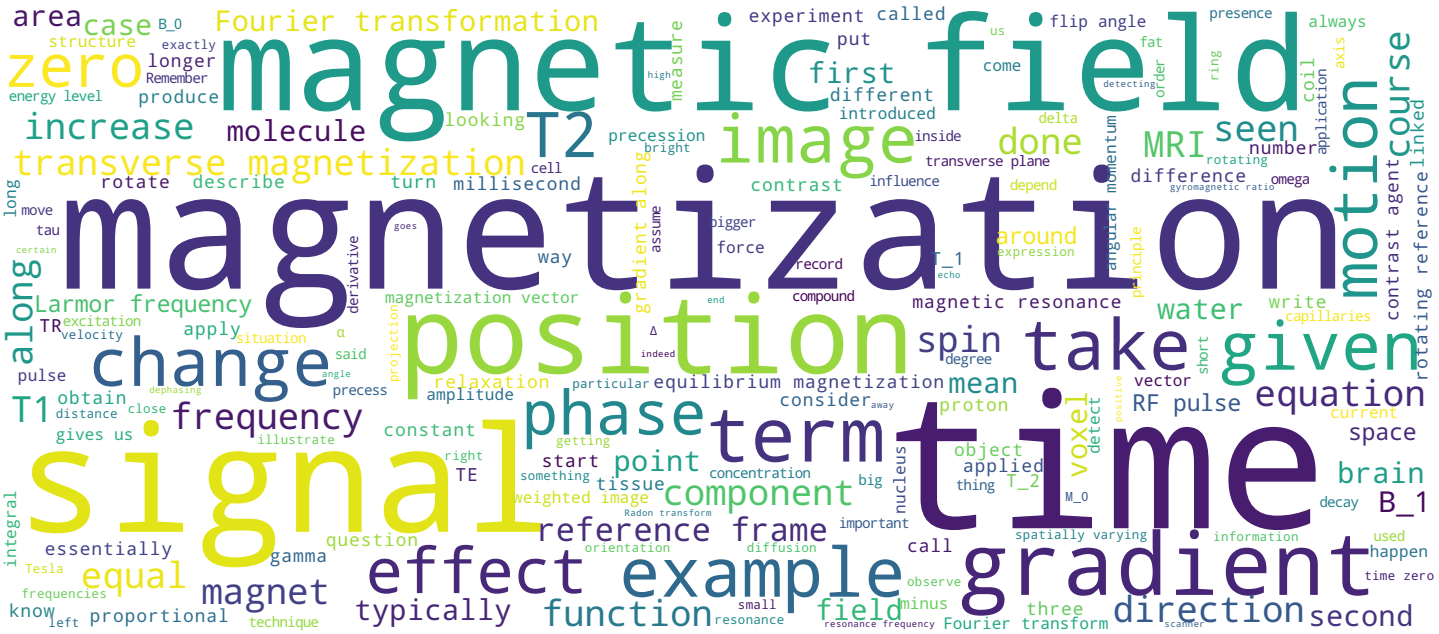
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

# 11.2 Basic principle of spatial encoding

## Fundamentals of Biomedical Imaging

Prof. Rolf Gruetter

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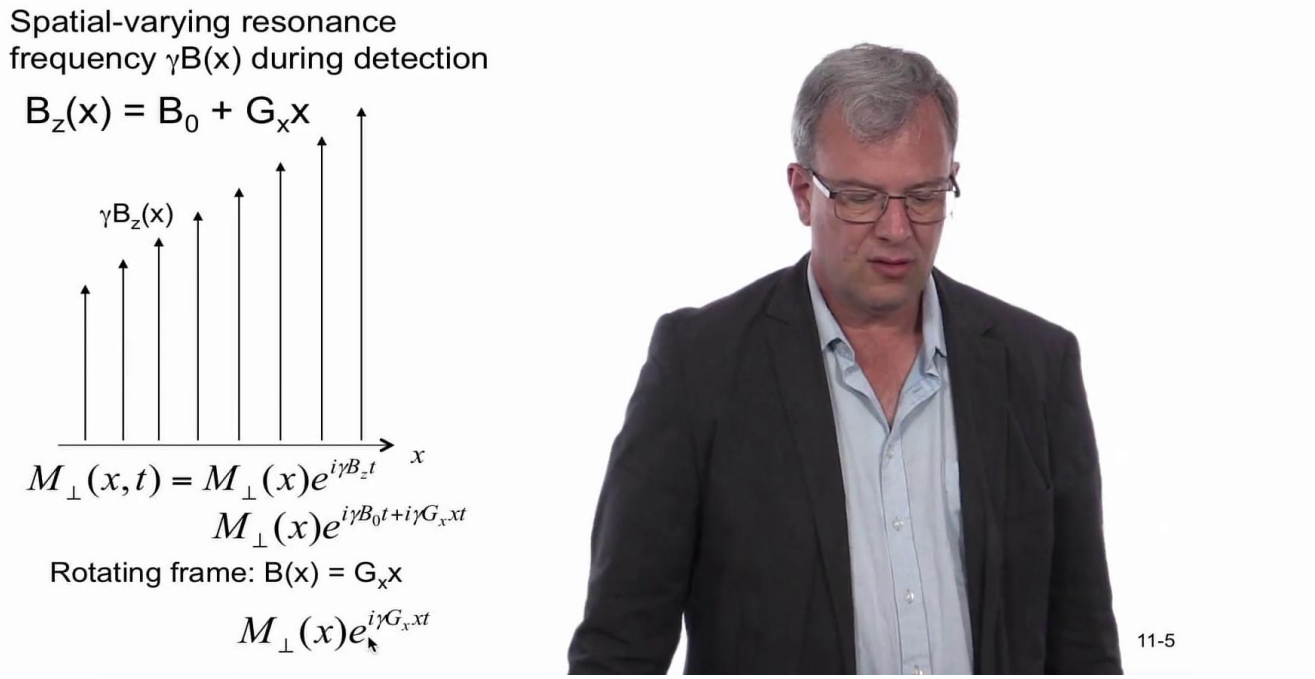
## 11-2. What is the basic principle of encoding spatial information ?

frequency encoding - 1D example




So now the question is what is the basic principle of encoding spatial information? We have during detection, in the presence of a gradient, we have a spatially varying resonance frequency. So the *z* field is equal to *B0 + Gx* times *x*. And so if we plot this as a function of *x*, we have a magnetic field here, and here. It increases, et cetera, so that's *γ Bz(x)*. Our transverse magnetization now will precess, it will be given by the transverse magnetization at position *x* times *e^(i γ Bz)* times *t*. So this is in general terms, this is how this is written here. We have the *B0* term and we have the gradient term. Now what we'll do is we'll put ourselves into the rotating frame to describe that so that we don't need to consider the *γ B0*. Remember, *γ B0* is much bigger than *γ Gx* times *x*, so we have now just a magnetic field that varies with *x*, so it's the *z* component. So now we can write the precession of the transverse magnetization, given by the transverse magnetization at position *x* times *e^(i γ Gx.x)* times time. So if we look here at this position, our magnetization vector will precess here with a certain precessional frequency.

Notes

Summary

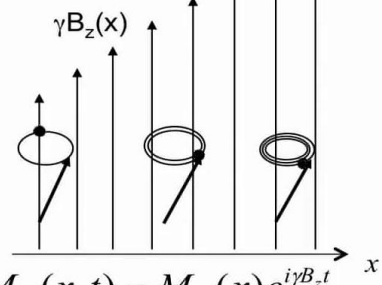
0m 04s

## 11-2. What is the basic principle of encoding spatial information ?

frequency encoding - 1D example

Spatial-varying resonance frequency $\gamma B(x)$ during detection

$B_z(x) = B_0 + G_x x$




$$M_\perp(x,t) = M_\perp(x)e^{i\gamma B_z t}$$

$$M_\perp(x)e^{i\gamma B_0 t + i\gamma G_x x t}$$

Rotating frame: $B(x) = G_x x$

$$M_\perp(x)e^{i\gamma G_x x t}$$

Detected signal = sum of all precessing magnetization:

$$\underbrace{S(t) \propto \int_{object} M_\perp(x,0)e^{i\gamma G_x x t}\,dx}$$

What does this resemble ?

$$S(t) \propto \int_{object} M_\perp(\omega,0)e^{i\omega t}\,d\omega$$

11-5

If you go at this position, the magnetic field is stronger so here, it's precessing faster. And finally, at this position we have an even stronger magnetic field and here the magnetization precesses faster. So this is what we are detecting. We have precessed different precessional frequencies and they depend on the position of the magnetization. Now the signal that one detects is given by the sum of all precessing magnetization. So what we now have to do is to get the signal as a function of time, we have to integrate over the object, here along *x*, the magnetization at time zero at a position *x*, times the evolution term here, so that's the phase evolution. And here we're basically saying that the real part is, for example, corresponds to the *x* component of the transverse magnetization at the imaginary, the *y* component. So we'll do this integration over position. So what does this equation resemble? Have you seen a similar equation already? Does it ring a bell? How about this equation, does that ring a bell? You have most likely come across this type of equation, and that's an inverse Fourier transform.

Notes

Summary

1m 30s

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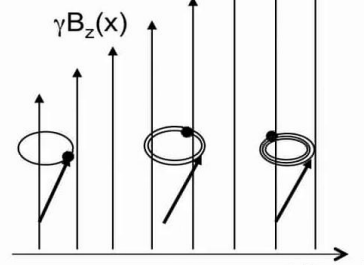
frequency encoding - 1D example

Spatial-varying resonance frequency γB(x) during detection

$B_z(x) = B_0 + G_x x$




$$M_\perp(x,t) = M_\perp(x)e^{i\gamma B_z t}$$

$$M_\perp(x)e^{i\gamma B_0 t + i\gamma G_x x t}$$

Rotating frame: B(x) = $G_x x$

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$$S(t) \propto \int_{object} M_\perp(\omega,0)e^{i\omega t}\,d\omega$$

**= Inverse Fourier Transformation !**

$$S(k) \propto \int_{object} M_\perp(x,0)e^{ikx}\,dx$$

11-5

Essentially looking at a signal that we are detecting, is the inverse Fourier transform of the magnetization as a function of frequency, and here frequency and position, they are linked through this equation, linearly linked through that. So this is a Fourier transformation of the object's magnetization that we detect. So we can rewrite this equation that we have here. I'll write it as S(k), which is proportional to the integral over the object magnetization again at time zero times *e^(i k x) dx*. And here we're using the term *k* for k-space, this is the conjugate or pair variable to position *x*, so here *x* and *k* are linked through Fourier transformation, they are a Fourier pair, like in this example, *ω* and *t* are the Fourier pair.

Notes

Summary

3m 03s

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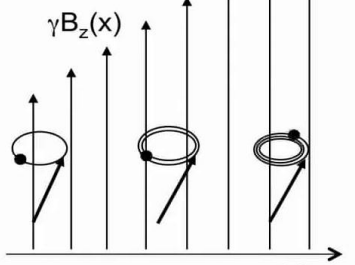


$$M_\perp(x,t) = M_\perp(x)e^{i\gamma B_z t}$$

$$M_\perp(x)e^{i\gamma B_0 t + i\gamma G_x x t}$$

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What does this resemble ?

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**= Inverse Fourier Transformation !**

$$S(k) \propto \int_{object} M_\perp(x,0)e^{ikx}dx$$

FT of S(t) (or S(k)) → M(x)

Reconstruction as in CT (in principle)

For 2D object:

$$M(x) = \int M(x,y)dy$$

=Radon Transform 11-5

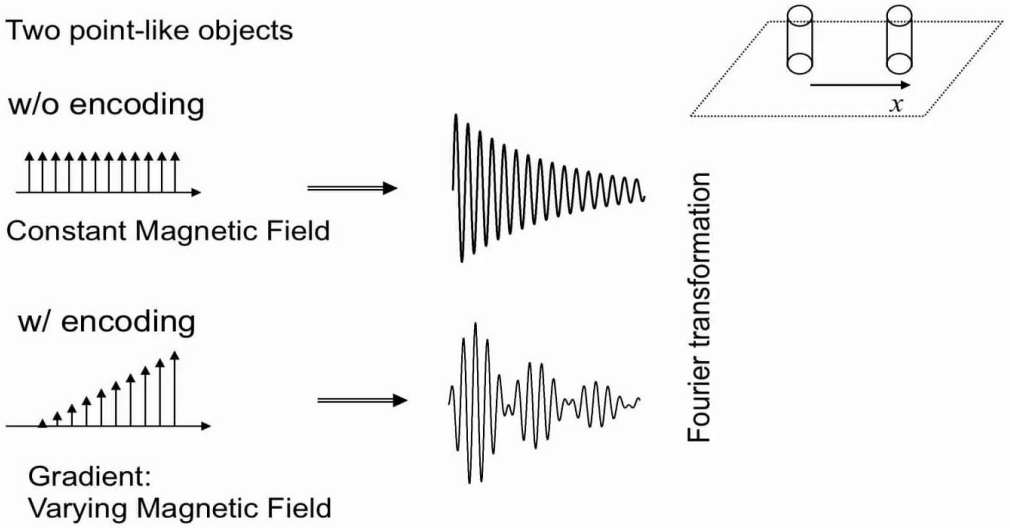
So if we take now the Fourier transformation of the signal that we record with time, or, we can also say as, that we record as a function of k-space, this gives us the magnetization along *x*. If we write this now for a 2D object, then the magnetization at a given position *x* that we observe, so we obtain Fourier transformation of this, we obtain *M(x)*, and this *M(x)*, if we look at the two-dimensional object, is then given by the integral of the magnetization in *(x, y)* along *y*, because we cannot distinguish, in the *y* direction, the position if we apply a gradient along *x*. Now this equation, does that look familiar? You have certainly seen it in the first part of this course. We have introduced it for computed tomography and this is a Radon transform. So in principle I could stop here and say, "Well, once," like I've said before, "once we see that the signal corresponds to that of a Radon transform, we know we can reconstruct an image with that projection." So in principle the reconstruction can be done here, as in computed tomography. However, this is not the general case and I will not stop here the lecture for this week, and we will go into the way that the images are reconstructed.

Notes

Summary

4m 01s

# Spatial Encoding: 1D example




11-6

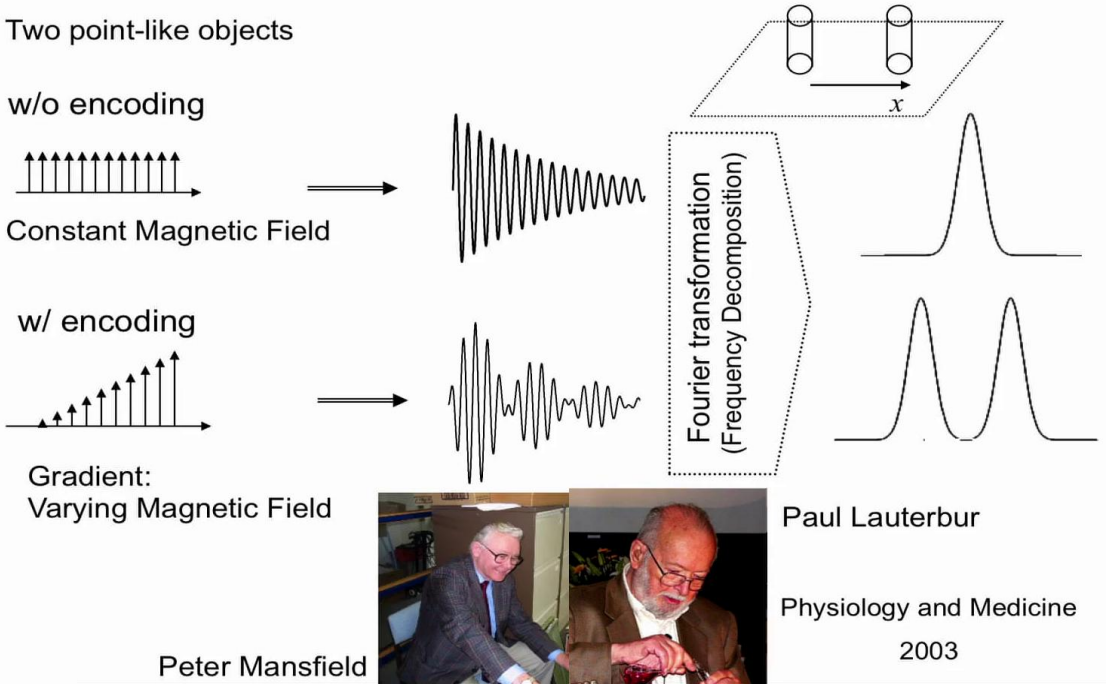
So first I want to come with a 1D example. And we'll take here, the example of two point-like objects, these could be capillaries, so they are very small dimension. Here they are. We'll give the position along *x* here, and without encoding, that is the magnetic field which is plotted here, is constant in position. We have a constant magnetic field in position, and we'll record a nice free induction decaying signal. The frequency is the Larmor frequency, and the decay is T2. Now, if we apply encoding, then the magnetic field varies linearly with space, and we'll take here the encoding along *x*, so we have applied a gradient along *x*, so it's a spatially varying magnetic field, and this produces now, an oscillation of two frequencies. We have two capillaries, two positions, we'll assume they're point-like so they're two very well-defined frequencies. This is gives a beating pattern of two frequencies, and that typically looks like this. Now if you imagine you have a more complex object, it is not so obvious just to see from the signal that's recorded, what is being observed. So what is done now is we apply the Fourier transformation, that is we decompose the signal into its frequency components.

Notes

Summary

5m 29s

# Spatial Encoding: 1D example




That's what Fourier transformation does for you. So we'll take the signal, Fourier transform it. Here we get a single resonance. We have a constant magnetic field, we have no information on the position, and in the presence of a spatially varying magnetic field, we're getting two peaks, two resonance positions two frequencies according to the position of these two capillaries in this example. And so these projections that we obtain, like I've said before, we could, in principle, reconstruct the image using back projection. And this was indeed, in the very first experiment done by Paul Lauterbur, a physical chemist, in the early '70s, this is actually how he produced the first images. He used MRI with back projection. The other person that is noteworthy to mention, also in the early '70s, that took a different approach, that was a physicist, Sir Peter Mansfield. Paul Lauterbur and Sir Peter Mansfield, they shared the Nobel Prize in Physiology and Medicine in 2003, for their groundbreaking work in making imaging possible with magnetic resonance.

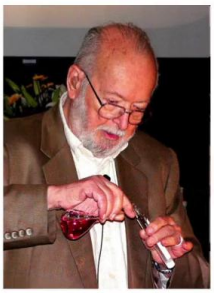
Notes

Summary

6m 48s

# Credits

**Sir Peter Mansfield (photo credit : University of Nottingham, School of Physics and Astronomy)**
http://physicsworld.com/cws/article/news/2003/oct/06/physicist-shares-nobel-prize-for-medicine



**Paul Lauterbur Nobel, Nobel Prize winners in Physiology or Medicine, 2003**
http://physicsworld.com/cws/article/news/2003/oct/06/physicist-shares-nobel-prize-for-medicine

Notes

Summary

8m 03s