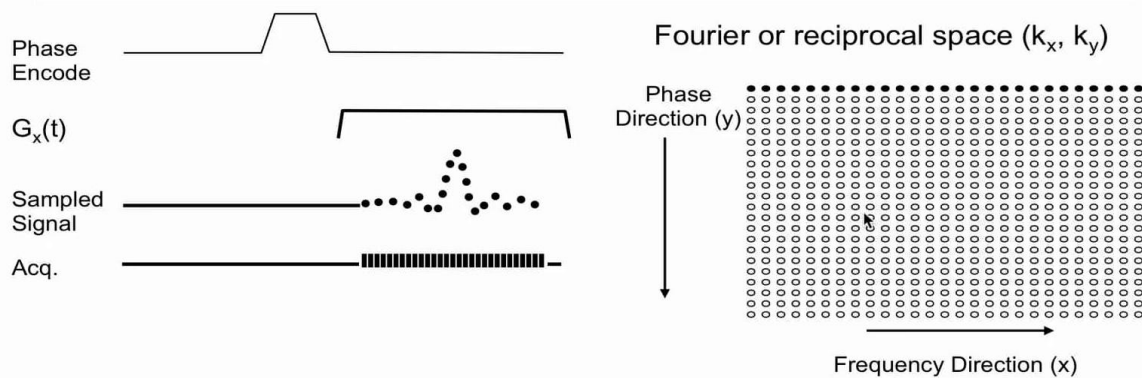




## 11-5. How is the spatial information encoded in MRI ?

scanning k-space (Fourier or reciprocal space) sequentially



11-14

Let's look at how this spacial information is encoded in MRI. We have introduced the fact that we are scanning Fourier, or reciprocal space, or k-space. This is the frequency direction along  $x$ -- I'll use the same variables and along  $y$ , the phasing code direction. So, along  $x$  we are looking at the gradient here. We are not going to bother again with the dephasing and rephasing, or the echo, we are just going to simplify here the diagram, and we have the phasing code gradient. This is our sample signal that we have here, we are acquiring at this point. So, if we apply the phasing code gradient, then we have a  $k_y$  that's non-zero, and we will encode  $k_x$  according to time here. So these are the points in  $k_x$ . So we have acquired here one line in k-space, or reciprocal space. We will repeat it with a different amplitude for the  $y$  gradient; then we encode the second line of k-space so the gradient is reduced, so  $k_y$  is reduced; the third line, and each time this is stored in a two dimensional matrix; fourth line, fifth line, sixth line seventh, eighth, et cetera, and as we acquire this.

Notes

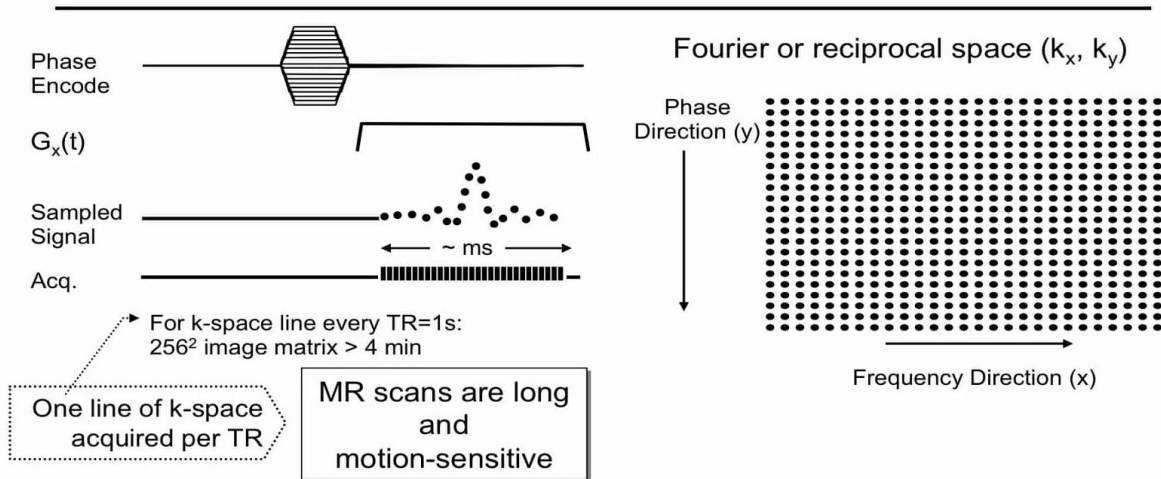
Summary



0m 05s

## 11-5. How is the spatial information encoded in MRI ?

scanning k-space (Fourier or reciprocal space) sequentially



11-14

And then we do this much faster, and this is the sound that you hear: "Bzz, bzz, bzz, bzz,..." As you hear the sound from the MRI, this is how the k-space matrix is being filled up. So, we don't have here the RF pulse-- we have not drawn this but, of course, we have an RF pulse, each time that we have to apply. And so this means, we have one line of k-space that's been acquired per  $TR$ ,  $TR$  being the time between RF pulses.  $TR$ , as we have seen, is determined by the longitudinal relaxation time, so it is typically on the order of seconds. The acquisition here, this is typically on the order of milliseconds. So this is very fast done-- these data points are fast acquired, and here every line is acquired every  $TR$  seconds. So, if we are looking for a  $256^2$  matrix, the re-dimension is in milliseconds-- that does not count-- but if we have 256 phasing code directions and we take typically  $T1$  of one second, our  $TR$  is on the order of seconds. Then an image  $256^2$  matrix, it takes roughly four minutes to acquire that data. Conclusion: MR scans are long and motion-sensitive. They are long because the acquisition has in large part to do with  $T1$ .

Notes

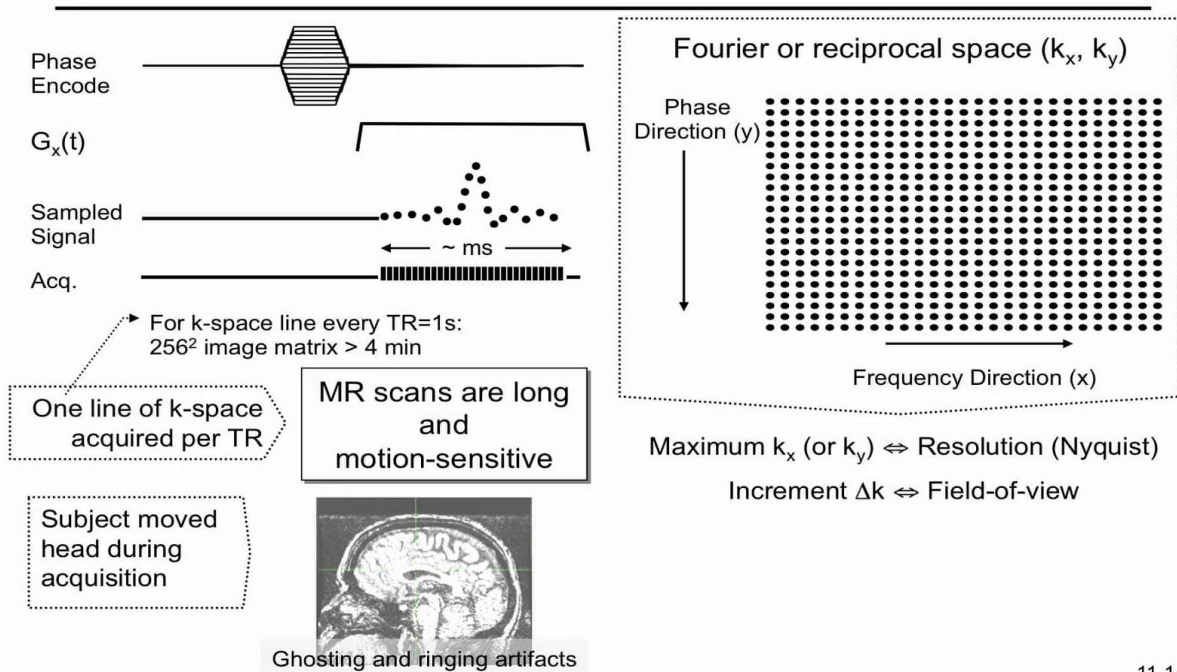
Summary



1m 27s

## 11-5. How is the spatial information encoded in MRI ?

scanning k-space (Fourier or reciprocal space) sequentially



11-14

$T1$  is not very short, and motion-sensitive because every time you acquire a k-space line here, you acquire information on the entire object. If this part of the matrix does not fit this part of the matrix, then, of course, they don't reconstruct very well, and this produces a mismatch and therefore motion artifacts. So, here is an example of a subject that moved during acquisition. [inaudible] this is a sagittal cut, the nose is here, the brainstem is here, so this is the brain and you can see here these bright areas, here these streaking artifacts, one has so called ghosting, so that signal that does not belong where it is being observed--there is no signal outside of the head-- or ringing artifacts that are being observed. So, there are few things we can say about the image matrix. So, the maximum  $k_x$  or  $k_y$  that one acquires, that determines the resolution, and the increment, the  $\Delta k$ , that is either  $\Delta k_x$  or  $\Delta k_y$ , through the Nyquist theorem determines the field of view. So if we have closely spaced samples, then one gets a big field of view, if one has a lot of samples, one gets a high spatial resolution.

Notes

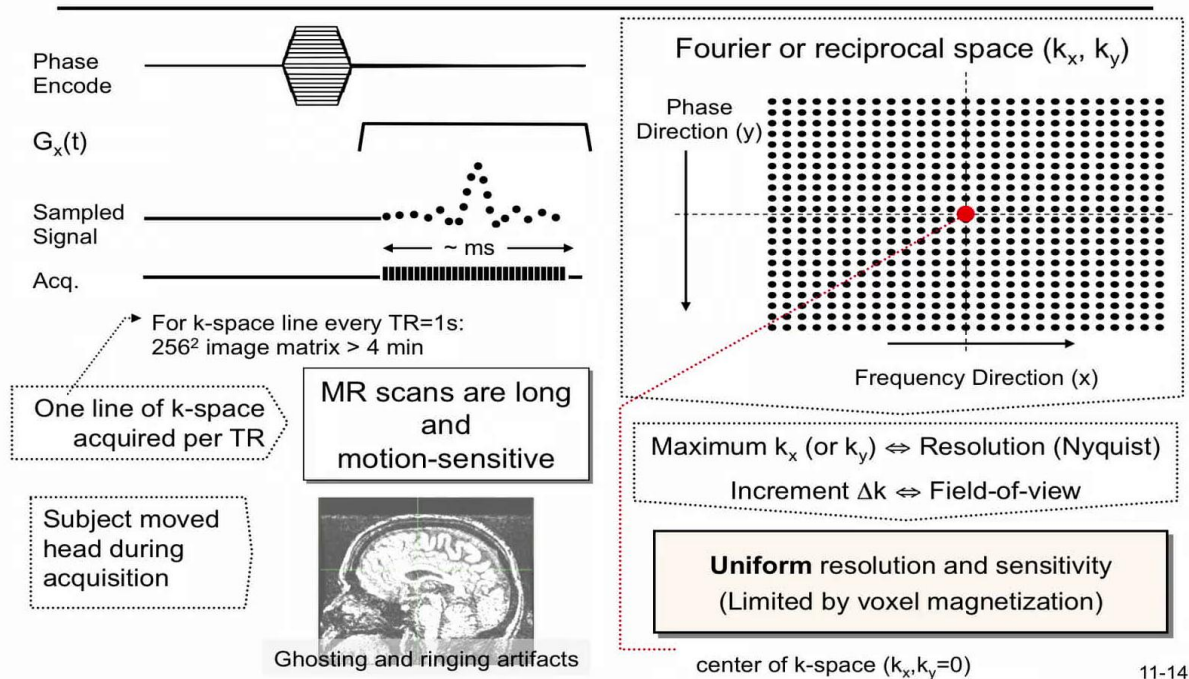
Summary



2m 54s

## 11-5. How is the spatial information encoded in MRI ?

scanning k-space (Fourier or reciprocal space) sequentially



So, this whole process, since what has been acquired, is the Fourier transform of the object. In this process of image reconstruction there is no influence on the resolution. The resolution is uniform across the entire image and also the sensitivity as such, once a signal is acquired, the reconstruction process induces a uniform sensitivity. So, in principle we could acquire an image with 8,192 pixels in one direction, and 8,192 pixels in the other direction-- we are not limited from the acquisition process to the size of the matrix. However, the limitation that we encounter is the voxel magnetization. As we have seen a few weeks ago, the sensitivity is maximal for thermodynamic equilibrium magnetization, and that is determined by the magnetization in the voxel-- that's the number of molecules et cetera, so there is the fundamental limit. We have in the k-space matrix we have one particular point to consider, and that's where  $k_x$  and  $k_y$  are equal to zero-- that's the center of k-space. At this point  $k_x$  and  $k_y$  are zero and this point is of particular interest-- that's where the image is maximal-- we have no dephasing in any direction of the magnetization.

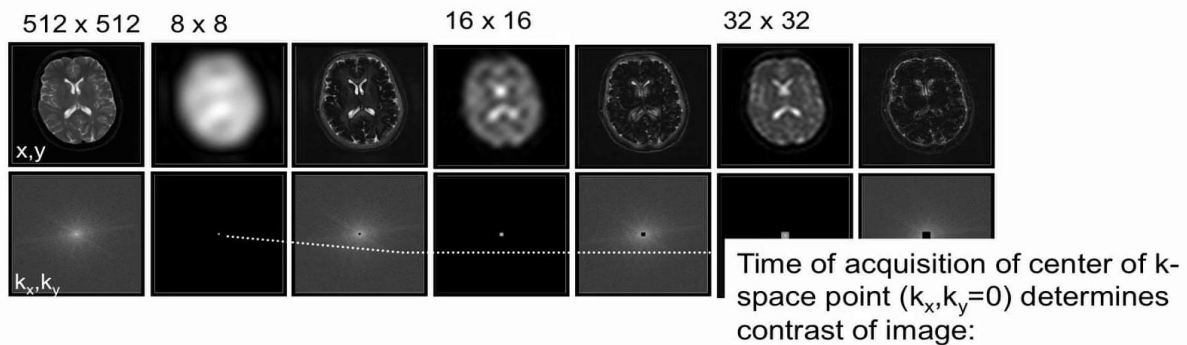
Notes

Summary





# What are some effects of incomplete sampling ? of Fourier space (k-space)



$$S(0,0) = \iint_{\text{object}} M_{\perp}(x,y) dx dy \Big|_{k_x=0, k_y=0}$$

11-15

Let's look at the properties of the k-space matrix. Here is an image of an MRI, the data acquired at 512 x 512 spatial resolution, that's the reconstructed image, that's the 2D Fourier transform, that's x, y space, and this is k-space here,  $k_x, k_y$ . If we now take the center eight points, eight by eight, and reconstruct this, we are getting a very blurry image. If we take the image and we eliminate those central eight points, we get more or less sharper features of the object. We do the same thing now with just taking the first 16 points at the center, reconstruct, and here we eliminate the central 16 x 16 points, and we are starting to see the edges here and here we are starting to see some of the structure. 32 x 32, and it's already quite amazing how much contrast we have at this point, 32 x 32, that's a very small matrix compared to 512. We are starting to see already structures here, in this brain scan, and eliminating the first central 32 points gives edge features. Now, the one thing that we have to bear in mind is that the center k-space point determines the contrast of the image. And this comes from the center of the k-space, that's  $S(0,0)$ ,  $k_x, k_y = 0$ , is given by this expression here with  $k_x = 0$  and  $k_y = 0$ .

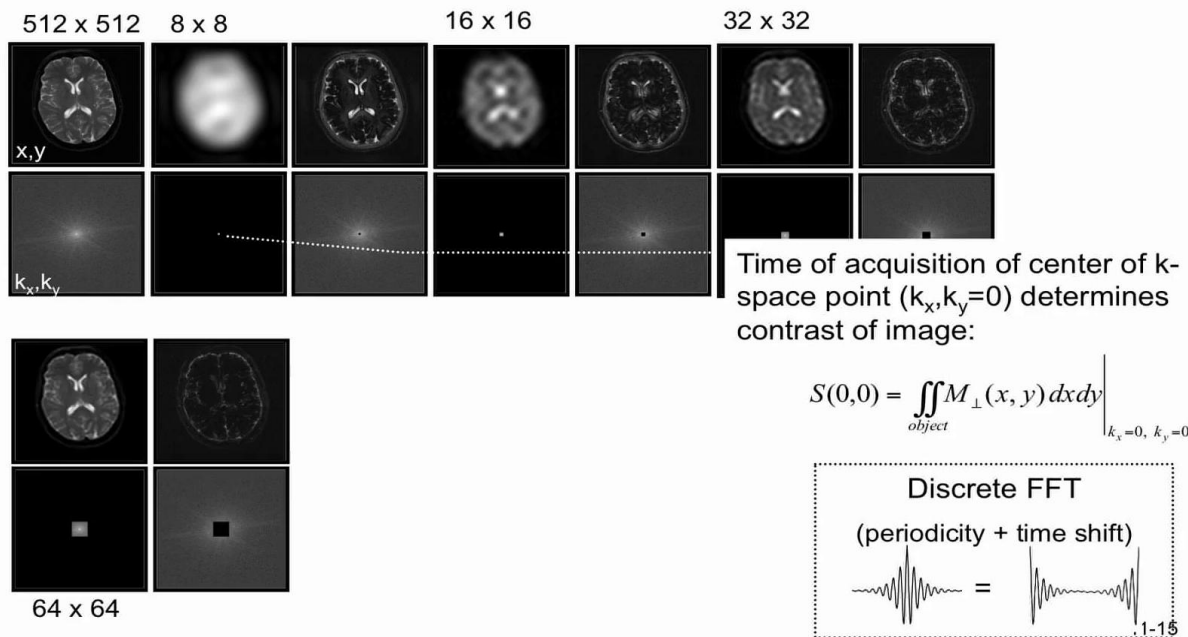
Notes

Summary



6m 09s

# What are some effects of incomplete sampling ? of Fourier space (k-space)



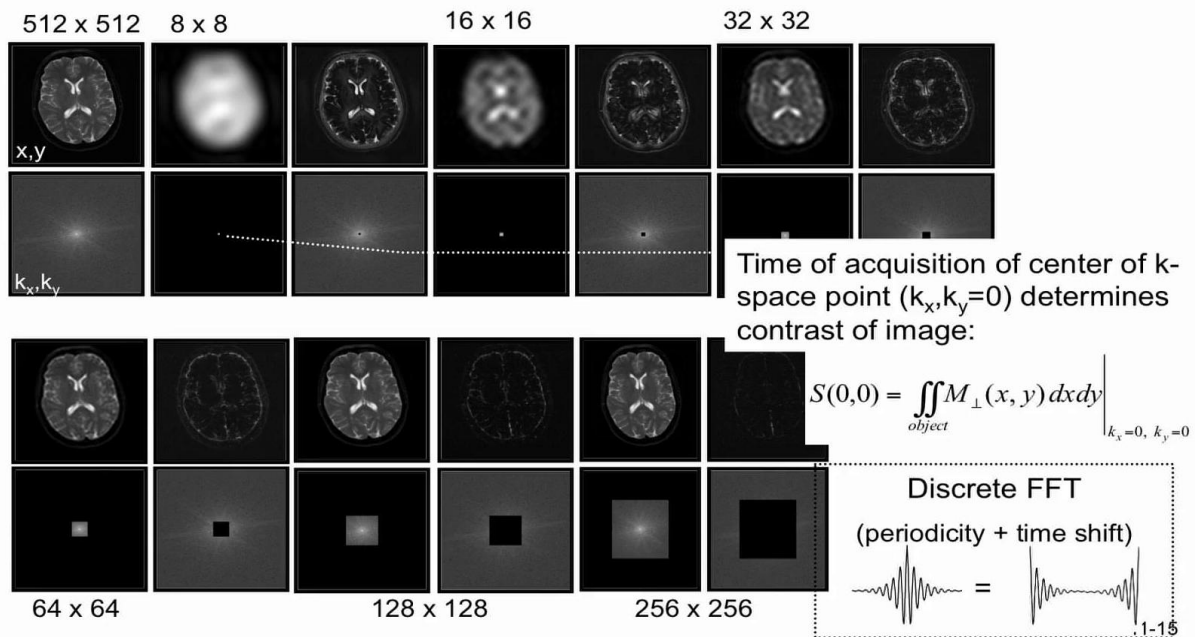
So this is the Fourier transform of the magnetization-- that's what we are detecting. If we look at the discrete Fourier transformation, and the discrete Fourier transformation we employ the fact here that it is periodic over the sample time period and we can do a time shift. With a time shift--so here is the signal-- that's the echo that we record we can in the discrete Fourier transformation this signal is going to be the same signal here in that magnitude as this signal, so we've taken this half, shifted it to the left and that half, as we shift, we fold it around and put it on this side. And now we have seen, before, for the Fourier transformation, that the signal at time zero is proportional to the magnetization. Here is our time signal, zero, so this signal here is proportional to the magnetization and therefore is proportional to equilibrium magnetization and the effects of relaxation. Okay, et's go on; we have 64 x 64, the image gets sharper and we are getting more and more, outside of the 64 x 64 are very sharp features, the high frequency component, 128 x 128, really now only seen the sharp edges and here the image is almost the same quality as this image here, and we are left with very sharp features.

Notes

Summary



# What are some effects of incomplete sampling ? of Fourier space (k-space)



So, what this means in terms of data acquisition, if your subject moves here close to the central k-space points you are going to have a very big impact on the image, whereas, if it moves here at the edges, where there is a very little signal out here, the impact is much less. So crossing through zero k-space point defines a contrast and also the magnitude of the motion artifacts.

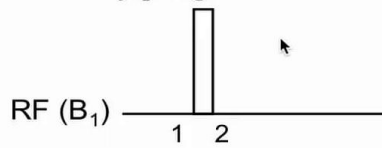
Notes

Summary





$$\omega(\mathbf{r}, t) = \gamma(B_0 + \mathbf{G}(t) \cdot \mathbf{r})$$

flip angle  $\alpha$ 

Magnetization at time points specified:

1:  $(0, 0, M_z)$ rotated by RF pulse by  $\alpha^0$  about x:2:  $(0, M_z \sin \alpha, M_z \cos \alpha) \equiv (0, M_y, \dots)$ [now only consider  $M_{xy}$ ]Precesses with  $B = \gamma G_y y - \gamma G_x x$ 

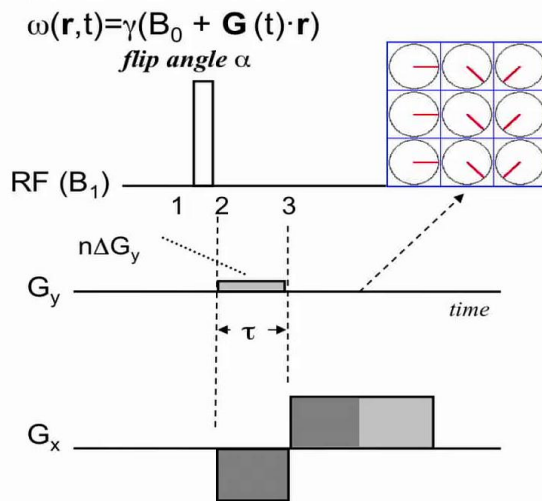
11-16

I want to summarize the spatial encoding with the gradients. I'll try here a depiction that is more on a mathematical basis. So, we will have here the RF pulse-- that's done by the  $B1$ . We have a flip angle  $\alpha$ , and we will look now at the magnetization at the time point specified, magnetization at time one. That's just z magnetization. This is being rotated by the RF pulse by  $\alpha$  degree about x, so at the time point two we have rotating about x in rotating plane, so the y component is  $M_z \sin \alpha$  and the z component is  $M_z \cos \alpha$ . And now we'll just be interested in what's happening in transverse plane, so we'll forget about the longitudinal component. We'll just consider now the transverse magnetization. This transverse magnetization will precess depending on the gradient with  $\gamma G_y y$  or  $\gamma G_x x$ . So the precessional frequency here is given by  $\gamma(B_0 + \mathbf{G}(t) \cdot \mathbf{r})$ . So I will take the gradient along y, we'll apply it here for a certain amount of time, with an amplitude of  $n \Delta G_y$ , and we have the gradient along y, first applied negative.

Notes

Summary





Magnetization at time points specified:

1:  $(0,0,M_z)$

rotated by RF pulse by  $\alpha^0$  about x:

2:  $(0, M_z \sin \alpha, M_z \cos \alpha) \equiv (0, M_y, \dots)$

[now only consider  $M_{xy}$ ]

Precesses with  $B = -\gamma G_y y - \gamma G_x x$

3:  $M_y(\sin[-\gamma(n\Delta G_y y + G_x x)\tau], \cos[-\gamma(n\Delta G_y y + G_x x)\tau])$

$\equiv M_y [\sin(-\phi_x), \cos(-\phi_x)]$ ,

with  $\phi_x = \gamma G_x x \tau$  (rotation by angle  $-\phi_x$ )

inverting gradient, i.e.  $B = +\gamma G_x x$ :

after another  $\tau$ , **rotates by angle  $+\phi_x$**

$\Rightarrow$  maximal signal at  $TE=2\tau$  ( $\Delta G_y=0$ )

11-16

We will define here the time points, we will define this time point, number three, and this duration between two and three shall be  $\tau$ . So, at this time point between two and three the magnetization in the transverse plane, we're just looking at the transverse plane, is the  $M_y$  magnetization that we had here, this magnetization, times  $\sin$  of the phase and for the x component and for the y component  $\cos$  of the phase-- that we have here. So, this is the phase  $x$ , so if we take influence of the gradient we will call this phase. So, now we will have the inverted gradient, so now the B field becomes  $+\gamma G_x x$ . During this time we have the precession according to the position in our area and so we will have a rotation, so rotation of the magnetization along  $x$  by an angle  $+\phi_x$ . So, what we have here we have written here-- we have not considered the effect of the  $y$  gradient, just considered the effect of  $x$  gradient. This gives the rotation by  $-\phi_x$ . If we have now inversion of the polarity of the gradient, this rotates the magnetization by an angle  $+\phi_x$ . So, this gives us a maximal signal at  $TE = 2\tau$  that's this time here.

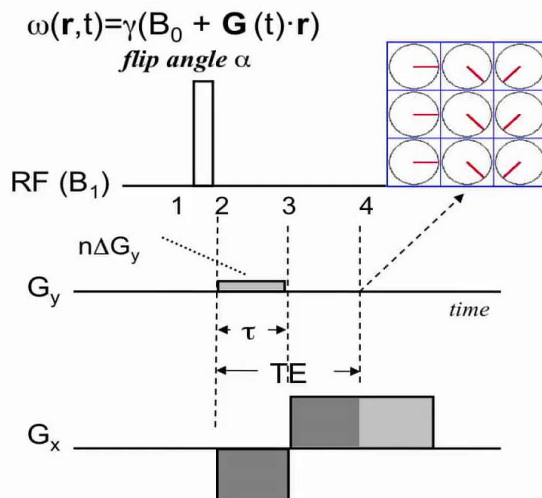
Notes

Summary



# Summary: Spatial encoding with gradients

Phase encoding, echo formation + 2DFT



Magnetization at time points specified:

1: \$(0, 0, M\_z)\$

rotated by RF pulse by \$\alpha^0\$ about \$x\$:

2: \$(0, M\_z \sin \alpha, M\_z \cos \alpha) \equiv (0, M\_y, \dots)\$

[now only consider \$M\_{xy}\$]

Precesses with \$B = -\gamma G\_y y - \gamma G\_x x\$

3: \$M\_y (\sin[-\gamma(n\Delta G\_y y + G\_x x)\tau], \cos[-\gamma(n\Delta G\_y y + G\_x x)\tau])\$

\$\equiv M\_y [\sin(-\phi\_x), \cos(-\phi\_x)]\$,

with \$\phi\_x = \gamma G\_x x \tau\$ (rotation by angle \$-\phi\_x\$)

inverting gradient, i.e. \$B = +\gamma G\_x x\$:

after another \$\tau\$, **rotates by angle \$+\phi\_x\$**

\$\Rightarrow\$ maximal signal at \$TE=2\tau\$ (\$\Delta G\_y=0\$)

4: \$M\_y (0, 1, \dots) = (0, M\_z \sin \alpha, M\_z \cos \alpha)\$

\$\rightarrow\$ **Echo formation**

$$\begin{aligned} \text{Signal } S(\tau, t) &\propto M_z(t) = \iint m(x, y, t) dx dy \\ &= \iint m(x, y, 0) e^{i\gamma(n\Delta G_y y)\tau} e^{i\gamma G_x x(\tau+t)} dx dy \\ S(k_x, k_y) &\propto \iint m_1(x, y) e^{-ik_x x} e^{-ik_y y} dx dy \end{aligned}$$

MRI measures the **2D Fourier transformation** of the object

(measuring the 2<sup>nd</sup> dimension requires time!)

11-16

And we'll assume here for this argument that \$\Delta G\_y = 0\$. And now we are looking at the fourth time point. For \$\Delta G\_y = 0\$ the magnetization is now given by \$M\_z \sin \alpha\$ for the \$y\$ component and \$M\_z \cos \alpha\$ for the \$z\$ component, where we have echo formation. So, we can work in these equations fully for the mathematically for the effect of the gradients. Now, the signal at time \$\tau\$ or \$t\$ it is for the phasing code gradient, we have the influence of the phasing code gradient, magnetization at the start and influence of the [inaudible] gradient, so we have \$T\$ here, \$\tau\$ here so it's \$\tau + T\$ is the influence of the gradient, and we can rewrite this as the signal in \$k\$-space, again, with appropriate variable transformation. So mathematically MRI-- this is the key take-home message-- MRI measures the 2D Fourier transformation of the object. That's what is being registered. What is also key is that measuring the second spatial dimension, that requires time, and this time is strongly influenced by \$T1\$ [at hand], which are typically on the order of seconds. So, this is very complicated, I understand, so understanding phase encoding is not a very easy topic.

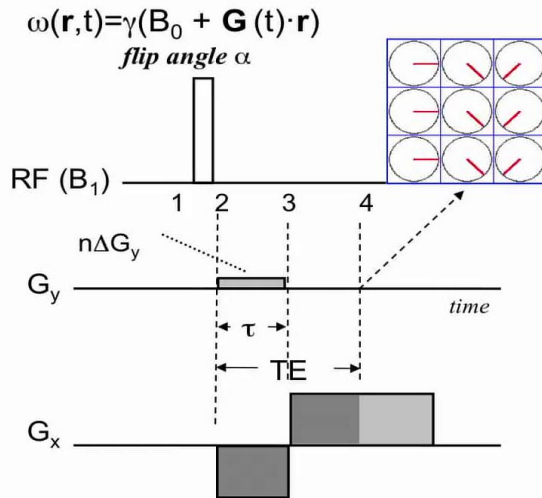
Notes

Summary



# Summary: Spatial encoding with gradients

Phase encoding, echo formation + 2DFT



Magnetization at time points specified:

1:  $(0,0,M_z)$

rotated by RF pulse by  $\alpha^0$  about x:

2:  $(0, M_z \sin \alpha, M_z \cos \alpha) \equiv (0, M_y, \dots)$

[now only consider  $M_{xy}$ ]

Precesses with  $B = -\gamma G_y y - \gamma G_x x$

3:  $M_y [\sin[-\gamma(n\Delta G_y y + G_x x)\tau], \cos[-\gamma(n\Delta G_y y + G_x x)\tau]]$

$\equiv M_y [\sin(-\phi_x), \cos(-\phi_x)]$ ,

with  $\phi_x = \gamma G_x x \tau$  (rotation by angle  $-\phi_x$ )

inverting gradient, i.e.  $B = +\gamma G_x x$ :

after another  $\tau$ , **rotates by angle  $+\phi_x$**

$\Rightarrow$  maximal signal at  $TE = 2\tau$  ( $\Delta G_y = 0$ )

4:  $M_y(0,1,\dots) = (0, M_z \sin \alpha, M_z \cos \alpha)$

$\rightarrow$  **Echo formation**

$$\begin{aligned} \text{Signal } S(\tau, t) &\propto M_{\perp}(t) = \iint m(x, y, t) dx dy \\ &= \iint m(x, y, 0) e^{i\gamma(n\Delta G_y y)\tau} e^{i\gamma G_x x(\tau+t)} dx dy \\ S(k_x, k_y) &\propto \iint m_{\perp}(x, y) e^{-ik_x x} e^{-ik_y y} dx dy \end{aligned}$$

MRI measures the **2D Fourier transformation** of the object

(measuring the 2<sup>nd</sup> dimension requires time!)

11-16

We will try to rehash the process of phase encoding under a different perspective at the beginning of next week to look at this with a slightly different optic so we can digest this factor and understand how spacial information is being encoded.

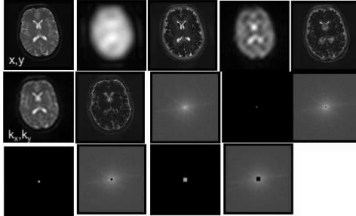
Notes

Summary

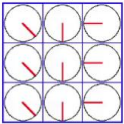




**Unidentifiable source (internet)**



**Walter Block and Franck R. Korosec, PhD, University of Wisconsin**  
<http://slideplayer.com/slide/5880663/>



## Animation

<https://www.cis.rit.edu/htbooks/mri/chap-7/g3-13.htm>

[illegible]



Summary

