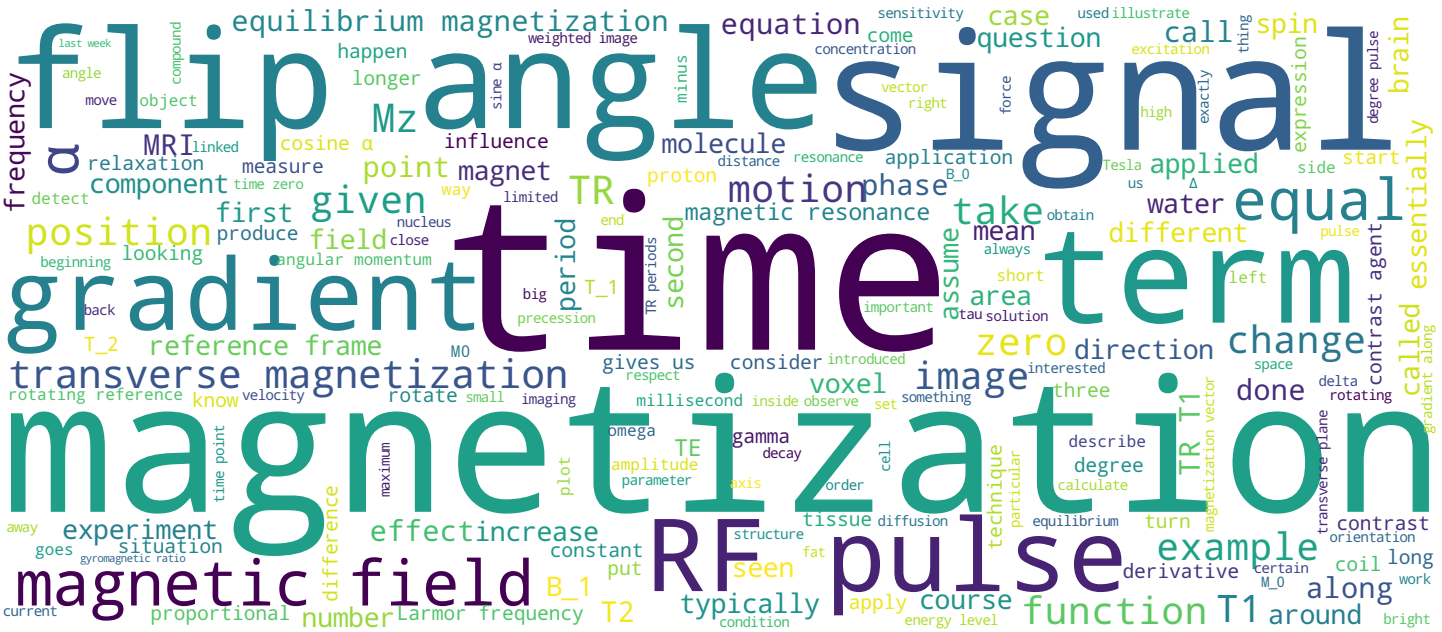
10.2 Optimal SNR

Fundamentals of Biomedical Imaging

Prof. Rolf Gruetter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10-2. When is SNR (sensitivity) optimal ?

Situation: RF pulses α applied every TR seconds n times
Question: M_{xy} (=signal) maximal ?
⇒ Calculate the optimum ***flip angle***
α=f(TR)

Now we have introduced magnetic resonance as a very insensitive technique. We are limited by the equilibrium magnetization, which is limited by the Boltzmann distribution, and we have clearly established that this technique is not suitable for detecting the signal from just a few molecules. But we have to get a decent signal to be able to measure it. And this is typically done by the approach of averaging. So multiple acquisitions are being acquired, be it for spectroscopy, for NMR, be it for MRI, this is generally the case. So what one does is one applies the same experiment over and over again, by averaging obtains a sensitivity, an SNR that is increased. So we have the situation in this case, that RF pulse is, or flip angle α, are applied every *TR* seconds, and they're applied this *n* times. And now the question is at what point, under what conditions is the transverse magnetization, which is proportional to the signal, maximal? So we have two parameters. We have the flip angle α and the repetition time *TR*. And what one now is interested is calculating the flip angle α as a function of *TR*. It is clear that if one waits so long that one has equilibrium magnetization, so 5 times *T1*, *e^-5* is equal to 0.01, 5 times *T1* gives us the equilibrium magnetization.

Notes

Summary

0m 04s

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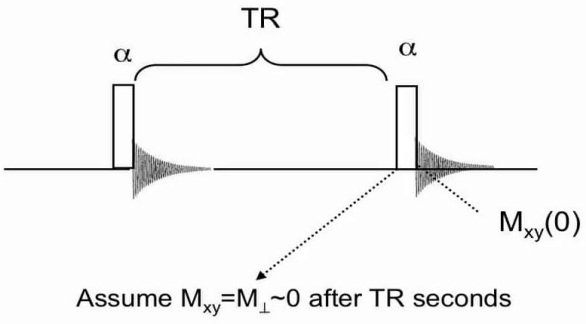

10-6

At that point, the optimal flip angle is 90 degrees. However, if our *TR* is very short, and we always apply a 90-degree flip angle, then we will not get any signal, we've seen already at the beginning. At the end of last week's course we have seen an example thereof where we get no signal. So this is the strategy here. We want to figure out what is the good choice of the flip angle as a function of *TR*. So here is the experiment. We have an RF pulse flip angle *α*. We detect the signal. We have the RF pulse *α* applied again. We detect the signal here again, and we'll call here this signal at the beginning of this second RF pulse, we'll call this *M* the transverse magnetization at time zero, that is our signal that we wish to optimize. This time here is equal to *TR*, we'll call this *TR*. And now we will assume that before the application of the RF pulse, any transverse magnetization has decayed away in the time of *TR* second. This is usually a good approximation and it shall suffice for the calculation here. So we'll assume that we have no transverse magnetization and we just have magnetization along z at this time point.

Notes

Summary

1m 43s

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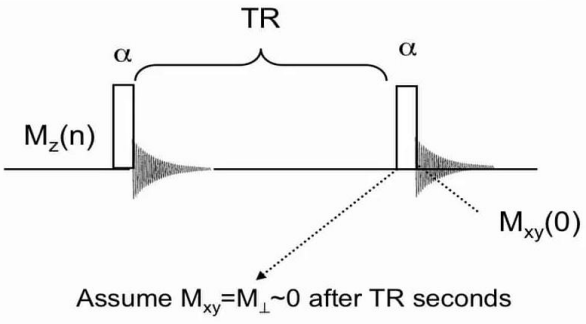

$$M_z(t) = M_0(1 - e^{-t/T_1}) + M_z(0)e^{-t/T_1}$$

Immediately after n[th] TR: $M_z = M_z(n)$

Assume $M_{xy}=M_{\perp}\sim 0$ after TR seconds

10-6

So our z magnetization has a function of time here, this is again, the general solution of the differential equation for the z component, is given by this term here, which describes return to equilibrium magnetization, and this term here, which describes longitudinal coherence. Now let's assume that we are doing a number of these *α* degree pulses. Can be a thousand, ten thousand, a few hundred, or so on. And after these RF pulses and the *nth TR*, so let's say, we have done 100 *TR* periods, 100 RF pulses followed by 100 *TR* periods, we have a z magnetization at that point and we'll call that *Mz(n)*. So at 100 or 1000, doesn't matter what the *n* is, it has to be sufficiently big. In practice, it's actually 3 to 5, is already sufficient, but we'll see that later. So we'll call this magnetization here, just before this RF pulse after *n* TR periods, *n* applications of *α* RF pulses and TR periods, *Mz(n)*. Now a flip angle *α* is applied and so the *Mz* at time zero, that is the z magnetization here at this time, that would be here, is given by *Mz* before the RF pulse times *cosine α*. We've already seen that relationship, that's the magnetization that's left after a flip angle of *α*. Now during this period, we'll consider the *T1* recovery.

Notes

Summary

3m 07s

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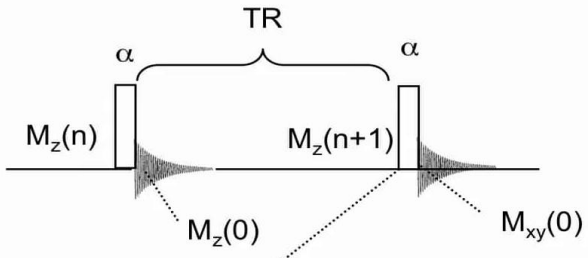

$$M_z(t) = M_0(1 - e^{-t/T_1}) + M_z(0)e^{-t/T_1}$$

Immediately after n[th] TR: $M_z = M_z(n)$

Assume $M_{xy}=M_\perp \sim 0$ after TR seconds

After RF Flip α $M_z(0) = M_z(n)\cos\alpha$

After T_1 recovery $M_z(n+1) = M_0 - \left[M_0 - M_z(n)\cos\alpha\right]e^{-TR/T_1} = M_0\left[1 - e^{-TR/T_1}\right] + M_z(n)\cos\alpha e^{-TR/T_1}$

In equilibrium (steady-state condition):
$M_z(n+1)=M_z(n)=M_z$

10-6

So *Mz(n + 1)* is equal to this term here. That's essentially using this equation here, by substituting *Mz(n)* here, times *cosine α*, as *Mz(0)*. So we'll take this term, substituting this equation, we have the terms rearranged here so that the exponential term is on this side. And so this *Mz(n + 1)*, that's the z magnetization just before the second RF pulse here in this scheme, and I've regrouped the terms here and we have *M0*, the return to equilibrium, and here's the longitudinal coherence term. Now we're going to apply a steady-state condition. We're going to assume that a sufficient number, *small n*, of RF pulses and TR periods, has been applied so that the *Mz(n + 1)* and the *Mz(n)*, are equal. We can assume that this happens at some point. At some point it will always recover to the same z magnetization and then we will call this magnetization equals *Mz(n + 1)* and we'll just call it *Mz*. So we have now here *Mz(n + 1)*, here we have *Mz(n)*. After *n* periods we'll now assume that these two are equal, so we'll set them equal, I've done this here, and we'll put the terms that contain *Mz* on the left side, and we're left with the terms that contain *M0* on the right side.

Notes

Summary

4m 46s

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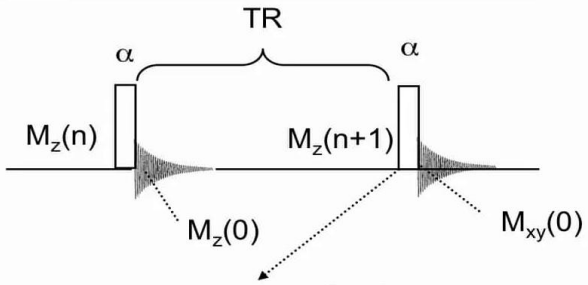

$$M_z(t) = M_0(1 - e^{-t/T_1}) + M_z(0)e^{-t/T_1}$$

Immediately after nth TR: $M_z = M_z(n)$

Assume $M_{xy}=M_\perp \sim 0$ after TR seconds

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In equilibrium (steady-state condition):
$M_z(n+1)=M_z(n)=M_z$

$$M_z\left(1 - \cos\alpha \cdot e^{-TR/T_1}\right) = M_0\left(1 - e^{-TR/T_1}\right)$$

$$\Rightarrow M_z = \frac{M_0\left(1 - e^{-TR/T_1}\right)}{1 - \cos\alpha \cdot e^{-TR/T_1}}$$

Equilibrium *transverse* Magnetization:
$M_{xy}(0)=M_z\sin\alpha$

$$M_{xy} = \frac{M_0\left(1 - e^{-TR/T_1}\right)\sin\alpha}{1 - \cos\alpha \cdot e^{-TR/T_1}}$$

10-6

And so we can now express the z magnetization as a function of the equilibrium magnetization, as a function of the time between RF pulses, the repetition time, and as a function of the flip angle here, the *cosine α* term. Now this is not yet our signal. This is just the equilibrium z magnetization that we have just before this RF pulse here, so we are now interested in the equilibrium transverse magnetization because it gives us the signal that we obtain. So the transverse magnetization here, after this RF pulse is equal to the z magnetization before the RF pulse times *sine α*. We'll set this in and now we have the equilibrium magnetization equation here. So that's our transverse magnetization at equilibrium is proportional to *M0*, is influenced by the flip angle, and influenced by the *TR/T1*. And so we have now *TR/T1* on one side, and the flip angle *α* that influences this equilibrium magnetization. This is a fairly fundamental equation here that describes the influence of the signal as a function of flip angle and time between RF pulses.

Notes

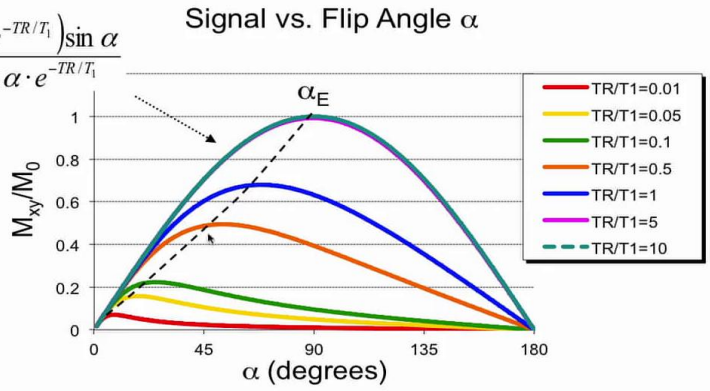
Summary

6m 22s

How does the signal depend on TR, T_1 and flip angle ?

Ernst Angle α_E

$$M_{xy} = \frac{M_0\left(1 - e^{-TR/T_1}\right)\sin\alpha}{1 - \cos\alpha \cdot e^{-TR/T_1}}$$



10-7

So now comes the question, this was a fairly complicated equation, and the question is, how does a signal depend on TR, time between RF pulses, T1, the T1 relaxation, and the flip angle *α?* So if you plot the signal versus flip angle *α*, and here I'm showing this equation again that we just derived on the previous slide. This is the same expression here, and now let's look at what we observe. So if *TR/T1* is 10, *e^-10* is essentially zero. So we have this is *M0 sine α* divided by 1. So we have *M0 sine α*, so our signal describes very nicely a sinusoidal function here. And the maximum is at 90 degrees, not surprisingly. Even for *TR/T1* equals 5, here *e^-5* is 0.01, we get essentially the same behavior. So this is not surprising. Then as we see, if we plot this function here for different *TR/T1's*, as *TR/T1* gets smaller and smaller, then we can see that the maximum shifts further and further away from 90 degrees. So here's this function plotted and I've tried to, with this dashed line, indicate where the maximum of the flip angle is. So at what choice of flip angle do we get the maximum transverse magnetization?

Notes

Summary

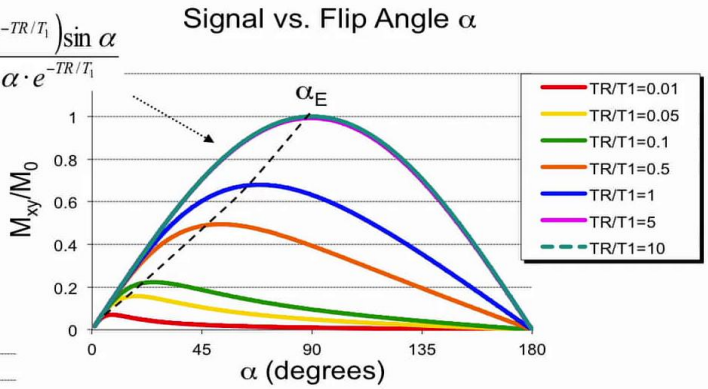
7m 45s

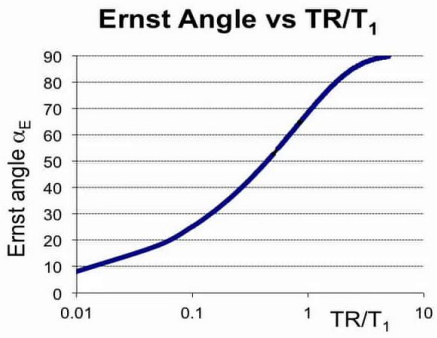
How does the signal depend on TR, T_1 and flip angle ?

Ernst Angle α_E

Richard Ernst
Physical Chemist
1991

$$M_{xy} = \frac{M_0\left(1-e^{-TR/T_1}\right)\sin\alpha}{1-\cos\alpha\cdot e^{-TR/T_1}}$$





M_{xy} (signal) → maximum at α_E

$dM_{xy}/d\alpha=0$: $\cos\alpha_E = e^{-TR/T_1}$

Where α_E = Ernst Angle

10-7

And here it's expressed normalized to *M0*. So if you look now at this flip angle, so we want to calculate the transverse magnetization, which is our signal, at what flip angle *α* is the maximum obtained? And this is done by taking the derivative of the transverse magnetization of this expression, with respect to the flip angle, and setting it to zero. I will spare you the math of doing this. This is a simple cumbersome exercise of multiple calculations. We'll just take the derivative of this term here, with respect to *α* and I'll give you the solution here, which is *cosine α* is equal *e^-TR/T1*, where *α* is called the *Ernst Angle*. So if we plot this now, we have the *TR/T1*, and notice here the logarithmic scale, the angle here in degrees. We have at a very high *TR/T1*, we have the not surprising 90 degrees, and as the *TR/T1* gets shorter and shorter, this asymptotically goes to zero. Of course, if *TR/T1* goes to zero, then *cosine α* goes to 1, that is a zero-degree flip angle. And this optimal flip angle is called the Ernst Angle. It's been named after Richard Ernst, a physical chemist from Zurich, who received for his work, among also this work, he received the Nobel Prize in Chemistry in 1991.

Notes

Summary

9m 09s

Credits

Richard Ernst
http://www.quotationof.com/bio/richard-ernst.html

Notes

Summary

10m 45s