ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

11.4 Encoding the 2nd dimension

Fundamentals of Biomedical Imaging

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gradient
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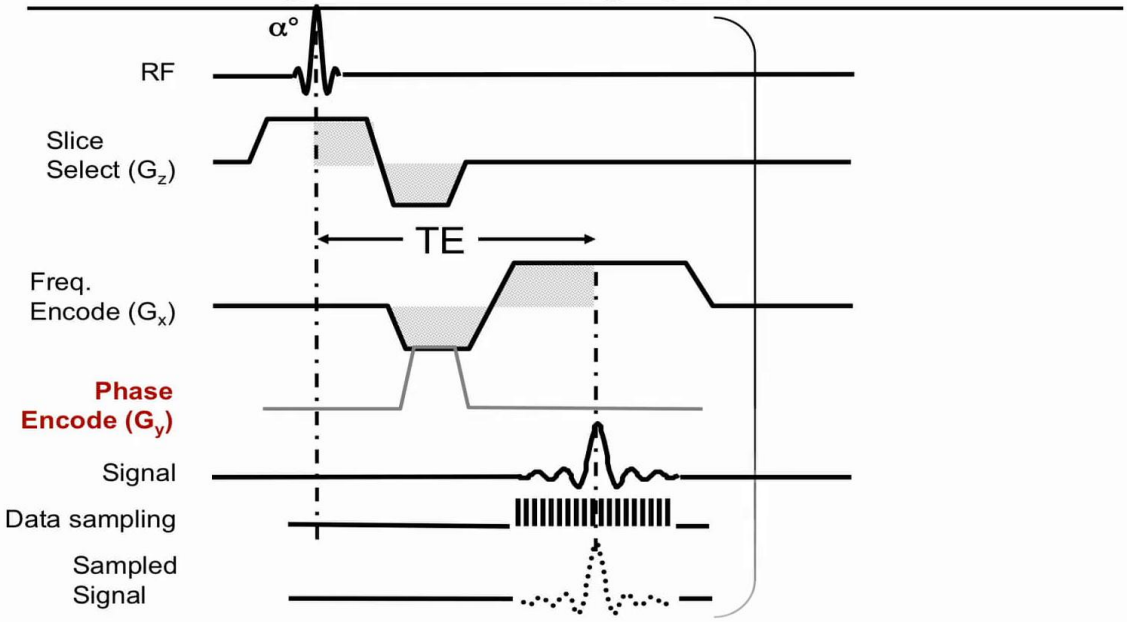
11-4. How is the 2nd dimension encoded ?

gradient echo imaging sequence



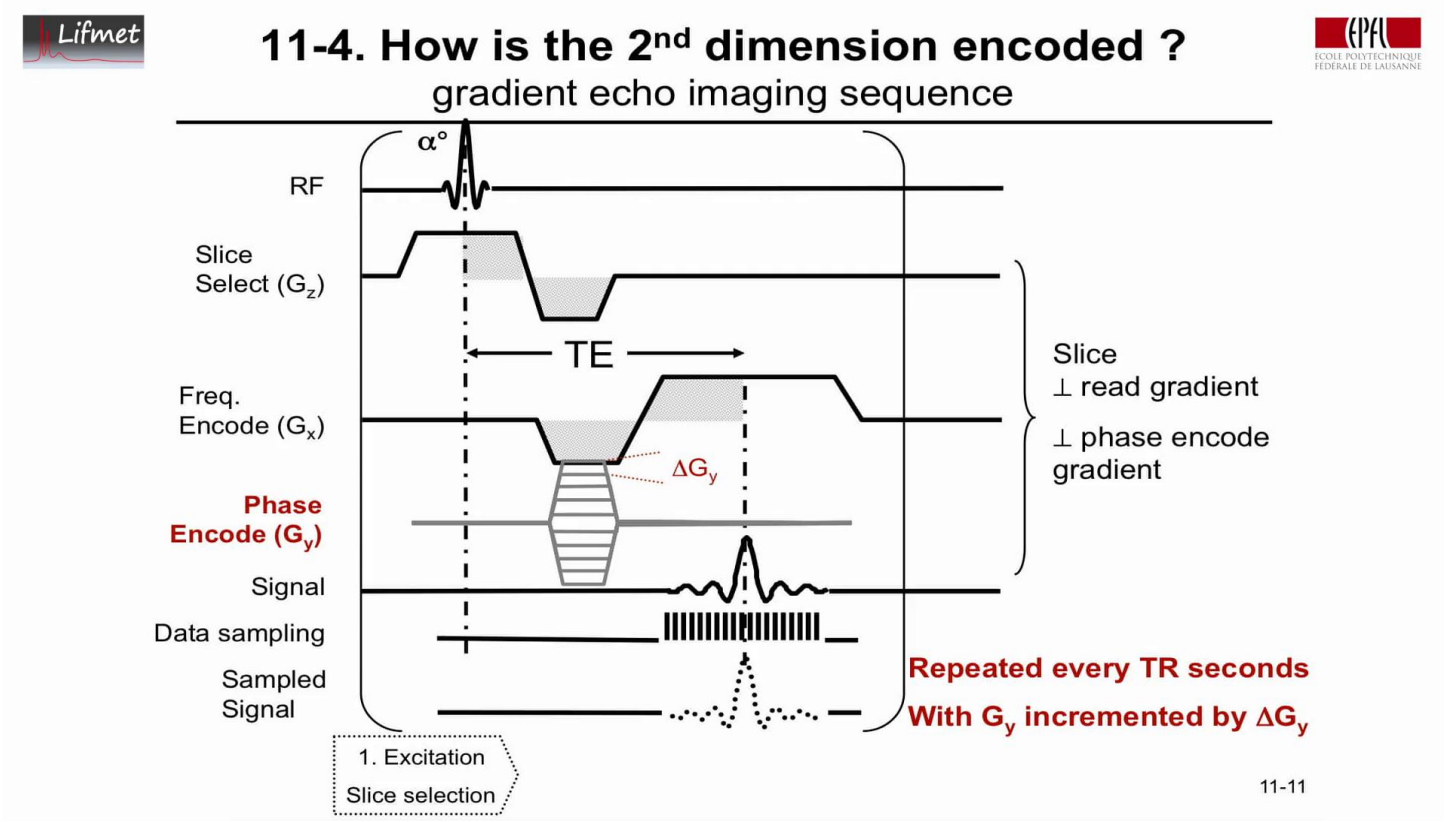
11-11

In the previous section, I introduced the principle of slice selection and frequency encoding with gradient switching, and I said, this is almost the essential-- the basic imaging sequence. What I haven't told you is, how do we get the second spatial dimension encoded? So, let's recap what we've seen. We have had an RF pulse, that flips the magnetization to transfer spin by α degrees. We have the slice select gradient with the rephase. I will not comment on here. The frequency encode, with echo formation. This is the condition of zero integral for the gradient, so this defines the echo time, TE. This is the signal that is produced. The data sampling is done over this time period, and this is what is the sample signal that arrives in our computer. Now, what we see here is that this produces us a slice selection-- that's what this does-- and an encoding of a projection, in one direction. But where's the second dimension? So, for the second dimension, we turn on the phase encode gradient here along Y. We just turn on the gradient at this position for a certain amount of time, with a certain amplitude. And this is now the entire experiment.

Notes

Summary

0m 04s

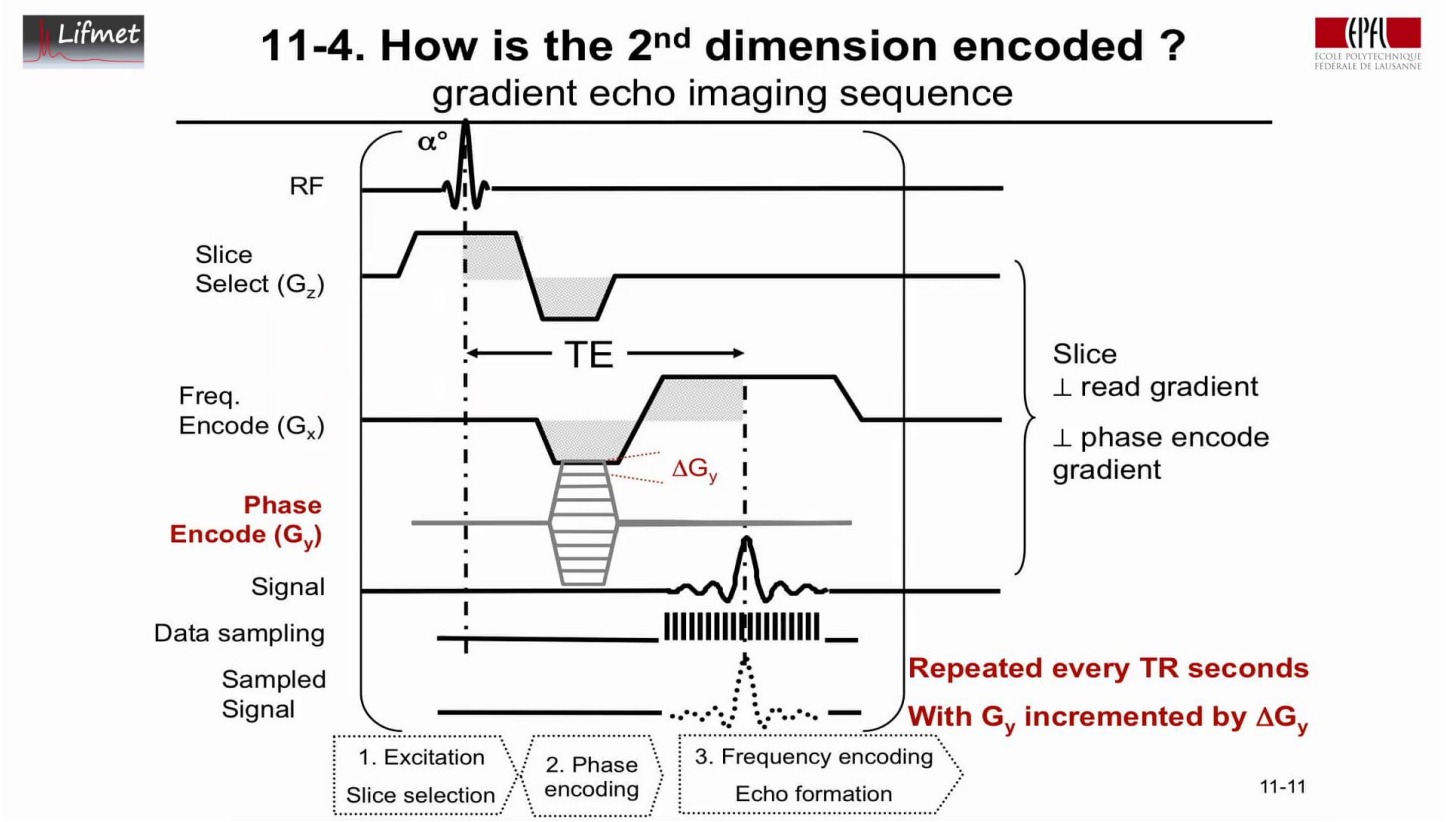


However, this gradient is applied-- this phase encode gradient is applied with a certain amplitude. That does not give us spatial information. So, this is repeated, with a changed amplitude. So, here, reduced by an amount ΔGz. Uh, Gy. And we repeat it again. We'll repeat it every TR seconds, that's the repetition time. We increment or decrement by ΔGy, and so forth. We go through zero, we go into the negative values, and so forth, until we have scanned the number of phase encode steps that we want. So, what we have here now is three gradients. The slice selection gradient, perpendicular read gradient, and the phase encode is perpendicular to both slice and read. This gives us a Cartesian encoding of the signal. So, we are distinguishing three phases of the imaging sequence. We have-- first part is excitation. Since all the spatial manipulations, the spatial encoding, is done on the transverse plane, we need to get the magnetization into the transverse plane. During excitation, typically there is slice selection that's being done. The next phase is phase encoding, so that is the encoding of the second dimension, and then we have frequency encoding and echo formation, that is the encoding of the first spatial dimension.

Notes

Summary

1m 26s



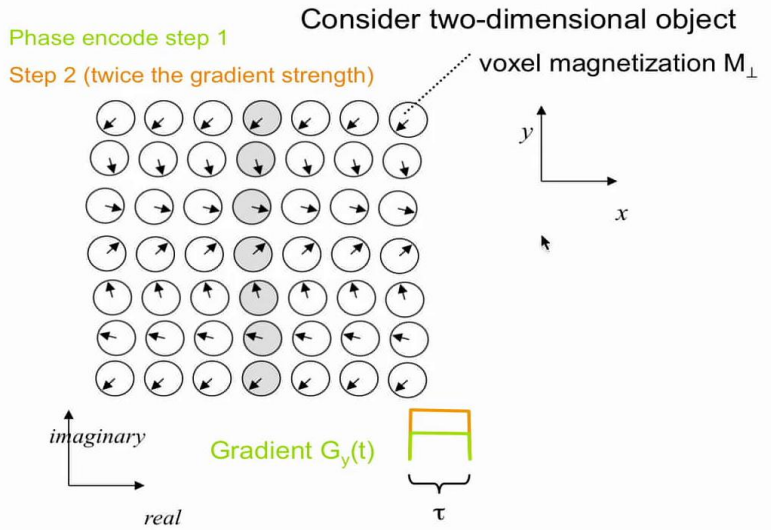
So, we have slice selection, frequency encoding, and phase encoding. All this together, repeated every TR seconds, with the phase encode gradient changing after every RF pulse-- this gives us the information to generate an image. Now, to understand how this phase encode gradient produces the information that allows us to reconstruct an image is not trivial, and I'm going to try, in the next following slides-- try to explain this. It is difficult to understand how this phase encoding gradient allows to encode the second spatial dimension.

Notes

Summary

2m 50s

How does the phase encoding gradient encode the 2nd spatial dimension ?



Phase of voxel magnetization $e^{i\phi}$:

11-12

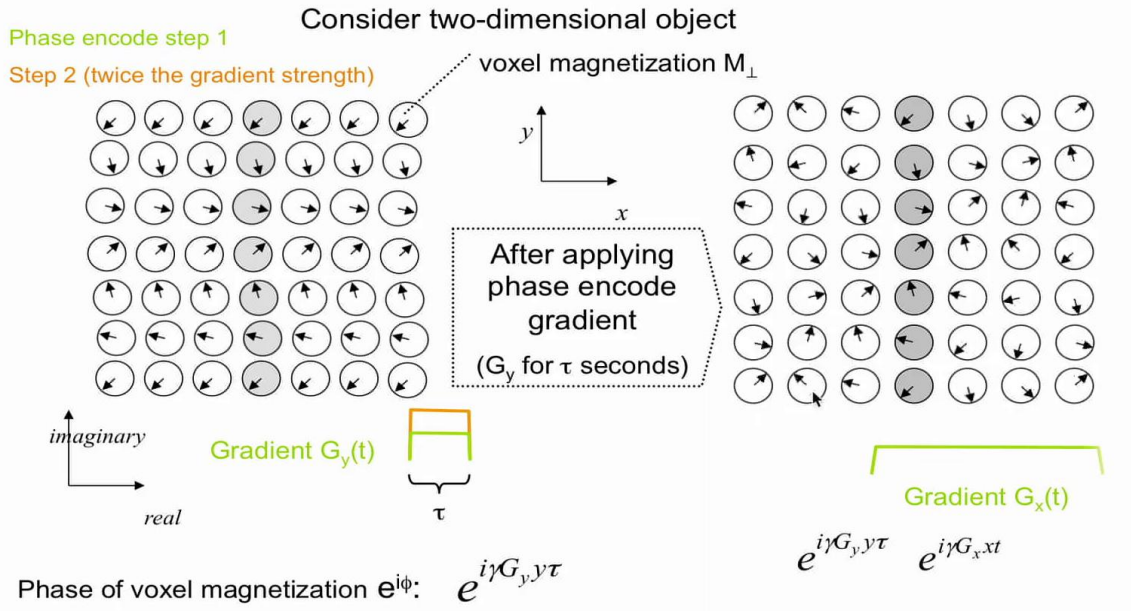
So, and that's exactly the question we're going to look at now. So, let's say we have a two-dimensional object, along X and Y. We're looking at the voxel magnetization-- the transverse voxel magnetization in our two-dimensional object. It's a seven by seven object, here. Now, if we do apply a phase encode gradient along Y, then we apply it for a certain duration, with a certain amplitude, then we have-- along Y, we have phases that change. For all the positions along X, they have the same position, so they have the same phase, but along Y, it depends on their position in Y. These guys have not changed, the others have changed. These turned to the right, and these turned to the left, below. Now, we'll repeat this with twice the gradient strength. So, this means that, if we repeat this experiment with twice the gradient strength, for each of the positions, the vector moves by twice the angle. Now we'll say we have applied this gradient for time τ, and we'll express this in imaginary and real compositions, in the complex plane. So, we'll say, the X magnetization is... real, and the Y magnetization is imaginary. These X and Ys, that's in magnetization space. This is the real space that pertains to here.

Notes

Summary

3m 33s

How does the phase encoding gradient encode the 2nd spatial dimension ?



Phase of voxel magnetization $e^{i\phi}$: $e^{i\gamma G_y y\tau}$

11-12

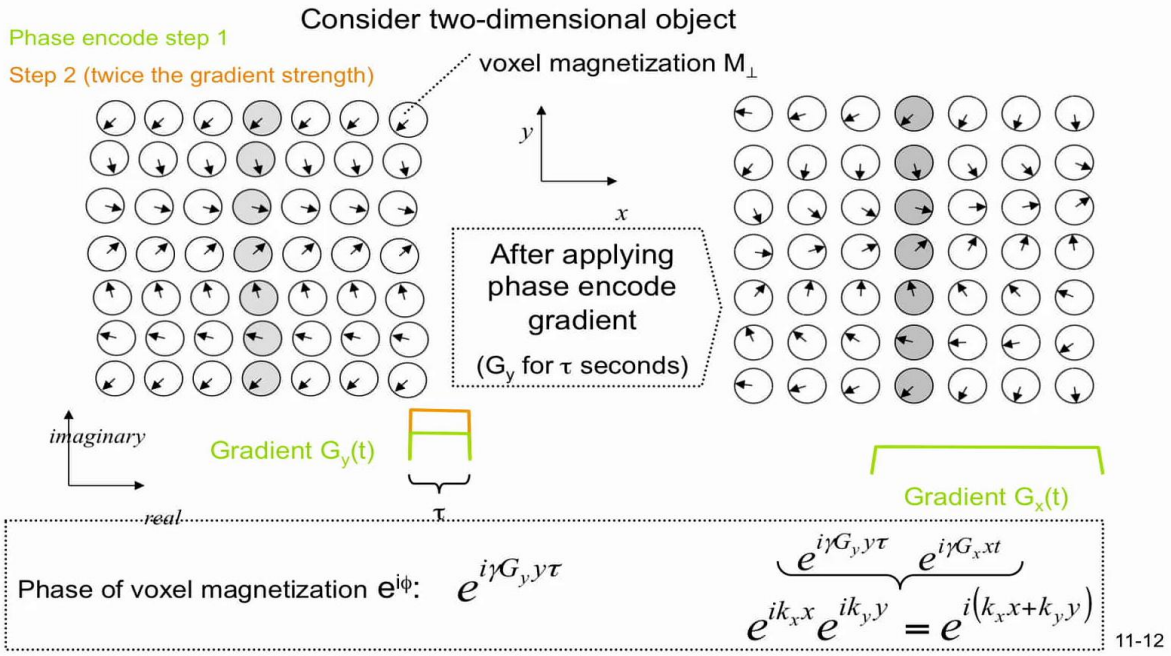
So, if we look at a given voxel, then its position-- its phase is given by the position along Y, the duration τ, and the gradient Gy. So, the phase here is given by this term here. That's what's happened after applying this gradient Gy for a duration τ. Now, after we have applied this phase encode gradient, Gy for τ seconds, we have this phase term here. I'll re-write this here. This is the situation that we have. And now, we want to look at what happens during the application of the read-out gradient. And here, we're not going to deal with the complexities of the need to generate an echo; we'll just start at what's happening at the time of the echo. So, at the time of echo, we will start with a phase evolution that's determined now-- because it's under the influence of an X gradient, that's determined by the magnitude of the gradient, the position, and the time variable τ. So, I'll plot this, and as the gradient is turned on along X, then we will see, now, that the magnetizations along Y... they have the same precessional frequency here, but along X, their precessional frequency is different. These turn clockwise, these turn counterclockwise, and here, on the resonance, they don't move.

Notes

Summary

5m 05s

How does the phase encoding gradient encode the 2nd spatial dimension ?

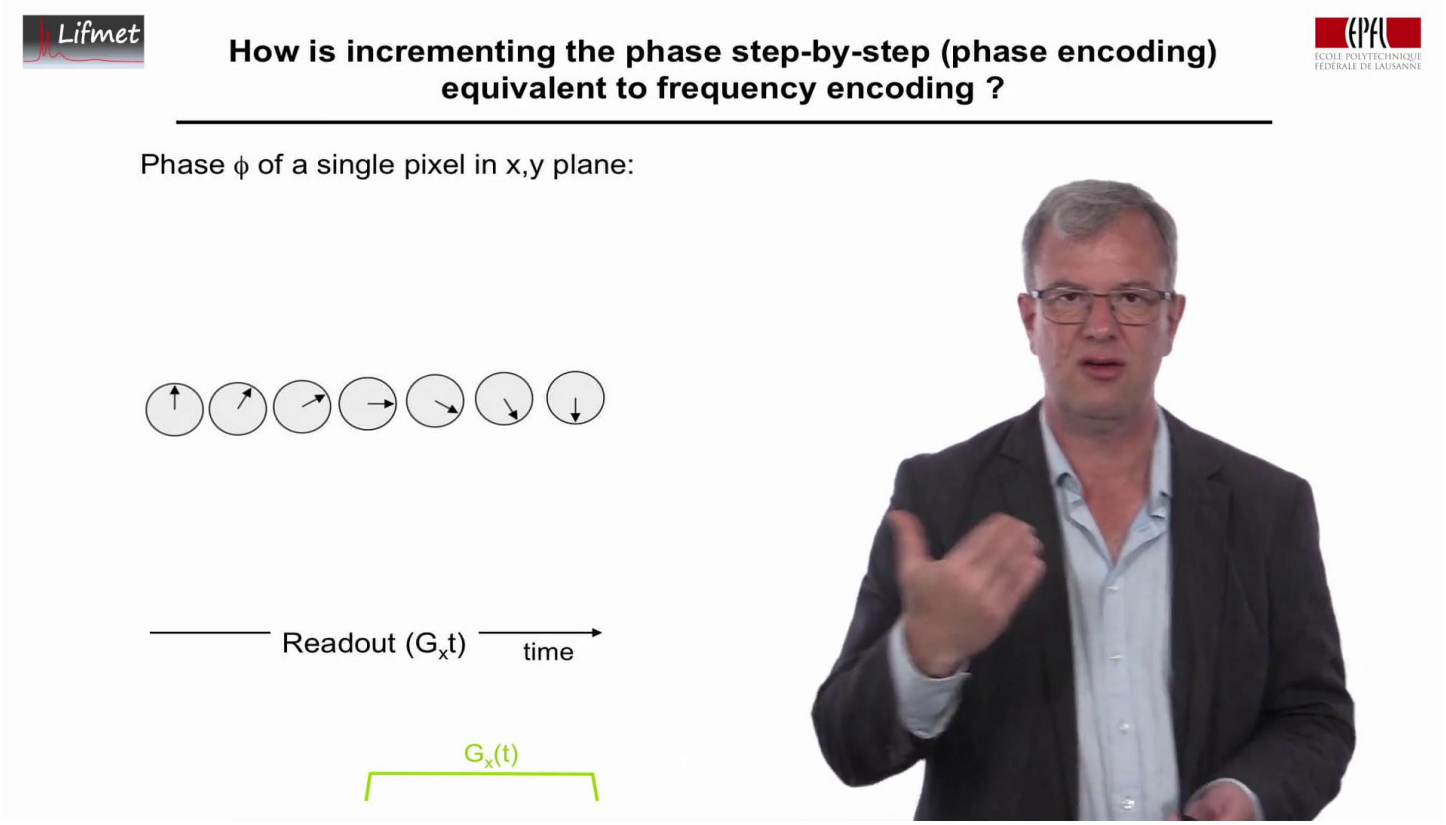


They just have the phase difference that was induced by the Y gradient. So, clearly, if we now Fourier transform this signal, this is what we are observing here, we will get the different frequencies as a function of X position. We'll now take these two terms here, and we'll re-write them, and we'll re-write them such that we introduce the variables Kx, Ky here, the Fourier pair variables to X and Y, to the position. So, this we can write as e ^ (Kx · x + Ky · y) · i. That's the... re-writing here, we have used here the definition for the K space vector in this case. So, that's the phase of our voxel magnetization, and that we have for a position X and Y that is given.

Notes

Summary

6m 32s



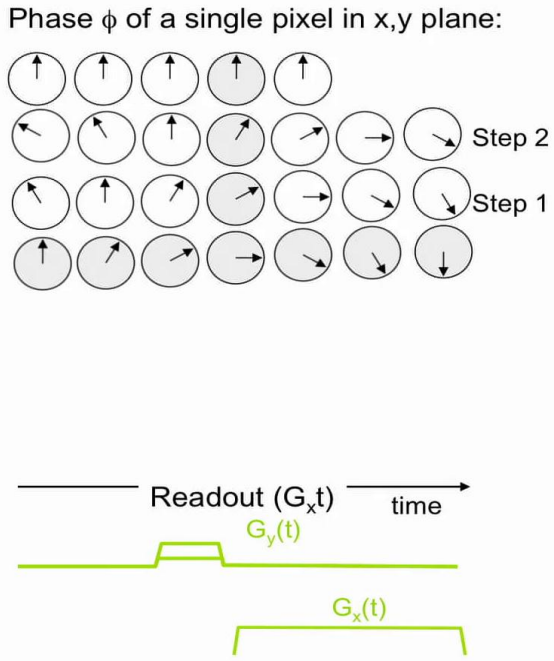
Ok, so we have seen that applying a gradient for a short time along Y, in the phase encode direction, produces a phase twist according to the position in Y. And then, applying the gradient along X induces precessional frequencies that are being recorded, that depend on the position in Y. And now, how can we understand how this generates the necessary information for image reconstruction? What we're now going to do is, we're going to look at the phase of a single pixel in the X-Y plane. Ok, we'll say we'll take the magnetization phase of a single pixel, at a given position-- you can consider it a pointwise object, a capillary or something-- just a pointwise object, idealized, with no dimensions. We'll just take a single pixel. And we'll plot, on the horizontal axis here, we will plot the time. During that time, we have the readout, that is the gradient Gx is on, so we have Gx · T, is the phase, dependent on Gx · T. So, we'll start out here. We apply the gradient along X, and we will have, over time, as we apply the gradient, we have the phase of our magnetization that changes with time. This is simple precession. The phase increments, for this particular voxel-- remember, we're just looking at one single voxel, one single pixel in space.

Notes

Summary

7m 29s

How is incrementing the phase step-by-step (phase encoding) equivalent to frequency encoding ?



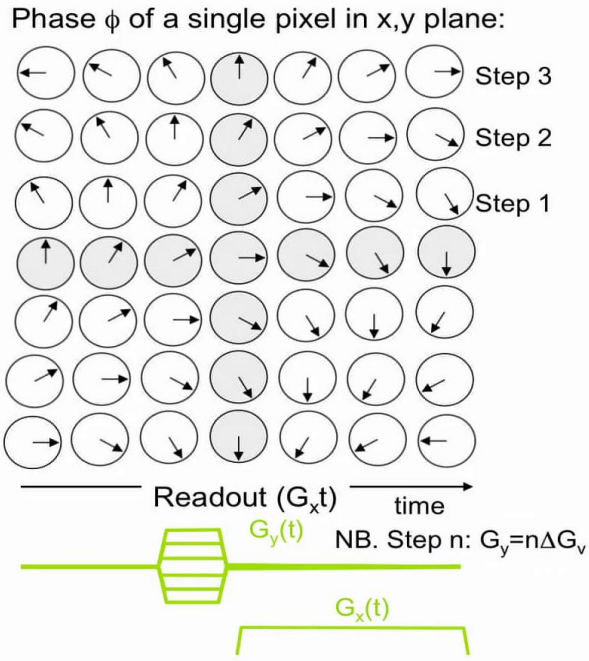
We're just considering that case. And so, now, we have the evolution of the phase with time. That's basically our representation of... precession. So, we can say we are digitizing, here, seven time points. We're looking at seven time points. Now, we'll do step one. We will apply gradient along Y. That gradient will induce a phase twist, according to the position of the voxel. So, after having applied this gradient, the magnetization of our voxel starts out at this position, and now, during the time where we apply the X gradient, there is the same phase twist that's applied on the magnetization of our voxel, of our pixel, that's happening. So, the relative phase of this, relative to this, is the relative phase... of this to this, but this twist here is due to the position in Y, to the application of the gradient along Y. We'll do the next step. So, we have twice the gradient amplitude along Y. So, the phase is twice as big here, to start with. We'll just draw the seven time points along X, and as we apply the gradient along X, then the magnetization develops along X, as we see here. We'll do the third step, and we'll have a phase evolution during the Y gradient.

Notes

Summary

9m 01s

How is incrementing the phase step-by-step (phase encoding) equivalent to frequency encoding ?

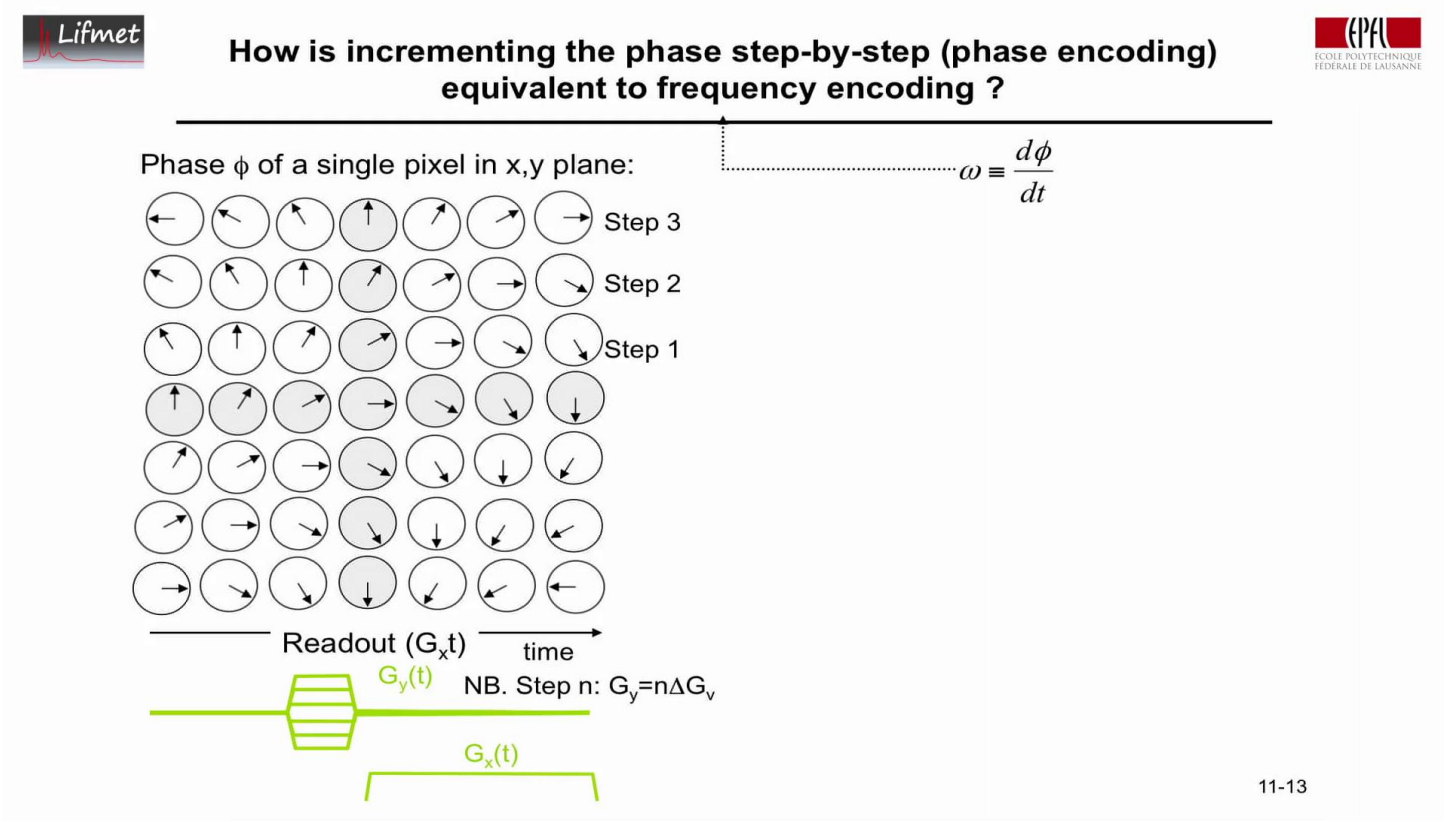


Now, for each of the end steps, the gradient that's applied here is proportional to ΔGy, so it increases... linearly with time. If we go in the opposite direction, we'll have the starting phase of the magnetization in the opposite direction, twice the opposite direction, and three times the opposite direction, again with the phase being twisted by the readout gradient. So, here we have no evolution along X, so we see the starting phase of the magnetization for each of the gradient applications along Y. Here was no gradient application along Y. Here, we have the pure effect of the X gradient. Now, the question here is, does the computer that will digitize this seven-by-seven matrix here-- will the computer be able to distinguish... whether these seven points have been recorded sequentially, or these seven points have been recorded sequentially? Look at the magnetization vector here, and the magnetization vector here, and you will see that those... this particular column and this particular row, they are-- in terms of magnetization vector, they are indistinguishable. We cannot distinguish their positions. So, from a standpoint of computer-- the computer is a stupid machine.

Notes

Summary

10m 26s

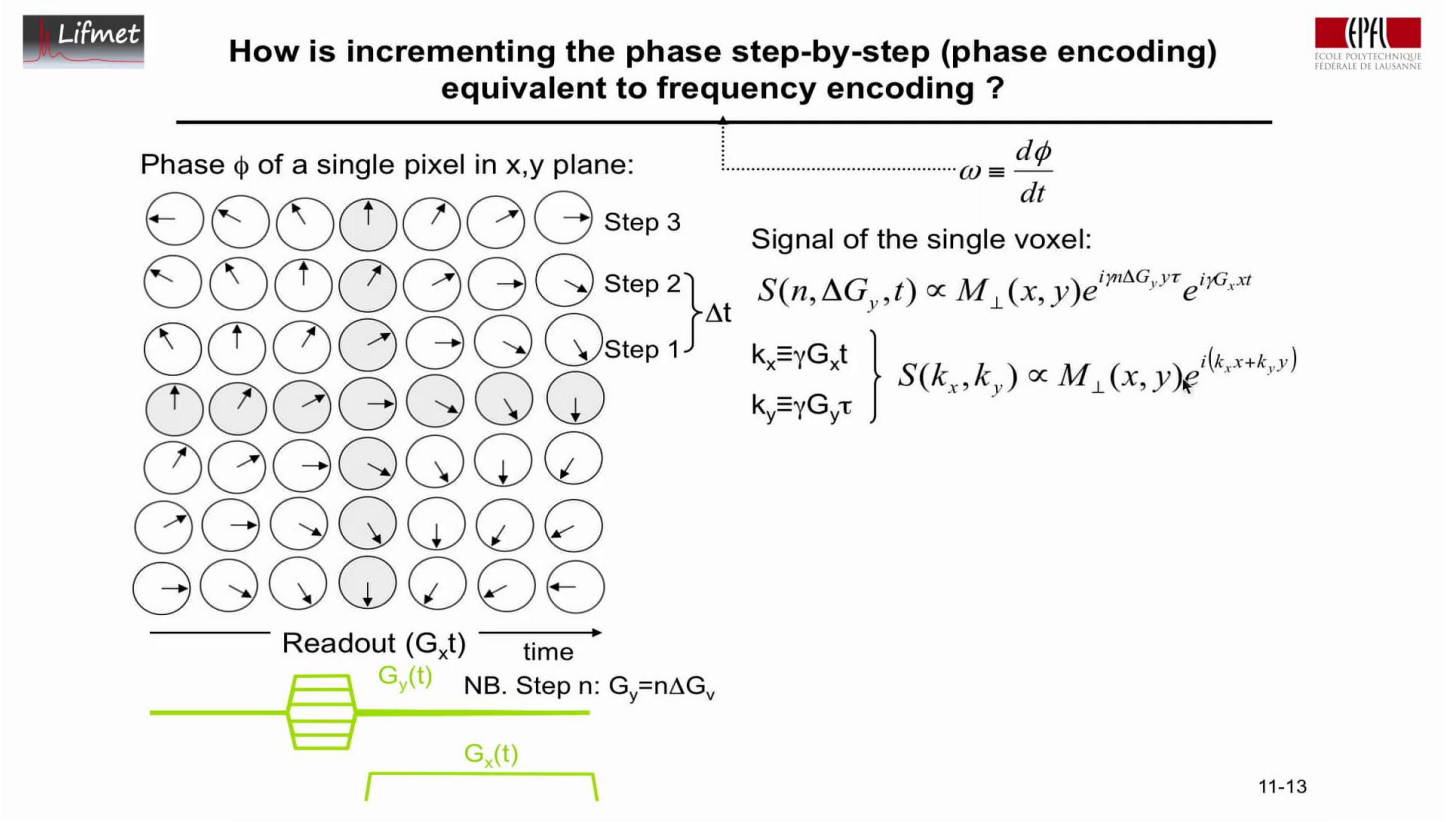


At least, those computers that we are talking about here. It does not know how the data was acquired. It just gets the data. If it now examines this data set, it will not be able to say from this seven-by-seven matrix, we've been encoding these seven time points sequentially, or we've been encoding these seven time points sequentially. So, in essence, when you store the data in the computer in this fashion, then the way the data is stored is, the role of phase encode and readout gradient are interchangeable. They can no longer be distinguished. And so, the phase encoding amounts, essentially, to frequency encoding, which is what we're doing in the X direction, in the same way. Now, this comes from the simple fact that, if we have a change in phase with time, like here, this corresponds to a precessional frequency that we have seen. That's very simple, but if we now introduce the second dimension, also a time component, then this phase evolution here will also correspond to a frequency-- a precessional frequency. Except that it's not real time, it is the time variable introduced by phase encode steps one, two, three, et cetera.

Notes

Summary

12m 03s



So, if we look at the signal of the voxel we have here, we can attribute this to a time ΔT, so we can introduce, along Y, a second time component. Or, in general, our signal is given by the nth gradient, phase encode gradient N, the increment of the phase encode gradient, ΔGy, which is constant-- this is proportional to the transverse magnetization at the beginning, at the position X and Y. That's this particular XY. This is the effect of the phase encode gradient, so it's γ · N · ΔGy, is this amplitude Gy, and e ^ (i · γ · Gx · X · T), that's the evolution during the readout gradient. Again, here we have not considered echo formation. We are assuming that we're starting at the echo. The principle here is more important than exact detail. So, this is our signal of the individual voxel. Now, we'll introduce the definition of two variables, Kx-- that's γ · Gx · T, and Ky, which is γ · Gy · τ. These are the phases-- the phase terms, proportional to the phase terms. With these definitions, we can rewrite our signal as a function of Kx and Ky. We have the transverse magnetization, that's this term, and now the phase term here changes, and you can verify this gives us the (Kx · X + Ky · Y) · i.

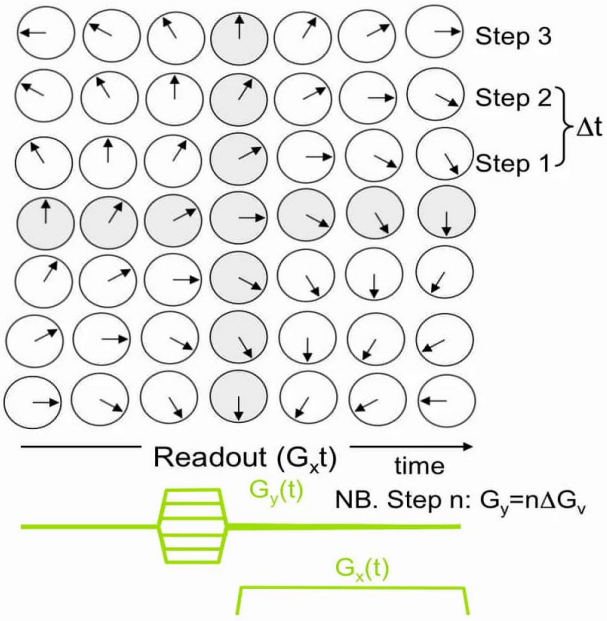
Notes

Summary

13m 22s

How is incrementing the phase step-by-step (phase encoding) equivalent to frequency encoding ?

Phase ϕ of a single pixel in x,y plane:



$$\omega \equiv \frac{d\phi}{dt}$$

Signal of the single voxel:

$$S(n,\Delta G_y,t) \propto M_\perp(x,y)e^{i\gamma n\Delta G_y y\tau}e^{i\gamma G_x xt}$$

$k_x \equiv \gamma G_x t$, $k_y \equiv \gamma G_y \tau$

$$S(k_x,k_y) \propto M_\perp(x,y)e^{i(k_x x+k_y y)}$$

Signal of the entire object :

$$S(k_x,k_y) \propto \iint_{object} M_\perp(x,y)e^{i(k_x x+k_y y)}dxdy$$

What does this resemble ?

$$M_\perp(x,y) \propto \iint_{object} S(k_x,k_y)e^{-i(k_x x+k_y y)}dk_x dk_y$$

MR image generation: FT of the signal

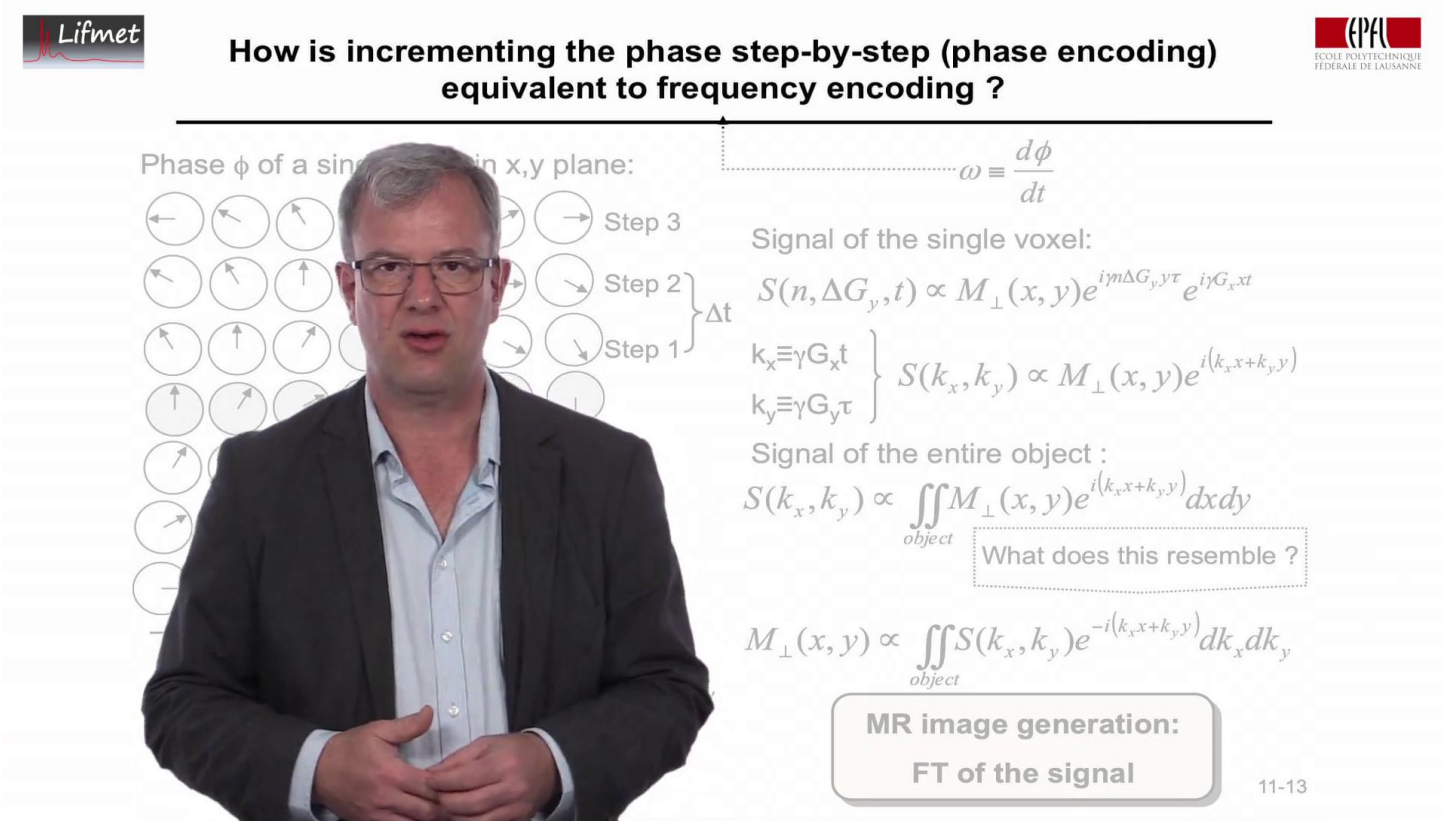
11-13

That's our signal of our voxel with the transformation of the variables. Here, this is what we relate to the experiment. These are mathematical variable transformations. Ok, so far this is the signal for a single pixel. Pointwise object. How about the signal for an entire object? Well, we're going to have to integrate along X and Y, so we'll do a two-dimensional integral across our object, in X and Y. Our signal along the Kx, Ky is given by-- is proportional to the transverse magnetization at that position, times e ^ i · (Kx · x + Ky · y). Well... what does this resemble? Maybe you've come across similar expressions already? Well, this is the Fourier transformation of the object. Or, to obtain the MR image, we have to do the Fourier transformation of the signal. Remember, this is the signal we record. This is with the variable transformation. We have a Fourier representation, to get this from this, we have to do the Fourier transformation, so we get the magnetization by doing the Fourier transformation of the signal here in K space.

Notes

Summary

15m 00s



So, K space and real space, that's like Fourier space and real space. It's like reciprocal space, that's the K space, the term here. X and Y are the real space, but the image-- the process of acquiring the image-- with MRI, essentially what we are acquiring is the Fourier transformation of the object. And to obtain from the data that's stored in the computer-- to obtain the image, what one has to do is a two-dimensional Fourier transformation.

Notes

Summary

16m 25s