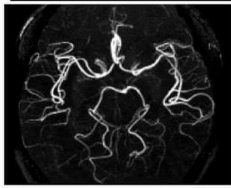




## 13-1. How does Bulk Motion affect the Rephased Signal ? (Blood Flow)



Phase  $\phi$  of the magnetization:

$$M_{\perp}(t) = M_{\perp}(0)e^{i\phi(t)}$$



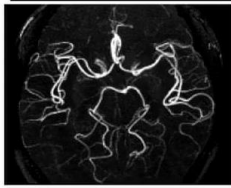
We've seen in Lecture 11 that MRI, magnetic resonance imaging, is a technique that's very sensitive to motion. There we introduced it as a problem in the sense that it creates corrupted data sets. Now, usually when there is a problem with magnetic resonance then there is usually also an application, and this is what I want to talk about first-- that is, the effect of motion on the signal, and, in particular, here on the rephased signal. And we're first going to talk about bulk motion, and later we're going to talk about random motion today. So, what is bulk motion? What are we concerned here with, or what are we interested in? Here is an example of an angiogram, it's blood flow. We have the blood vessels depicted here-- this is an MR angiogram-- obtained completely non-invasively, a depiction of the blood vessels here in the brain. What is bright are the blood vessels and the stationary magnetization, that doesn't move, has been suppressed from this image. So let's look at the phase of the magnetization to understand how this can be registered. The phase of the transverse magnetization, as a function of time,  $M_{\perp}(t)$ , is given by the transverse magnetization at time zero,  $M_{\perp}(0)$ , times  $e^{i\phi(t)}$ .

Notes

Summary



## 13-1. How does Bulk Motion affect the Rephased Signal ? (Blood Flow)



Phase  $\phi$  of the magnetization:

$$M_{\perp}(t) = M_{\perp}(0)e^{i\phi(t)}$$

$$\phi(t) = \int_0^t \gamma G_x(t') x(t') dt'$$

(Gradient along x)

$$\phi(t) = \int_0^T \gamma G_x x(t) dt - \int_T^{2T} \gamma G_x x(t) dt$$



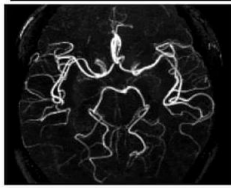
This is in a complex plane, so we denote, as usual, the real part, for example, is the  $x$  component of the magnetization, and the imaginary part of this complex number is the  $y$  component of the magnetization. Now, here we have written  $\Phi(t)$ , so it's a time-dependent phase, and the question is, what is this phase? This phase, in very general terms, we have seen, is given by the integral of the gradient times the position with time-- there's a  $\gamma$  in there, of course, to get the phase-- and this particular example, this equation here is for the case of a gradient along  $x$ . Here  $t'$  denotes the integration variable, just to distinguish it from the time  $t$ , which is this variable here-- this is the integral from  $0$  to  $t$  here. So this is the phase that we obtain, that's the general expression. Now, I'm going to-- and you'll see shortly why-- I'm going to now write the phase of  $t$ , this integral, I'll split it in two integrals, so there's two time periods-- one from zero to big  $T$ , where the gradient along  $x$  (we'll stick with  $x$  here) is along  $x$  is  $G_x$ -- and then we have a subsequent time period from time  $t$  to  $2t$ , here, where the gradient is minus  $G_x$ .

Notes

Summary



## 13-1. How does Bulk Motion affect the Rephased Signal ? (Blood Flow)



Phase  $\phi$  of the magnetization:

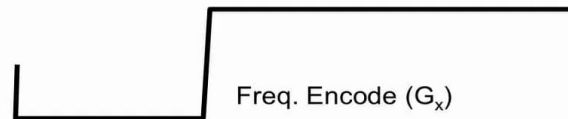
$$M_{\perp}(t) = M_{\perp}(0)e^{i\phi(t)}$$

$$\phi(t) = \int_0^t \gamma G_x(t') x(t') dt'$$

(Gradient along x)

$$\phi(t) = \int_0^T \gamma G_x x(t) dt - \int_T^{2T} \gamma G_x x(t) dt$$

$$x(t) = x_0 + vt$$



13-2

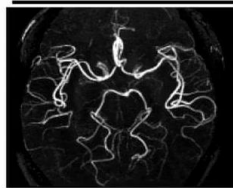
So we're basically depicting an experiment where the gradient is first positive, and then, at time  $t$ , switch to a negative value. And this is what we're looking at. Now here this is just-- negative is first, and then positive, it doesn't make a difference what the sign here is; the sign really is not important. So this is the gradient along  $x$ , what we see, the phase evolution as a function of time at *time*  $2T$  is given here by this expression here. So, small  $t$  here is equal to big  $2T$ , 2 times big  $T$ . Now, what we're going to express here is the position of the magnetization that we have in the voxel. Normally we would describe it as a stationary magnetization, that's what we've dealt with so far in Lecture 11. Now, here we're going to assume that this magnetization in our voxel is moving at a constant velocity  $v$ . So the position as a function of time is now no longer constant but it's given by  $x_0$ -- the original position-- plus  $v$  times  $t$ . And now we'll write the substitute  $x$  of  $t$  in the above integral-- so we'll substitute this expression in here-- and we'll obtain these two integrals.

Notes

Summary



## 13-1. How does Bulk Motion affect the Rephased Signal ? (Blood Flow)



Phase  $\phi$  of the magnetization:

$$M_{\perp}(t) = M_{\perp}(0)e^{i\phi(t)}$$

$$\phi(t) = \int_0^t \gamma G_x(t')x(t')dt'$$

(Gradient along x)

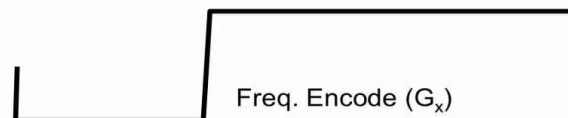
$$\phi(t) = \int_0^T \gamma G_x x(t)dt - \int_T^{2T} \gamma G_x x(t)dt$$

$$\vdots$$

$$x(t) = x_0 + vt$$

$$\phi(2T) = \underbrace{\int_0^T \gamma G_x (x + vt)dt}_{\gamma G_x \left( xt + v \frac{t^2}{2} \right) \Big|_0^T} - \underbrace{\int_T^{2T} \gamma G_x (x + vt)dt}_{\gamma G_x \left( xt + v \frac{t^2}{2} \right) \Big|_T^{2T}}$$

$$\boxed{\phi(2T) = \gamma G v T^2}$$



13-2

We can now see that this integral here and this term here, they are the same, they are constant in time, their interval is  $t$ , so these two terms will subtract out, the stationary magnetization due to stationary magnetization, and that is the condition of echo formation. So I'll just be concerned with the term that is dependent on the velocity-- well, here we've kept actually the time, the constant term, in here-- and we'll take this term here, here's the integral. So here's the constant term, this is negative, this is positive, same value from 0 to  $T$  and from  $T$  to  $2T$  so the difference in  $t$  here is  $T$ , so these two terms cancel. So we're just concerned with this term here, and this term here, and you can verify here very easily that these terms now no longer cancel out to zero. And the consequence is-- do the integration here, I won't do it in all details-- that we get a phase at time  $2T$ , big  $T$ , which is given by  $\gamma G$ , the gradient, times the velocity, times this time squared. So we have a phase now of the magnetization that is changed depending on the velocity in the direction of the gradient, and the gradient strength, as well as the duration of the gradient application.

Notes

Summary



# 13-1. How does Bulk Motion affect the Rephased Signal ? (Blood Flow)



Phase  $\phi$  of the magnetization:

$$M_{\perp}(t) = M_{\perp}(0)e^{i\phi(t)}$$

$$\phi(t) = \int_0^t \gamma G_x(t')x(t')dt'$$

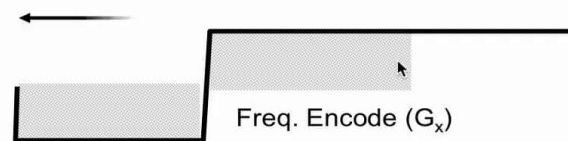
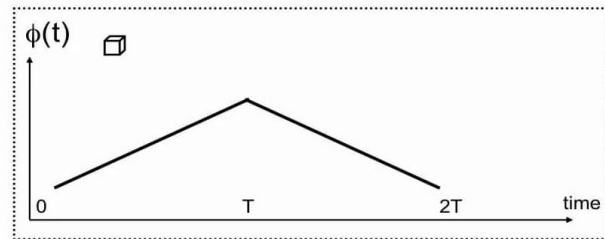
(Gradient along x)

$$\phi(t) = \int_0^T \gamma G_x x(t)dt - \int_T^{2T} \gamma G_x x(t)dt$$

$$x(t) = x_0 + vt$$

$$\phi(2T) = \underbrace{\int_0^T \gamma G_x (x + vt)dt}_{\gamma G_x \left( xt + v \frac{t^2}{2} \right) \Big|_0^T} - \underbrace{\int_T^{2T} \gamma G_x (x + vt)dt}_{\gamma G_x \left( xt + v \frac{t^2}{2} \right) \Big|_T^{2T}}$$

$$\phi(2T) = \gamma G v T^2$$



13-2

So we'll look now, graphically, at the implication of this. We'll first plot the phase as a function of time. We'll first consider a stationary voxel-- so no magnetization moving, here's the phase on the vertical axis; on the horizontal axis we have time. And as we move through time with the gradient, the phase goes first up, with a constant slope-- that's the constant gradient-- and it then comes down after another time big  $T$ . So we have the times here indicated: zero is the beginning where the gradient is turned on, the big  $T$  is the time where the gradient polarity that is the sign of the gradient's changed, and  $2T$  is two times this time. So this is for stationary magnetization; this is the case we've discussed extensively in Lecture 11 and a little bit in Lecture 12. We have, of course, this condition that the phase at the end is zero here-- the phase comes back to the same value as it was here. This is given by the fact that [inaudible] for the stationary magnetization, the integral of the gradient, here, the total area, is zero; or this area is equal to this area, and this is echo formation. So this then defines the echo time,  $TE$ .

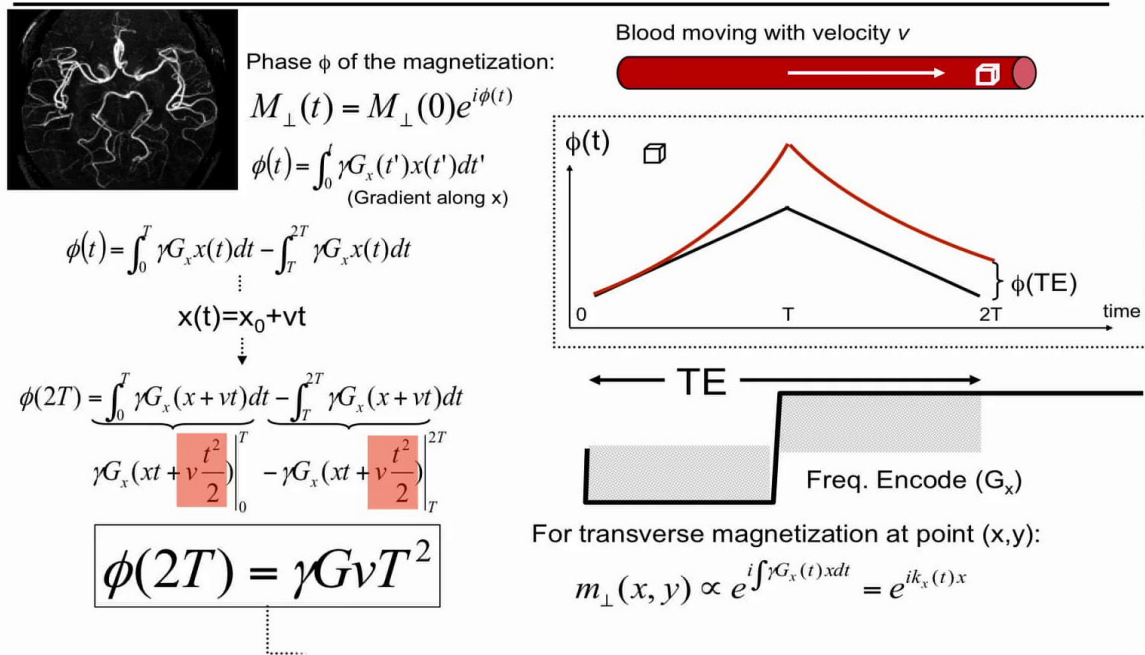
Notes

Summary





# 13-1. How does Bulk Motion affect the Rephased Signal ? (Blood Flow)



13-2

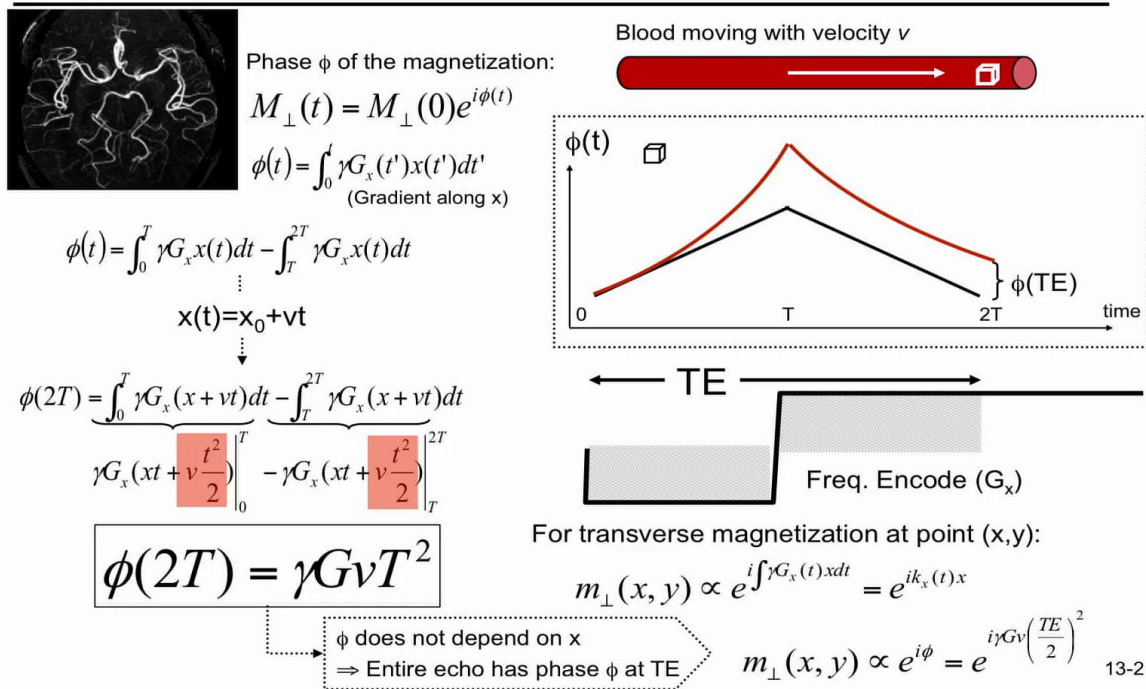
Now, if we consider blood moving with velocity  $v$ -- so here's the blood, we have our magnetization in the voxel, and that is now moving with velocity to the right. As this happens, as it moves, we are accruing a phase, and then we're losing the phase again, but at the end we have a difference in phase compared to stationary magnetization. And this difference in phase here,  $\Phi(T E)$ , this phase is dependent on the velocity, so it is a measure of the velocity of the moving blood in the direction of the gradient. So in the integration it comes from these two terms here, that don't cancel out; in this integral this stationary magnetization, the phase cancels out. So if we look at the transverse magnetization at a point (x, y), then the magnetization at this point is proportional to  $e$  to the  $i$  times the integral of the  $\gamma G$  times the position--here it's gradient along x, so that's this expression here, that we are using here-- and this is equal to the k-space; that's the normal image formulation that we've had for stationary magnetization. Now, the phase that is induced by the motion does not depend on the position.

Notes

Summary



# 13-1. How does Bulk Motion affect the Rephased Signal ? (Blood Flow)

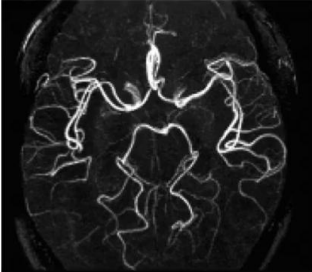


So that means, basically, for a given velocity, in the direction of the gradient, the phase in the voxel will change by the same amount of radians, or degrees, independent of its position. So this is position-independent, so it's a direct encoding of the velocity, and that means that the entire echo, if all the magnetization's moving at this velocity, in the direction of the gradient, has a phase  $\Phi(TE)$ . So we can now write the magnetization at a position in voxels, is proportional to  $e^{i\Phi}$ -- and  $\Phi$  here is given by the expression on the left-- in this expression I've substituted  $2T$  with  $TE$ , so now we have the term here. So, for obtaining images like this, this is one way to obtain encoding of velocity in the voxel. For such an image one can obtain the encoding of the velocity-- not just the velocity but also the direction; it's along the gradient-- by simply recording the change in phase compared to stationary magnetization with appropriate reference scans. So we see that motion affects the phase of the voxel. So this is now coherent motion, we've assumed that the entire voxel, the magnetization in the entire voxel, moves in a certain direction. Now what we are going to be concerned with is random motion. This will be the next section.

Summary







**"SANTRAL SINIR SİSTEMİ ANATOMISI" Dr.ŞERİFE LEBLEBİSATAN**  
<http://slideplayer.com/slide/4020649/>

[illegible]

Summary



