

8-5. Why use a Rotating frame of reference to describe the motion of magnetization ?

Rotating frame: A reference frame which rotate about z of the laboratory frame at frequency ω_{RF}

Why use a rotating reference frame ?

$$\frac{d}{dt} \vec{M} = \vec{M} \times \gamma \vec{B}$$



8-14

Now, we are going to explore the motion described by the Larmor equation. And for understanding the motion of magnetization, it is important to introduce a concept on how to simplify the description of the motion-- how to make it more accessible understanding what exactly happens. And this simplification is based on using a rotating reference frame. And why do we want to do this? Well, first I want to go and explain what is a rotating reference frame. A rotating reference frame is a frame that rotates about the z axis in the laboratory frame-- so that's our reference frame will rotate around the z axis-- we'll assume the magnetic field is vertical in this case. So why would we use a rotating reference frame? We've seen that the equation of motion, the Larmor equation, gives us a rotation around the magnetic field. So if we have a magnetization that is not parallel to the magnetic field, then it will rotate around that magnetic field just like the bicycle wheel that we have seen rotating. This is similar to a carousel-- so we are on a carousel, and if we look at a carousel, the motion of people on the carousel from the outside-- it's fairly complicated because we have any motion on the carousel superimposed by the motion of the carousel itself.

Notes

Summary



8-5. Why use a Rotating frame of reference to describe the motion of magnetization ?

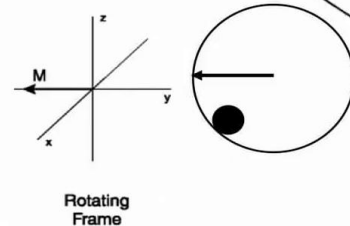
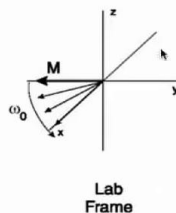
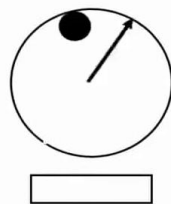
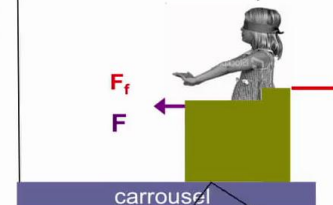
Rotating frame: A reference frame which rotate about z of the laboratory frame at frequency ω_{RF}

Why use a rotating reference frame ?

$$\frac{d}{dt} \vec{M} = \vec{M} \times \gamma \vec{B}$$



I am immobile but feel a force \vec{F}_f
That pushes me against the chair ...



8-14

That's what we see from the outside. And if we now put ourselves on the reference frame on the carousel anything that happens on the carousel with respect to the carousel is stationary. What we do in classical physics, I'll do the analogy here, is that we are sitting-- let's say we are sitting on a chair without motion, then we have a force, that's the centripetal force towards the center-- that's the force of the chair that pushes us and maintains the circular motion that we have. But we feel from our inertia that there is a force that pushes out against the chair. This is not a physical force, it's purely the inertia that our visual system senses; it is a fictitious force. So we have, similarly to the motion on the carousel, we have the magnetization vector here it turns here in the lab frame, and if we are in the rotating frame, the magnetization vector is stationary. So we can, just like in the carousel, if we have our magnetization that is precessing around the magnetic field B zero, we can put ourselves into a reference frame that rotates with the magnetization and there the magnetization becomes stationary, just like the person here on the carousel.

Notes

Summary





So what are the consequences of using a rotating reference frame? I'll start here with making the analogy to classical physics. In this video you see kids throwing a ball at each other, first seen with a camera from above-- the ball goes pretty much in a straight line-- and then when the camera is put on the merry-go-round you see that the ball goes in a very curved motion which seems odd.

Notes

Summary

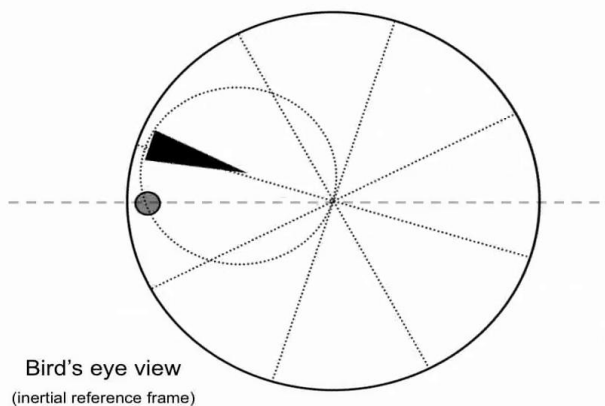


3m 01s

What are the consequences of using a rotating reference frame ?

Fictitious forces

Situation: A pendulum swings above a rotating disk with amplitude ($R\sin\theta$) equal the disk radius and period equal to that of the disk



I will show in the following, with an animation, this effect. And basically the situation we're considering here is that we have a pendulum that swings with an amplitude equal to the disk radius and a period that's equal to that of the disk that rotates below. So here's the disk. The triangle indicates the motion of the disk. Now we have the pendulum swinging-- its period is exactly to the period of the disk. That's the equivalent of the merry-go-round. So we're throwing the pendulum in a straight line-- this is the bird's-eye view, it's in the inertial reference frame, everything is the way we normally see it. Now what we're going to do is we're going to put ourselves into the rotating reference frame of the carousel. And to help your eyes see the motion I have indicated here the coordinate system by the spokes on the carousel. We'll take away the motion seen from the outside and now train your eye-- do you see what motion you see with respect to a carousel? Here it is. So it is indeed a circular motion. Our object, seen from the carousel, undergoes circular motion. And just to convince you that I'm not cheating here in any way, I'll put back the horizontal line-- so seen from the outside, we still see the motion of the object of the pendulum swinging back and forth in a straight line.

Notes

Summary



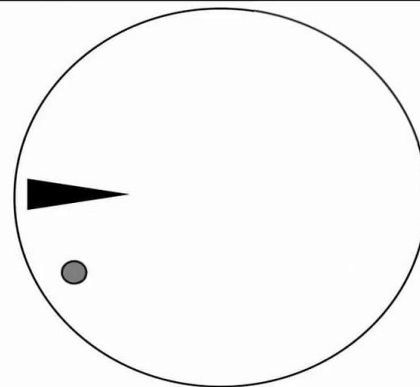
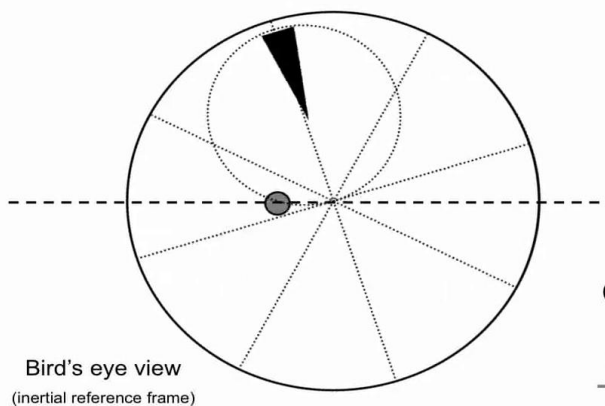
3m 25s

What are the consequences of using a rotating reference frame ?

Fictitious forces

Situation: A pendulum swings above a rotating disk with amplitude ($R\sin\theta$) equal the disk radius and period equal to that of the disk

Observation: While the pendulum swings along a straight line in the inertial reference frame it moves along a circle in the rotating reference frame.



Still need **proper description of motion**
Need to add 2 **fictitious** forces:

Centrifugal Force

$$-m\vec{\omega} \times \vec{\omega} \times \vec{r}$$

Coriolis

$$-2m\vec{\omega} \times \vec{v}$$

8-15

But from the carousel-- seen from the carousel, from the merry-go-round-- we see it as circular motion. So that's essentially the motion that is seen if our reference frame that describes our motion is equal to that of the merry-go-round, the carousel, or the rotating reference frame. Now what we have done here is we have just changed the mathematical framework on describing the motion. We go from straight motion to circular motion depending on whether we describe the motion in a reference frame that rotates. That's a purely mathematical description of the motion. And we still want to have a proper description of the motion, and to do that we want to look at the motion in our merry-go-round-- we want to add all the forces that act on our ball, or on our object-- and in the end we still want to have a proper calculation of the acceleration-- that's Newton's second law. So to get to the proper acceleration here-- to describe this motion here, to get the proper acceleration-- we have to add two fictitious forces: centrifugal force and the Coriolis force. Now bear in mind, those two forces have nothing to do with physics, as such.

Notes

Summary

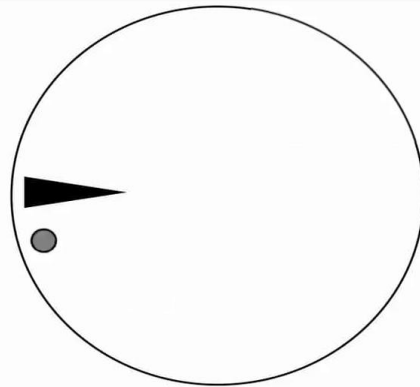
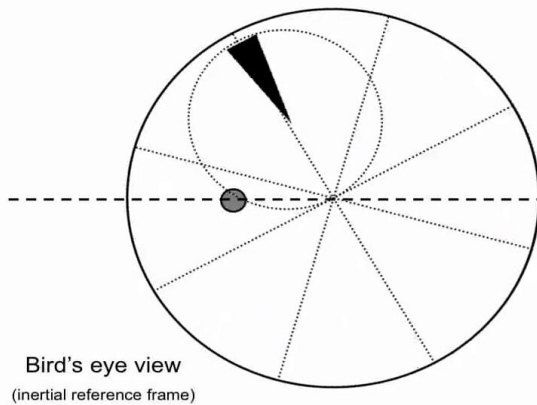


What are the consequences of using a rotating reference frame ?

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8-15

They are the consequence of the fact that we mathematically change our reference frame from an inertial frame-- that's a typical non-moving reference frame-- to one that rotates around the axis z. These forces have to be added so that we can describe in the reference frame that rotates-- this one here-- that we can still arrive at calculating the proper accelerations with Newton's second law.

Notes

Summary



What is the equation of motion for magnetization in the rotating reference frame ?

Larmor frequency in reference frame rotating
with ω_{RF} : $\Omega = \omega_L - \omega_{RF}$



8-16

Now you might ask why am I transgressing into classical physics-- what is the link to magnetic resonance? Actually the link is there. We have a mathematical description in a reference frame that rotates. In classical physics, we need to subtract or add two fictitious forces and, as you will see in the next slide, we have to do a similar thing for the proper description of the magnetization in the rotating reference frame. Except that we're not going to deal with forces. We are rotating in the rotating reference frame, not in a physical reference frame; it's a reference frame linked to the magnetization. So how does the equation of motion for the magnetization change if we change the reference frame? So we'll now say, we're going to put ourselves into a reference frame that rotates with ω_{RF} -- that's in radians per second. And in that reference frame we observe the magnetization rotating with a Larmor frequency, a *big omega* here which is given by the Larmor frequency γB_0 minus this ω_{RF} -- that's the angular velocity of the rotating reference frame.

Notes

Summary



6m 47s

What is the equation of motion for magnetization in the rotating reference frame ?

Larmor frequency in reference frame rotating with ω_{RF} : $\Omega = \omega_L - \omega_{RF}$

$$\Omega = \gamma \Delta B$$

$$\Rightarrow \Delta B = \Omega / \gamma = B_0 - \omega_{RF} / \gamma$$

[lab frame: $\omega_{RF}=0 \Rightarrow \Omega=\omega_L$ ($\Delta B=B_0$)]

$$\frac{d\vec{M}}{dt} = \vec{\Omega} \times \vec{M}$$

(fictitious) magnetic field ω_{RF}/γ is progressively subtracted from B_0

8-16

We can now associate to this *omega*-- this observed Larmor frequency, [inaudible] can observe-- we can relate this to a magnetic field and we'll call this magnetic field *delta B*. So *big omega* is equal to *gamma delta B*. Or, if we now stick this in here, and we use *omega_L* equals *gamma B_zero*, the *delta B* is given by *B_zero*-- the external magnetic field-- minus *omega* of the rotating frame divided by *gamma*. In the lab frame, where our rotating frame does not rotate-- that's the lab frame-- *omega_RF* is zero, and the Larmor frequency that we observe in that reference frame is equal to *omega_L* and *delta B* is equal to *B_0*. So now what we want is, in our rotating reference frame, independent of the frequency with which I take my reference frame to rotate at, I want the Larmor equation to be true. For it to be true, I have to increasingly subtract a fictitious magnetic field. And this magnetic field is this term here: *omega_RF* over *gamma* that we have to subtract from *B_0*.

Notes

Summary



8m 09s

What is the equation of motion for magnetization in the rotating reference frame ?

Larmor frequency in reference frame rotating with ω_{RF} : $\Omega = \omega_L - \omega_{RF}$

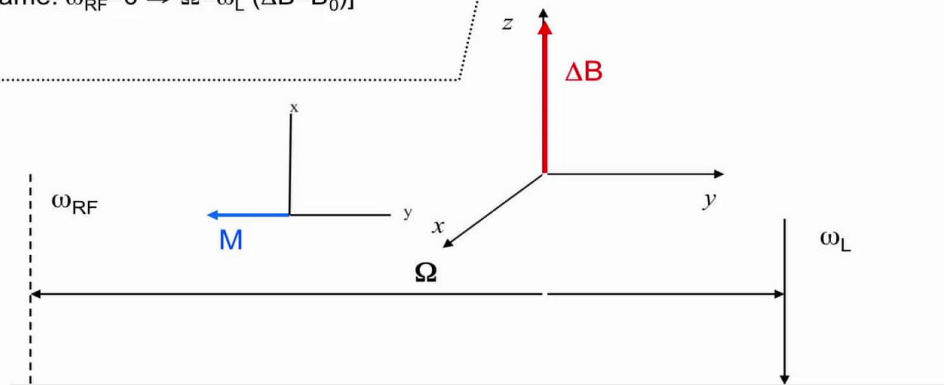
$$\Omega = \gamma \Delta B$$

$$\Rightarrow \Delta B = \Omega / \gamma = B_0 - \omega_{RF} / \gamma$$

$$[\text{lab frame: } \omega_{RF}=0 \Rightarrow \Omega=\omega_L (\Delta B=B_0)]$$

$$\frac{d\vec{M}}{dt} = \vec{\Omega} \times \vec{M}$$

(fictitious) magnetic field ω_{RF}/γ is progressively subtracted from B_0



8-16

So this is not unlike the classical mechanics where we take our physical forces; once we're in the rotating reference frame we have to add or subtract-- depending on how you look at it-- the centrifugal force and the Coriolis force. Here, in order for the equation of motion to work, we have to subtract a fictitious magnetic field-- that's a consequence of the motion-- to make the equation of motion work; we still want this equation to be true in any of our reference frames. So how does this work? We'll look first at the lab frame. We are far away from the Larmor frequency so this is our rotating reference frame-- let's say it is at zero-- here's our Larmor frequency on the horizontal axis is frequency. Then this difference in frequency is given by ω here. And if we look in this reference frame at the motion we have a ΔB which is given by this term here. So if we say ω_{RF} , the reference frame frequency is zero, then the ΔB is equal to B_0 . Now what is the motion of magnetization that we observe? Here's the magnetization-- we'll say it is in the transverse plane, so xy -- and if this magnetization's a transverse plane then it will precess at a very high frequency dictated by $\gamma \Delta B$ around the magnetic field B_0 .

Notes

Summary



What is the equation of motion for magnetization in the rotating reference frame ?

Larmor frequency in reference frame rotating with ω_{RF} : $\Omega = \omega_L - \omega_{RF}$

$$\Omega = \gamma \Delta B$$

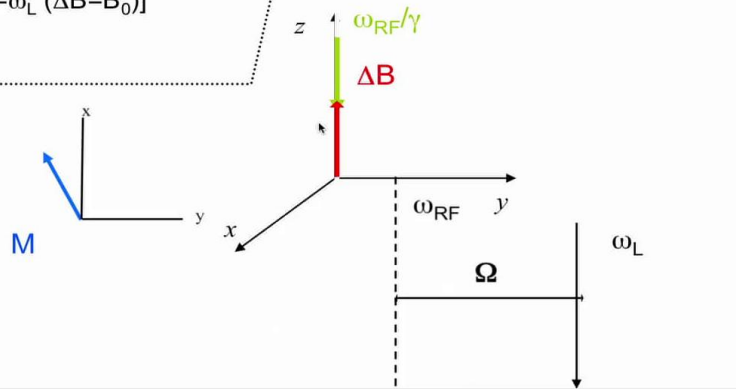
$$\Rightarrow \Delta B = \Omega / \gamma = B_0 - \omega_{RF} / \gamma$$

$$[\text{lab frame: } \omega_{RF}=0 \Rightarrow \Omega=\omega_L (\Delta B=B_0)]$$

$$\frac{d\vec{M}}{dt} = \vec{\Omega} \times \vec{M}$$

(fictitious) magnetic field ω_{RF}/γ is progressively subtracted from B_0

Off-resonance



8-16

This is what we call *off-resonance*. We're far away with our rotating frame from the Larmor frequency. Now let's get a little bit closer. And we'll go closer with our rotating frame frequency, and now the motion is slower-- we're starting to approximate the Larmor frequency. This is as if we have first started out observing a carousel-- it turns, and now we're starting, with our reference frame, starting to move in the direction of the rotation of the carousel. Then the carousel, from that reference frame, will turn at a slower frequency and this is what we see here with the magnetization. As we have increased our *omega RF*-- we have increased this term-- we have to subtract this fictitious field as a consequence of the rotating reference frame [it will become] *omega RF* divided by *gamma*. So our *delta B* is now reduced. If we go even closer, then we're now relatively close to the Larmor frequency the precession will be noticeably slowed down, and we have to subtract *omega_RF* as it's increased a bigger fictitious field *omega_RF* over *gamma*, and the *delta B* is now decreased because this motion here dictates the frequency of precession here.

Notes

Summary



What is the equation of motion for magnetization in the rotating reference frame ?

Larmor frequency in reference frame rotating with ω_{RF} : $\Omega = \omega_L - \omega_{RF}$

$$\Omega = \gamma \Delta B$$

$$\Rightarrow \Delta B = \Omega / \gamma = B_0 - \omega_{RF} / \gamma$$

[lab frame: $\omega_{RF}=0 \Rightarrow \Omega=\omega_L$ ($\Delta B=B_0$)]

For $\omega_{RF} = \omega_L$ $\Delta B = 0$ (on-resonance)

Off-resonance

(fictitious) magnetic field ω_{RF}/γ is progressively subtracted from B_0

$$\dot{\vec{M}} = \vec{\Omega} \times \vec{M}$$

$$\omega_{RF}/\gamma$$

$$\Delta B$$

On-resonance: $\Omega=0$

$$\omega_{RF} = \omega_L$$

$$\omega_L$$

y

8-16

And now if we go essentially on to the Larmor frequency then our *big omega* goes to zero, and our magnetization no longer turns-- I'm still showing a turning magnetization here, otherwise it's a very boring animation. But essentially, if our *omega RF* is equal to the Larmor frequency then our magnetization, we are rotating exactly at the frequency with which the magnetization rotates around B_0 and in that reference frame, the magnetization is stationary so it does not move. If the magnetization does not move, then it is stationary in that reference frame so the derivative for the magnetization with time is zero, so *omega* has to be zero, and that means that *omega_RF* is equal to *omega_L*, ΔB equals zero and this case we call *on-resonance* where *omega_RF* is equal to the Larmor frequency. That's the on-resonance situation. That's the situation as if you are on the carousel and now your reference frame is moving exactly with the motion of the carousel. And so I want to show this precession of the magnetization, the Larmor equation, with a simulated experiment of precession. It's a macroscopic experiment-- it simulates how spin precession works. We'll just take the view of the lab frame in this experiment that follows.

Notes

Summary



12m 32s

Ex. Flipping magnetization over in the rotating reference frame



8-17

This is an experiment which simulates the motion of precession-- the Larmor equation-- on a macroscopic scale. We have in this yellow ball, we have a magnet and that magnet will align itself based on the magnetic field. However, it is required that it has-- just like the nuclear spin-- that it has an angular momentum. If I don't have an angular momentum here, and I turn on the magnetic field, the ball will just go down as we have seen with the wheel experiment, the suspended bicycle wheel. So now what I will try to do is, I will try to give this ball a spin. I'll set it into circular motion. Nothing happens now, now I turn on the magnetic field and you can see it is turning around in one way, and if I stop the magnetic field it stops, it just turns around its axis, and if I invert the polarity of the magnetic field it turns the other direction. And those are the properties of the Larmor equation demonstrated here with this macroscopic experiment. We have today, in the course, talked about that the equilibrium thermodynamic magnetization is along z . And we want to look at how we can get the magnetization to change its orientation.

Notes

Summary



13m 58s

Ex. Flipping magnetization over in the rotating reference frame

Start with thermodynamic equilibrium magnetization M_0

Reference frame rotating with ω_L (on-resonance)

Apply *additional*, constant magnetic field with magnitude B_1 (in xy plane) for time τ

What motion can be observed for M ?

$$\frac{d\vec{M}}{dt} = -\gamma \vec{B}_1 \times \vec{M}$$



B-17

And this I want to do for the end of the lecture, we'll resume this description, but it's an example of describing the motion in the rotating reference frame. I want to illustrate how the magnetization can be manipulated. So if we're putting ourselves into a magnet we will develop a thermodynamic equilibrium magnetization M_0 . If we take the reference frame that rotates with the Larmor frequency, that's the on-resonance case as we have described it, then we have to-- In that reference frame, we have no net magnetic field so the magnetization is stationary-- we have no B_0 , the ω_{RF} over γ just cancels the B_0 term. Now, in this reference frame, we will apply an additional constant magnetic field with magnitude B_1 -- we'll give it the name B_1 -- and it shall be somewhere in the transverse plane along x or y . It doesn't matter whether it's along x or y for the purpose of this discussion, and we'll apply this additional magnetic field B_1 for a time τ . So, question: What motion can we observe for the magnetization in our rotating reference frame? Well, let's look at the equation of motion.

Notes

Summary



15m 24s

Ex. Flipping magnetization over in the rotating reference frame

Start with thermodynamic equilibrium magnetization M_0

Reference frame rotating with ω_L (on-resonance)

Apply *additional*, constant magnetic field with magnitude B_1 (in xy plane) for time τ

What motion can be observed for M ?

$$\frac{d\vec{M}}{dt} = -\gamma \vec{B}_1 \times \vec{M} \quad \rightarrow \quad M_0 \text{ precesses about } B_1$$

Magnetization rotates about B_1 with angular velocity γB_1



8-17

We have said it is always the same-- the derivative of the magnetization is equal to *minus gamma* the magnetic field times the magnetization, and the times here is the vector product. Since we are rotating in the reference frame that rotates with the Larmor frequency, the B_0 term does not exist in the equation and we've said the B_1 here is constant-- it is in the transverse plane so it is perpendicular to B_0 . So what motion can we observe for the magnetization if we start out with equilibrium magnetization? The motion is the same, and in this reference frame the magnetic field is constant, so the motion that we observe-- this is what we've discussed-- this equation says that the equilibrium magnetization will now start to precess about B_1 . So it will rotate about the vector B_1 at a constant angular velocity. So this angular velocity in radians per second γB_1 has the frequency of γB_1 over 2π and it has the period of 2π over γB_1 -- of course: this is the relationship between frequency and period.

Notes

Summary



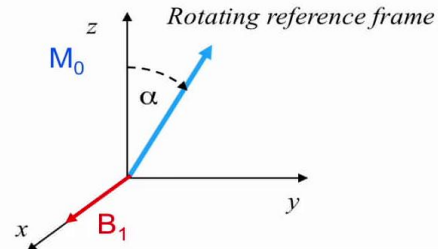
16m 50s

Ex. Flipping magnetization over in the rotating reference frame

Start with thermodynamic equilibrium magnetization M_0

Reference frame rotating with ω_L (on-resonance)

Apply *additional*, constant magnetic field with magnitude B_1 (in xy plane) for time τ



Definition Flip angle = angle of rotation α induced by B_1 applied for τ seconds

What motion can be observed for M ?

$$\frac{d\vec{M}}{dt} = -\gamma \vec{B}_1 \times \vec{M} \quad \vec{M}_0 \text{ precesses about } B_1$$

Magnetization rotates about B_1 with angular velocity γB_1

Frequency $\gamma B_1 / 2\pi$

→ period $T = 2\pi / \gamma B_1$

Special cases of α :

90°: Full **excitation** (all M_0 is rotated into transverse plane, xy, i.e. $M_0 \rightarrow M_{xy}$)

8-17

So let's visualize this-- we're in the rotating reference frame, we have our coordinate system in the rotating reference frame, we have this constant B_1 in this rotating reference frame, we have the equilibrium magnetization M_0 and now we have applied this B_1 for a period of τ . After this time τ , the M_0 -- the magnetization-- has precessed about B_1 by an angle α . If the B_1 is along x then M_0 will precess in a zy plane and after time τ it has undergone a rotation by the angle of α . This angle is important. As I'll define it here, this angle is called the *flip angle*-- it describes the angle of rotation induced by the... in this case by this B_1 field that was applied for τ seconds. What are the special cases of the flip angle α ? Special cases are 90 degrees. If α is 90 degrees, we call that *excitation*, or *full excitation*, that is, all the magnetization that is along z , here in this example, this equilibrium magnetization, is rotated into the transverse plane.

Notes

Summary

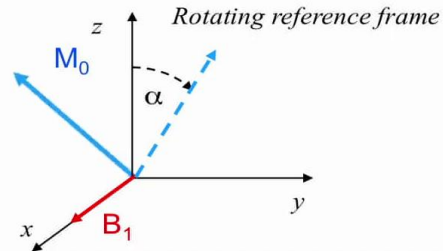


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Frequency $\gamma B_1 / 2\pi$

\rightarrow period $T = 2\pi / \gamma B_1$

Special cases of α :

90°: Full **excitation** (all M_0 is rotated into transverse plane, xy, i.e. $M_0 \rightarrow M_{xy}$)

180°: **Inversion** ($M_z \rightarrow -M_z$)

B_1 = **radiofrequency (RF) field** (why?)

8-17

So we rotate M_0 from being along z into magnetization in the transverse plane and we often call this M_{xy} for the transverse magnetization. So that's this case here. We have rotated the magnetization into the transverse plane. If we now double the duration so that we get an angle of 180 degrees then that is called an *inversion*. We go from an equilibrium magnetization that's along z like this to an equilibrium magnetization that's opposite: minus M_z in the *minus* z direction, and that is called an inversion. So that's the situation here after 180 degrees. And if we leave the B_1 on, then the magnetization will precess around B_1 for as long as we leave this B_1 field on. Now B_1 is called the *radiofrequency field*.

Notes

Summary





Carousel Cartoon Drawing

Free image from the internet



Unidentifiable source (internet)



The Coriolis Force

https://youtu.be/_36MiCUS1ro

Why? Can you picture this? Why is this called a radiofrequency field? It helps here to remind ourselves, we've said B_1 is stationary-- that does not change with time in the rotating reference frame that rotates with the Larmor frequency. What does that mean if I look at it from the outside? What form does the B_1 have? Well, my hand's already telling you it's rotating. So, in the lab frame, seen from the outside, we have actually, the B_1 is a time-dependent field. It is given by B_1 , the amplitude, and it rotates exactly with the Larmor frequency, ω_L . Now, why do we call this radiofrequency? Because, if we look at the Larmor frequency, we take for protons the gyromagnetic ratio which is 42MHz per Tesla , we calculate the Larmor frequency in Hertz and we'll find that this is typically on the order of 100MHz -- so it's perfectly in the range of radiofrequency. Actually, here it is in the range of, truly, the radiofrequency of 100MHz . And that's why the B_1 is called radiofrequency field.

Notes

Summary



20m 16s