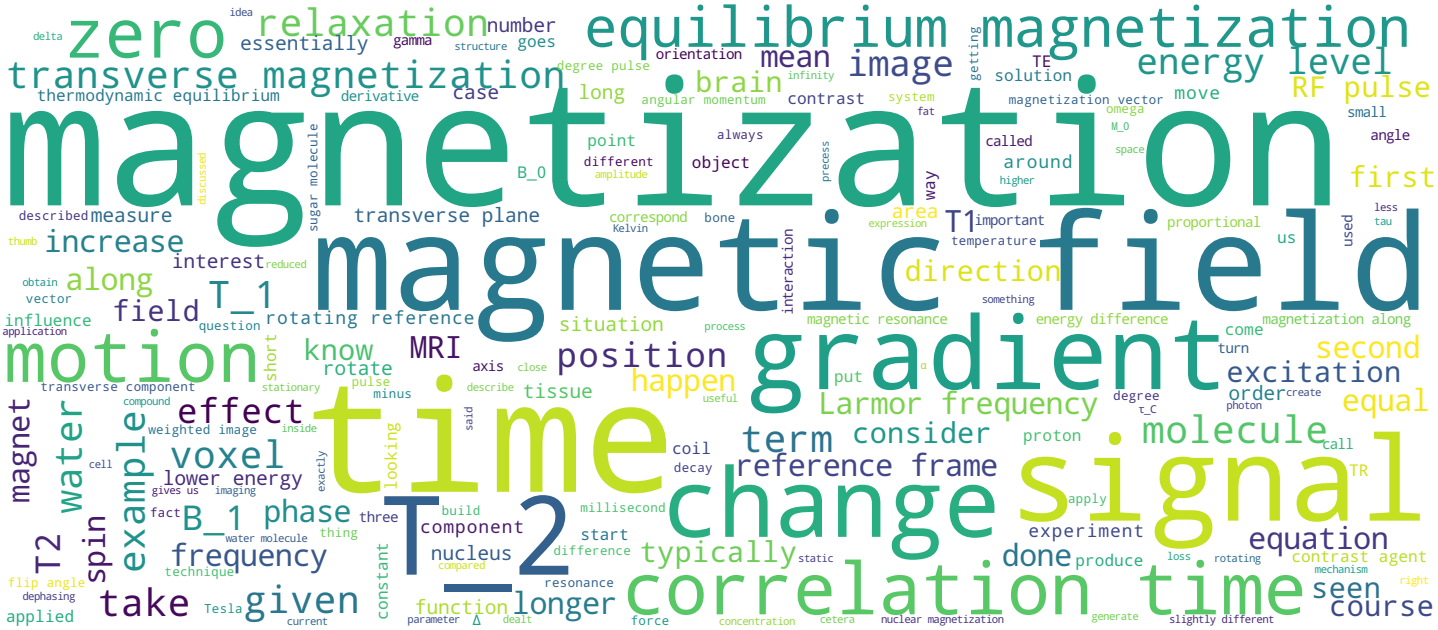
9.4 Mechanism of relaxation

Fundamentals of Biomedical Imaging

Prof. Rolf Gruetter



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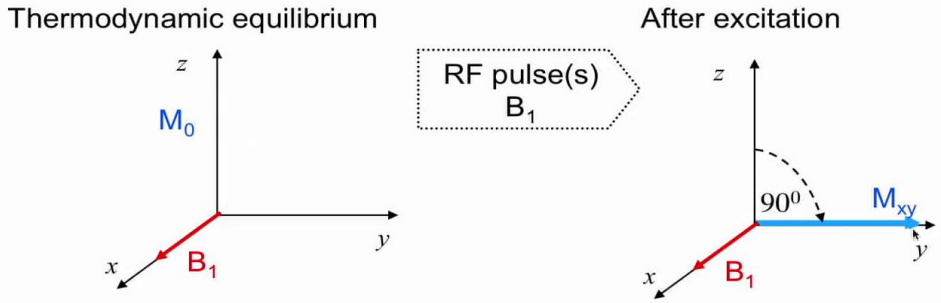
Video

9-4. How is the return to equilibrium M_0 governed ?

Relaxation



9-11

So far we have dealt with the fact that when we go into a magnet there's an equilibrium nuclear magnetization that develops. We have discussed on how we can get this magnetization into the transverse plane. Now, we know Boltzmann rules, Boltzmann is always right. So we know after we have a applied our *B_1* field, we tipped the magnetization transverse plane. It's not going to stay there forever. It will eventually at some point return to the equilibrium magnetization. This return to equilibrium magnetization is what I will discuss in this section. Let's talk about -- we have thermodynamic equilibrium. We have a magnetization, equilibrium magnetization along that. We'll apply now a *B_1* field on this magnetization. This is done by RF pulses or pulse, or multiple RF pulses mediated by the RF field *B_1*. Now, we look at the situation. What happens after excitation? Here's the *B_1*. After excitation, our magnetization is in the transverse plane. This is for excitation in 90-degree pulse. Now, in this situation, as you've noticed here, we have no longer a *z* magnetization. Magnetization is no longer present. We have a transverse magnetization.

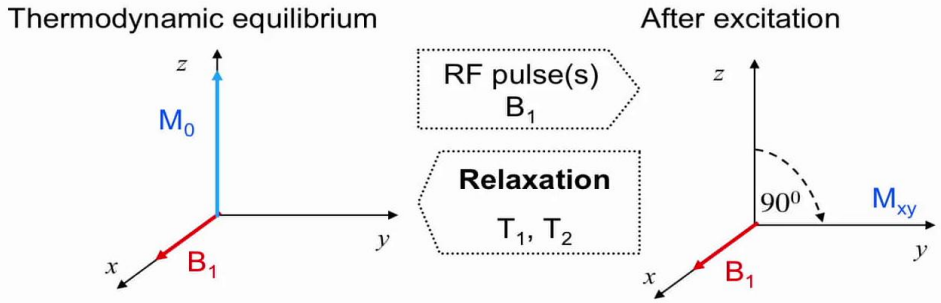
Notes

Summary

0m 04s

9-4. How is the return to equilibrium M_0 governed ?

Relaxation



Transverse magnetization:

(along x and y-axis, on resonance)

$$\frac{dM_x(t)}{dt} = -\frac{M_x(t)}{T_2} \qquad \frac{dM_y(t)}{dt} = -\frac{M_y(t)}{T_2}$$

Equations formally equivalent to linear attenuation coefficient (x-ray) (same solution)

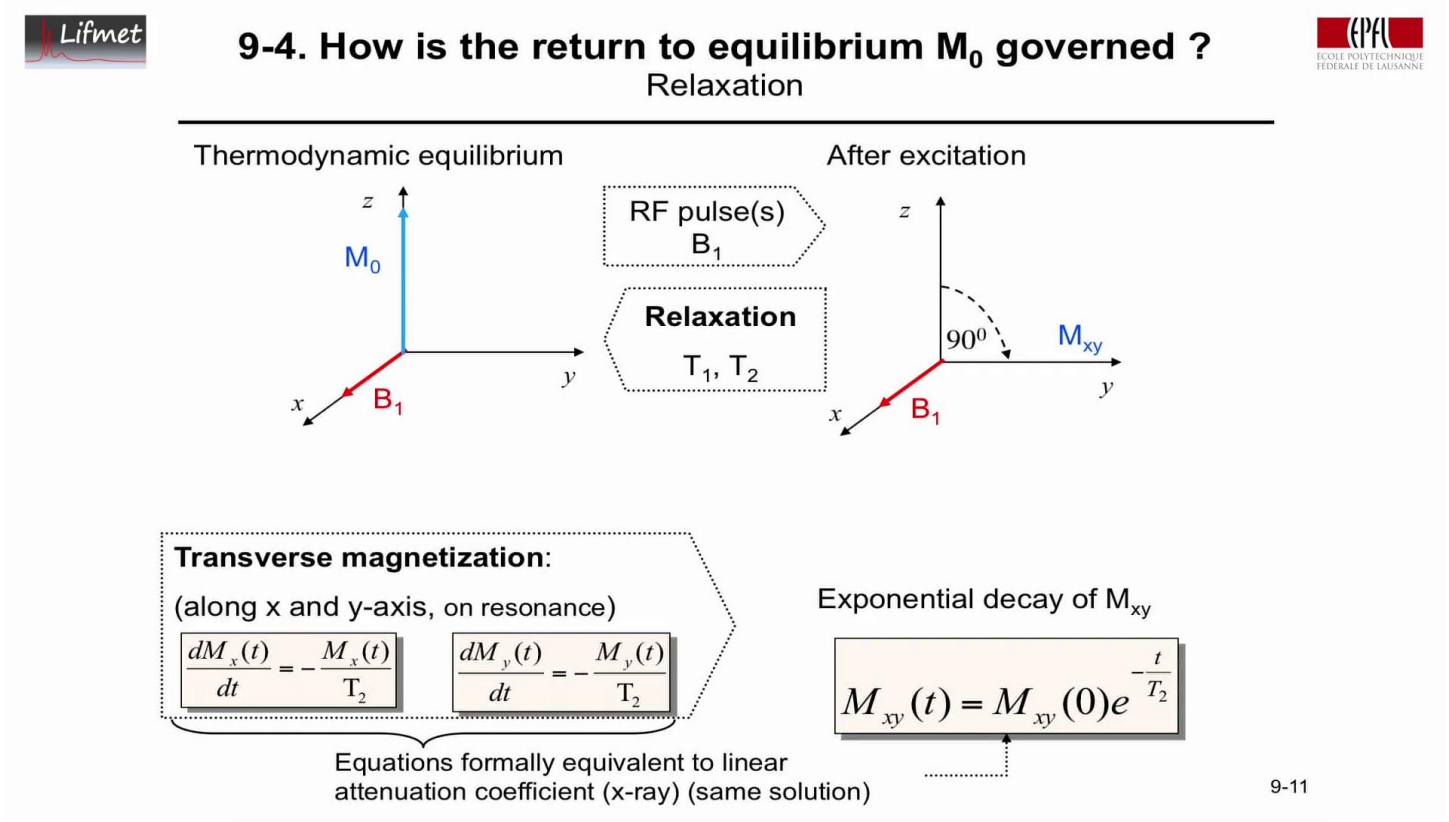
9-11

Now, what happens after this excitation? We know that the transverse component has to disappear and a longitudinal component has to build up. This is described by a process that's called relaxation. There are two time constants that are affiliated with it that we will discuss shortly. The transverse component decays, and the longitudinal component builds up again along *z*, that [inaudible] the equilibrium magnetization. This process of relaxation, the reestablishment of thermodynamic equilibrium magnetization has a phenomenological description. For the transverse magnetization, we'll consider here *x* and *y* axis, and we'll now look at, on the resonance that is, we are rotating with a Larmor frequency of the magnetization. We have the change in magnetization is proportional to the magnetization itself, same for the *y* magnetization. These two expressions are actually fairly similar to what we've had for the linear attenuation coefficient. So we have the same mathematical solution here as we have dealt there, and we are getting an exponential decay of the transverse magnetization.

Notes

Summary

1m 33s

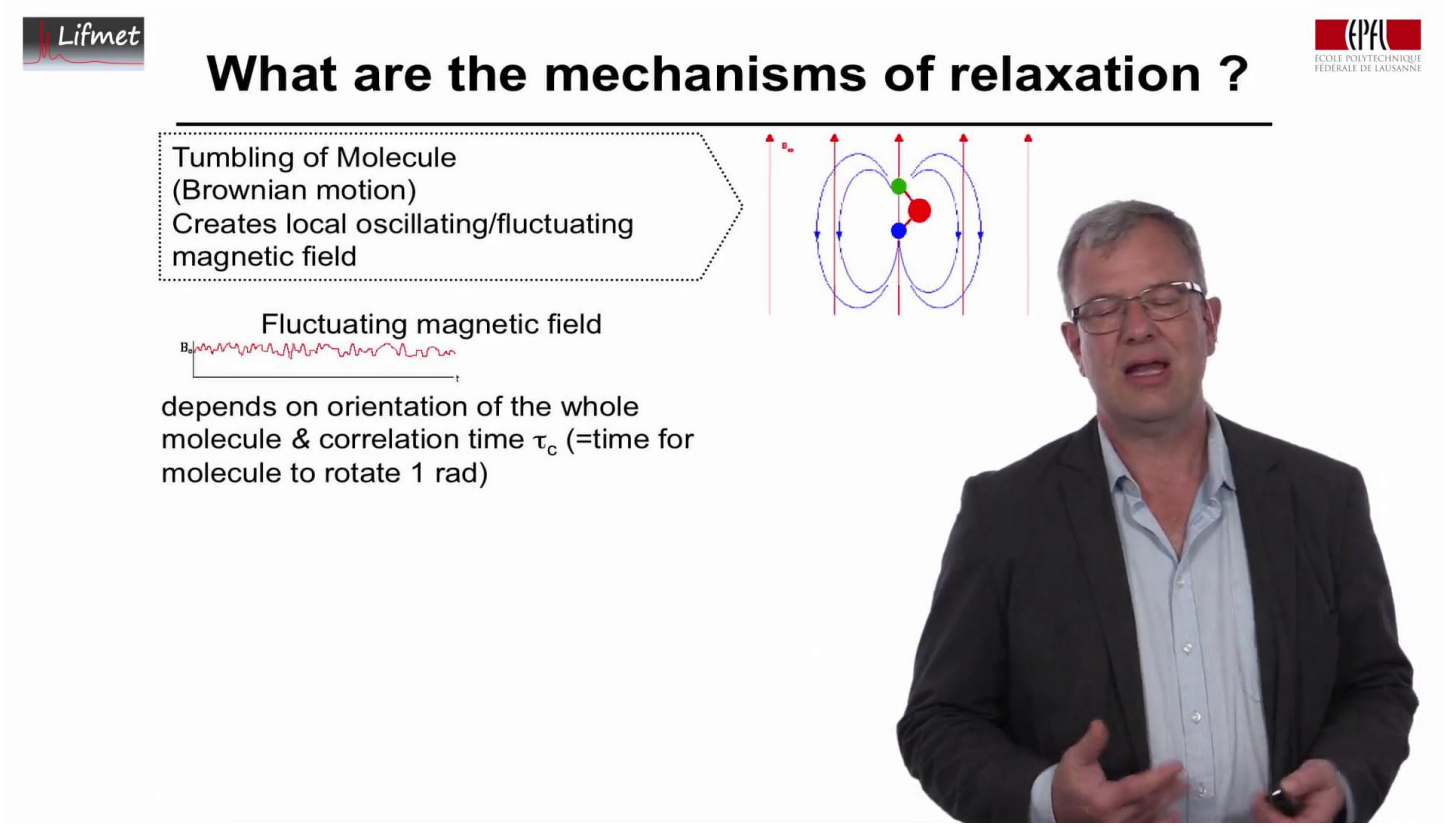


Our transverse magnetization – this is a solution of this equation, it's now right here *x y* instead of each individual component, which just means any transverse component is given by the transverse magnetization at zero times *e ^(- t / T_2)*. This is phenomenological *T_2*. There are a lot of calculations in trying to tease out from the theory how this *T_2* is being calculated. For us, it suffices to accept this as a phenomenological constant. I will in the following give you a rationale what influences this decay time *T_2*.

Notes

Summary

2m 49s



What are the mechanisms of relaxation? Essentially, what we're dealing with here is the fact that our molecules are not static, that they move. We have, for example, Brownian motion. Now, if you have a molecule that moves, the motion in the magnetic field will create local oscillating or fluctuating magnetic fields. They are small. You can easily picture this. Here's a molecule that kind of tumbles here. We have a magnetic field, and we can certainly envisage that this molecule on a local level, on atomic level, or molecular level has a non-constant magnetic field. There are some perturbations to the main magnetic field that are time-dependent. In addition to the *B_0*, which is on the order of Teslas, one gets a small fluctuating magnetic field. It is small, but it's randomly fluctuating that is acting on the nucleus. This field depends on the orientation of the molecule, and also, how it behaves is described by the correlation time, *τ_C*. The correlation time is defined as the time for a molecule to rotate by about 1 radian. It essentially is a way also to say it is a measure of how long the motion between two molecules, for example, is correlated.

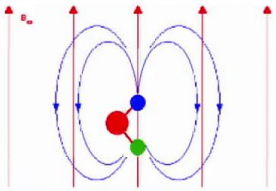
Notes

Summary

3m 29s

What are the mechanisms of relaxation ?

Tumbling of Molecule
(Brownian motion)
Creates local oscillating/fluctuating magnetic field

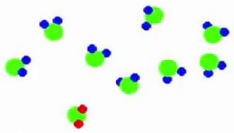


Fluctuating magnetic field

depends on orientation of the whole molecule & correlation time τ_c (=time for molecule to rotate 1 rad)

Sources of fluctuating magnetic field:

- Dipolar coupling between nuclei and solvent
- interaction between nuclear magnetic dipoles



Correlation function G $G(t) \propto e^{-t/\tau_c}$

Describes degree of correlation of motion t sec apart

Correlation time τ_c: $\tau_c = \frac{4\pi\eta r^3}{3kT}$

η : viscosity

9-12

Now, what are some sources of this fluctuating field? We have coupling between dipoles of the nucleus of interest and the solvent. So if we take, for example, glucose in water and the glucose molecule will interact with the water. Here's our nucleus of interest and in blue is the solvent, or we have interactions between nuclear magnetic dipoles. That's another source of this local fluctuating magnetic field. If you have a water molecule and another water molecule, then the two dipoles which are not perfectly aligned, and they will interact and create this local magnetic field. It is useful here to consider the correlation function, which is given by this correlation time. The correlation function is *e ^ (- t / τ_C)*. This function essentially describes the correlation of motion that is *t* seconds apart. It's dependent on this correlation time. What I'm getting at here is the parameter is the correlation time because that correlation time depends on a number of things that we know. One of them, viscosity. This is easily envisageable. If you take the sugar molecule in our blood, this sugar molecule will come more fairly fast. So the correlation time is short.

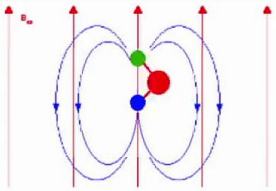
Notes

Summary

4m 53s

What are the mechanisms of relaxation ?

Tumbling of Molecule
(Brownian motion)
Creates local oscillating/fluctuating magnetic field



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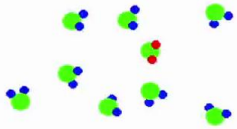


Correlation function G $G(t) \propto e^{-t/\tau_c}$

Describes degree of correlation of motion t sec apart

Correlation time τ_c: $\tau_c = \frac{4\pi\eta r^3}{3kT}$

η : viscosity
k: Boltzmann constant
r: size of molecule

9-12

If you take the same sugar molecule in honey--honey has a very high viscosity so there that sugar molecule will move less, so the correlation time will be longer. Then it makes also sense that, as temperature decreases, the correlation time increases because essentially at 0 Kelvin, we froze and we don't move at all. Finally, that it is somehow dependent on the radius of the molecule makes also sense. It takes just much more thermal energy to rotate the big macromolecule as compared to a small glucose molecule. This correlation time has some molecular effects in there, some properties of the molecule itself. Relaxation has in large part to do with microscopic motion, random motion that generates a randomly varying magnetic field, and this has a lot to do with molecular properties of the molecules.

Notes

Summary

6m 16s

What is the cause of loss of transverse Magnetization ?

fluctuating *microscopic* magnetic fields δB

T_2: phenomenological time constant

Range 10µs (bone)… several s (water)

"transverse relaxation", "T2 relaxation"

$$M_{xy}(t) = M_{xy}(0)e^{-\frac{t}{T_2}}$$

Cause:

Molecular dynamics and spin-spin interactions

Historically : "spin-spin" relaxation

→ loss of signal in xy plane

"Memory" relaxation

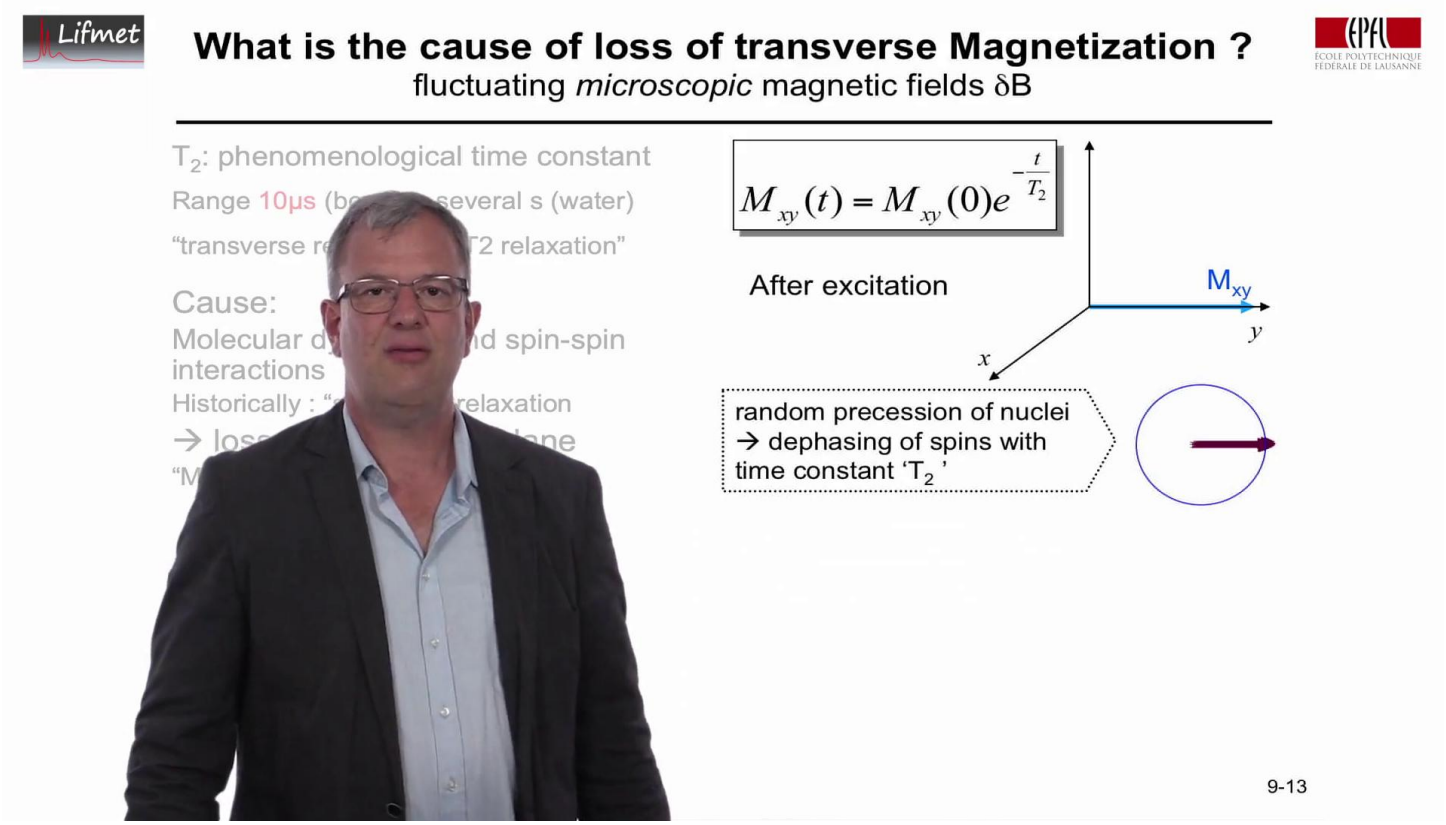
9-13

Now, how does a locally varying magnetic field that fluctuates cause a loss of transverse magnetization? We will deal with *T_2* largely as a phenomenological time constant. But in this little example I want to give you some rationalization on how this *T_2*, this decay time for the transverse magnetization and how that is influenced by molecular events. First, to give us an idea what we're talking about, the *T_2* ranges from typically on the order of microseconds, 10 microseconds in bone to several seconds in water. *T_2* is also called the transverse relaxation, *T_2* relaxation. The underlying cause is molecular dynamics and spin-spin interactions. If you have nearby spins and depending on how the molecule moves, this will influence the magnitude of this relaxation constant. It has also been called historically "spin-spin relaxation" because it is based, to a large extent, on interactions between spins. This leads to a loss of signal in the *x y* plane. It's also called "memory relaxation." How does this lead to this loss in magnetization that we phenomenologically described by the exponential decay? Let's look at a situation after excitation.

Notes

Summary

7m 22s



We have a transverse magnetization, we have now [afflicted] around *x* with the *B_1* field. We've tipped it to magnetization along *y*. If now you can see that your voxel as composed of little sub-voxels and in each of the sub-voxels, your magnetic field that the nucleus spins experience is identical for all in nuclear sub-voxels. Nothing of interest will happen. They will all precess with the same Larmor frequency, and this magnetization vector is preserved. But now, we have random local magnetic field. Imitate your voxel in image. Imagine that you've cut it into tens of thousands sub-voxels, tiny little pockets where in each of the pockets you have a magnitization. A tiny magnetization associated with it, so sub-voxels, if you will. Now in your mind assigned to each of these sub-voxels a slightly different external magnetic field *B_0*. Now, what you will get is that in your voxel, these magnetization will now precess at slightly different Larmor frequencies. If they have slightly different Larmor frequencies even if you put ourselves into the rotating reference frame of the main Larmor frequency, some of them will precess faster, and some of them will precess slower in that rotating reference frame.

Notes

Summary

8m 56s

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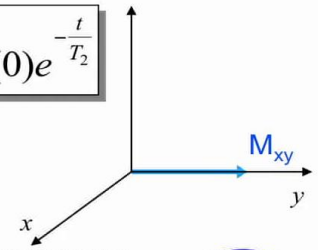
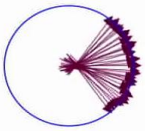
"Memory" relaxation

After excitation



random precession of nuclei → dephasing of spins with time constant 'T_2'



Rule of thumb for tissue water:
The less "tissue" (bone, solutes, proteins, membranes) is in contact with bulk water, the longer bulk water T_2

Phase ϕ accrued over τ_c:

$\phi = \delta B \tau_c$

$$\iiint_{voxel} \rho(\vec{r}) e^{i\delta B(\vec{r},t)\tau_c} dV \rightarrow 0$$

9-13

If they don't precess exactly at the same frequency, that means between these magnetizations, an angle will build up. And this is in this looping animation here that we have this build-up of the angle between the magnetizations that produces... ...the dephasing of the spins and therefore, a reduction as they lose the same orientation. The effective transverse magnetization that we measure is reduced. We have a rule of thumb for water. That means the less the water is in contact with tissue, be it bone, be it membranes, be it proteins, et cetera, the longer is the bulk water *T_2*. The purer the water, the longer is the *T_2*. The longest *T_2* that you can have for water is degassed, demineralized water. How does this T2 relate to the correlation time? Any of these things increases a correlation time of water. If you look over a voxel, we have the density of our spins times the local fluctuation in the magnetic field with time times the correlation time. If our correlation time increases, then this term here increases over *τ_C*, and so we get a faster differential precession along with differential precession until the molecules reorient and this goes to zero faster.

Notes

Summary

10m 23s

What is the cause of loss of transverse Magnetization ?

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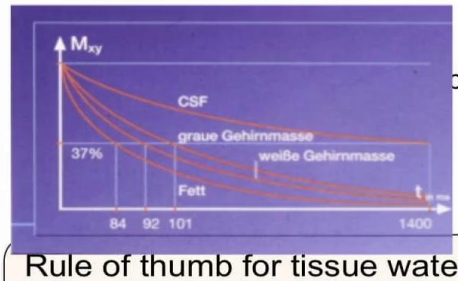
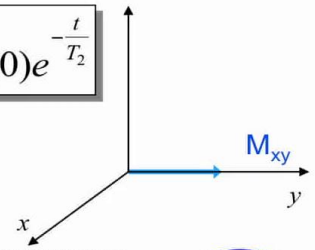
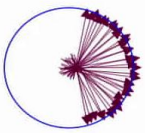
$$M_{xy}(t) = M_{xy}(0)e^{-\frac{t}{T_2}}$$



After excitation



random precession of nuclei → dephasing of spins with time constant 'T_2'



Rule of thumb for tissue water:
The less "tissue" (bone, solutes, proteins, membranes) is in contact with bulk water, the longer bulk water T_2

Phase ϕ accrued over τ_c:

$\phi = \delta B \tau_c$

$$\iiint_{voxel} \rho(\vec{r}) e^{i\delta B(\vec{r},t)\tau_c} dV \rightarrow 0$$

τ_c large (immobile spins):

large phase differences ⇒ short T_2

bone, membranes, proteins are MR-"invisible" 9-13

And here's another major conclusion. If your object of interest, your molecule of interest has a long correlation time, it accrues a lot of phase difference over this correlation time. This results in a short *T_2*. That is important and the conclusion is this one, this is the take-home message here. Bone, membranes, proteins, for intents and purpose of biomedical imaging, they are invisible. That means, although they carry magnetization, it decays so fast that we cannot practically measure it. Here's an example. We have a transverse magnetization in cerebrospinal fluid of the brain. There is hardly any proteins. This decays the slowest. The *T_2* is 1.4 seconds. We have grey matter. It's 100 milliseconds, 90 milliseconds is white matter and that, for other reasons, has an even shorter *T_2*. These are some examples here for the relaxation that they are, because of the differential environment and the interactions at the water molecule has on a molecular level, these interactions are different for different tissues. And that is a source for changes in *T_2* between different tissues.

Notes

Summary

12m 08s

How does M_z return to equilibrium ?

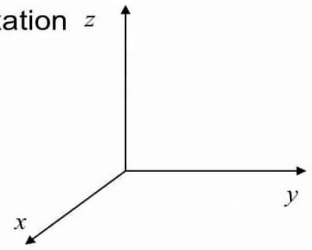
Longitudinal relaxation T_1

After decay of M_{xy} by T_2: $M_z \rightarrow M_0$

Longitudinal Relaxation

(along z-axis) $\frac{dM_z(t)}{dt} = -\frac{M_z(t) - M_0}{T_1}$

After T_2 relaxation



9-14

Now, we have said, for relaxation to occur and for the thermodynamic equilibrium magnetization to be re-established, we have to re-establish the magnetization along *z* to get to the equilibrium magnetization. And so, there is also something happening with the longitudinal magnetization, the magnetization along *z*. So, we consider that we have excited our magnitization to the transverse plane. The transverse magnetization, *M_ x_y* has decayed by T2, and now our *z* magnetization, which is zero, has to be reconsittuted and approach the equilibrium magnetization. And this is described by the longitudinal relaxation which happens along the *z* axis, and this is governed by this differential equation here. So we have the time derivative of the *z* magnetization component is given by something minus the *z* magnetization times the constant. So, if we have the derivative of something equals to something of that derivative and a constant, it makes sense to consider an exponential solution here as well. Let's first, however, look at what happens after *T_2* relaxation. We have no *z* magnetization, so it is zero.

Notes

Summary

13m 28s

How does M_z return to equilibrium ?

Longitudinal relaxation T_1

After decay of M_{xy} by T_2: $M_z \rightarrow M_0$

Longitudinal Relaxation

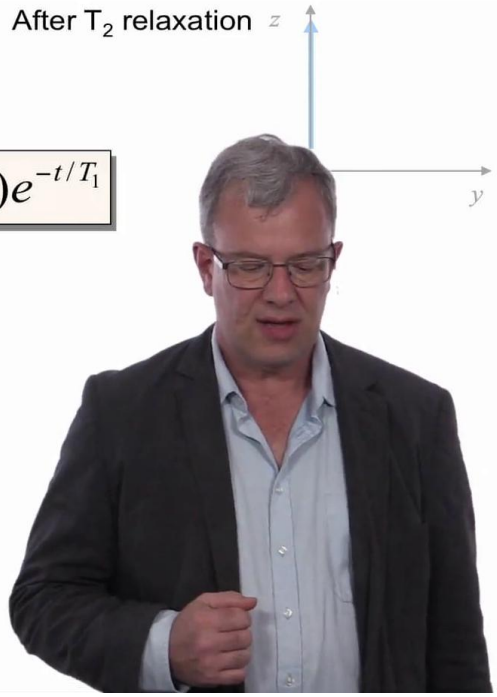
(along z-axis) $\frac{dM_z(t)}{dt} = -\frac{M_z(t) - M_0}{T_1}$

$$M_z(t) = M_0(1 - e^{-t/T_1}) + M_z(0)e^{-t/T_1}$$

Mechanisms: Incoherent molecular fluctuations on the order of the Larmor frequency ω_L
possibility of energy transfer → matching frequency

Historically : spin-lattice relaxation
(heat lost to the surroundings)
T_1 ~ 0.5-5s (water)

After T_2 relaxation



9-14

The solution to this equation here is given by this solution here. You can verify by plotting this term in here, doing the derivation that this is indeed the solution. If we said, *t = 0*, then we have *M_z (0) = M_z (0)*, so that works. If we set *t* to infinity, *t / T_1*, this goes to zero, this goes to zero, then we have *M_0*, so it's the right behavior. What this means is that after we have had *T_2* relaxation, we have no nuclear magnetization in our sample. And as *T_1* relaxation, this equation here on the left works, the *z* magnetization is reconstituted until the equilibrium *z* magnetization. Now, what are the mechanisms for *T_1* relaxation? They again have a lot to do with molecular motion, its incoherent fluctuations, and it has to do a lot with how these fluctuations, the motion, the molecular fluctuations are close to the Larmor frequency. Remember, a change in thermodynamic equilibrium magnetization is associated with the change in energy transfer between the energy levels in the higher and lower energy diagram. It has therefore been also historically called spin-lattice relaxation because one can consider the spin pool as a thermally isolated pool and the heat is lost to the surroundings.

Notes

Summary

14m 51s

How does M_z return to equilibrium ?

Longitudinal relaxation T_1

After decay of M_{xy} by T_2: $M_z \rightarrow M_0$

Longitudinal Relaxation

(along z-axis) $\frac{dM_z(t)}{dt} = -\frac{M_z(t) - M_0}{T_1}$

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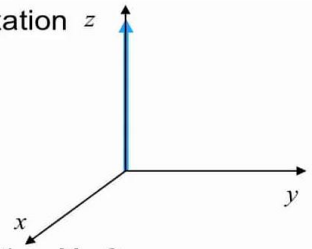
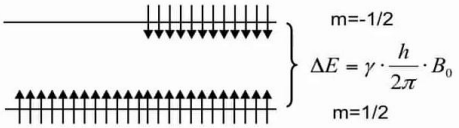
T_1 ~ 0.5-5s (water)

After T_2 relaxation



After 90^0 excitation: M_z=0
⇒ population distribution corresponds to T=∞:



9-14

Remember, if we have a 90-degree pulse, we have equal distribution of the spins in upper and lower energy level. This corresponds technically to a temperature of infinity, so the spin system has to cool down. Incidentally, this by the way has been used, manipulation of nuclear magnetization has been used at a very low temperature to further cool a sample by working with heat and exchange between the lattice and the spin system. To give an idea what the *T_1* is, it's typically between 0.5 and 5 seconds. The longest is for water, 5 seconds value that's pure, degassed, demineralized water. We have at the time, after 90-degree magnetization, we have no *z* magnetization. So the population distribution is equal in the two energy levels. This corresponds to a temperature of infinity. We draw the two energy levels here, lower and higher energy level, here's our energy difference governed here, and this would be at 0 Kelvin, we have a mole in the lower energy state. And we'll take here an extreme drawing that is our equilibrium magnetization that we have at the beginning. We start out with equilibrium magnetization, and when we do a 90-degree flip, we have equal population in the higher and lower energy level, which corresponds to infinite temperature.

Notes

Summary

16m 21s

How does M_z return to equilibrium ?

Longitudinal relaxation T_1

After decay of M_{xy} by T_2: $M_z \rightarrow M_0$

Longitudinal Relaxation

(along z-axis) $\frac{dM_z(t)}{dt} = -\frac{M_z(t) - M_0}{T_1}$

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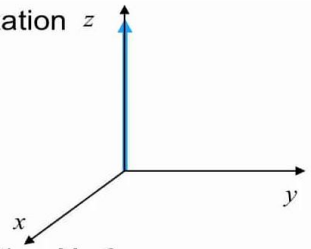
After T_2 relaxation



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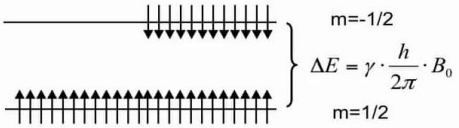
⇒ population distribution corresponds to T=∞:

$\frac{N_1}{N_2} = e^{-\frac{\Delta E}{kT}}$



Boltzmann distribution re-established by energy (photon) transfer from spins to system (lattice).

Most efficient when energy levels of system and nuclear spins match, i.e.
$\omega\tau_c$~1 ⇒ T_1 minimal

(bone: $\omega\tau_c$>>1 ⇒ T_1~ s to min)

9-14

Now, we wait, *T_1* relaxation is happening, and the spins that are in excess in the higher energy level eventually return back to the lower energy level until the Boltzmann distribution is re-established. Then we have thermodynamic equilibrium magnetization, which is dictated by the Boltzmann distribution here. And *1 / 82 = e ^ (-Δ E / k T)*. Now, this change in energy level, the spins that are in the higher level that are in excess, they have to go to the lower energy level. In doing so, they have to give off energy. This is essentially done by transfer of energy from the spins to the system, and the system here is the lattice. It is done by photon transfer, but it is done thermally. This energy transfer is most efficient if the motion of the system, that is, frequency with which the system fluctuates, the energy fluctuates, matches the energy difference here between the energy levels. We want to give off the quantum of photon. We can only give it off the energy in form of exactly, this energy difference. This happens most effectively if the motion has that frequency in it as well. If the motion of the molecule largely does not correspond to the frequency of energy difference here, this *Δ E* here, then where do the photons go?

Notes

Summary

17m 55s

How does M_z return to equilibrium ?

Longitudinal relaxation T_1

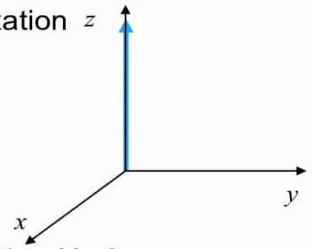
After decay of M_{xy} by T_2: $M_z \rightarrow M_0$

Longitudinal Relaxation

(along z-axis) $\frac{dM_z(t)}{dt} = -\frac{M_z(t) - M_0}{T_1}$

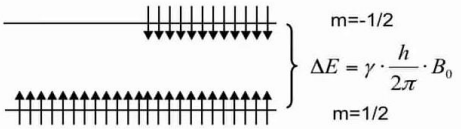
$$M_z(t) = M_0(1 - e^{-t/T_1}) + M_z(0)e^{-t/T_1}$$

After T_2 relaxation



After 90^0 excitation: $M_z=0$

$\Rightarrow$ population distribution corresponds to T=∞:



Mechanisms: Incoherent molecular fluctuations on the order of the Larmor frequency ω_L

possibility of energy transfer → matching frequency

Historically : spin-lattice relaxation

(heat lost to the surroundings)

T_1 ~ 0.5-5s (water)

$\frac{N_1}{N_2} = e^{-\frac{\Delta E}{kT}}$

Boltzmann distribution re-established by energy (photon) transfer from spins to system (lattice).

Most efficient when energy levels of system and nuclear spins match, i.e.

$\omega\tau_c \sim 1 \Rightarrow T_1$ minimal

(bone: $\omega\tau_c >> 1 \Rightarrow T_1$~ s to min)

Rule of thumb for water:

The less "tissue" is in contact with bulk water (bone, solutes, proteins, membranes), the longer bulk water T_1

9-14

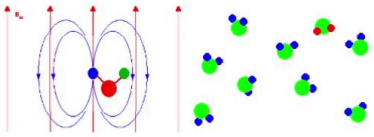
They cannot be easily given off. That's the intuitive description here. This basically means that the Larmor frequency times the correlation time should be roughly 1. Then we have the most efficient transfer. The most efficient transfer means that we have a minimal *T_1* relaxation time. Now, if we have solid state like bone or plastics, or cement, or anything, or teeth, then the correlation time is very long. The ω τ*_C* is much larger than 1. Then we have a *T_1* on the order of seconds to minutes. You take water and you freeze it down to a few Kelvin of temperature. You measure the *T_1*. In that situation, that *T_1* is in the order of tens of thousands of seconds. It's very long because there is no motion close to the Larmor frequency of our magnetization. To sum up here the relaxation of *T_1*, there's also a rule of thumb that is useful for us to know. It is very similar to what we had for *T_2*. Basically means the purer the *T_2*, the purer the water is, so there's less other molecules at the water interact, the longer is the *T_1*. The more there are like membranes, proteins, et cetera with which the bulk water can come in contact, the shorter is the *T_1*. That's the rule of thumb for tissue that is useful to remember.

Notes

Summary

19m 32s

Credits



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Notes

Summary

21m 10s