



Course material

Course:

Digital Signal Processing II

Video:

2.1.1.a Linear time-invariant filters

Concepts (extracted from automatically generated subtitles):

Linear combination. Signal processing. Time invariance. First requirement. Block diagram. Structure of the processing device. Linear time. Linear combination of outputs. Wah pedal. Kind of processing. Signal x . Interesting effects. Kinds of rock music. Louder note. Very reasonable requirement.



[to video sequence search](#)
(within Digital Signal Processing II.)

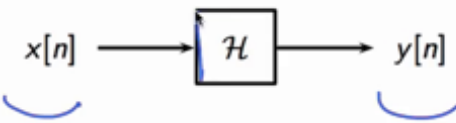


[to video](#)

Center for Digital Education. More educational support material here:

<https://exoset.epfl.ch/booc/>

A generic signal processing device



$$y[n] = \mathcal{H}\{x[n]\}$$

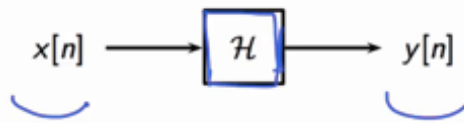
In general, when we talk about signal processing, we imagine a situation where we have an input signal x of n , an output signal y of n , produced by some sort of black box here that manipulates the input into the output. We can write the

notes

summary



A generic signal processing device



$$y[n] = \mathcal{H}\{x[n]\}$$

relationship mathematically like so, where y of n is equal to some operator h

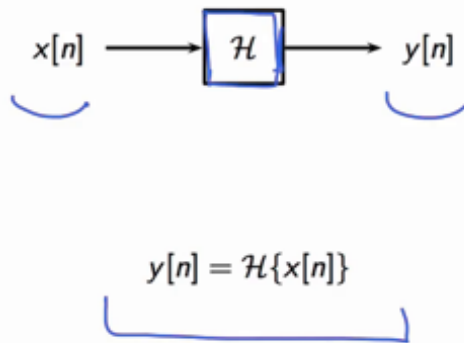
notes

summary

0m 10s



A generic signal processing device



that is applied to the input x of n . Already when we draw this block diagram

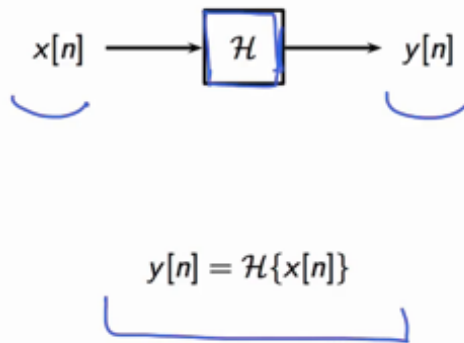
notes

summary

0m 13s



A generic signal processing device



we are making assumptions on the structure of the processing device in the sense that we consider a system with a single input and a single output. We

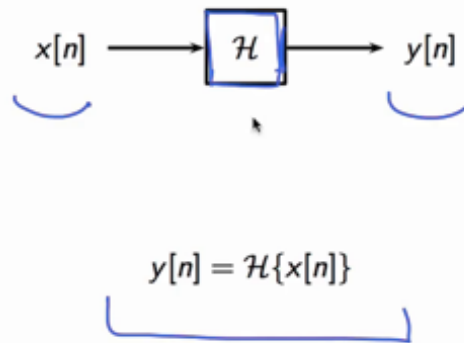
notes

summary

0m 25s



A generic signal processing device



could imagine a system with multiple inputs or multiple outputs. But even with these limitations, the possibilities for what goes inside this block here are

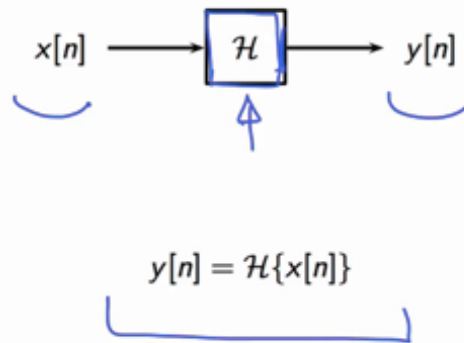
notes

summary

0m 34s



A generic signal processing device



pretty much limitless and unless we impose some structure on the kind of processing that happens into this block, we will not be able to say anything

notes

summary

0m 43s



Linearity

$$\mathcal{H}\{\alpha x_1[n] + \beta x_2[n]\} = \alpha \mathcal{H}\{x_1[n]\} + \beta \mathcal{H}\{x_2[n]\}$$

particularly meaningful about the filtering operation. So the first requirement that we impose on a filter is linearity. Linearity means that if we have two

notes

summary

0m 49s



Linearity

$$\mathcal{H}\{\alpha x_1[n] + \beta x_2[n]\} = \alpha \mathcal{H}\{x_1[n]\} + \beta \mathcal{H}\{x_2[n]\}$$


inputs and we take a linear combination of said inputs, well the output is a linear combination of outputs that could have been obtained by filtering each

notes

summary

1m 1s



Linearity

$$\mathcal{H}\{\underbrace{\alpha x_1[n]}_{\uparrow} + \underbrace{\beta x_2[n]}_{\uparrow}\} = \underbrace{\alpha}_{\uparrow} \underbrace{\mathcal{H}\{x_1[n]\}}_{\uparrow} + \underbrace{\beta}_{\uparrow} \underbrace{\mathcal{H}\{x_2[n]\}}_{\uparrow}$$

sequence independently. This is actually a very reasonable requirement. For instance,

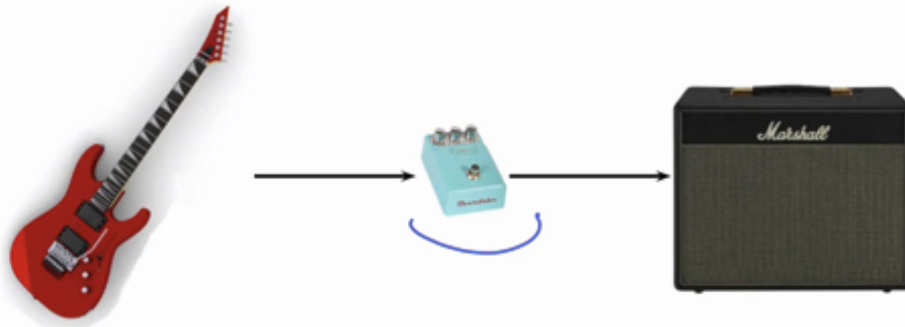
notes

summary

1m 9s



(Non) Linearity



take a situation where your processing device is an amplifier and you connect a guitar to your amplifier. Now if you play one note and then you play the same note louder, you expect the amplifier to produce just a louder note. Similarly, if you play one note and then another note and then you play two notes together, you expect the amplifier to amplify the sum of two notes as the sum of two independent amplifications. Now note that this is not necessarily the case in all situations. For instance, in some kinds of rock music you want to introduce some distortion and so you add a fuzz box that will distort the signal nonlinearly to create very interesting effects but that belong to a completely different category of processing.

notes

summary

1m 20s



Time invariance

$$y[n] = \mathcal{H}\{x[n]\} \iff \mathcal{H}\{x[n - n_0]\} = y[n - n_0]$$

The second requirement that we impose on the processing device is time invariance. Time invariance in layman terms simply means that the system will behave

notes

summary

2m 25s



Time invariance

$$y[n] = \mathcal{H}\{x[n]\} \iff \mathcal{H}\{x[n - n_0]\} = y[n - n_0]$$

exactly in the same way independently of when it's switched on. Mathematically we can say that if y of n is the output of the system when the input is x of n , well

notes

summary

2m 37s



Time invariance

$$\underbrace{y[n]} = \mathcal{H}\{\underbrace{x[n]\}_{n_0}\} \iff \mathcal{H}\{x[n - n_0]\} = y[n - n_0]$$

if we put a delayed version of the input inside the system x of n minus n_0 zero,

notes

summary

2m 46s



Time invariance

$$\underbrace{y[n]} = \underbrace{\mathcal{H}\{x[n]\}} \iff \underbrace{\mathcal{H}\{x[n - n_0]\}} = \underbrace{y[n - n_0]}$$

what we get is the same output delayed by n_0 . And again we can use a guitar

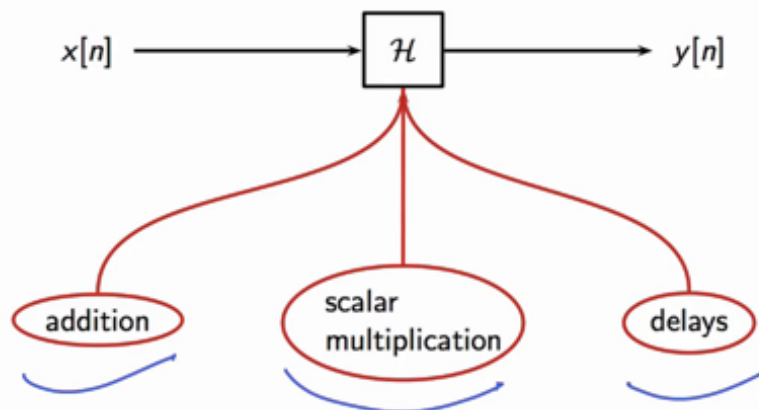
notes

summary

2m 50s



Linear, time-invariant systems



amplifier as an example. If I turn it on today, well I expect it to amplify the notes exactly in the same way that it amplified them yesterday. But again some types of guitar effects exploit time variance to introduce a different flavor to the music that's being played. For instance the wah pedal is a time varying effect that will change the envelope of a sound in ways that are not time invariant. So what are the ingredients that go into a linear time invariant system? Well linear ingredients, addition which is a linear operation, scalar multiplication, another linear operation and delays. Another requirement that is not mandatory but makes a lot of sense if you want to use a linear time variant system in real time is that the system be causal. By that we mean that

notes

summary

3m 1s



Linear, time-invariant systems

$$y[n] = H(x[n], x[n-1], x[n-2], \dots, y[n-1], y[n-2], \dots)$$

with $H(\cdot)$ a linear function of its arguments

the system can only have access to input and output values from the past. In that case you can write the input-output relationship as follows. The output is a linear functional of past values of the input and past values of the output.

notes

summary

4m 2s

