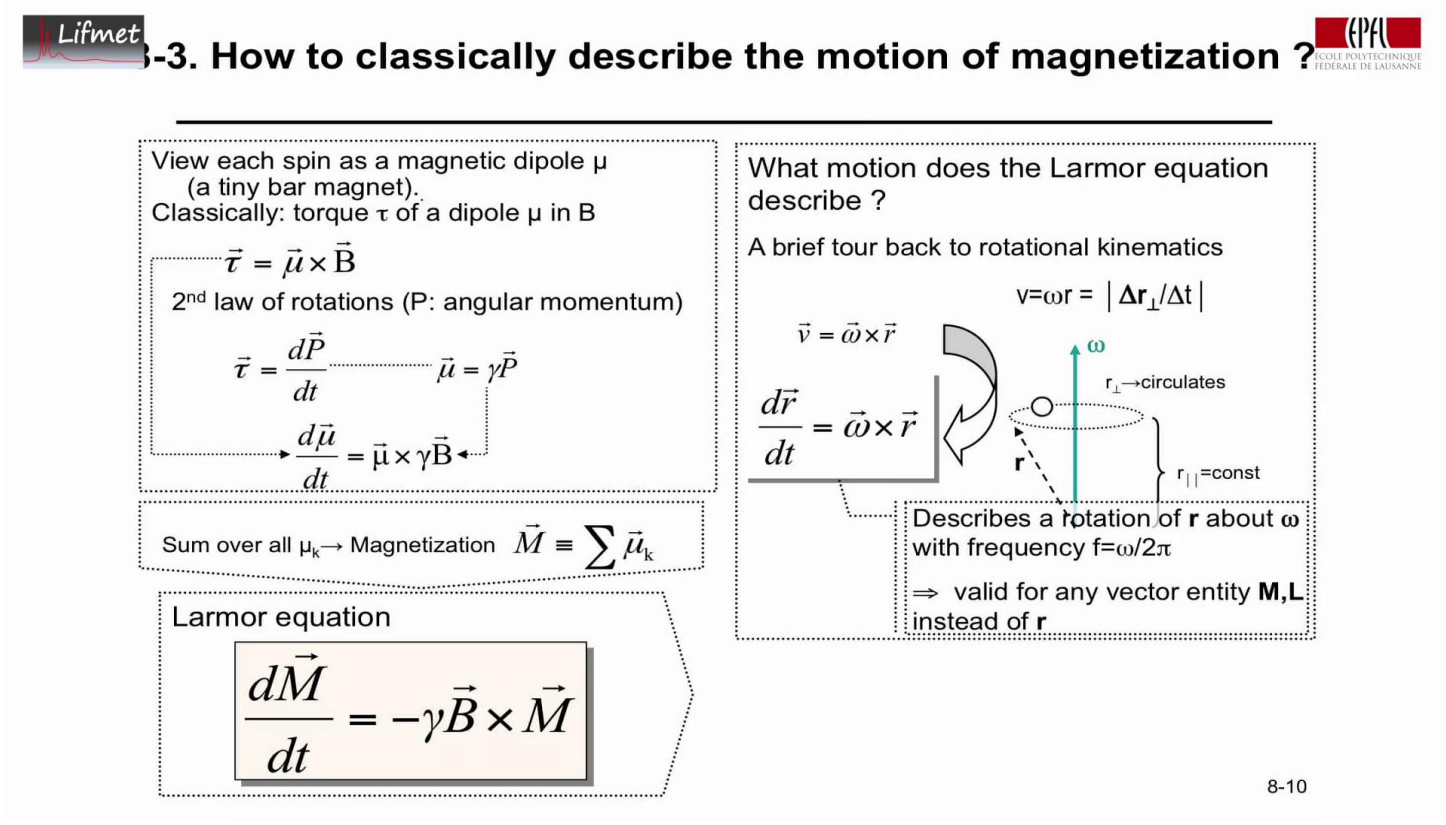
8-10

We've had, for a circular motion, we had the relationship between velocity, angular velocity and the radius of our circular motion. And if you look at this in vector form, we had velocity is equal to the vector product of the angular velocity with the position of our object. Now I can rewrite this equation for our circular motion by replacing the definition of velocity, which is the derivative of the position with time, and that is equal to the vector product of *omega* cross *r.* Now this motion, we have seen in the first year course, this describes the rotation of the vector *r* about the vector *omega* with a frequency of *omega* divided by *2π.* So we have here the angular velocity vector. I will draw now the position vector *r,* here. I'll put the tip of the position of our object in motion. That's our object-- so the vector *r* describes the position of this object. And that's exactly the motion that we see. We have the component of the position perpendicular to *omega*, which moves in a circular motion, it rotates.

Notes

Summary

3m 16s



And the component that's parallel to *omega* is stationary, it is constant because we can see here, we can always decompose the vector *r* into one that's parallel and perpendicular to *omega*, the one that's parallel, the vector product is null so that component does not change with time. So that's essentially that motion that we classically have for the position vector. Now in this equation here, I can substitute the position vector *r* with any quantity, be it angular momentum *L*, be it the magnetization vector, and I'll have an equation that is formally equivalent to what we have here. I'll substitute *r* with *M*, I'll substitute *omega* with minus *gamma B* and you can see that mathematically, the two equations are perfectly homologous, analogous, they describe the same motion for now, for a different vector. So if the *r* does--around *omega*-- does a motion like this, then the *M*--the magnetization-- will do a motion around *B* in the same fashion.

Notes

Summary

4m 33s

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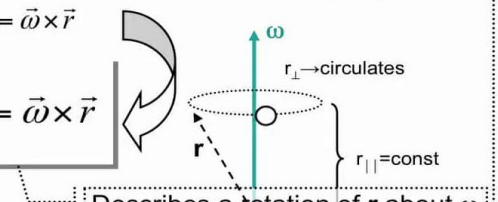


Describes a rotation of **r** about ω with frequency f=ω/2π

⇒ valid for any vector entity **M**,**L** instead of **r**

Precession of **M** about **B** with frequency $\gamma B/2\pi$

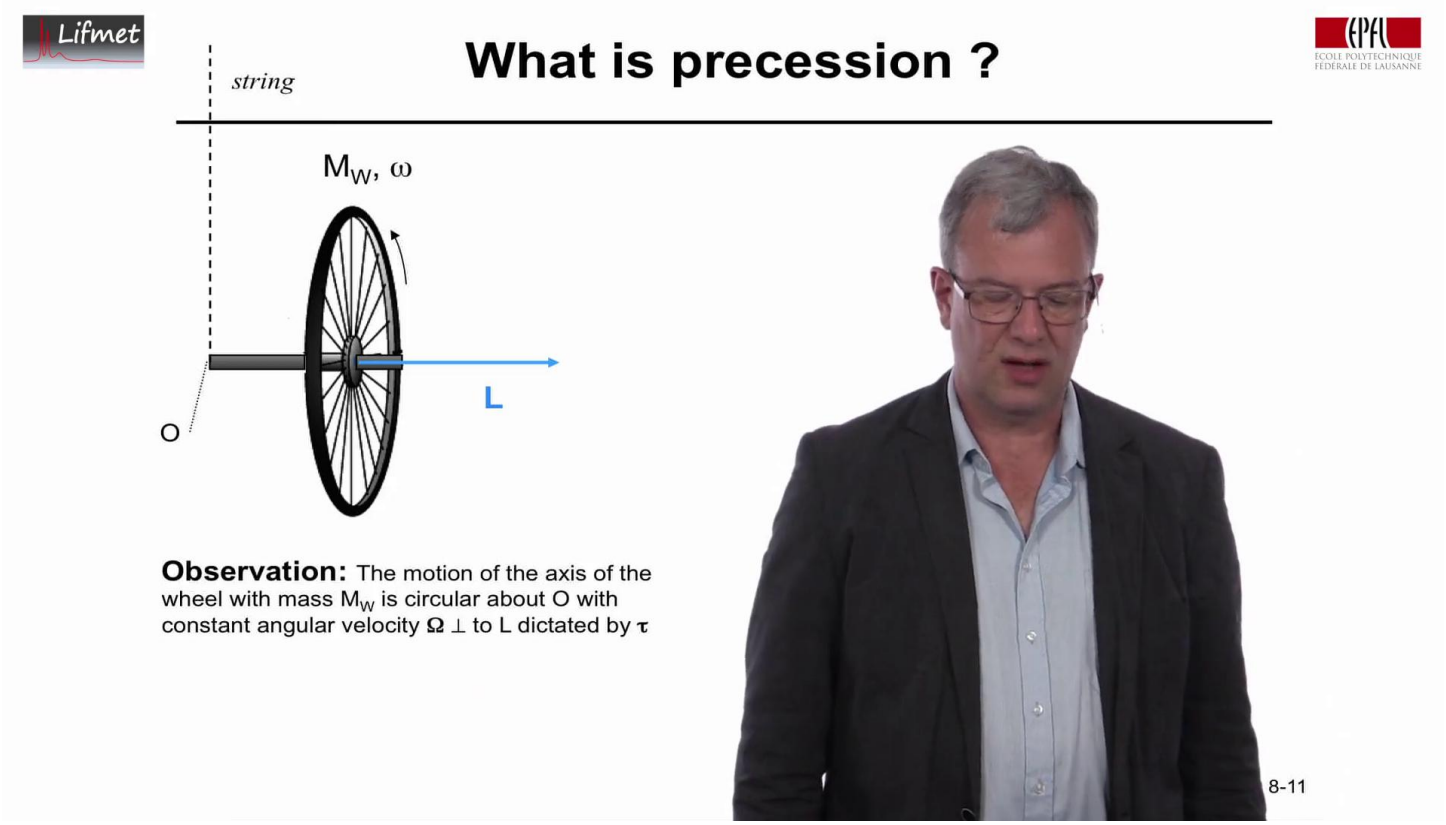
8-10

And this motion we call *precession* so what we have now is the Larmor equation describes the precession of the magnetization around the magnetic field *B* with a frequency that's given by *gamma B* over *2π*. So this is important, this motion of precession of the magnetization about the magnetic field, this relationship is very important. We're going to use it over and over again in the next weeks.

Notes

Summary

5m 42s



So what is precession? And here I'll go back again to classical mechanics. We'll take a very simple experiment. We'll have a wheel that is suspended on a string. The wheel has a certain mass and the wheel turns with a certain angular velocity. So if it turns with this angular velocity the wheel will develop, here, an angular momentum *L*, here indicated by the blue vector. The wheel is suspended at the point *O* and we can now make some observations in the experiment, and this we will demonstrate.

Notes

Summary

6m 08s

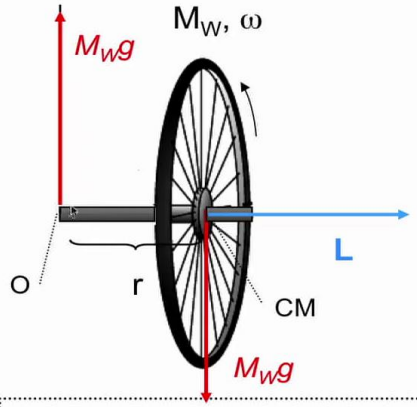
Okay, as mentioned in the course, we have an analogy in the motion of magnetization with classical mechanics and this is the experiment I want to illustrate this. We have the wheel suspended here at the yellow wire. The yellow wire will maintain its position here. I will give it a spin so it will develop an angular momentum and from the equations of motion that we've discussed from classical mechanics, there will be a torque from this string tension that will force the angular momentum into a motion of precession. We've established in classical mechanics that this motion of precession, of the angular momentum vector corresponds to one of precession. So I will now spin it this way so the angular momentum points in this direction. So here we have the wheel now turning and now when I cut the wire here, we can see that there is now the motion-- the angular momentum vector is now going on a circular motion. That is the motion of precession. This motion does require the presence of an angular momentum. If there is no angular momentum associated with the wheel we of course have-- it just falls down as such. So I can also start this experiment this way. I don't need the wire. Again, this demonstrates motion of precession.

Notes

Summary

6m 46s

What is precession ?



From Newton's 2nd law (rotations):

$$\frac{d\vec{L}}{dt} = \vec{\tau} = -M_W \vec{g} \times \vec{r}$$

Observation: The motion of the axis of the wheel with mass M_W is circular about O with constant angular velocity $\Omega \perp$ to L dictated by τ

$$\frac{d\vec{L}}{dt} = \vec{\Omega} \times \vec{L}$$

What is the value of Ω?

8-11

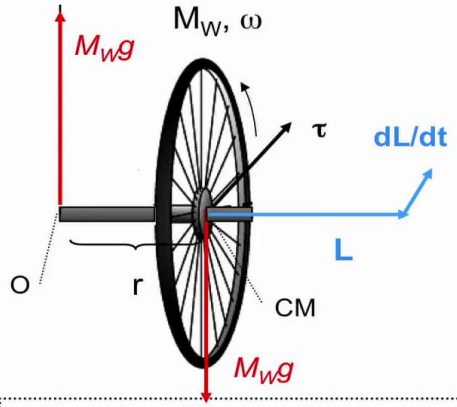
Now as we have seen in the demonstration the motion of the axis of the wheel is circular about the point *O* and it has a constant angular velocity, which is vertical to the angular momentum. Here's the center of mass of our wheel and we will now calculate the motion here. Now the motion that we've seen for the angular momentum is circular so we know that the precession equation-- that is here again now for the angular momentum-- is valid in describing the motion of the angular momentum. Now we want to calculate here the precession frequency because it bears some analogy to the magnetization. So we have our angular momentum. Our center of mass is a distance *r* away from the suspension point. We have the mass here, that's one fourth. The wheel does not fall as we've seen in the experiment so we have the string tension, which is the same, is the weight of the wheel upwards. Now we can calculate the derivative of the angular momentum, which is equal to the torque. We have only one torque that acts on here. Whether we use point *O* or the center of mass as a reference point we'll end up with the same equation, so I won't make the distinction here.

Notes

Summary

8m 16s

What is precession ?



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$$\frac{\vec{L}}{L} \cong \frac{\vec{r}}{r}$$

$$\frac{d\vec{L}}{dt} = -\frac{M_W r \vec{g}}{L} \times \vec{L}$$

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What is the value of Ω?

8-11

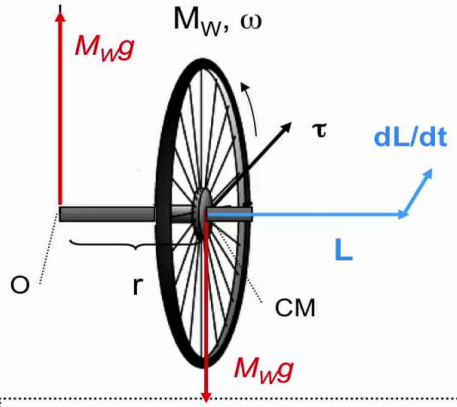
It is *r* cross force, so this minus if I put the force before the *r*. That's the definition of torque. So we have the torque in this direction and this gives us the orientation of the change of angular momentum. Now I'll rewrite the equation here by dividing by the norm of the vector *r* and multiplying by its length. So length here, length here, but I'll put it here. Now, the vector *r* divided by its length is basically a vector that has unit length so it's a unitary vector. And I will use the fact that the axis, the orientation of the axis of the wheel and the orientation of the angular momentum are largely colinear so I can replace *r over r* with *L over L*, which is what we are now going to do here. And this gives us this equation here. So we now have replaced this term here by this term and then we get this equation. If I regroup this then we have the derivative of the angular momentum is some term times the angular momentum and this is essentially our precession frequency. I'll use the definition of the angular momentum, so it's *I omega*, *I* as being the moment of inertia, and then we have the precession frequency is equal to the radius times the mass times gravitational acceleration--the *g*-- divided by *I omega*. So that's our precession frequency here derived.

Notes

Summary

9m 33s

What is precession ?



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$$\frac{d\vec{L}}{dt} = -\frac{M_W r g}{L} \times \vec{L} \qquad L = I\omega \qquad \frac{\vec{L}}{L} \cong \frac{\vec{r}}{r}$$

⇒ Precession frequency $\Omega = \frac{r}{I\omega} M_W g$

Observation: The motion of the axis of the wheel with mass M_W is circular about O with constant angular velocity Ω ⊥ to L dictated by τ

$$\frac{d\vec{L}}{dt} = \vec{\Omega} \times \vec{L}$$

What is the value of Ω?

Precession frequency increases with

1. mass M_W of the wheel
   → **gyro**magnetic ratio γ
2. gravitational pull g
   → magnetic field B_0

Just like a spinning **Gyro**scope in gravity

$$\frac{d\vec{M}}{dt} = -\gamma \vec{B}_0 \times \vec{M}$$

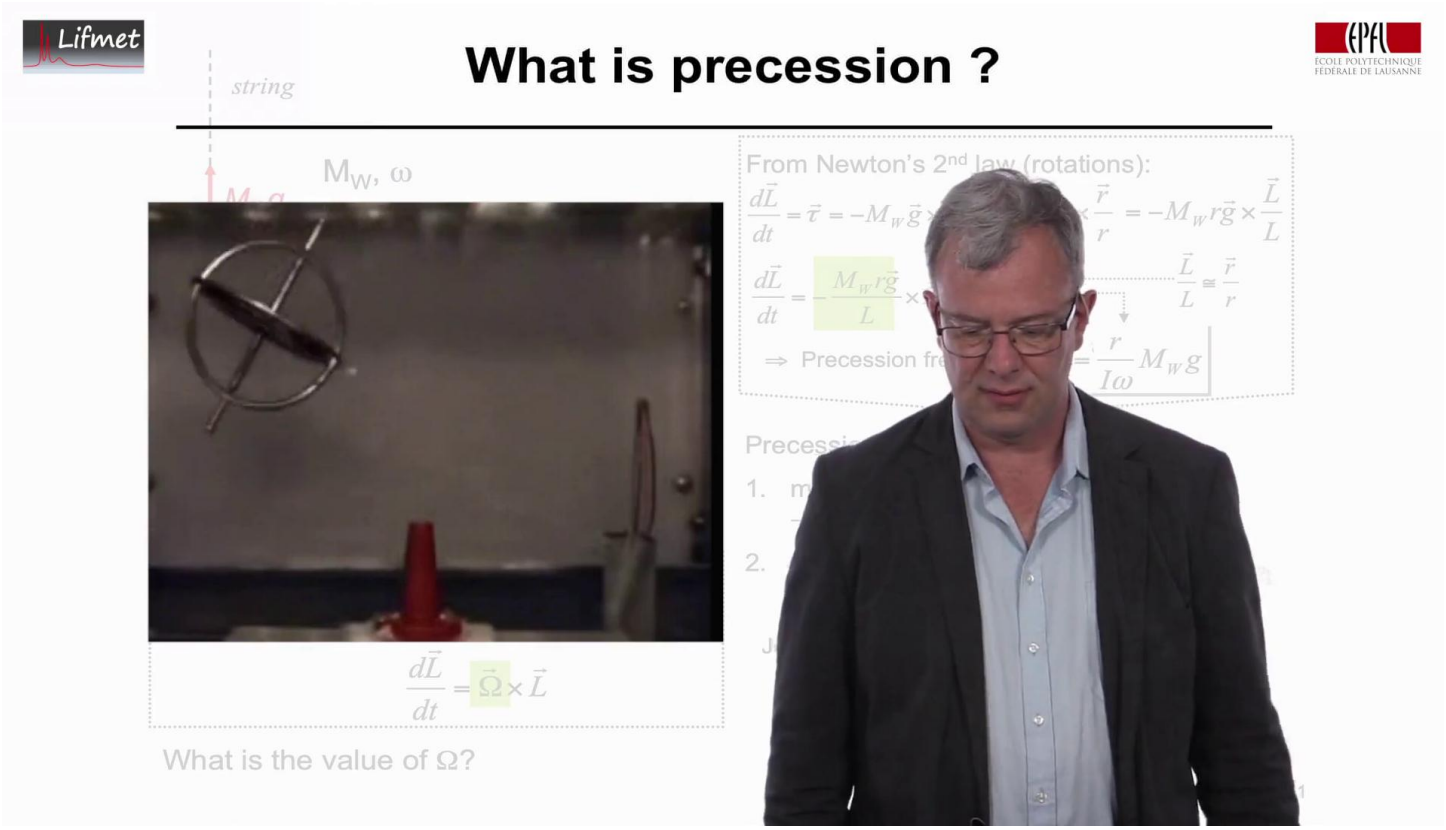
8-11

Compare this to the equation that we have here-- we have the *omega* here, we have this value-- and to the equation of the magnetization. It is basically our magnetization does a motion just like a spinning top. And hence, we can look at now some factors that increase-- if you look at the wheel, if the mass of the wheel increases the precession frequency increases. Well, this is linked to-- plays the same role as-- the gyromagnetic ratio and hence, because the motion is similar to that of a gyroscope, the *gamma* is called *gyromagnetic ratio.* And then we have also the gravitational pull, if we have more gravitational pull, our wheel turns faster so the gravitational pull takes the role of the external magnetic field.

Notes

Summary

11m 08s



So this is illustrated in this video that we are seeing. So we are first seeing-- what we are seeing is a gyroscope, a spinning top rotating just like we have here on the animation, rotating first vertically. It's put in an elevator and this elevator is then let fall in freefall to the ground. The first experiment is not very interesting. We see that the gyroscope detaches from the support. The second experiment that we are looking at-- the second gyro-drop-- here we have the gyroscope precessing just like we've seen for the wheel and that gyroscope-- the minute it is no longer exposed to gravitation, the minute there is no longer a torque exerted on the gyroscope, it stops immediately precessing. That's as if we had magically turned off the gravitation--same effect-- or if we had turned off the magnetic field.

Notes

Summary

12m 01s

Credits

Unidentifiable source (internet)

Free Animated GIFs
http://www.gifs-paradise.com/science/gifs/image/animated-gifs-science-06/

EM. Purcell, E. G. Bowen, and H. I. Ewen
https://homepages.warwick.ac.uk/~phscz/cargese2012/material/Cargese_2-Life_Hi_Re.pdf

Gyroscope in Free-Fall « Zero-G »
https://youtu.be/IIPQ1F-gHdw

Notes

Summary

12m 56s