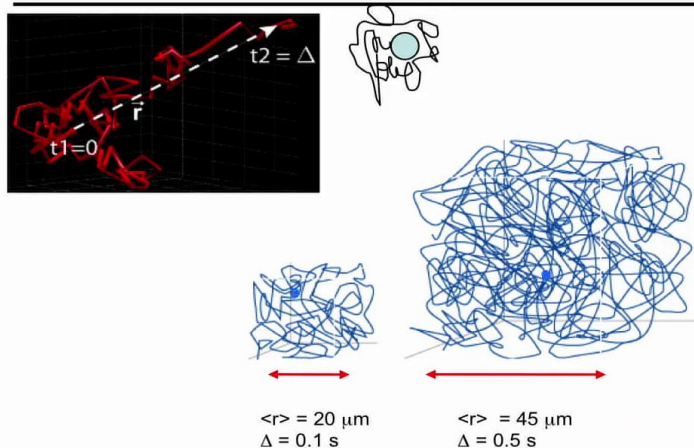


13-2. How does self-Diffusion influence the MR signal ?



We have seen that our magnetization is the result of the sum of the individual dipole moments of all the molecules that are present in our voxel. Now the molecules, in general, and certainly our body temperatures, the molecules are moving around, they are diffusing around. So we have diffusion, we have motion of the molecules during the MR experiment. And I want to first discuss the nature of this motion, and then we will go into the description on how this affects the MR signal. So here we have a random path, we have a molecule, and this will undergo some random path. Or in more general terms, we start at a time zero, the molecule is here, and after time Δ it will be at a certain distance displaced from its original origin. This is the effect of self-diffusion. So if we look at here an example, we have at time Δ , which is 100 ms, and in this case, this gives us a mean displacement of the molecules of roughly 20 microns. If one waits for a longer time, of course, the molecules will diffuse further away, and here we have, then, a distance of 45 microns if we wait half a second. So the molecules will diffuse further away, on average than with longer time.

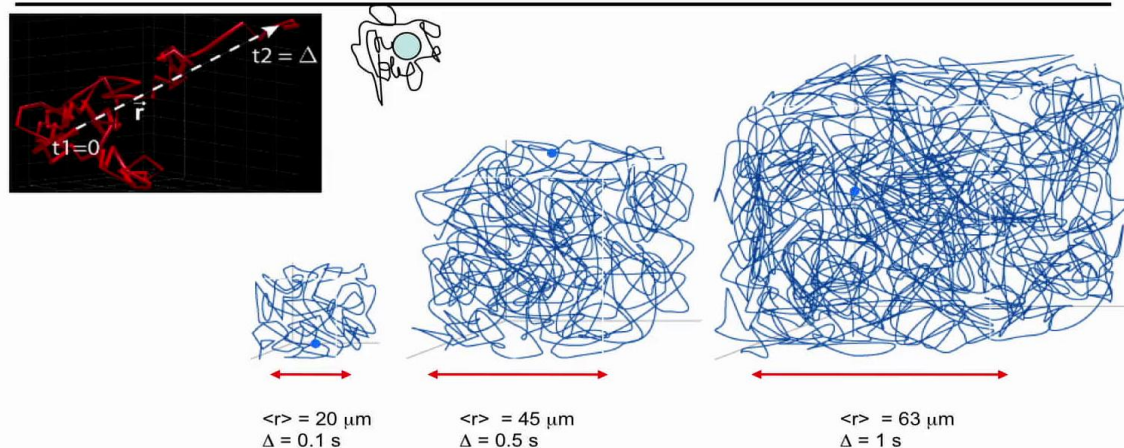
Notes

Summary



0m 04s

13-2. How does self-Diffusion influence the MR signal ?



Einstein random walk:

$$\langle r \rangle = \sqrt{6D\Delta}$$

D : self diffusion coefficient

$\langle r \rangle$: root mean square displacement after Δ seconds

13-3

And here, if we take an example of 1 s, then the mean displacement is 63 microns, and this gives a larger extent of the motion. This random motion diffusion was described by Einstein. It's the Einstein random walk, and this mean displacement is given by the square root of 6 times a constant D times the time *big* Δ . D is called here the *self-diffusion coefficient*, and r is the root means square displacement after Δ s. So the self-diffusion coefficient depends on a number of parameters, it depends also on the size of the molecule, how easily does it diffuse, smaller molecules move more than bigger molecules, and we're going to illustrate this in the following experiment with two different types of fluids to illustrate the effect.

Notes

Summary





So what I'm going to do is I'm going to take some colored alcohol here, pour it in, and we can see how it starts to distribute the color. And here we have a colored water solution. I'll pour it in. And now we can see the clear difference. On the alcohol side, it's already distributed, so fast diffusion. The color has already distributed, whereas with the water we can see this is going to take quite some time until the color is evenly distributed. We can see that now several minutes have lapsed, and we can see that on the alcohol side it's uniformly blue, so it's very well diffused, whereas for the water colored solution that we put in, it still hasn't mixed with the uncolored water, so it's very slow diffusion on this side. This illustrates the effect of molecule size on the diffusability of the molecule.

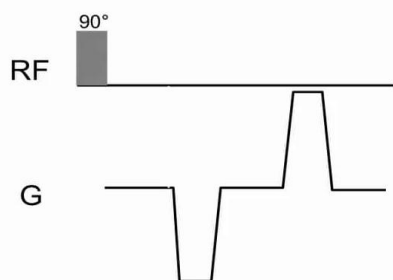
Notes

Summary

2m 37s



What is the effect of random motion on magnetization phase ? when applying pulsed gradient



13-4

Okay, so we've seen that molecules move, we've seen it in the experiment with the colored dyes. They move, this motion can depend on molecule size. But now the question is how does this affect the magnetization, in particular the phase? We've seen that bulk motion affects the phase in a coherent manner, which allows us to determine the velocity of a molecule. And we'll go here and we'll stick with the same principle, we're going to look at the magnetization phase in complex plane. And we're looking here now at an experiment, we'll do a 90-degree pulse, we'll just, for simplicity, assume it's a 90-degree pulse, and now we'll turn on the gradient in any direction, x, y, or z, you can call it whatever you want, it doesn't really matter. What is important here is it's the same strength in both cases, one's positive, one's negative, or negative and positive, and the duration is the same. So the area is the same for the two gradient pulses that are applied. As we have seen before, this is a-- in terms of magnetization phase for molecules that don't move-- is a dumb experiment. The effect of the gradient is nil, there will be no net phase accrued after these two gradients for a stationary molecule.

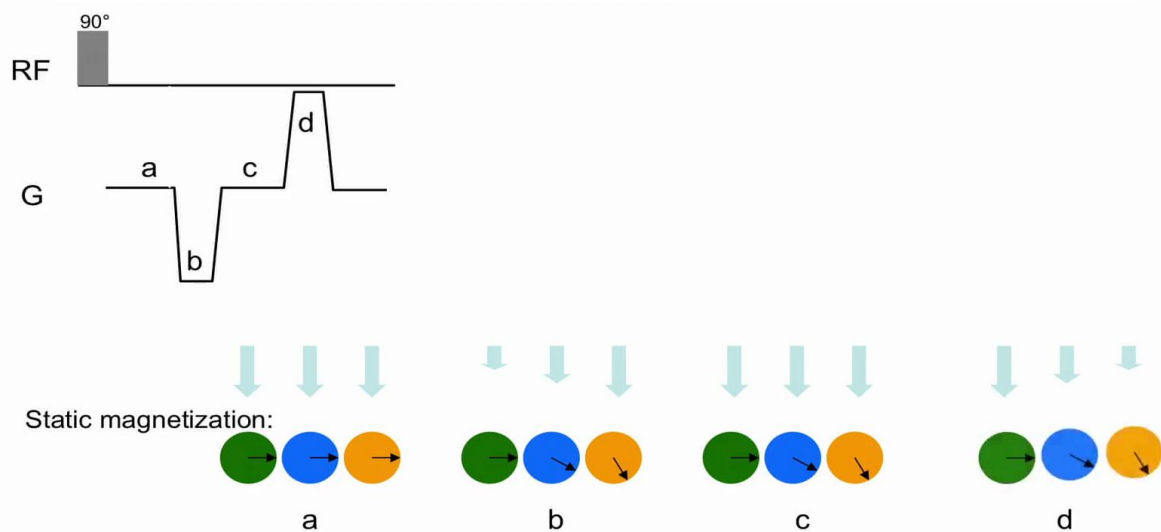
Notes

Summary



3m 38s

What is the effect of random motion on magnetization phase ? when applying pulsed gradient



13-4

So let's look at the stationary case, graphically. We'll have the after the 90-degree pulse, we have the magnetization all in phase. The 90-degree pulse flips them all, and they are all colinear. And we have just plotted here three positions along the gradient. Now during the gradient period indicated by the letter *b* here, so we turn on the gradient. What now happens is that the magnetizations undergo a phase evolution depending on their position, and that's indicated here by the three positions. Then comes the period *c*, and this period *c*, we just wait a certain amount of time. So nothing happens, we still have the same phase, and we're neglecting relaxation here in the consideration. And now finally comes period *d*, we turn on the gradient with opposite polarity. So we turn it on, and now what happens is since the gradient has opposite polarity, the three magnetization positions-- the magnetization at the three positions-- will turn with a same precession velocity, the Larmor frequency but in the opposite direction, and after the duration of this gradient-- so this area is equal to this area, or the same amplitude, the duration is the same-- they will all be aligned, again, as in the original position.

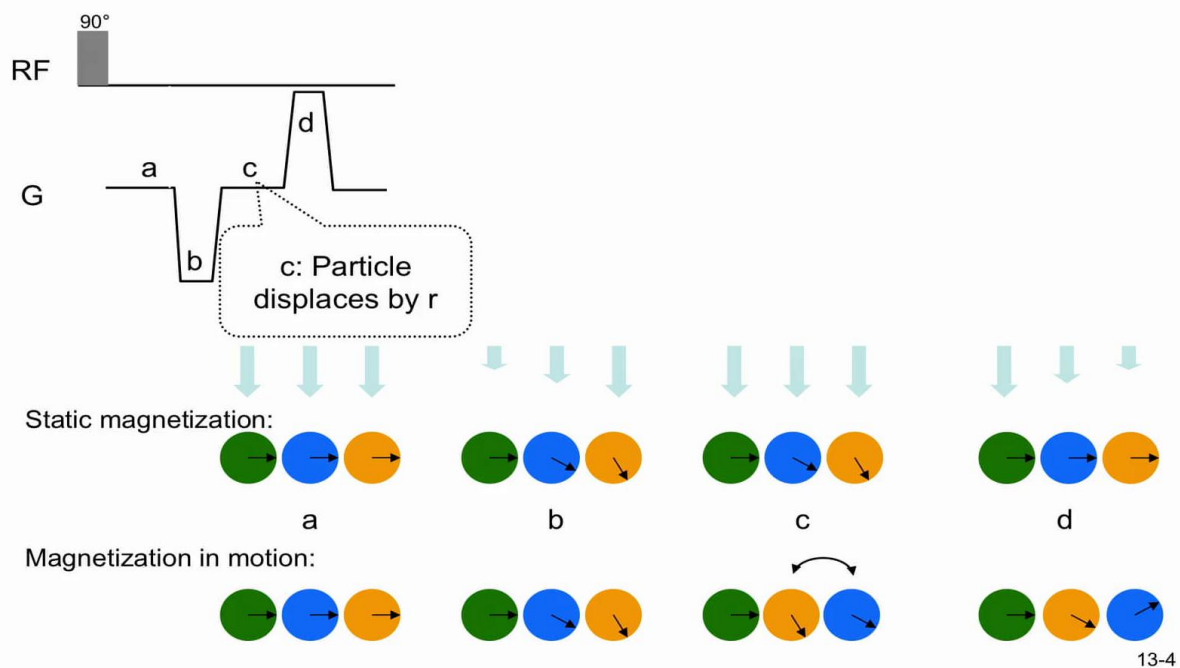
Notes

Summary



4m 50s

What is the effect of random motion on magnetization phase ? when applying pulsed gradient



And this is what I said, this is the dumb experiment, there's nothing interesting going on here. We have echo formation, of course, which is a nice feature. Now let's look at what happens if we allow the magnetization to undergo motion. So we start out with the magnetization stationary, here. We turn on the gradient pulse, so we have the encoding, so we have the different phase. In this case, the experiments are the same. And now during period c, we will assume that there is motion of the molecules. So, as you can see, the colors have changed. So the blue molecule is now where the orange was, and the orange is where the blue was. So we are assuming here, that the molecules are being displaced, and in this case, a very simple situation to illustrate the principle, we've just interchanged the position of two of our magnetizations here. So these are the two that we have interchanged. Now what happens when we turn on the positive gradient, we change the polarity. Now the blue one will rephase with the precession velocity corresponding to this position, but it has not had initial phase that corresponds to this phase, so it will actually overturn here, whereas the orange one, which had a bigger phase to start with from this position, now will only go back according to the blue position here so it will not completely turn back.

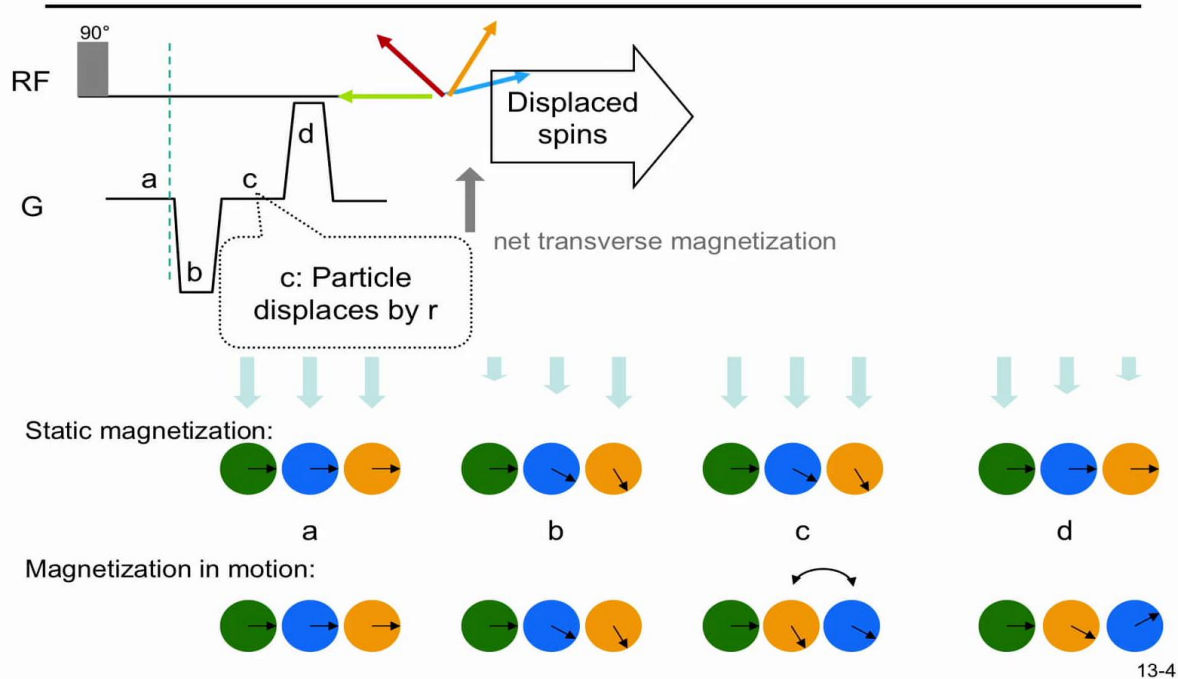
Notes

Summary



6m 07s

What is the effect of random motion on magnetization phase ? when applying pulsed gradient



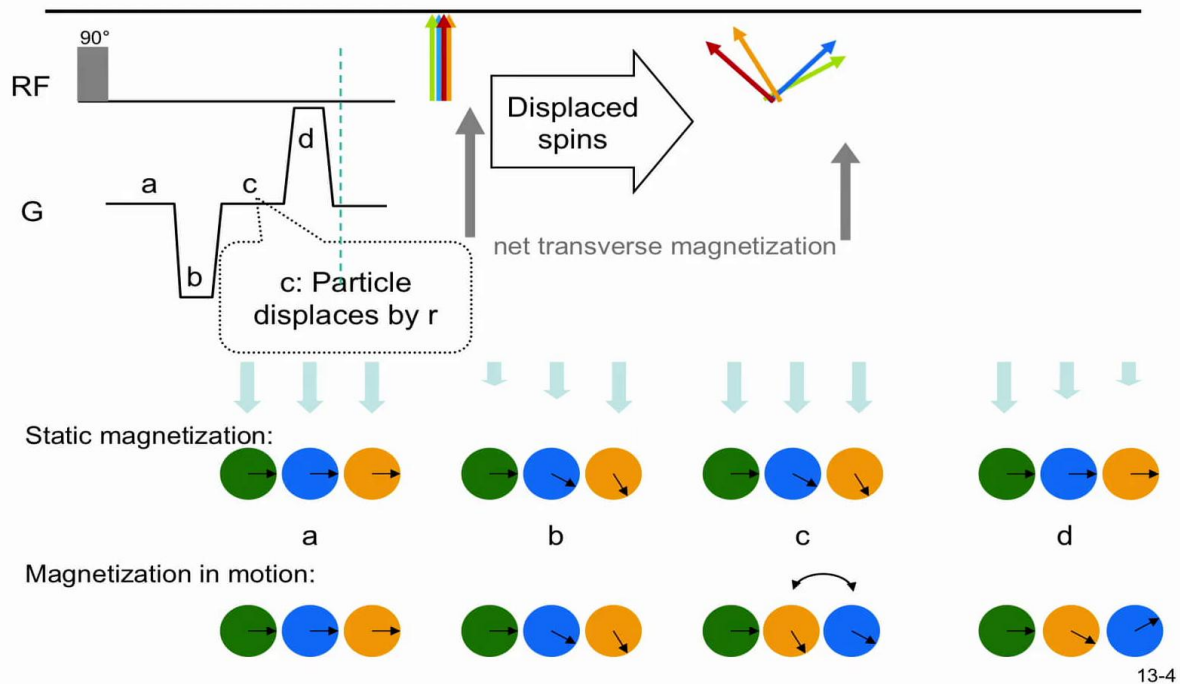
Now, can you guess what the effect is on the net magnetization? Consider here, there's no motion, we have nice echo, we have maximum amplitude. Here we have the magnetization vectors are no longer colinear. And since what we are observing the total magnetization is the sum of all magnetizations in our voxel, the voxel magnetization is reduced. So as a consequence, the effect of diffusion, which happens here during time period c , is to reduce the signal amplitude. We have assumed here in this whole procedure, that the time c is long compared to the gradient. This is not likely strong here, we'll assume this is very short, this is very short, and this is relatively long, so we'll only consider motion during the time period c . So now I want to do this with magnetization vectors. The animation will start here in condition a , so we have all the magnetization vectors lined up, they are colinear, and the net transverse magnetization is maximally. As we go through the gradient, we have dephasing, according to their positions. Then we will look at displaced spins, so we have them colinear at the beginning, we have also dephasing, so this is the same.

Notes

Summary



What is the effect of random motion on magnetization phase ? when applying pulsed gradient

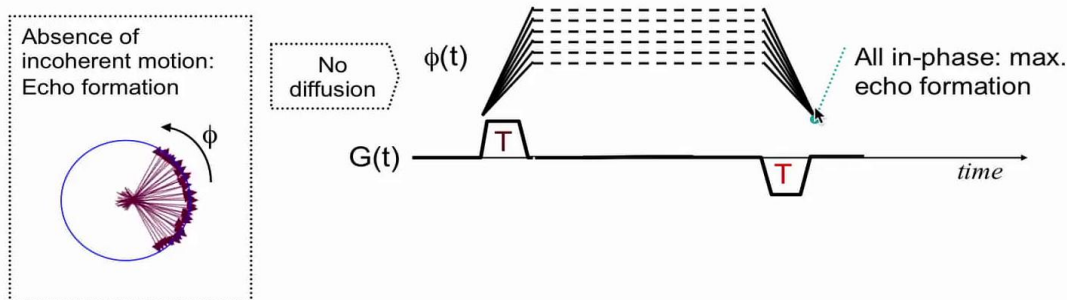


Now we'll assume that for the displaced spins, for these guys, that they will change position among each other, so similar to the case here, so they will be returning back but now with different precessional velocities. Here, we'll consider the static magnetization, they are not exchanging position. So when we do now, and this has happened now for these spins here, here nothing has happened, here they have changed position. And now we'll turn on the positive gradient. For the static magnetization we come back to the maximum magnetization possible, but for those who have undergone diffusion, they no longer end up colinear, and now if we do the vector addition of these vector sums here of these magnetizations, we end up with a net magnetization that's not equal to four times here, and so the transverse magnetization is reduced. So this is the consequence of random motion on the MR signal, there is an amplitude reduction, this is fair to say.

Notes

Summary





13-5

So let's look at this magnetization-- the effect of diffusion on magnetization, I'll take another view on this. We have time here, we turn on the gradient, one's positive, one's negative. And we'll consider now, for a time T , both with the same amplitude but positive and negative, and we'll look at the effect on signal amplitude. So we have reduction of signal amplitude, and then when the second gradient is turned on-- now it's the letter T is turning on the red-- then we have an echo formation. We have, again, dephasing during the first time period, then when we turn on the second gradient, we have rephasing that is in the absence of incoherent motion, that is for static magnetization. So how do we display this in terms of phase? So we'll plot the phase here, for six isochromats. The phase is being built up according to the six different positions during the first gradient time. So this is here. Then it stays constant during the time here between the gradients. And when we turn on the negative gradient, the slope is equal but opposite, so they all come back to the same phase. And so here that means if the phase is the same for all magnetization vectors, we have maximum amplitude, and echo formation, maximum echo amplitude.

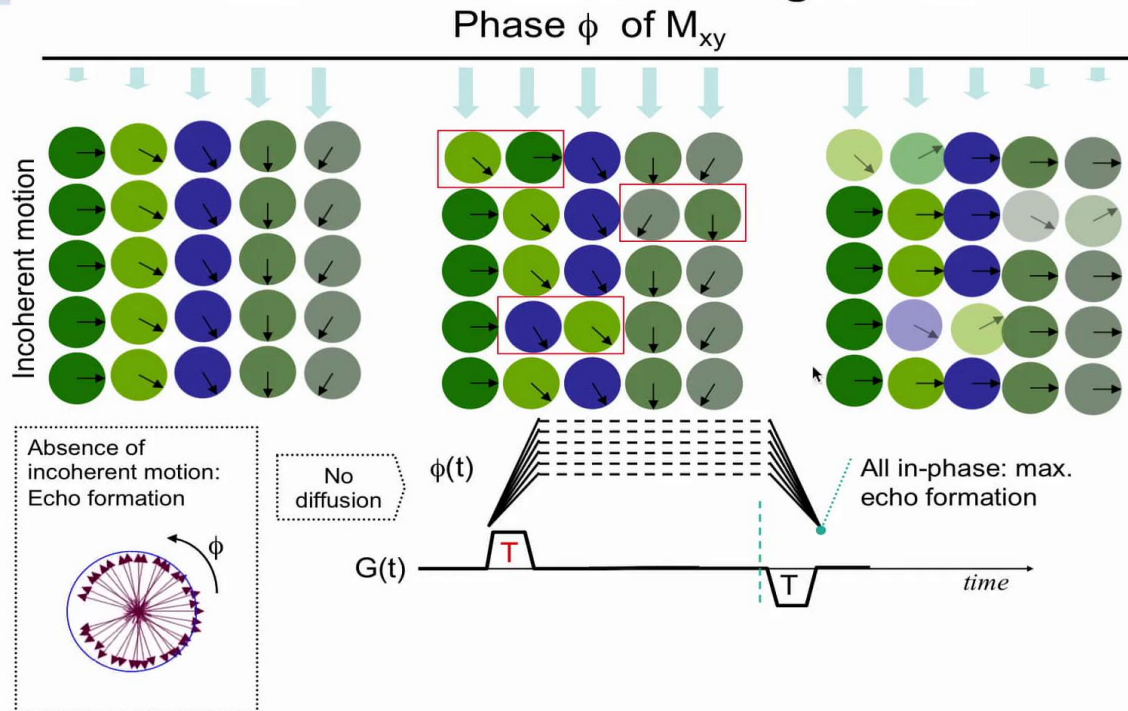
Notes

Summary



9m 57s

Ex. Effect of Diffusion on Magnetization



13-5

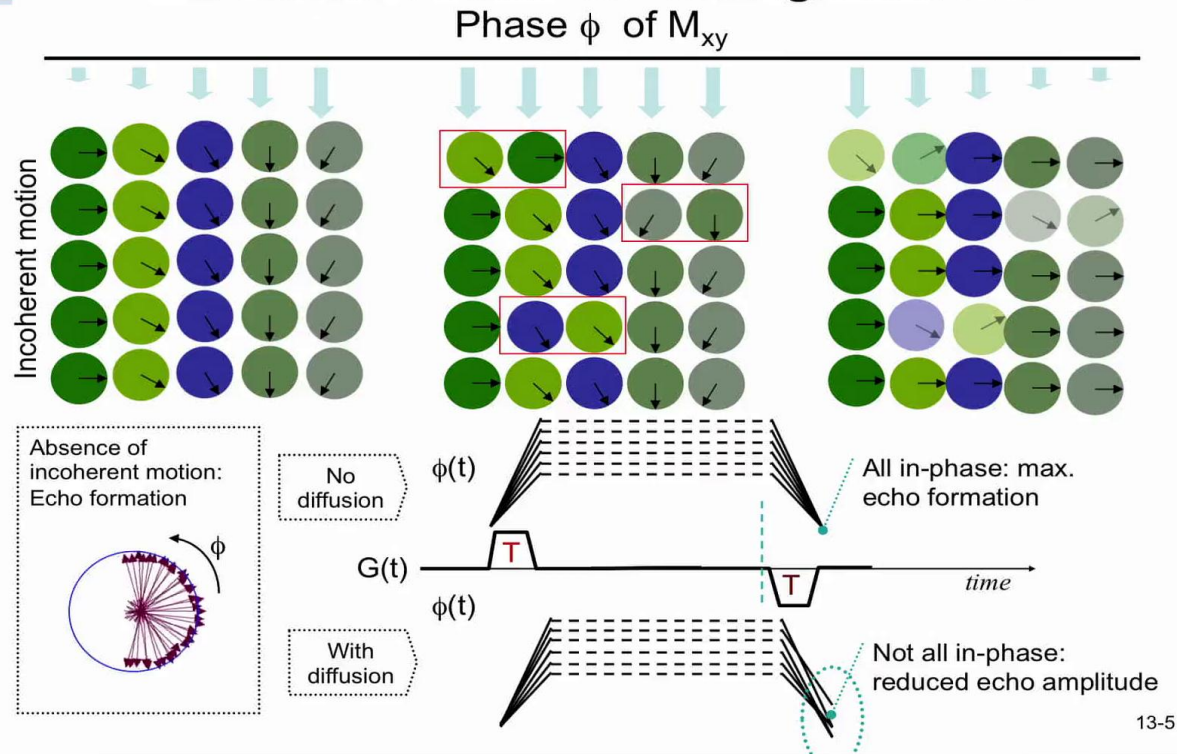
So now let's look at the effect of incoherent motion, and here I'll plot a matrix of 5x5 voxels. We apply a gradient in the horizontal direction, so now, according to that position, they are undergoing a differential phase. This is during the first gradient. And now what we have during the period between the gradients, and here we'll assume these gradients are very small in duration, this period is very long by comparison, and again, this is relatively short. So now we have no gradient during the time between the gradients, but we will assume during this time, that some molecules undergo diffusion. Of course, we cannot draw 10^{23} molecules so I've just taken three pairs here that during this process we'll have assumed that they have exchanged positions to illustrate the case. This is all in the direction of the gradient, which is in the horizontal direction. It doesn't matter what you call this direction, x, y, or z, whichever, it's just the direction of the gradient. So now these three pairs have exchanged positions, and when we now turn on the negative gradient, what is the effect that we see? According to the position that they have now, the phase is turned back to its original position.

Notes

Summary



Ex. Effect of Diffusion on Magnetization



So for the static magnetizations, we can see here they're all colinear, these guys here, these guys, these guys, et cetera. They're all colinear, so that is for them is maximum signal amplitude. But for these magnetizations, which have undergone diffusion in this animation, their magnetization vectors no longer are colinear, and therefore, if you now sum all the 25 magnetization vectors here, the net magnetization, the amplitude has been reduced. So if I draw now this phase as a function of time with diffusion, so we'll look at the phase, it's a build up, and then when we go down with a negative gradient, the slope is no longer the same for all these six isochromats because they have exchanged positions. So we end up at the end of the second gradient, with a magnetization and phase dispersion that is no longer the same, and therefore we have at this point-- because they're not all in phase-- we have a reduced echo amplitude. So this is the graphical intuitive explanation of what happens during diffusion.

Notes

Summary



How is the effect of diffusion on the MR signal described ?

Mathematical description

Degree of echo signal reduction

1. Strength of the diffusion process (D)



13-6

So, now how do we describe the effect of diffusion on the MR signal in a more quantitative way? We'll do this here, and we will have a consideration on what it is that reduces the signal. Now we have to think about here, the reduction of the signal is a fact that the molecules are displaced by a certain distance during a time between the two gradients. This distance of displacement is the sole mechanism by which the amplitude is reduced because when we're refocused with a gradient, they are at different position and the difference in phase compared to the initial phase is what reduces the signal. So what are the factors that reduce the signal amplitude? One is, of course, the strength of the diffusion process. If the diffusion process is very strong, that is the molecules diffuse easily, then we have a large displacement, so it's the diffusion process, and this is the diffusion constant.

Notes

Summary



13m 48s

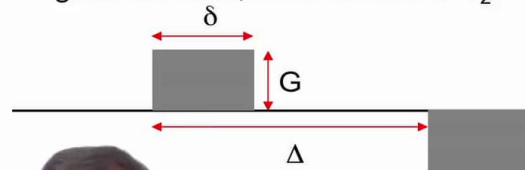
How is the effect of diffusion on the MR signal described ?

Mathematical description

Degree of echo signal reduction

1. Strength of the diffusion process (D)
2. Delay between dephasing and rephasing gradient (Δ)
3. Area of the dephasing gradient (strength G , duration δ)

gradient echo, i.e. sensitive to T_2^*



13-6

Now if we look at an experiment with a positive gradient here, of amplitude G , and a negative gradient of same amplitude G , the duration of the gradient is *small* δ , and they are separated by this time *big* Δ . Then what is the next factor that influences the distance at the beginning and at the end of the experiment, the mean distance, that's the time between the dephasing and rephasing gradient, that's the time *big* Δ . And finally, the phase is also dependent on the strength of the gradient. It's precisely not just the strength but it's the integral of the gradient. So if we have a strong gradient for a long time period, the magnetization vectors spread out much more. And then we apply the same gradient, so for the same position the phase difference between original position and the end position is going to be bigger, and therefore the signal is stronger attenuated. So it's the area of the dephasing gradient that influences, as well, the signal reduction. Now I'm going to-- this is the example of a gradient echo that's sensitizing the diffusion process-- now I'm going to give you the mathematical formulation of this but not the derivation of this.

Notes

Summary



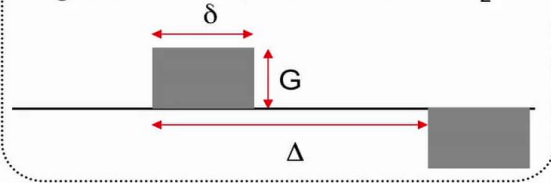
How is the effect of diffusion on the MR signal described ?

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3. Area of the dephasing gradient (strength G , duration δ)

gradient echo, i.e. sensitive to T_2^*



Attenuation of the signal (echo amplitude) due to diffusion in the direction of G

$$S(b) = S_0 e^{-bD} \quad b = (\gamma G \delta)^2 (\Delta - \delta / 3)$$

13-6

We're just going to concern ourselves with the result of this process. So the attenuation of the signal, due to diffusion in the direction of the gradient-- of course, if there's diffusion perpendicular, we won't be able to measure that, there's no influence on the signal phase-- is actually described by a signal in the absence of diffusion, S_0 -- that's this term here-- times $e^{(-bD)}$. That's a simple equation, [an empirical] equation, it works very well for a free liquid that is like the color dyes that we have seen. D is the diffusion process, b is an experimental parameter, and b is the so-called *b factor* or *b value*, and here is the exact value given. So it's first the accrued phase, so it's gradient times duration times γ , squared, times *big* Δ minus *small* δ over 3. So if the *small* δ is small, the duration of the gradient is small compared to this time, then this is essentially the duration, this is the gradient area, that's this area here, squared. That's the b factor and, like I said, I don't want to go through the mathematical derivation of this expression here, we'll just satisfy us with the equation here.

Notes

Summary



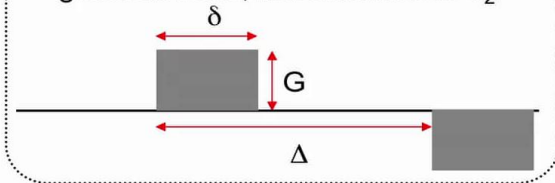
How is the effect of diffusion on the MR signal described ?

Mathematical description

Degree of echo signal reduction

1. Strength of the diffusion process (D)
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gradient echo, i.e. sensitive to T_2^*

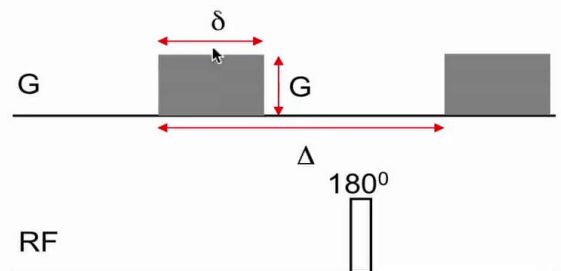


Attenuation of the signal (echo amplitude) due to diffusion in the direction of G

$$S(b) = S_0 e^{-bD} \quad b = (\gamma G \delta)^2 (\Delta - \delta/3)$$

D : **apparent diffusion coefficient (ADC)**

Equivalent sequence (spin echo, i.e. sensitive to T_2)



13-6

So b is the parameter that the operator can choose. We can choose the *big* Δ , we can choose the *small* δ , and we can choose the gradient strength. That's the experimental parameter that we can choose, and D describes the diffusion process. Now in this case, it's not the self-diffusion coefficient in magnetic resonance, it is called the *apparent diffusion coefficient* or *ADC* because it's an apparent diffusion that one measures and it is not necessarily the true diffusion of the molecule. Although, in the case of free water, for example, we can very well quantitatively measure this ADC, and from the ADC measurement-- because this changes with temperature-- we can actually determine, to a very good precision, the temperature of the water sample. Now the sequence that I've shown you here-- that's diffusion weighting, it introduces this weighting into the signal-- can also be done with a spin echo. So we have the two gradients, now they are positive because between them we've inserted a 180-degree pulse. So, as you remember, for static magnetization, positive gradients, the effect of 180-degree pulse is to refocus the static magnetization here.

Notes

Summary



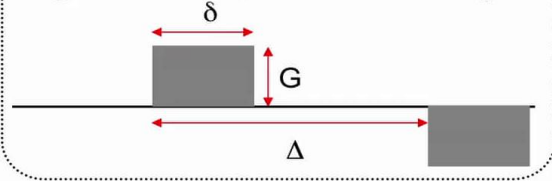
How is the effect of diffusion on the MR signal described ?

Mathematical description

Degree of echo signal reduction

1. Strength of the diffusion process (D)
2. Delay between dephasing and rephasing gradient (Δ)
3. Area of the dephasing gradient (strength G , duration δ)

gradient echo, i.e. sensitive to T_2^*

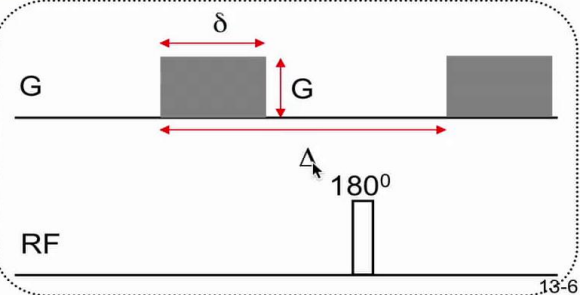


Attenuation of the signal (echo amplitude) due to diffusion in the direction of G

$$S(b) = S_0 e^{-bD} \quad b = (\gamma G \delta)^2 (\Delta - \delta/3)$$

D : apparent diffusion coefficient (ADC)

Equivalent sequence (spin echo, i.e. sensitive to T_2)



13-6

So that would be the equivalent sequence with a spin echo, above we have the gradient echo, and the spin echo has the advantage, it is only dependent on T_2 , and T_2 is always longer than T_2^* , so one can use longer diffusion times, *big* Δ , with this approach.

Notes

Summary



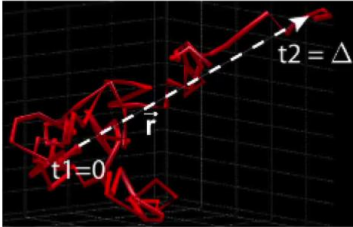


Fig. 1 from Patric Hagmann et al. Radiographics 2006; 26: S1

Notes

Summary

19m 13s

