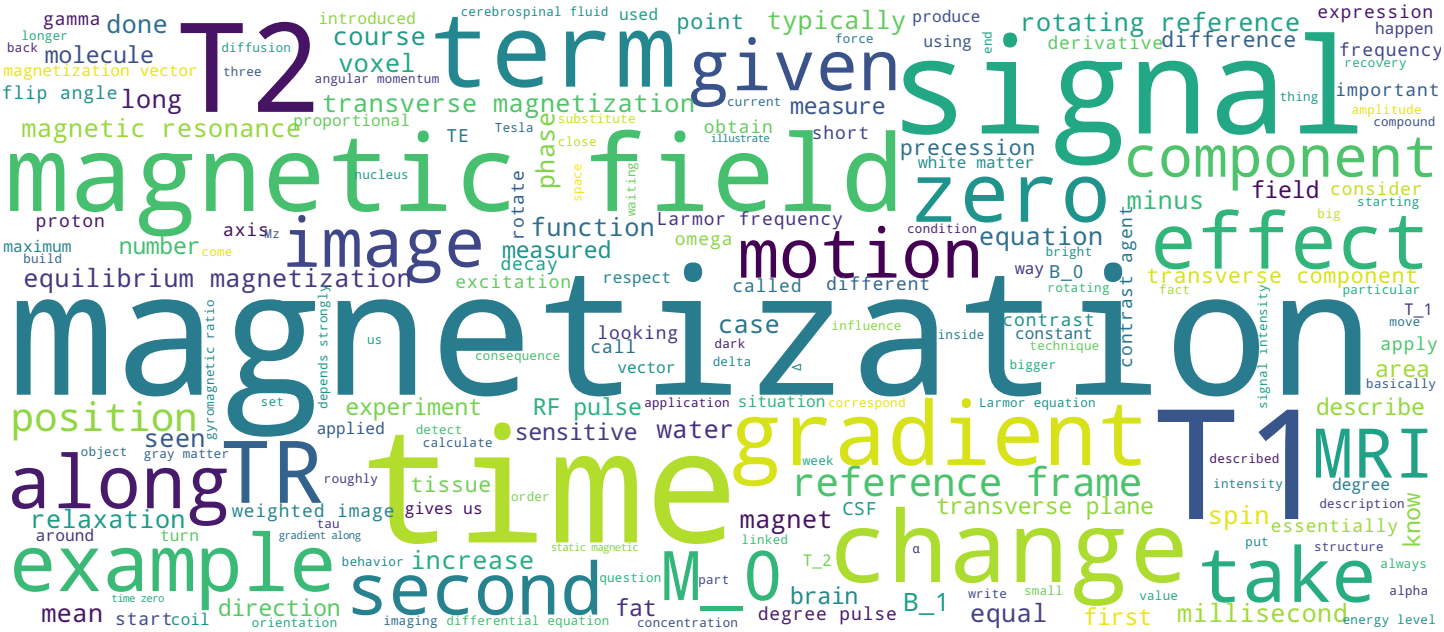
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

9.5 Bloch equations, the basic MR signal and intermediate summary

Fundamentals of Biomedical Imaging

Prof. Rolf Gruetter



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9-5. What equations describe the change in magnetization ?

Bloch Equations

add relaxation terms (T_1, T_2) to the fundamental Eq of motion of magnetization:

$$\frac{dM_z(t)}{dt} = \gamma[M_x(t)B_y(t) - M_y(t)B_x(t)] - \frac{M_z(t) - M_0}{T_1} \quad \text{along z}$$

$$\frac{dM_x(t)}{dt} = \gamma[M_y(t)B_z(t) - M_z(t)B_y(t)] \quad \text{along x}$$

$$\frac{dM_y(t)}{dt} = \gamma[M_z(t)B_x(t) - M_x(t)B_z(t)] \quad \text{along y}$$

$$-\gamma\vec{B}\times\vec{M}$$

9-15

So now we will look at a description of the changes in magnetization. And this will bring us to a full description of the changes in the longitudinal and transverse components. What we have to do, we've already seen in the previous lectures-- in the previous weeks-- we have seen the Larmor equation, which describes the precession of magnetization around either a static magnetic field or a rotating magnetic field like the *B_1*. Now what we have to do here is we just have to add the terms in the differential equations for relaxation-- that is *T1* and *T2*. So for the *z* magnetization we will have this term that's *along z*. For the *x* magnetization we have this term. And for the *y* magnetization we have this term. Now this is a fairly complicated set of equations here. As you can see, there are couple of differential equations but if we look at this as a whole-- this expression here-- this is nothing but the vector product of *gamma B* with *M*. So this describes the precession of the magnetization around the magnetic field *B*, and here *B* is a general vector. Now we have the relaxation *along z* that we have introduced this week.

Notes

Summary

0m 04s

9-5. What equations describe the change in magnetization ?

Bloch Equations

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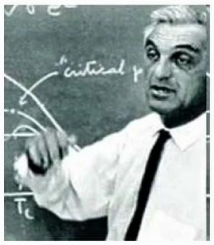
$$\frac{dM_z(t)}{dt} = \gamma[M_x(t)B_y(t) - M_y(t)B_x(t)] - \frac{M_z(t) - M_0}{T_1} \quad \text{along z}$$

$$\frac{dM_x(t)}{dt} = \gamma[M_y(t)B_z(t) - M_z(t)B_y(t)] - \frac{M_x(t)}{T_2} \quad \text{along x}$$

$$\frac{dM_y(t)}{dt} = \gamma[M_z(t)B_x(t) - M_x(t)B_z(t)] - \frac{M_y(t)}{T_2} \quad \text{along y}$$

$$-\gamma\vec{B}\times\vec{M}$$



Felix Bloch
Physics
1952

Substituting $\Omega = -\gamma B_0 + \omega_{RF}$ ($B_0 = B_z$ is not time-dependent) yields:

Rotating reference frame

$$\gamma[M_x(t)B_1^y(t) - M_y B_1^x(t)] - \Omega M_y(t) - \gamma M_z B_1^y(t)$$
$$\gamma M_z(t)B_1^x(t) + \Omega M_x$$

$$\left(\gamma\vec{B}_1 + \vec{\Omega}\right)\times\vec{M}$$

B_1 : RF field in rotating frame

9-15

So we have the change in magnetization *along z*, change in magnetization *along x* and *y*. So these are the relaxation terms. And this gives us the full set of differential equations-- the behavior-- so we have, on one hand, we have precession around *B*-- that's these terms here-- and we have relaxation now incorporated. And these three equations here are the Bloch equations. They're named after a Swiss physicist, Felix Bloch, who was awarded the Nobel Prize for his discovery of the nuclear magnetic resonance effect. He was awarded Nobel Prize in Physics in 1952. If we substitute the term for the rotating reference frame, so we'll substitute the *big omega*, which is given by minus *gamma B_0* plus *omega RF*, then we obtain these equations for the term that's substituting these three equations with this term and this is the rotating reference frame, or in other words, it's *gamma B_1* plus *omega* in the rotating reference frame that rotates with the *B_1* cross *M*. So that would be in the description in the rotating reference frame.

Notes

Summary

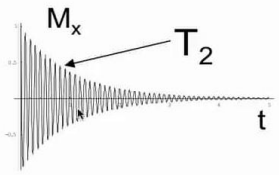
1m 31s

What characterizes the basic MR signal ?

Free induction decay: Precession and relaxation (after RF pulse)

Transverse magnetization

$$M_{xy}(t) = M_{xy}(0)e^{-i\omega t}e^{-t/T_2}$$



9-16

Now what characterizes the basic MR signal? We'll start out with the transverse magnetization and here I'll describe it as a complex function. So the transverse magnetization in complex terms is given by the transverse magnetization times zero, times *e* to the minus *i-omega-t*, times *e* to the minus *t/T2*. This is relaxation, this is precession. Here we're using the complex notation-- we're using the real part, for example, to represent the *x* axis and the imaginary part to represent the *y* axis. And what does this signal look like? This expression-- this is the *x* component. The envelope, that is the amplitude, decays with *T2*, that means the envelope decays with *T2* and we see oscillations here in the real part, that's the *cosine omega t* term. Likewise, for the *y* term-- that would be the imaginary part of this equation-- has a similar function except it's *sine omega t* in this case, but it has also the *T2* as the decaying amplitude-- the *T2* effect.

Notes

Summary

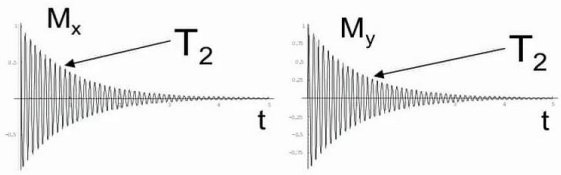
2m 53s

What characterizes the basic MR signal ?

Free induction decay: Precession and relaxation (after RF pulse)

Transverse magnetization

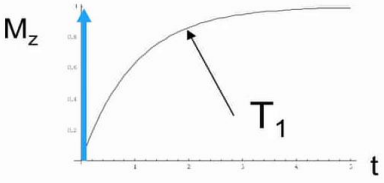
$$M_{xy}(t) = M_{xy}(0)e^{-i\omega t}e^{-t/T_2}$$

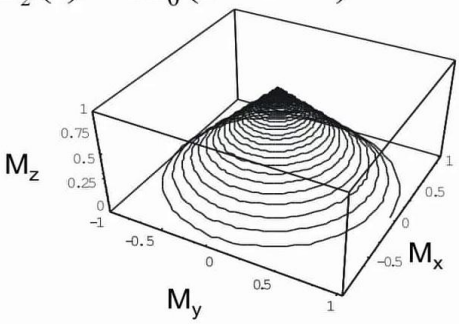


Longitudinal magnetization

(after 90⁰ RF excitation)

$$M_z(t) = M_0(1 - e^{-t/T_1})$$





9-16

For longitudinal magnetization, and here I'll just stick to the case where we have a 90 degree full excitation, so the magnetization has been flipped into the transverse plane, then the *z* magnetization-- the longitudinal magnetization-- is equal to *M_0* times 1 minus *e* to the minus *t/T1*. So in the end, what we have is a complex description of the motion. We have the *z* component which recovers towards *M_0.*, we have the *x* and the *y* component, which basically together means that the transverse component rotates in the transverse plane-- according to the Larmor equation-- and also decays exponentially. So this could be described by a behavior like this and this is fairly complicated. So if you look at the *z* magnetization, it builds up in an exponential function. So it goes from zero to *M_0*. This-- how fast it recovers to *M_0* is described by *T1*. So we have the equilibrium *z* magnetization like this. We'll start out essentially at zero magnetization and then what happens with time is that the *z* magnetization recovers with *e* to the minus *t/T1* towards *M_0*, that's what the magnetization vector will do over time.

Notes

Summary

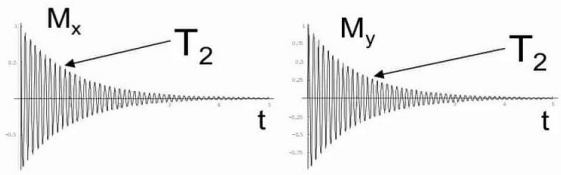
4m 07s

What characterizes the basic MR signal ?

Free induction decay: Precession and relaxation (after RF pulse)

Transverse magnetization

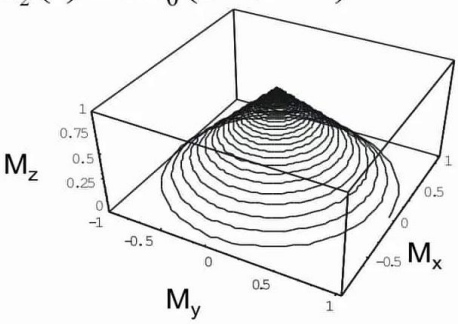
$$M_{xy}(t) = M_{xy}(0)e^{-i\omega t}e^{-t/T_2}$$

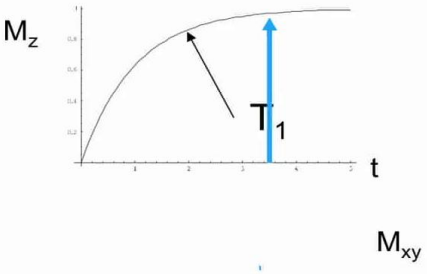


Longitudinal magnetization

(after 90⁰ RF excitation)

$$M_z(t) = M_0(1 - e^{-t/T_1})$$





NB. M can never exceed $M_0 \Rightarrow T_2 \leq T_1$

9-16

Now if we look at the transverse component *Mxy*-- I'll here not distinguish between *x* and *y*, I'll just take this as the transverse component-- then this component decays while the *z* component increases. Of course, *Mxy* precesses depending on the choice of our rotating reference frame. So we have also a condition here that we can never exceed the equilibrium magnetization *M_0* and this is a constraint that if we look at the norm of the vector-- the total magnetization vector-- it cannot be bigger than *M_0* and this typically implies that *T2* is not bigger than *T1*. It is typically much smaller, but certainly it does not exceed *T1*. Now, I want to give you an example.

Notes

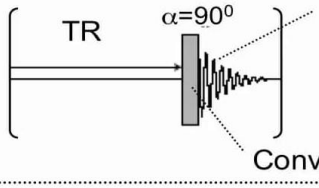
Summary

5m 41s

How can T_1 changes be measured ?

repetitive pulsing

Multipulse experiment with RF pulses applied every TR seconds



Converts M_z to M_{xy}

$$M_z(t) = M_0(1 - e^{-t/T_1}) + M(0)e^{-t/T_1}$$

This is a preparatory example for what is to come next week, and this is how can we be sensitive to *T1* changes? Or how can we measure *T1* changes precisely? And here we'll consider a so-called *multipulse experiment.* So the experiment is fairly simple, an RF pulse that is a *B_1* field is applied every *TR* seconds. So here is our scheme, we apply this every *TR* seconds. So here's the time *TR.* Then we have a flip angle *alpha* applied here and we'll set this to 90 degrees. And what this pulse does, it converts the *z* magnetization to transverse magnetization and then the signal is measured. So at the beginning, the signal is proportional to the *z* magnetization and therefore it is sensitive to *T1* effects. So now we'll take the basic differential equation that describes the behavior of the *z* magnetization. We have the recovery to *M_0* and we have the starting magnetization *M_0*, that *M* of time zero that decays with *T1*. At time zero, we can say that the magnetization, the *z* magnetization, is *M_0 cosine alpha* and then we can write this in this form that is basically we are using the fact that we have used a 90 degree pulse here.

Notes

Summary

6m 40s

How can T_1 changes be measured ?

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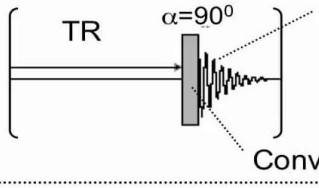
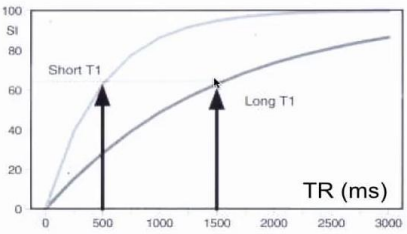
Multipulse experiment with RF pulses applied every TR seconds



$$M_z(TR) = M_0(1 - e^{-TR/T_1})$$

The effect of T_1 (and T_2) on the signal depends on how it is measured



$$M_z(t) = M_0(1 - e^{-t/T_1}) + M(0)e^{-t/T_1}$$

$$M(0) = M_0 \cos\alpha$$

$$M_z(t) = M_0(1 - e^{-t/T_1})$$

9-17

So *M_0 cosine alpha*, since cosine of 90 degrees is zero, then we have this term is zero, so our *z* magnetization is just this term here, which is written here again. So, if I express this now as a function of the repetition time, so we'll repeat our 90 degree pulse every *TR* seconds, then our *z* magnetization as a function of *TR* is given by the equilibrium magnetization times 1 minus *e* to the minus *TR/T1*. Now here what is crucial is we have an effect of *T1* as well as an effect of-- the same holds true for *T2* but the effect of *T1* on the signal depends strongly on how it is measured. So let me give you the example here. If we take a short *T1* component and we measure every 500 milliseconds, we get the signal intensity that is like this. If we have a long *T1* component, but we measure with 1.5 seconds *TR*, we get the same pixel intensity. So unlike the CT units-- the Hounsfield units in an MRI-- we *cannot* attribute a signal intensity in a pixel to a specific value of *T1* or a specific biophysical mechanism *because* we need to know how this has been measured.

Notes

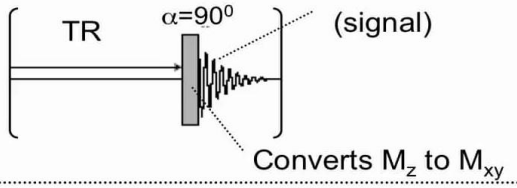
Summary

8m 16s

How can T_1 changes be measured ?

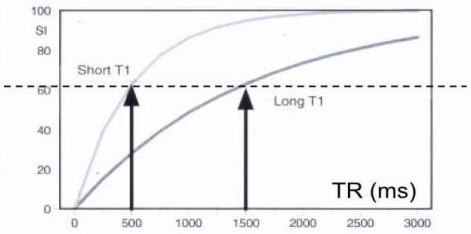
repetitive pulsing

Multipulse experiment with RF pulses applied every TR seconds



$$M_z(TR) = M_0(1 - e^{-TR/T_1})$$

The effect of T_1 (and T_2) on the signal depends on how it is measured



$$M_z(t) = M_0(1 - e^{-t/T_1}) + M(0)e^{-t/T_1}$$

$$M(0) = M_0 \cos\alpha$$

$$\rightarrow M_z(t) = M_0(1 - e^{-t/T_1})$$

Optimal TR to detect changes in T_1 ?

Use noise error propagation calculation (Lesson 1)

$$\frac{\partial M_z(t)}{\partial T_1} = \frac{t}{T_1^2} e^{-t/T_1} \equiv F$$

9-17

So in this particular example, if we had measured at 1.5 seconds, this component will give the same intensity as if we had measured at 500 milliseconds for a short *T1* component. So this is essential here that the pixel intensity, as such, doesn't have a direct biophysical interpretation. Now if we are doing this multipulse experiment, so we apply a 90 degree pulse every *TR* second, at what point would be the most sensitive to changes in *T1*? And for imaging that's important, that would give us the maximum contrast. To approach this, we will use the error propagation calculation that I introduced in lesson one of part one of this course. The same procedure here applies. So first we calculate the derivative of the *z* magnetization with respect to *T1*. This just tells us how strong does the *z* magnetization vary if one were to vary *T1*. So where is the *z* magnetization? How much does it vary in response to variations to *T1*? So that's just the derivative of this term with respect to *T1* and this is the expression that we obtain. And we'll call this function *big F*. Of course we know, if we measure with *TR* it essentially equals zero.

Notes

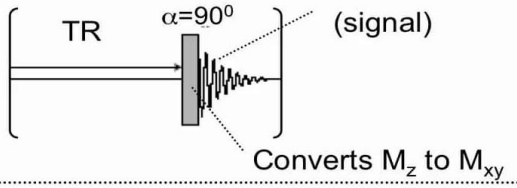
Summary

9m 43s

How can T_1 changes be measured ?

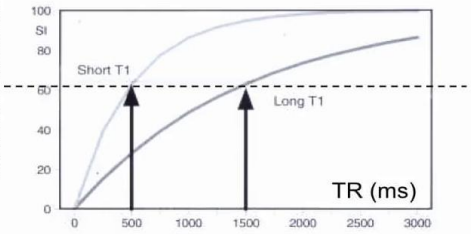
repetitive pulsing

Multipulse experiment with RF pulses applied every TR seconds



$$M_z(TR) = M_0(1 - e^{-TR/T_1})$$

The effect of T_1 (and T_2) on the signal depends on how it is measured



$$M_z(t) = M_0(1 - e^{-t/T_1}) + M(0)e^{-t/T_1}$$

$$M(0) = M_0 \cos\alpha$$

$$\rightarrow M_z(t) = M_0(1 - e^{-t/T_1})$$

Optimal TR to detect changes in T_1 ?

Use noise error propagation calculation (Lesson 1)

F=max:

$$\frac{\partial M_z(t)}{\partial T_1} = \frac{t}{T_1^2} e^{-t/T_1} \equiv F \qquad \frac{dF}{dt} = 0 = \frac{1}{T_1^2} e^{-t/T_1} - \frac{t}{T_1^3} e^{-t/T_1}$$

$$0 = \frac{1}{T_1^2} e^{-t/T_1}\left(1 - \frac{t}{T_1}\right) \quad t=TR_{opt}: \quad TR_{opt} = T_1$$

9-17

The magnetization cannot recover so it will be essentially zero. So we are not sensitive to changes in *T1*. And if we write 5 times the biggest *T1* that is present in our tissue then everything has recovered to *M_0* and we're again, not sensitive to changes in *T1*. So somewhere between zero and infinity-- for the time *T*-- we have a maximum of this function. And to find the maximum of this function *F*, we'll take the derivative of *F* with respect to the time *T*, which is here the *TR*. We'll set the *F* to *dt* equals zero, we want to find the maximum, we do the chain rule for derivation, so that's what we're interested in and then we obtain this expression here. If we simplify what we've obtained here, this is not zero, this is not zero so only what is in brackets can be zero. And *t* equals the *optimal TR*, if the *optimal TR* is equal to *T1*. So this is a not surprising result, we have seen similar things. We're always dealing with exponential functions and typically the change in the decay constant or, here, the *T1* is optimally detected if we measure at an exponential parameter that corresponds to this decay time, or the *T1* here.

Notes

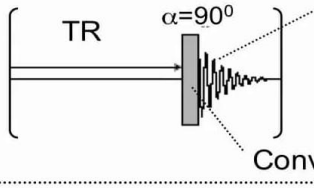
Summary

11m 04s

How can T_1 changes be measured ?

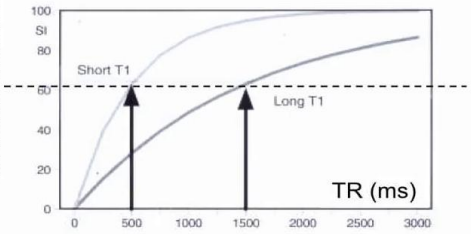
repetitive pulsing

Multipulse experiment with RF pulses applied every TR seconds



$$M_z(TR) = M_0(1 - e^{-TR/T_1})$$

The effect of T_1 (and T_2) on the signal depends on how it is measured



$$M_z(t) = M_0(1 - e^{-t/T_1}) + M(0)e^{-t/T_1}$$

$$M(0) = M_0 \cos\alpha$$

$$\rightarrow M_z(t) = M_0(1 - e^{-t/T_1})$$

Optimal TR to detect changes in T_1 ?

Use noise error propagation calculation (Lesson 1) F=max:

$$\frac{\partial M_z(t)}{\partial T_1} = \frac{t}{T_1^2} e^{-t/T_1} \equiv F \qquad \frac{dF}{dt} = 0 = \frac{1}{T_1^2} e^{-t/T_1} - \frac{t}{T_1^3} e^{-t/T_1}$$

$$0 = \frac{1}{T_1^2} e^{-t/T_1}\left(1 - \frac{t}{T_1}\right) \quad t=TR_{opt}: \quad TR_{opt} = T_1$$

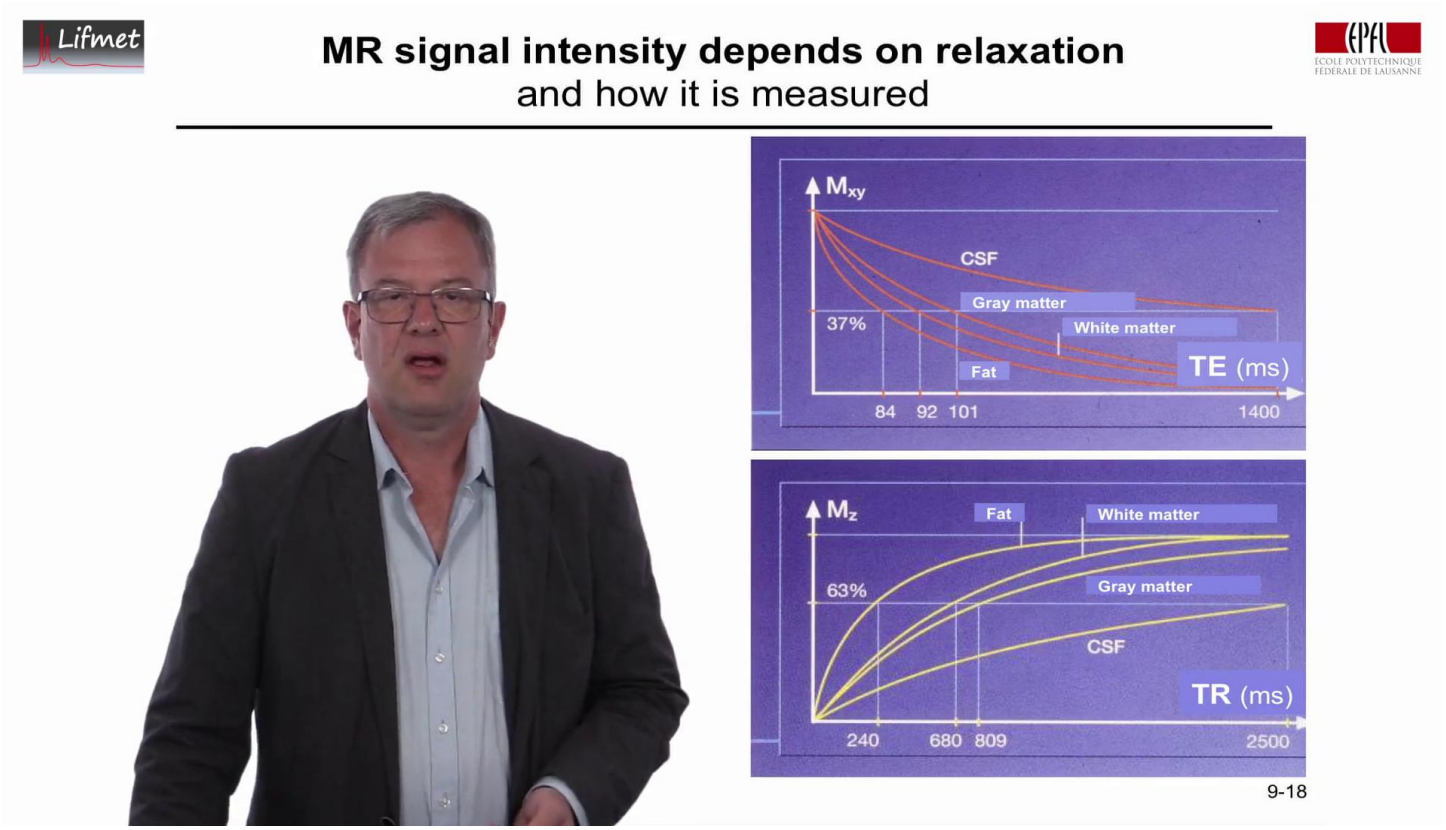
9-17

So in other words, if we have a rough idea what the *T1* is, we should be setting for a series of 90 degree pulses that we apply every *TR* seconds. We should be setting this *TR* to roughly *T1* to be most sensitive to changes in *T1*. And here is a message that is dear to me that I want to stress.

Notes

Summary

12m 26s



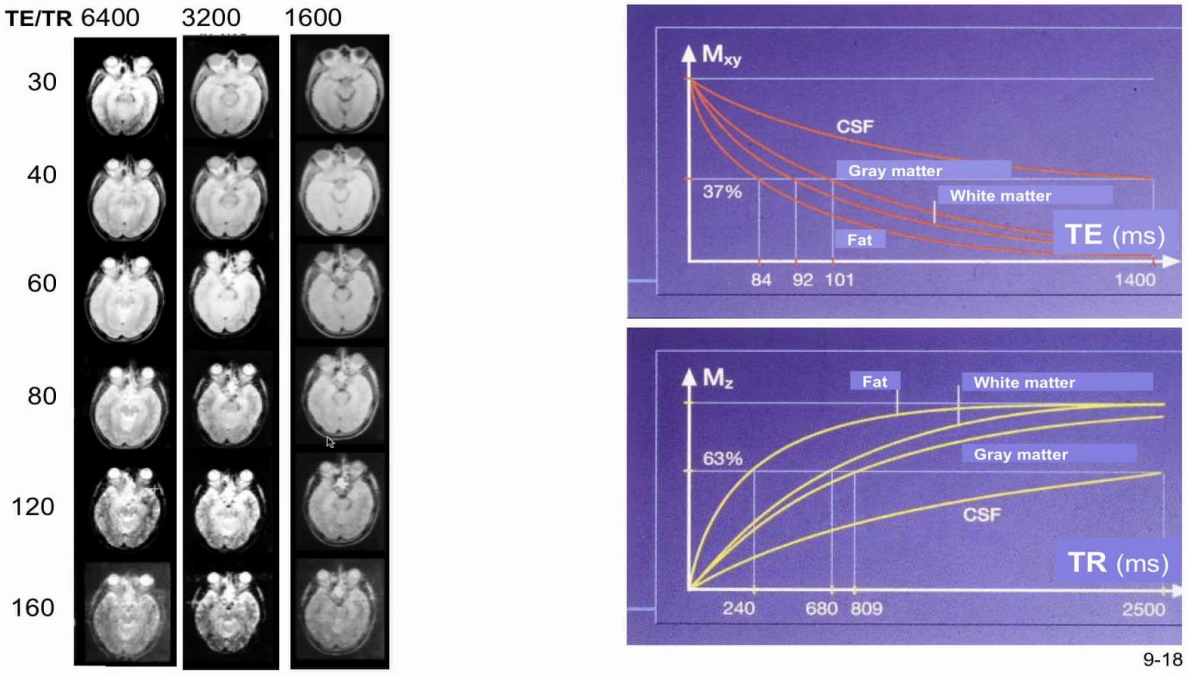
The intensity in our images-- the MRI images that are being measured-- depend strongly on the relaxation, but in a way, that depends strongly on how this signal intensity is being measured. So let's look at the transverse component. We have transverse magnetization. This is cerebrospinal fluid, we have gray matter, white matter and fat. And we can see for CSF-- cerebrospinal fluid which contains very little cell membranes, very little proteins-- the *T2* is very long, it's about 1.4 seconds in this example. Gray matter is at 100 milliseconds and white matter is at 90 milliseconds. And then we have fat here, so 37% of the decay signal gives us the *T2* here. If we look now at the recovery-- here's the 90 degree pulse and the recovery, then fat goes the fastest, white matter goes second fastest then gray matter then CSF. And CSF here in this example has a *T1* of 2.5 seconds. So what we see here is now typically we would do a T2-weighted image if we want to see changes in *T2*. We would wait a certain time after the excitation pulse and detect the signal.

Notes

Summary

12m 52s

MR signal intensity depends on relaxation and how it is measured



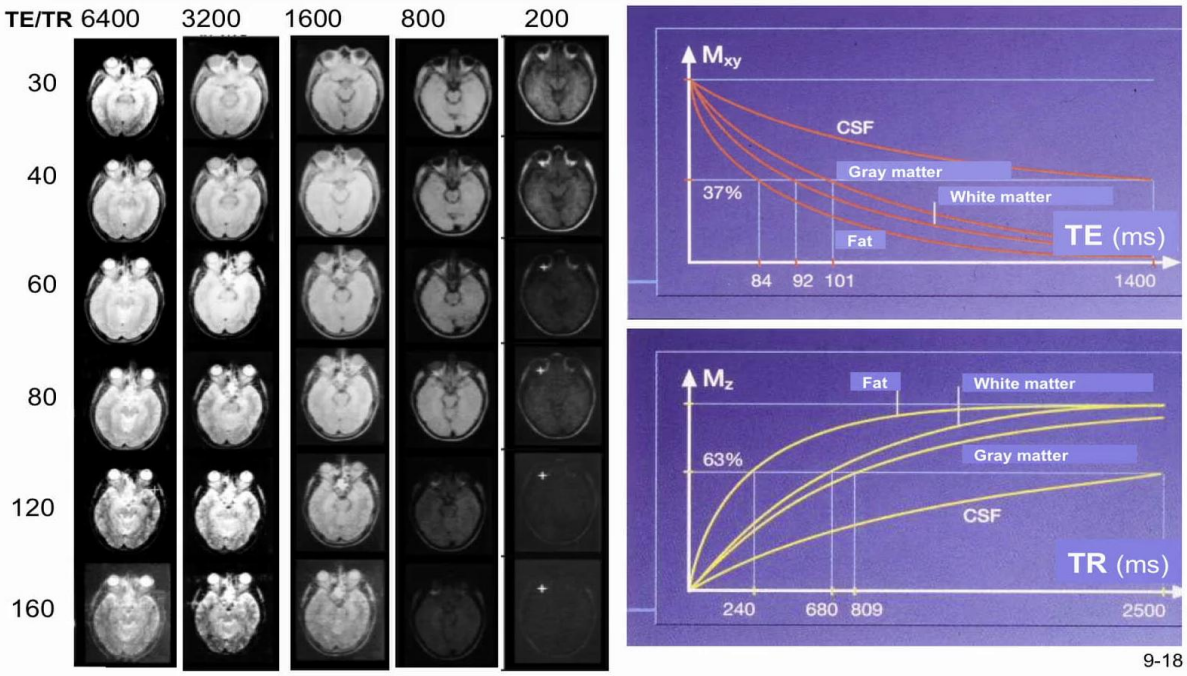
Now CSF will be the brightest because it has the longest *T2*, it has very little protein in it so it has long *T2*. Conversely, CSF has long *T1* but here we're deducing the waiting by waiting not the full time, we're waiting a shorter time than full relaxation. So actually, the signal from the cerebrospinal fluid will be lower compared to the signal from gray matter and white matter. So even though it has long *T1* and a long *T2* in T1-weighted images the CSF will be dark and in T2-weighted images it will appear bright compared to the other tissues. So let's look at an example here. We have on the vertical axis we are plotting the time after excitation and the *TR* on the horizontal axis. And so we have a very long *TR* here, we go from 30 to 160 milliseconds. Look at the eye bulbs which are primarily just water, how bright they get with a longer time after excitation. As we start reducing the *TR*, we're starting to have increasing effects of *T1* and we're starting also to see that the images are getting darker. That is a consequence of not being over here but being more and more closer to this situation.

Notes

Summary

14m 11s

MR signal intensity depends on relaxation and how it is measured

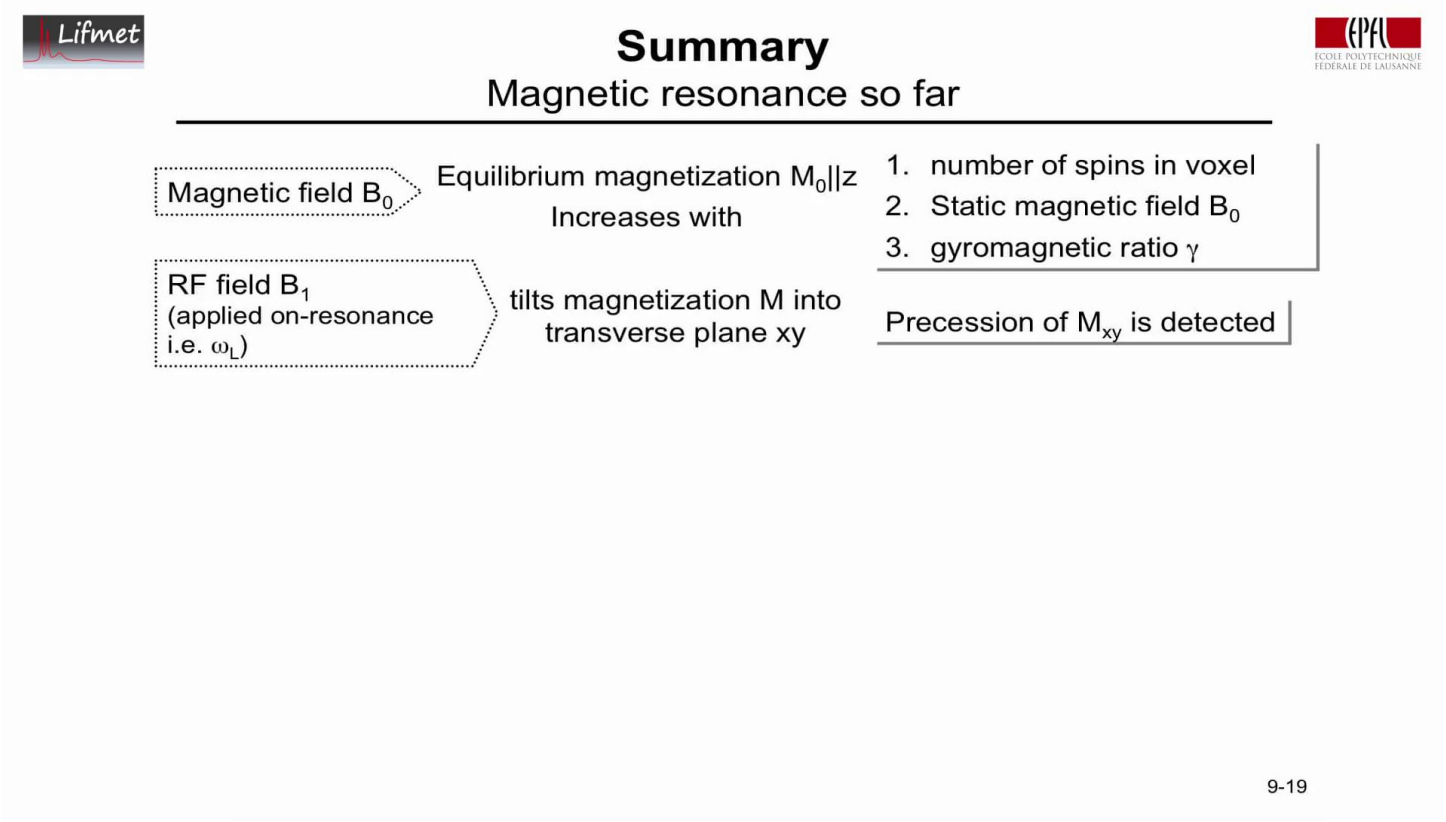


We're here at 800 milliseconds and here is 200 milliseconds. We can clearly see the fat here becoming bright at short echo time and short *TR*. The fat is becoming bright, the CSF here is dark, the eyes are dark-- that is consistent with these values here. And what is also important here is a good choice of parameters allows to depict certain pathological conditions. A bad choice of parameters is detrimental to detection of any signal. So if one is to choose a long echo *T*-- time after excitation-- and a long *TR*, then we are basically seeing no signal and that is not a good choice. So in order to interpret MRI signals, one *has* to know the experimental conditions that are being done and this is one of the reasons why MRI is such a complex technique to use in clinical practice but it is also an advantage because we have a lot of mechanisms and means to manipulate image contrast.

Notes

Summary

15m 36s



Now I want to conclude this week's lecture with a summary on magnetic resonance. What do we know so far? So we have established last week that we need a magnetic field *B_0*. This establishes an equilibrium magnetization *M_0* that is parallel to the *z* axis. And this equilibrium magnetization increases with the number of spins in the voxel, it increases with the static magnetic field *B_0* and it increases with the gyromagnetic ratio *gamma*. So that's one thing that we need. We need to be in a strong magnetic field to have an equilibrium magnetization. Then we need a RF field--*B_1*-- which is typically applied on the resonance that is close to that Larmor frequency, *omega L*. And this *B_1* field allows us to tilt the magnetization into the transverse plane, so we're tilting the *z* magnetization into the transverse plane. Here the plane is in *xy*. The signal detection process of magnetic resonance imaging is done by precession. We're looking at the transverse component of the magnetization that precesses around the magnetic field *B_0*. That is the detection process.

Notes

Summary

16m 46s

Summary

Magnetic resonance so far

Magnetic field B_0	Equilibrium magnetization $M_0 \| z$ Increases with	1. number of spins in voxel 2. Static magnetic field B_0 3. gyromagnetic ratio γ
RF field B_1 (applied on-resonance i.e. ω_L)	tilts magnetization M into transverse plane xy	Precession of M_{xy} is detected
T_2 and T_1 relaxation	exponential decay of M_{xy} exp. return of M_z to M_0	reflect molecular environment source of contrast

1. Only mobile spins (e.g. water) are detected
2. M_0 reflects amount of nuclei and thus water content
   [Water content varies 70-100ml/100g in body (poor contrast)]
3. Effect of T_1 and T_2 changes on image contrast depend strongly on experimental parameters (RF pulse timing and flip angle)

9-19

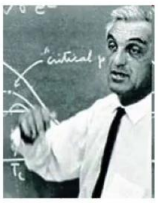
Then we have established this week that we have relaxation for *T2* that decays and for *T1* which builds up. So we have an exponential decay of the transverse magnetization and exponential return of *Mz* to *M_0* that's described by *T1* and *T2* relaxation. These time constants *T1* and *T2* reflect the molecular environment in which the molecule finds itself and they are a major source of contrast in MRI. So now some take home messages. In MRI, we can only detect mobile spins. So typically of major interest here is water. Immobile nuclei-- nuclei that are part of a molecule that's immobile-- are not detected. The equilibrium magnetization reflects the number of nuclei and thus the water content in tissue. Now, water content typically varies between 70 and 100% or 70 milliliters per 100 gram to 100 milliliters per 100 gram in our bodies. And so it is actually not a good source of contrast. To detect tissue contrast, we rely on differences between *T1* and *T2*. Those *T1* and *T2* differences, how they manifest themselves on the image contrast depends strongly on the experimental parameters. Examples that we have seen, are RF pulse timing or generally pulse timing, and another example here is the flip angle *alpha* by which the magnetization is tilted into the transverse plane.

Notes

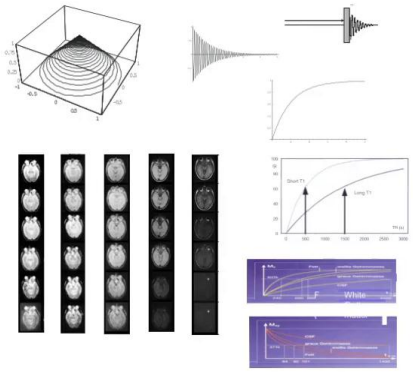
Summary

18m 17s

Credits



Felix Bloch
http://sjogodni.org.ua/publ/86-5



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So these are the key messages. This is what we have so far dealt with: magnetic resonance. Next week we will talk about spectroscopy and this makes a link between MRI and the chemical applications of magnetic resonance.

Notes

Summary

20m 08s