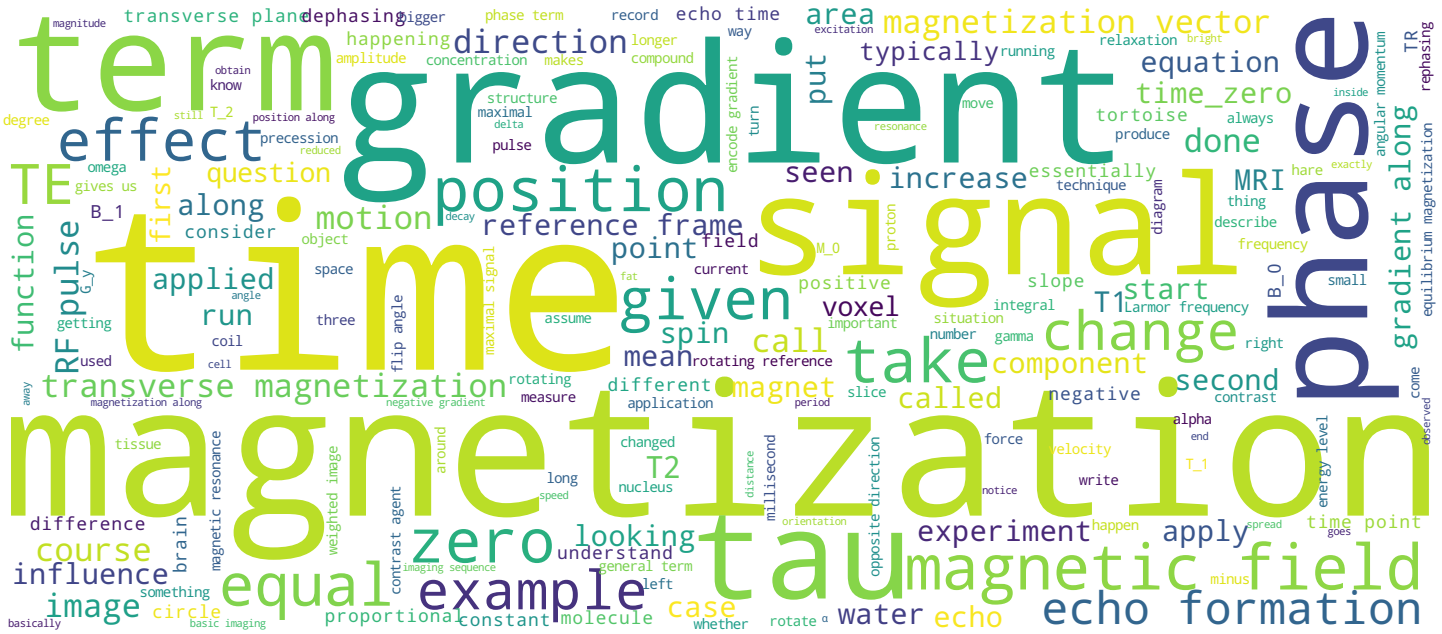
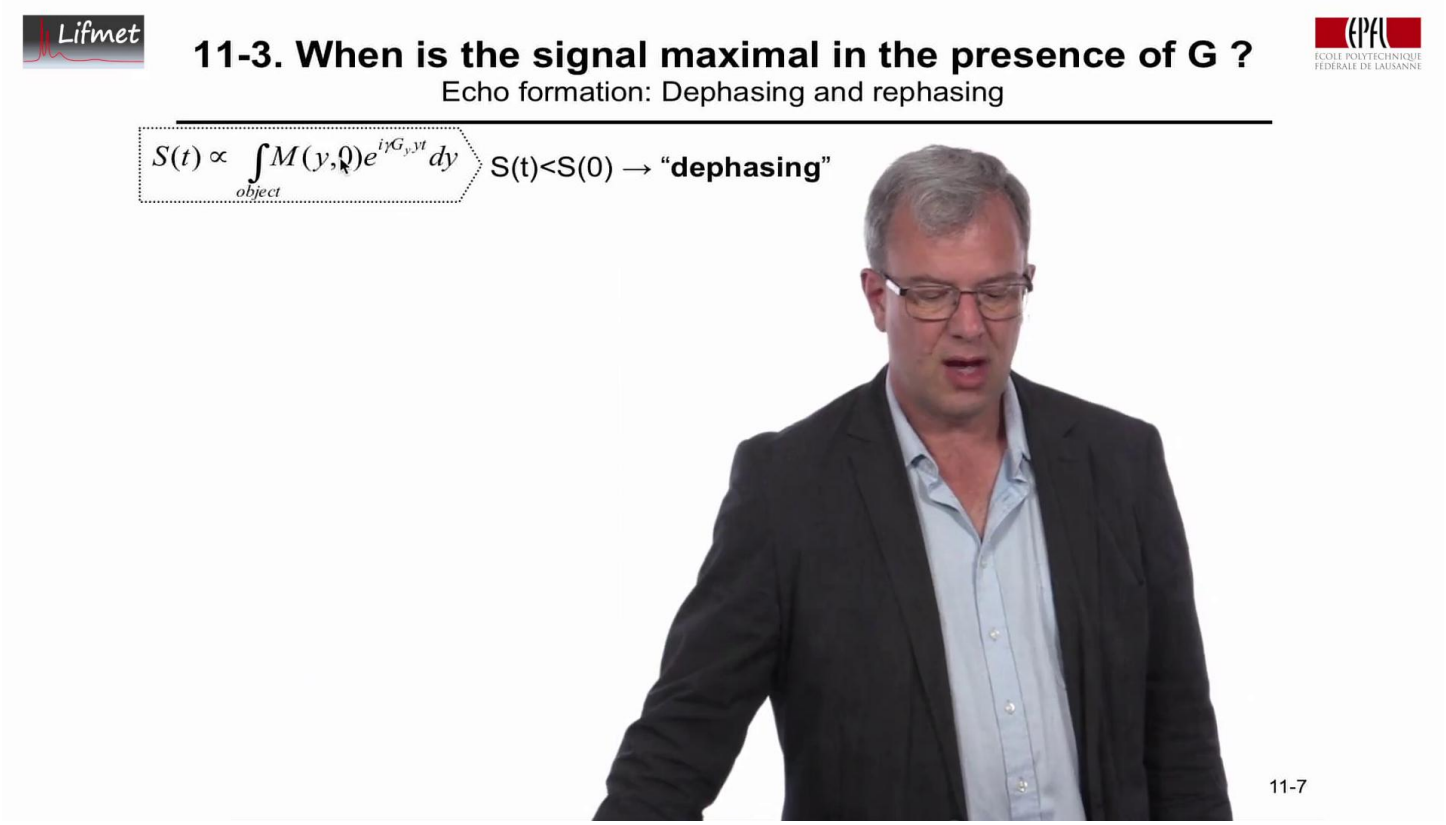
11.3 Echo formation

Fundamentals of Biomedical Imaging

Prof. Rolf Gruetter


magnetization
time
signal
gradient
term
phase
tau
magnetic field
echo formation
position
effect
equal
example
zero
looking
experiment
apply
transverse magnetization
change
take
given
spin
mean
call
voxel
applied
run
motion
reference frame
point
direction
area
magnetization vector
typically
equation
time_zero
done
seen
increase
MRI
start
T1
second
component
called
negative
magnet
different
water
echo
T2
nucleus
influence
course
image
circle
difference
brain
magnetic resonance
weighted image
contrast agent
proportional
constant
molecule
case
rotate
whether
left
general term
imaging sequence
basic imaging
basically
something
tissue
around
long
changed
speed
distance
millisecond
opposite direction
orientation
force
velocity
write
notice
energy level
time point
spread
goes
alpha
end
T_1
minus
contrast
period
application
measure
rotating
negative gradient
flip angle
maximal signal
assume
three
space
object
decay
consider
along
question
first
TE
pulse
RF
function
used
cell
coil
getting
angle
away
magnetization along
gives us
omega
B_1
G_y
degree
still
T_2
position along
precession
reduced
obtain
know
magnitude
transverse plane
dephasing
bigger
phase term
record
echo time
happening
amplitude
concentration
makes
longer
compound
way
excitation
relaxation
bright
running
put
encode gradient
turn
structure
maximal
delta
pulse
resonance
move
tortoise
always
produce
inside
angular momentum
TR
rephasing
gradient along
equilibrium magnetization
come
observed
exactly
hare
thing
describe
frequency
small
B_0
diagram
technique
essentially
slope
integral
situation
number
gamma
right
Larmor frequency
contrast
current
fat
M_0
proton
field
positive
rotating reference
slice
called
negative
second
component
change
take
given
spin
mean
call
voxel
T1
start
MRI
increase
seen
done
produce
tortoise
always

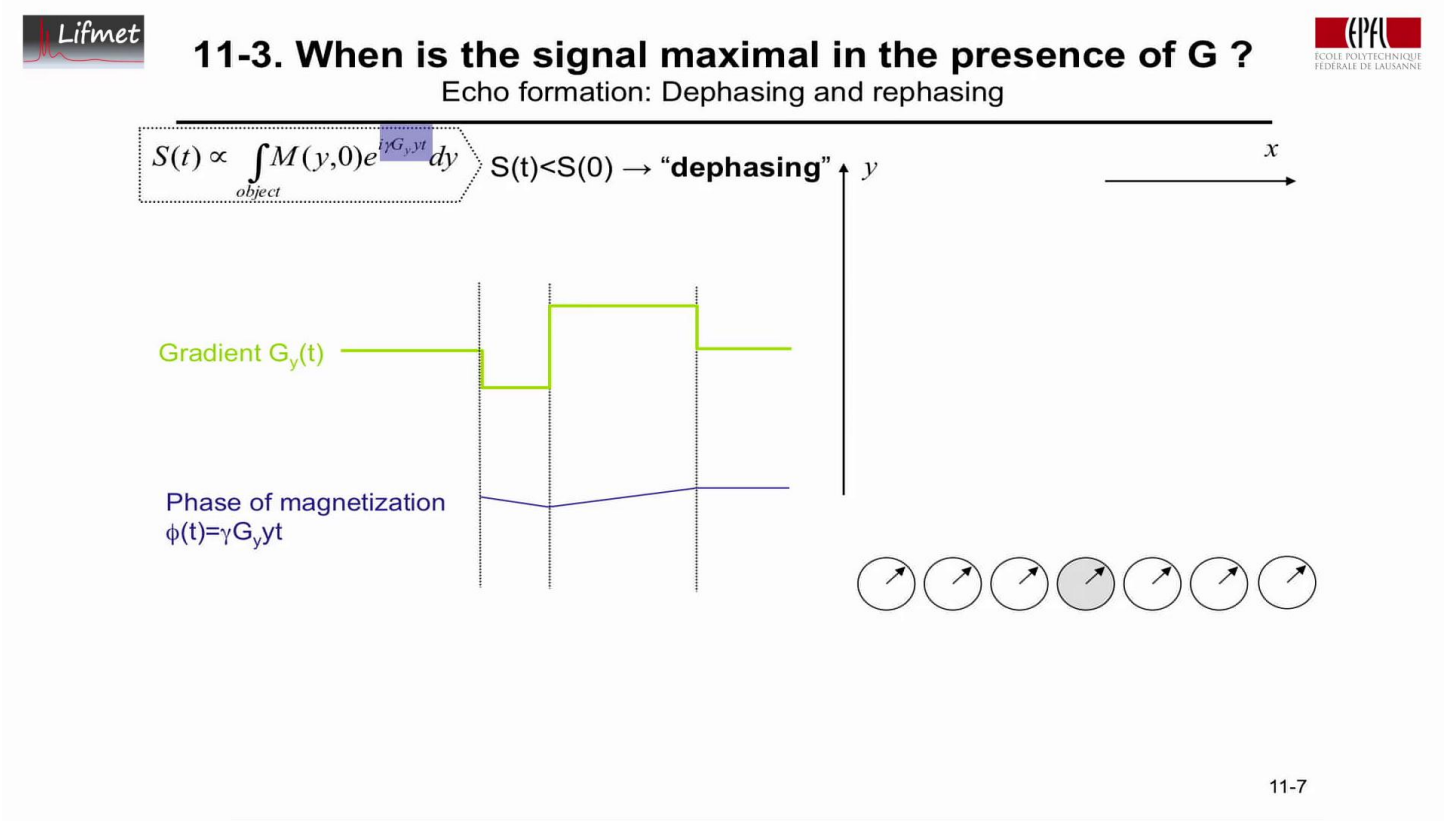


To understand the basic imaging sequence-- to understand how basic information is spatially encoded with MRI-- we need to deal with the question on what is the effect of the gradient on the signal, and how do we make, for example, the signal maximal under the influence of a gradient? Let's first look at the signal as a function of time. So we have an object, we have a gradient along *y* here-- now I'll use *y*, just because it pleases me; we don't always have gradient along *x*-- so we have the magnetization along *y*. This is the equivalent equation that we had two slides earlier. If we now look at this equation then we can see that for-- this is our complex numbers, so again, the real part is *x*, the imaginary part represents the *y* component of the magnetization or of the signal-- and if we look at this equation here then we can see that at *time_zero*, this term is 1 and then we are getting the signal as a function of time is the magnetization, it's constant. However, if we apply the gradient and we look at times bigger than zero, then we have, increasingly, one magnetization vector corresponding to a positive *y* position, moving in one direction, the other magnetization rotating in the other direction or in a complex plane.

Notes

Summary

0m 04s

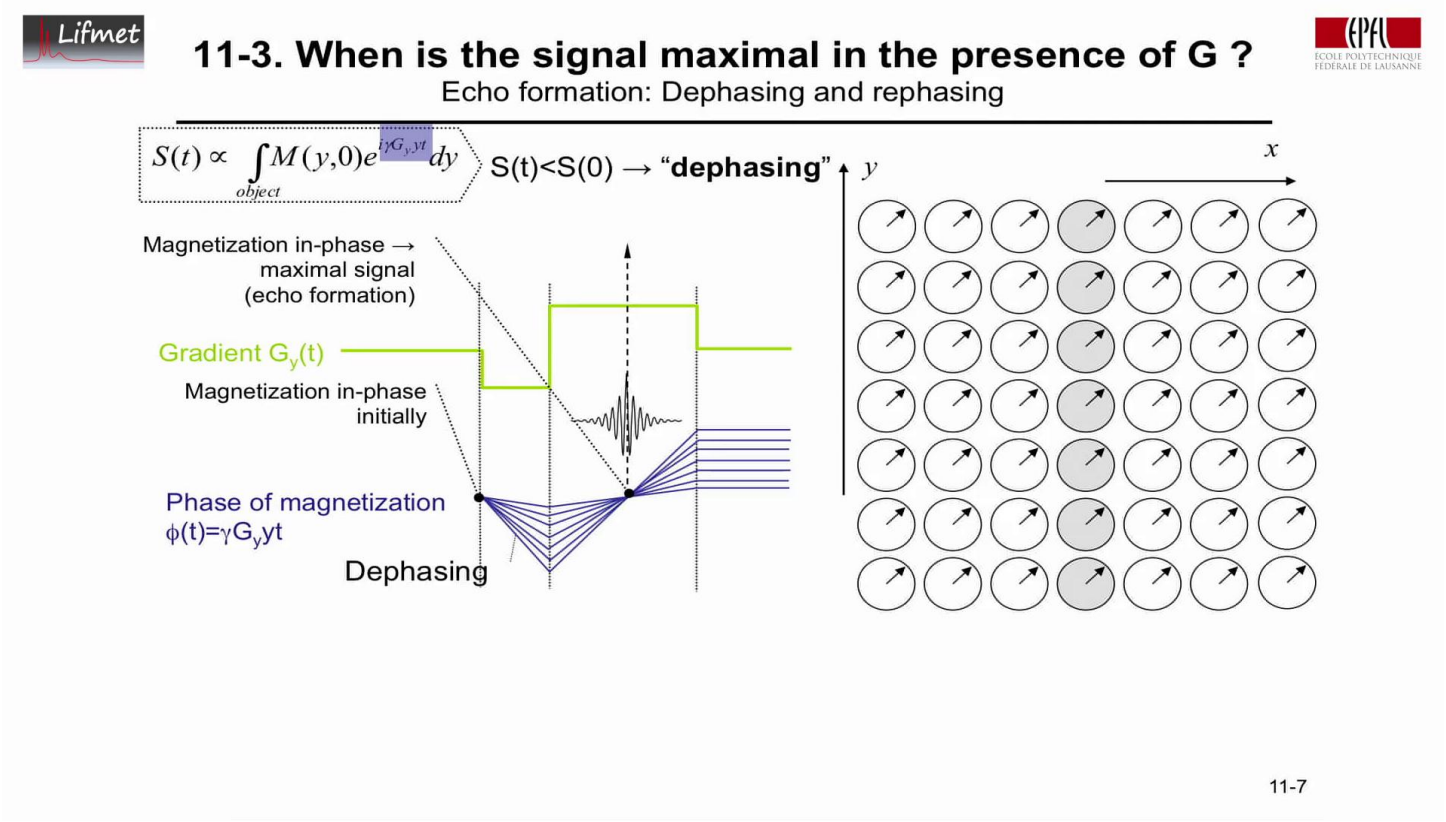


Now the complex numbers according to their position along *y* will have a different phase and they will no longer add co-linearly. And this is called *dephasing*. So our signal is maximal at the beginning. So let's look at a gradient-- and I will here take a gradient along *y*. I'll have it at zero, then I'll have it at a negative value, and then I'll put it to the same value that is positive here. We're going to consider these three time points here where we switch the current in the gradient coil and we're now looking at the phase of the magnetization-- that is this term here, over time, starting here at *time_zero*, where we have this magnetization here in phase. So we're looking at the phase: that's this term here just before the *i*. So if we take one position along *y* then we are getting a phase that evolves this way, I will plot here... In terms of position, we have *y* and *x*. So we'll have this position-- we're looking at one position in *y* so we'll take this position, *y* is small, and here we have all the magnetization at *time_zero* are in phase so they're all co-linear.

Notes

Summary

1m 25s

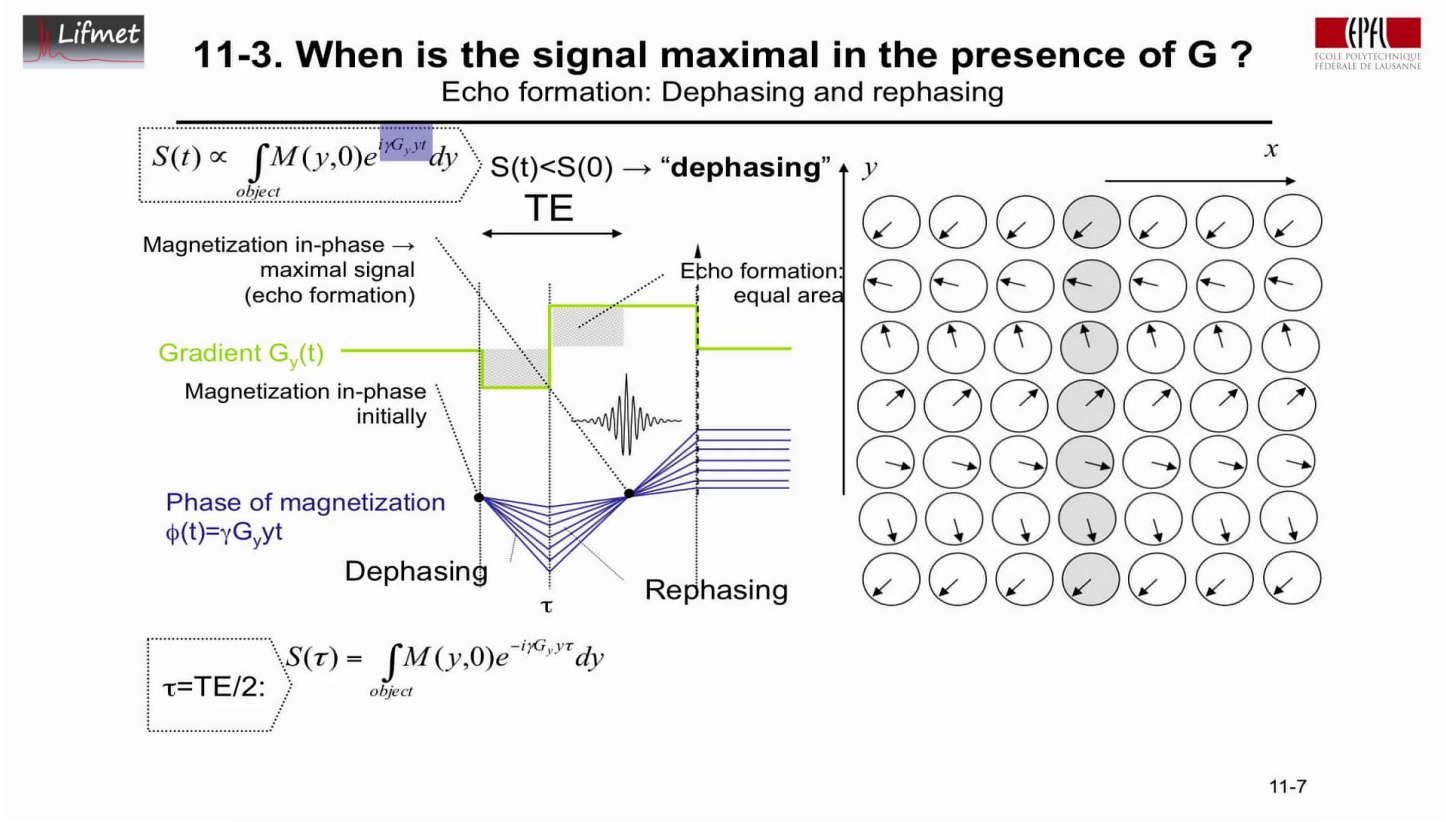


Now we'll take the next position in *y* which is a bit further away, so *G_y y* is bigger, so the phase is bigger, the slope is bigger here and here the slope is inverted because we invert the *sin* of *G_y*. For the third position, we have this-- and we'll just continue equidistantly, that's the fourth, and that will be the fifth here, the sixth, this position, and the seventh, at this position. And now what we see is-- because here the slope is proportional to position, and here the slope is also proportional to position but the slope here is equal to the negative of that slope, so since this time here equals to this time, and so we'll have a point here where all the magnetizations are again in phase-- they have a phase of zero, and this creates a maximal signal, and that's what we call *echo formation* and what you would typically see in the computer-- the digitized signal that's observed is something like this; it's an echo. That's what is called an echo. So if we look at the phases, we have dephasing. Dephasing means that the magnetization vector's spread out in the transverse plane, they no longer act co-linearly so signal decreases, and then, when they all come together again, we call this phase of the evolution *rephasing*.

Notes

Summary

2m 49s

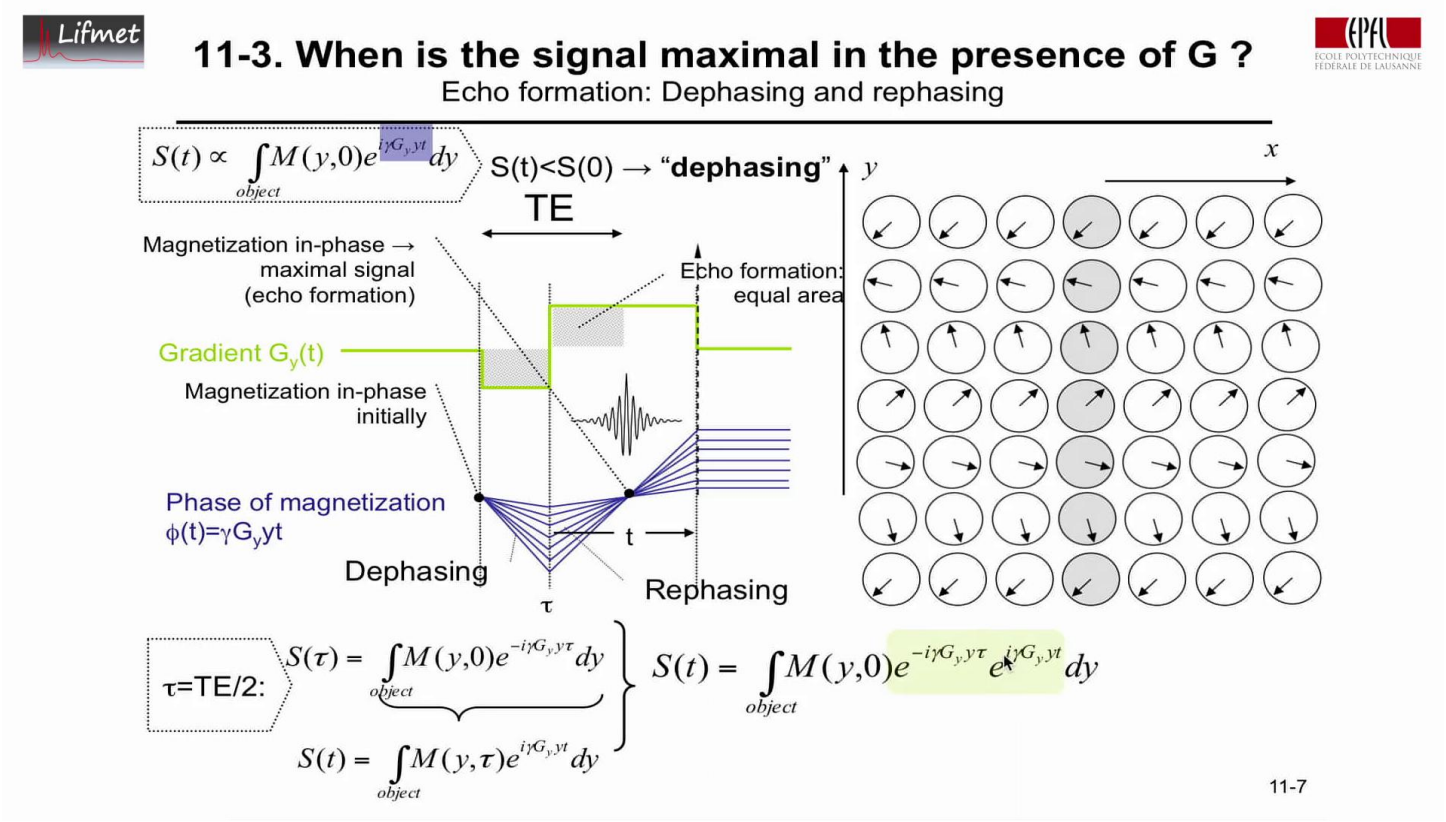


And so, if we look, what happens over time, then we have rephasing, here we have echo, all is in phase, now we have dephasing-- you can see they're not in phase-- now they are reversing their orientation and they are in phase again. So we have dephasing-- the magnetization vector's spread out, then they come together again-- here they're again together. And this is happening if the area of the gradient here equals the area of the gradient here or if the integral sum of all the gradient over time here equals to zero, in more general terms. So if we have equal area, positive and negative, then we have echo formation. And this time here, from here to here, is typically called the *time of echo*, or *echo time*, *TE*. And if we go on afterwards, then of course we have dephasing and now the vectors are all dephased and they are no longer producing a measurable signal after a certain amount of time. So if we call this time here *tau* then from here to here *tau* is equal to *TE/2* in our particular example. So we can write the magnetization at this time point is equal to the transverse magnetization at *time_zero* times the phase term and this is now being done by integrating along *y*.

Notes

Summary

4m 17s

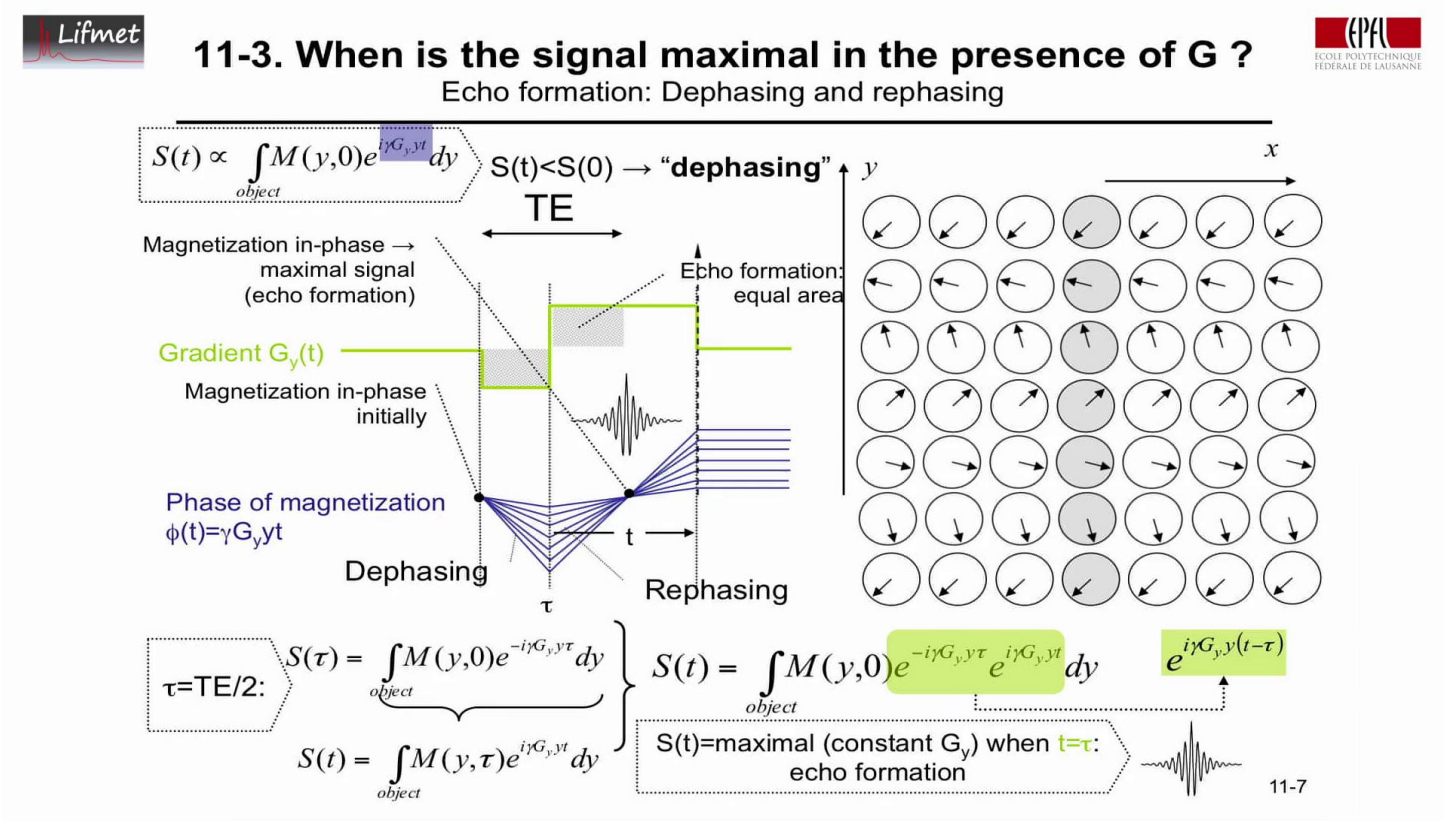


We have the minus here because the gradient is negative, so *G_y* is the magnitude of the gradient. The gradient during time *t*-- time *t* is this time here... that denotes this time here-- the signal during time *t* is proportional to the transverse magnetization: that's the sum of the transverse magnetization over the object. So we sum along *dy*, it's the magnetization along *y* at *time tau* times this term here. Here it's subpositive because we have changed the *sin* of *G_y*. And now we'll put this together-- that is, we'll insert this into here-- the magnetization at the time *tau* is proportional to this. We'll put this in here, and in the equations I have just assumed that they are equal-- this is really a proportionality; then we have the magnetization at *time_zero* here-- the transverse magnetization times these two terms. So this term we can understand: that's the magnetization rate after the RF pulse and here, what about these terms? So I will look at these two terms and I will re-write them into the exponent and so we now have *e* to the *i gamma G_y y* times *t* minus *tau*.

Notes

Summary

5m 43s



And so this means-- this term here, if you look at this integral, with the same argument that we had here-- if this phase term is zero, then we have maximal signal and that is happening when *t* equals *tau* and so we have echo formation and this is given at this point. So this is what is being observed. Okay, so this is a graphical, mathematical demonstration that, in a simple experiment, where we have a gradient that's negative and then we put it to positive, that after two times this period *tau* we echo formation how it occurs. Now this is not easy to understand and one has to really think about it, how to visualize that, and I'm going to now try to take a different approach.

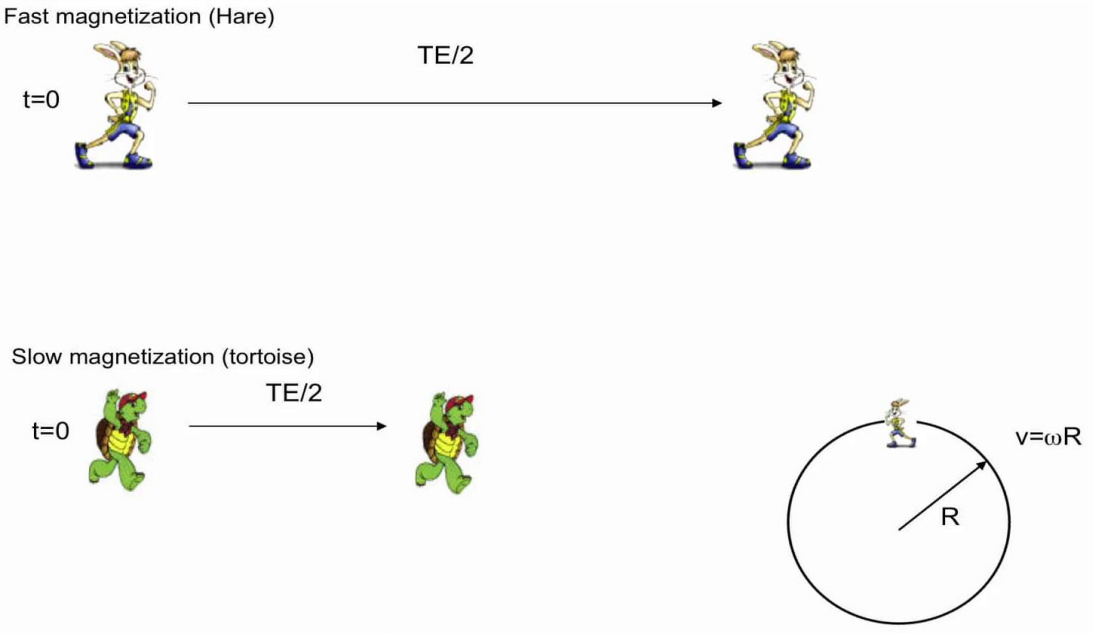
Notes

Summary

7m 00s

Echo formation

Vectors tips (or the hare and tortoise) running on a circle



So let's try to look at echo formation here, under a different viewpoint. And what we're going to do here is we're going to suppose that our magnetization vectors are represented by-- the position of our magnetization vectors are represented by a tortoise here, and a hare. And we're going to make them run on a circle because they're magnetization vectors-- the tortoise and the hare represent the tips of magnetization. And they're going to be running at a certain velocity-- that is, they are going to have-- this velocity is given by angle of velocity times radius of the circle. So this is like track and field-- instead of having an oval, we're running in a circle. So, at *time_zero* they're both at the same position. Now we'll have the hare running fast in one direction, and the tortoise running slow in the other direction. So that is the precession of fast and slow magnetization. At *time_zero* they're both up here, so they're here, and now we'll have them run for a time of *TE/2* seconds, or milliseconds if you want. Since the hare runs faster, he will have run a bigger distance than the tortoise, which is much slower.

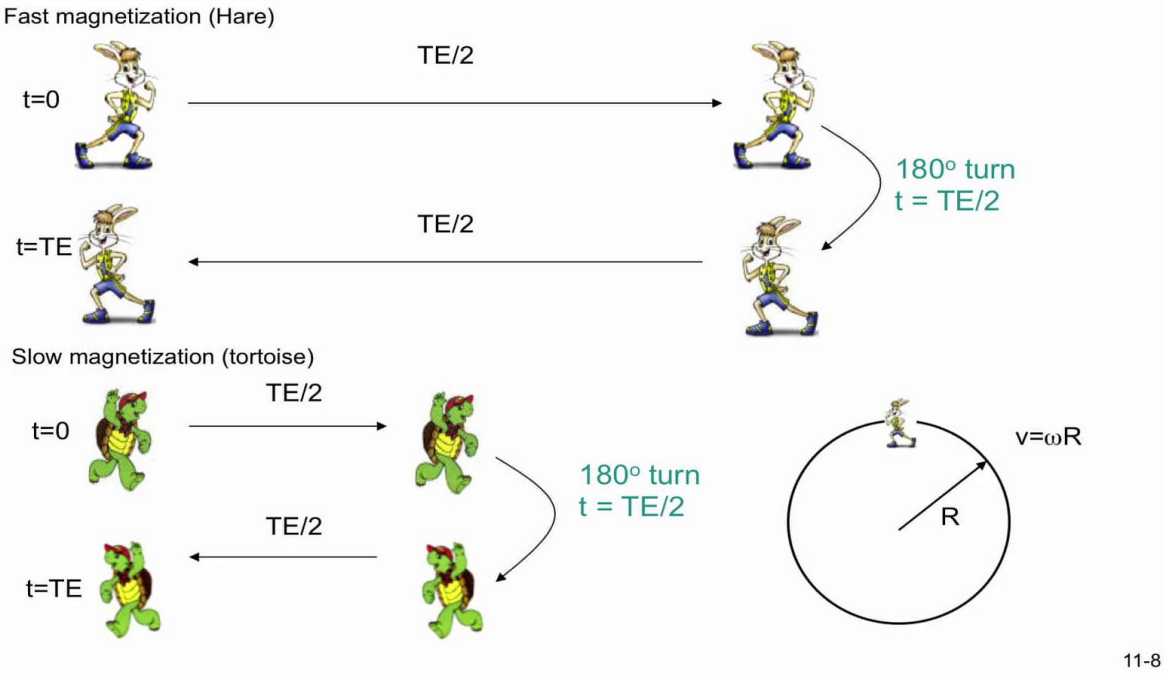
Notes

Summary

7m 56s

Echo formation

Vectors tips (or the hare and tortoise) running on a circle

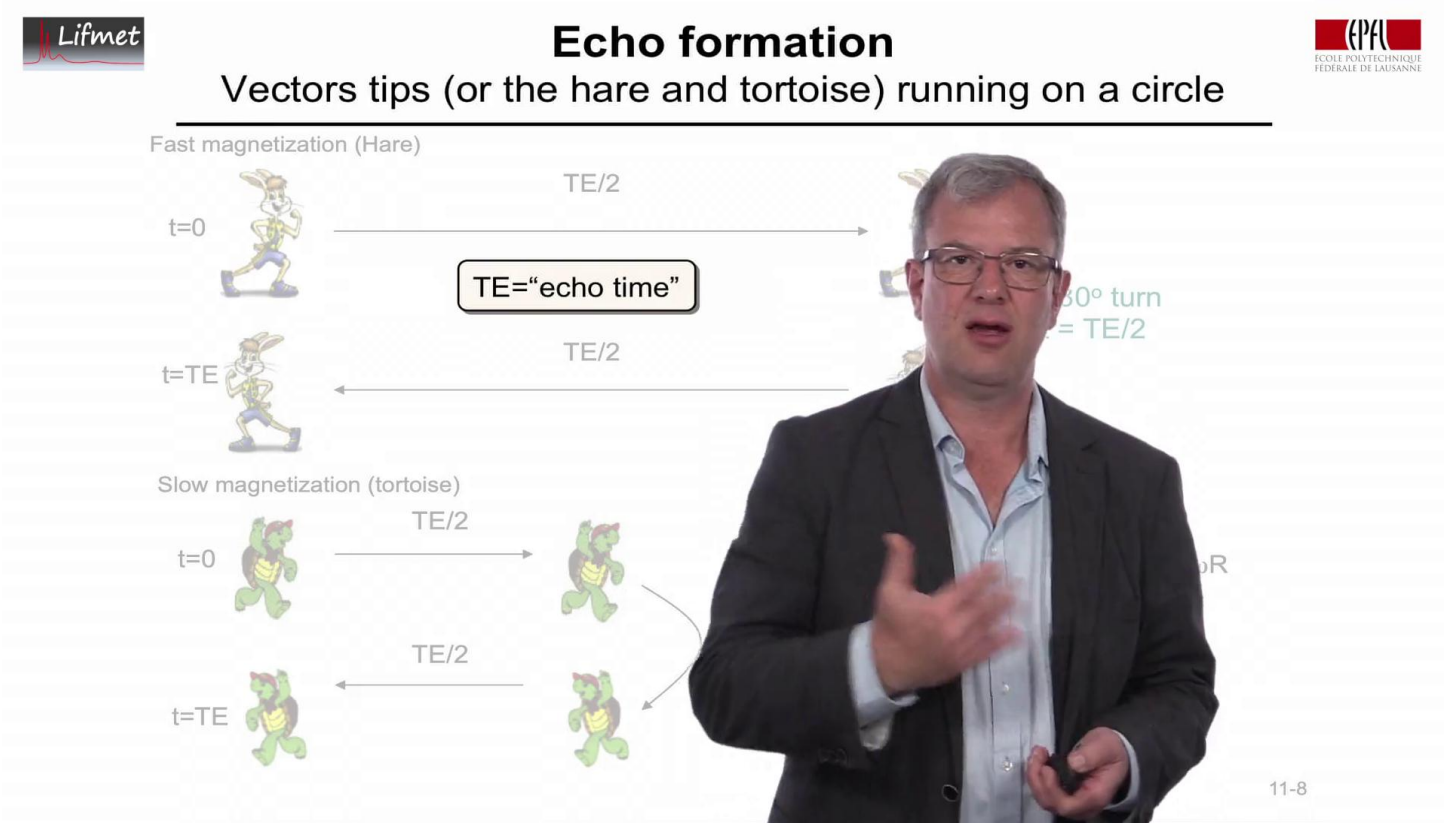


And here, for simplicity, we'll assume that the tortoise is running in the opposite direction as the hare-- so tortoise has run until here, the hare is until here, they have run in this direction. Now, after *TE/2* seconds we give the command to reverse the direction-- so that is a 180-degree turn. We're telling them, instead of running as you've been running, now you run in the opposite direction, still on the circle, but with the same velocity. So this is a 180-degree turn, turn around, and now we're running back to-- we're letting them run again for the same amount of time. Now, the distance that they're going to run on the circle, for the same amount of time, since they are running at the same speed, the distance traveled is going to be the same but in the opposite direction. So, after time *TE*-- after *TE* seconds-- they're both going to end up again at the same position, like this. Okay, so is this now clearer? So we have had a tortoise slowly running on a circle and a hare running fast on the circle. We've said these represent the tips of the magnetization vectors, of two magnetization vectors.

Notes

Summary

9m 11s



Now the argument that I've made here is assuming two different velocities for tortoise and hare-- so two different precessional frequencies-- that's the *omega* here. Now we can imagine what we have in our organ of interest, we have literally hundreds of thousands of different tortoises and hares and they're all running at different speeds and different directions, etc. I have made no assumption whatsoever on the specific circumstances on the speed of the tortoise or the hare, so this is valid for any two pairs that we compare that they will arrive after time *TE* at the same original position that they had. So this is, in general, the case. The only thing that we've imposed is, they're always running at the same speed and after half of the time, they reverse the direction. That's the influence of the gradient. And so the time *TE* is also called the *echo time* and that's what we will refer to very often from now on. So I hope this is now a bit clearer, how echo formation is done under the influence of a switched gradient where we change the polarity. There we changed-- with the polarity of the gradient we changed the direction in which the magnetization vector, at a given position, is precessing.

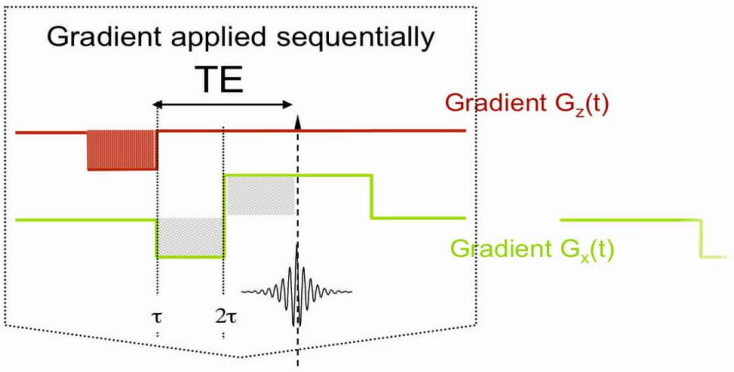
Notes

Summary

10m 29s

Is it important when a gradient is applied ?

gradient applied at different time has the same effect on magnetization phase



11-9

To understand better the diagrams I am going to be showing after this slide I have to address one question, and that is: Is it important when exactly a gradient is being applied? What is its influence when the gradient is being applied, on the signal? We'll consider here an experiment where we have a gradient along *x*, it goes from negative to positive-- doesn't actually matter exactly what we're doing here, but we'll take the same example as we had before, so if these two areas are equal in magnitude, then we will have, at this time point, we will have echo formation. And this time here we call the echo time, *TE*. And we will put a note here, the start of this time counting here as the time *tau*. And this time here will be *2 tau*. Now I will take a gradient along *z* and I will switch it just before the green gradient along *x*, so this gradient here is applied just before, it's turned off at the time *tau*. This is its effect on the phase. So that is what we call the sequential application of two different gradient orientations. Now we'll take the same experiment again.

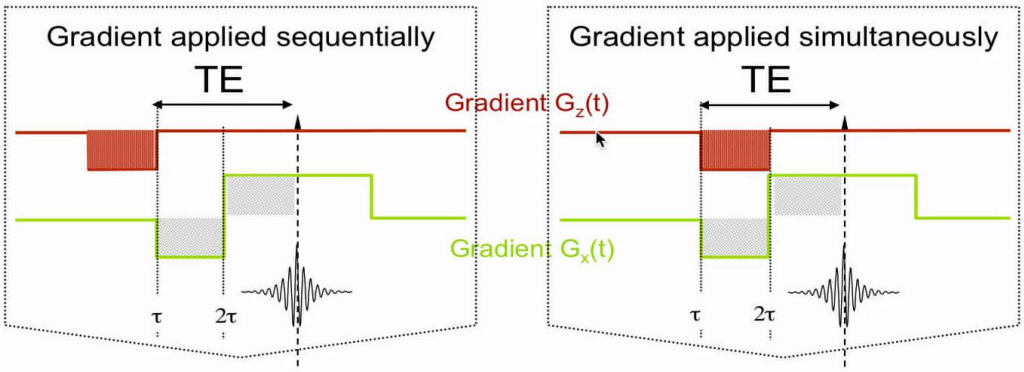
Notes

Summary

12m 02s

Is it important when a gradient is applied ?

gradient applied at different time has the same effect on magnetization phase



Question: Is there a difference in effect on echo ?

$$M(\tau) = M(0)e^{-i\gamma G_z z \tau} \qquad M(\tau) = M(0)$$

11-9

We have the gradient along *x*, we have echo formation, and we'll apply now this same gradient along *z* but we'll apply it at the same time as we apply the negative lobe of the *x* gradient. So everything is identical except the timing for the red gradient. Here's again the echo time, *TE*, and this is what we call the simultaneous application of the gradient. And now the question is: Is there going to be a difference on the echo that we record here? Does it make a difference? Can we tell from the signal that we record here, whether this gradient's been applied here, or whether it's been applied here? So let's look at the magnetization in the left example after time *tau*-- it's given by the magnetization at *time_zero*, times *e* to the *gamma G_z z* times *tau*-- this is the effect of this gradient here-- and on the right side, the magnetization at the time *tau* is still the magnetization at *time_zero*, we have not done anything to the phase of the magnetization. This is always done in the rotating reference frame so we don't consider the effect of *B_0*. It makes it easier to write less terms.

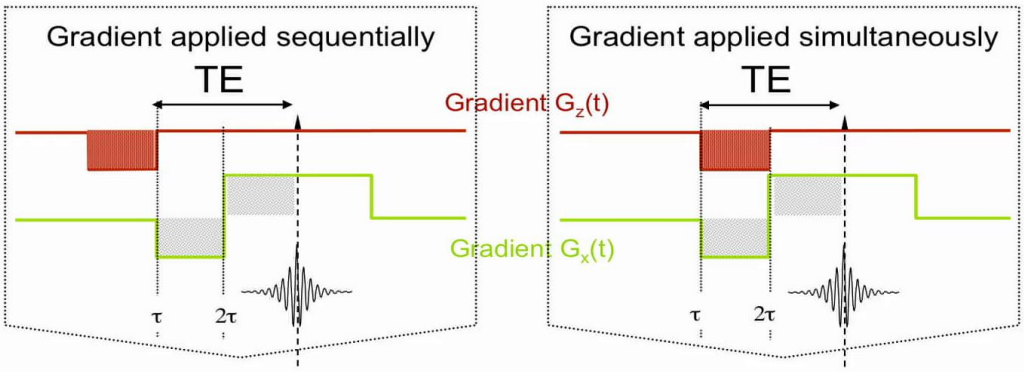
Notes

Summary

13m 22s

Is it important when a gradient is applied ?

gradient applied at different time has the same effect on magnetization phase



Question: Is there a difference in effect on echo ?

$$M(\tau) = M(0)e^{-i\gamma G_z z\tau}$$

$$M(2\tau) = M(\tau)e^{-i\gamma G_x x\tau} = M(0)e^{-i\gamma G_x x\tau}e^{-i\gamma G_z z\tau}$$

$$M(\tau) = M(0) \qquad \gamma(\vec{G}\cdot\vec{r})\tau$$

$$M(2\tau) = M(\tau)e^{-i\gamma(G_x x+G_z z)\tau} = M(0)e^{-i\gamma G_x x\tau}e^{-i\gamma G_z z\tau}$$

11-9

At times *2 tau* we have the magnetization at times *tau* here, that we start with-- we're always talking about transverse magnetization of course-- times *e* to the *minus gamma G_x* times *x* times *tau*. We have here the time here, from here to here, is *tau*. So that's the magnetization at times *2 tau*. And we can now substitute *M* of *tau*-- magnetizaton of time *tau*-- into this equation here, we get this term that is over here. On the right-hand side, the magnetization that is now recorded times *2 tau* is the magnetization at times *tau* times *e* to the *gamma G_x x* plus *G_z z* times *tau*. And what we have used here is just a general term-- that's the magnetic field-- the *B_0*-- in the presence of a gradient, now we take the *B_0* out because we're in a rotating reference frame, but the variation of the magnetic field in space is given by *gamma* times the scalar product of the gradient vector times position. And here the gradient vector has components along *x* and *z*. So we can rewrite this now: *M_z m* of *tau* is *M_0*.

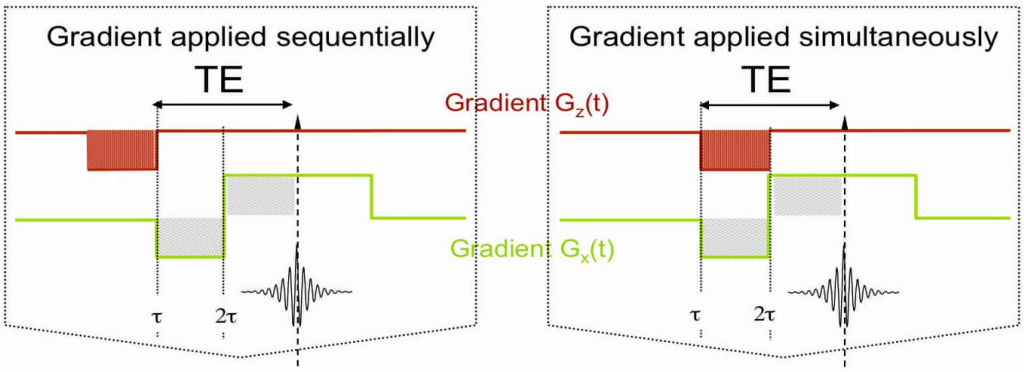
Notes

Summary

14m 33s

Is it important when a gradient is applied ?

gradient applied at different time has the same effect on magnetization phase



Question: Is there a difference in effect on echo ?

$$M(\tau) = M(0)e^{-i\gamma G_z z\tau}$$

$$M(2\tau) = M(\tau)e^{-i\gamma G_x x\tau} = M(0)e^{-i\gamma G_x x\tau}e^{-i\gamma G_z z\tau}$$

$$M(\tau) = M(0) \qquad \gamma(\vec{G}\cdot\vec{r})\tau$$

$$M(2\tau) = M(\tau)e^{-i\gamma(G_x x+G_z z)\tau} = M(0)e^{-i\gamma G_x x\tau}e^{-i\gamma G_z z\tau}$$

Application of two orthogonal gradients simultaneously or sequentially generates the same phase for M_{xy}

11-9

We'll write these terms here and we'll split them according to *x* and *z*. And now what we will notice is this term here, and this term here. Upon inspection, we notice that these two terms are identical. So the magnetization *M* of *zero* in both cases is the same-- we start out here, we don't know yet what's happening so the magnetization at that point has to be the same. The conclusion here is the application of two gradients that are orthogonal-- say *x* and *z* in this particular example, could be any two directions-- simultaneously or sequentially, doesn't matter, it produces the same phase for the transverse magnetization subsequently. So what this means, basically, is that we have the magnetization described here or described here. These two diagrams, in terms of what's happening to the magnetization-- the transverse magnetization-- to its phase, are identical. So if we have difficulty understanding the diagram on the right we can always suppose that this, on the left, is happening.

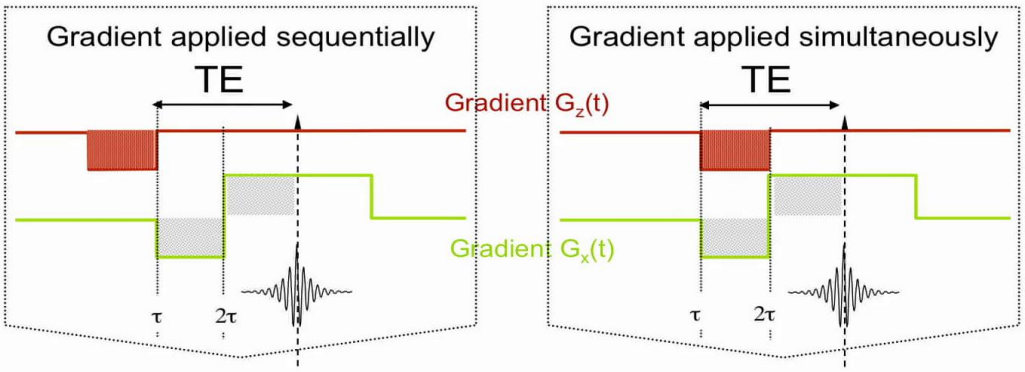
Notes

Summary

15m 42s

Is it important when a gradient is applied ?

gradient applied at different time has the same effect on magnetization phase



Question: Is there a difference in effect on echo ?

$$M(\tau) = M(0)e^{-i\gamma G_z z\tau}$$

$$M(2\tau) = M(\tau)e^{-i\gamma G_x x\tau} = M(0)e^{-i\gamma G_x x\tau}e^{-i\gamma G_z z\tau}$$

Application of two orthogonal gradients simultaneously or sequentially generates the same phase for M_{xy}

$$M(\tau) = M(0) \qquad \gamma(\vec{G}\cdot\vec{r})\tau$$

$$M(2\tau) = M(\tau)e^{-i\gamma(G_x x+G_z z)\tau} = M(0)e^{-i\gamma G_x x\tau}e^{-i\gamma G_z z\tau}$$

$$M(TE) = M(2\tau)e^{i\gamma G_x xTE/2}$$

$$M(TE) = M(0)e^{-i\gamma(G_x x+G_z z)\tau}e^{i\gamma G_x xTE/2} = M(0)e^{-i\gamma G_z z\tau}$$

11-9

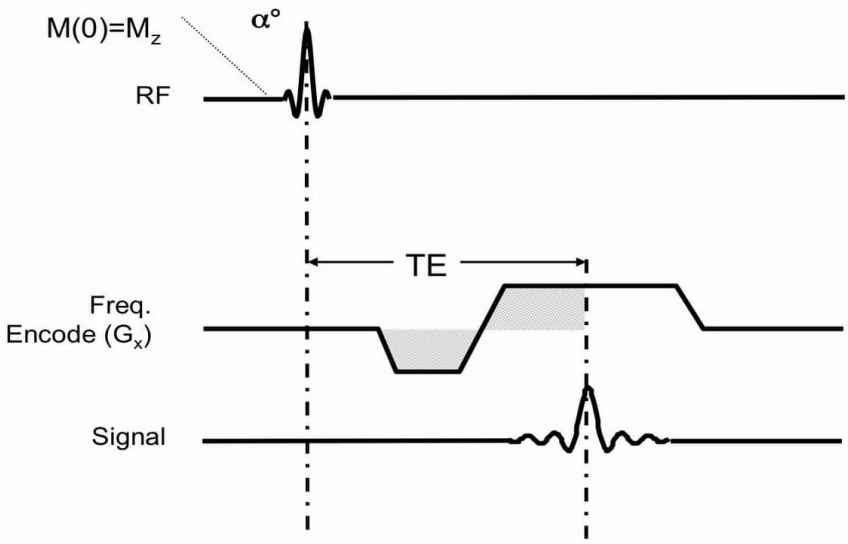
So at time *TE*, the magnetization is given by the magnetization at *2 tau* times this term here, and we can write this now, we may put in the magnetization of *2 tau* into this term here, and then we get the magnetization of *time_zero* times *e* to the *i gamma G_z* times *z* times *tau*. The effect of *x* of course, the gradient is nullified exactly at *TE*, so we only have the influence of the *z* gradient. Okay, so this is just to bear in mind if you see the diagram where the gradients are in two different directions, are applied simultaneously. If this creates conceptual problems you can always think of that you have much more space to draw them sequentially like on the left-hand side if that is easier to understand. I didn't do it because it makes the display of what is to come very crowded and very difficult to read.

Notes

Summary

16m 51s

What are the basic elements of the Gradient echo sequence ?



11-10

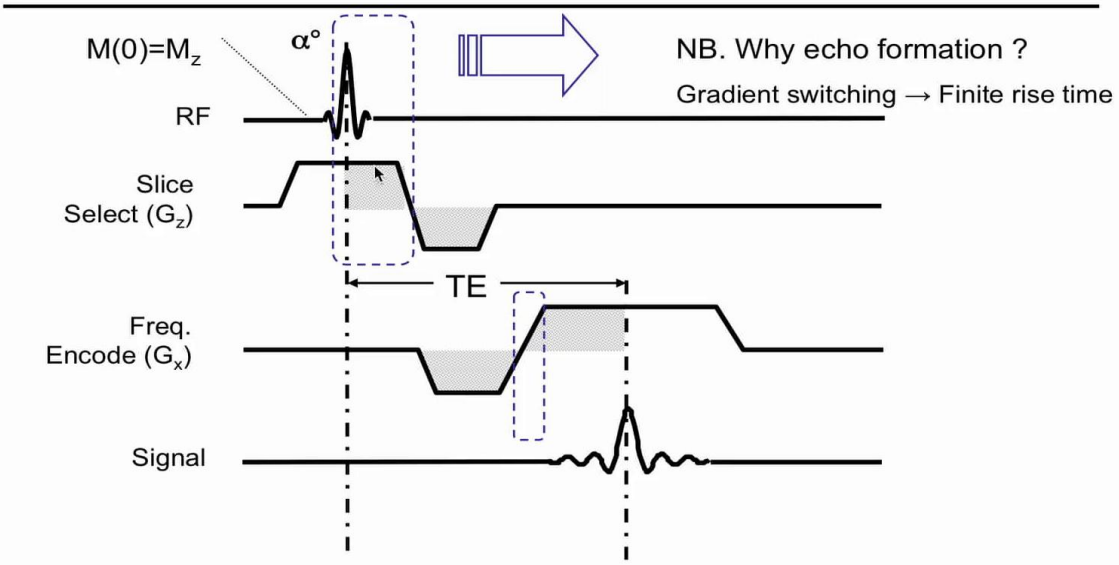
So now I will introduce the basic elements of the so-called *gradient echo sequence*. We'll start out, of course, with a RF pulse-- this is what we do with the RF pulse, we flip the longitudinal magnetization by *alpha* degrees into the transverse plane. We start out with a magnetization at *time_0* which is just the *z* component before the RF pulse. This defines one time point here. We will now have the frequency encode gradient, we have it negative and then positive, so that's essentially the situation we've been discussing in the previous slides. The negative area here equals to the positive area here. When this is matched then we have, at this time point, we have echo formation, and that is the echo time. So this is what we observe here-- we have the signal as a function of time and we have echo formation exactly at echo time. Now this is an experiment that we've discussed implicitly-- I have not drawn the RF pulses-- now we want to do slide selection, and as we've discussed earlier in the course, this is done by applying, at the same time as the RF pulses, to apply a gradient, and here we'll call this gradient the *z* gradient.

Notes

Summary

17m 45s

What are the basic elements of the Gradient echo sequence ?



11-10

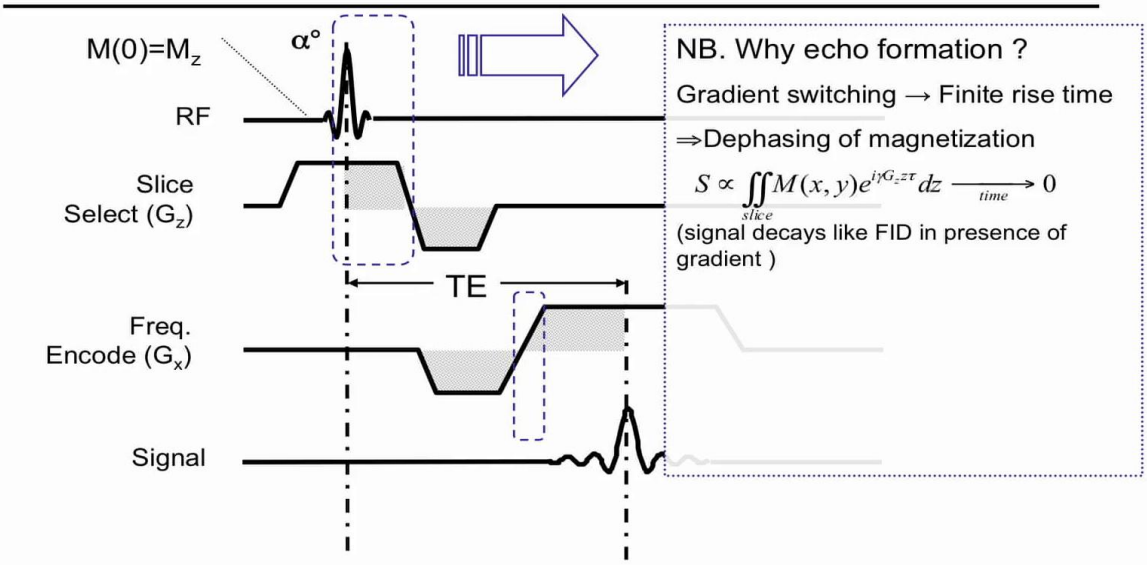
So what this *alpha* pulse now does-- *alpha degree* pulse-- it will flip the magnetization by *alpha* degrees into the transverse plane at the Larmor condition defined by the radio frequency of the RF pulse. And we have a positive gradient afterwards and a negative gradient subsequently. Now why do we have to deal with this gradient switching in the slide selection? Wouldn't it be just enough to turn on the gradient, we do the slide selection, and we're done with it? Well, the situation is also-- Here you can see. that previously we have assumed the gradient switched on instantaneously, and here you can see in this gross exaggeration, that the gradient switching takes some time. And this has to do with the question of why do have to have echo formation? We have, after the RF pulse, the gradient is on.

Notes

Summary

19m 07s

What are the basic elements of the Gradient echo sequence ?



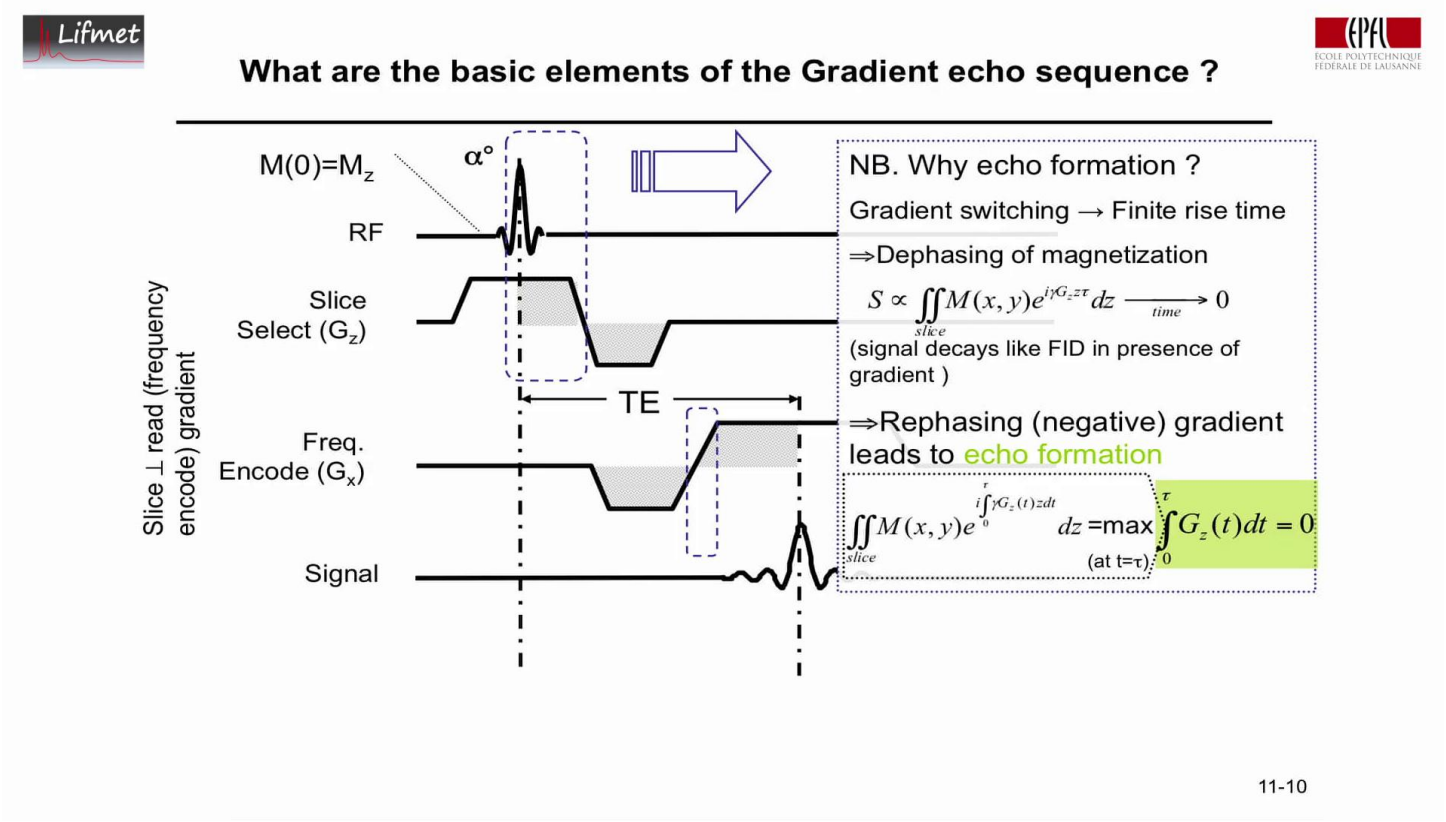
11-10

So the magnetization is in a transverse plane, it will spread out, if we turn it off, the magnetization will stay spread out. If it's spread out, that means it's spread out according to the position along *z*, the magnetization vectors no longer act co-linearly, and therefore the signal is reduced. What this negative gradient here does it takes into account this dead time here that we have and makes them run in the opposite direction, the magnetization vectors, and with a matching condition, they will be all co-linear and we will have maximal signal. So, let's look at the effect if we don't turn on this negative gradient, the signal is given by the transverse magnetization across the slice, times *e* to the *i gamma* and *G_z* times *z* over *tau* integrated over *z*. So this term, with time, will eventually go to zero-- that's dephasing. So we can not record the maximal signal, the signal decays, and it actually decays rather fast. It is a decay that is almost like a FID, it's not *T 2*, it's the effect of the dephasing from the different positions along the slice.

Notes

Summary

20m 09s

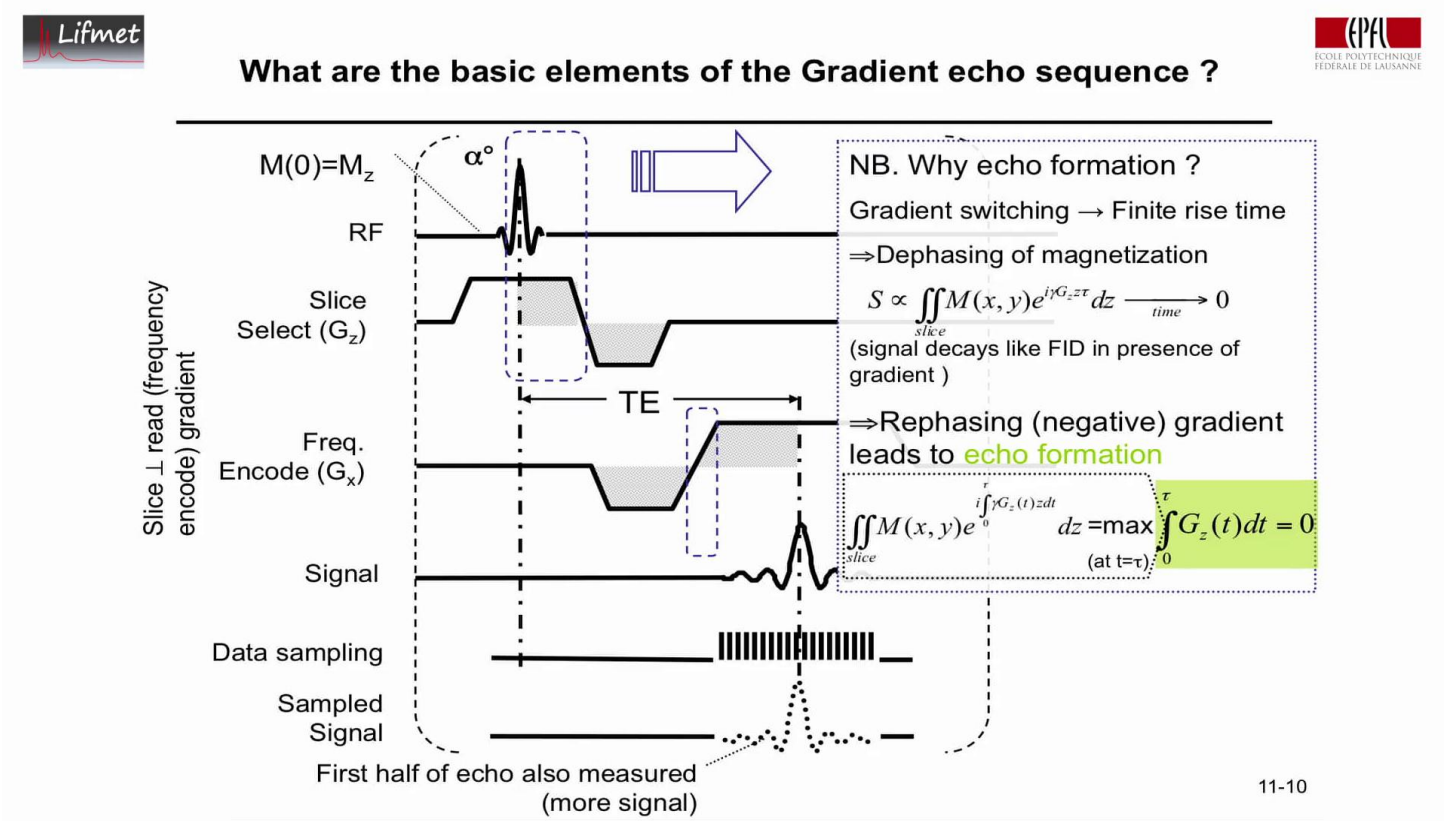


We can rephase this term, we can make the magnetization co-linear again by rephasing, that is, by applying a gradient with opposite polarity-- here it's strong negative-- and this leads to echo formation. And if we write this in general terms, and for this slice, if we integrate over this slice, we have the magnetization, times *e* to the-- in general terms the phase is here given by phase of time, the phase is given by the integral of... for a given position of *gamma G_z T* over time. And we want this to be maximal-- that's what gives us echo formation. So this phase term, this term must be maximal and there is actually a nice condition for that and that is that the integral of the gradient, from zero to a certain time of the gradient is equal to zero. So that basically is what's depicted here-- this area equals to this area, that's the same thing as saying it's the integral over time is equal to zero, and that leads to echo formation. So in this example we have the slice select, the slice orientation, the slice gradient is perpendicular to the read, or frequency encode gradient, so *x* and *z* are presumed to be orthogonal.

Notes

Summary

21m 22s



Whether we call them *x* or *z* or *y* and *z* and so on, whatever we call them, doesn't matter-- the main condition here is that they are orthogonal. Of course, at some point, the ADC is opened up, the data is sampled, that is done here, and then the recorded signal that is recorded in the scanner looks like this. What is crucial here is we don't just record from the maximal echo and then onward, we record also the first half of the echo. This is done because this is-- at very little cost of time, one can increase the sensitivity because there's more data points that are being sampled. So this is your basic imaging sequence, the basic elements, and this describes what's happening every *TR* seconds.

Notes

Summary

22m 51s

Credits

Creative commons pictures
http://www.coloring-book.info/coloring/

That's your basic imaging sequence.

Notes

Summary

23m 43s